Threshold Selection for SNR-based Selective Digital Relaying in Cooperative Wireless Networks

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***Abstract*—This paper studies selective relaying schemes based on signal-to-noise-ratio (SNR) to minimize the end-to-end (e2e) bit error rate (BER) in cooperative digital relaying systems using BPSK modulation. In the SNR-based selective relaying, the relay either retransmits or remains silent depending on the SNRs of the source-relay, relay-destination, and source-destination links. Different models assuming the availability of different sets of instantaneous and average SNR information at the relay are studied. For each model, the optimal strategy to minimize the e2e BER is a different threshold rule on the source-relay SNR, if the link SNRs are uncorrelated in time and space. Approximations for the optimal threshold values that minimize the e2e BER and the resulting performance are derived analytically for BPSK modulation. Using the derived threshold the e2e BER can be reduced significantly compared to simple digital relaying. By studying the performance under different models, it is shown that knowledge of the instantaneous source-destination SNR at the relay can be exploited. The gain from this knowledge is higher when the average source-destination SNR is large. However, knowledge of the instantaneous relay-destination SNR at the relay does not change performance significantly.**

***Index Terms*—Multihop communication, cooperative diversity, threshold based digital relaying, selective digital relaying, SNR based selective relaying.**

I. Introduction

In wireless networks, multihop relaying improves average link SNRs by replacing longer hops with multiple shorter hops. Relay transmissions are also used to induce cooperative diversity, which can increase system reliability without relying on multiple antennas. Several relaying schemes to realize cooperative diversity have been proposed in [1] and [2]. Cooperative relaying schemes are classified as digital or analog depending on the level of signal processing performed at the relay. In digital relaying the relay detects and then retransmits the detected signal, whereas in analog relaying the relay amplifies and retransmits the received signal. This work focuses on digital cooperative relaying.

In digital cooperative relaying, if the relay detection is correct, the destination receives the signal through two branches (from the source and the relay) thereby achieves diversity by combining them. However, if the relay has a detection error, the effective SNR at the destination after combining is significantly reduced. This phenomenon is called *error propagation*. The e2e performance of simple digital relaying, where the relay always retransmits, is limited by error propagation.

The relay can forward the data selectively in order to reduce the probability of error propagation. One measure that can be used for forwarding decisions is the link SNR. If the received SNR at the relay is low, the data is likely to have errors and hence the relay discards the data. In many wireless applications, relaying schemes might incorporate channel coding techniques. In this case, other measures of reliability that are extracted from the received signal at the relay can be used in conjunction with SNR [3].

If the reliability information is extracted from the received data, the relay is required to perform channel estimation, demodulation, and then error detection for each data block before making a forwarding decision. These operations cause additional delay and extra power consumption even if the relay eventually decides not to transmit. In cellular systems, the amount of power consumed by the terminals in receive mode is less significant compared to that in transmit mode. However, these two power levels are comparable in low power devices such as battery powered sensor nodes [4]. In SNR-based selective relaying, the relaying decisions are simpler and remain the same for a time duration in the scale of the channel coherence time in the network. Thus, when the source-relay SNR is low, the relay can be put into sleep mode. More importantly, sensor networks can adopt uncoded transmission or avoid decoding at the relay due to resource constraints [5], [6]. Hence, in networks that include nodes with a wide range of computation and communication capabilities, SNR-based relaying can be desirable in order not to isolate the nodes with scarce power and limited computational capability. SNR-based selective relaying is especially suited for applications where either uncoded transmission is used, or the relaying and channel coding are required to be transparent to each other, or the delay and the power consumption incurred for extracting the reliability information from the received data are significant.

Manuscript received April 4, 2007; revised September 19, 2007, November 26, 2007, and January 19, 2008; accepted March 14, 2008. The associate editor coordinating the review of this paper and approving it for publication was A. Stefanov. A short version of this paper appeared in the proceedings of the IEEE Wireless Communications and Networking Conference (WCNC), Hong Kong, 2007. This work was supported by Wireless Tech. Labs, Nortel Networks. John Thompson acknowledges the support of the Scottish Funding Council for the Joint Research Institute with the Heriot-Watt University which is a part of the Edinburgh Research Partnership.

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Digital Object Identifier 10.1109/T-WC.2008.070359

In this paper we address the design of SNR-based relaying policies for cooperative two-hop networks employing uncoded BPSK signaling. These polices minimize the e2e BER and lead to threshold rules for the source-relay link. The relay transmits only if the source-relay SNR is above this threshold. The choice of the threshold has considerable impact on the e2e performance of the cooperative diversity schemes. For instance, consider a relay detection threshold value of zero. This protocol is akin to simple digital relaying and its diversity order is equal to one [7]. On the other hand, for a very high threshold setting, the system degenerates to one path channel, which is the source-destination channel and dual diversity is not realized.

We formulate the selection of the optimal threshold as a simple decision problem from the relay's point of view. Four models that differ in the amount of SNR information available at the relay are considered. In the first model, Model 1, the relay makes decisions based on the instantaneous source-relay SNR, the average relay-destination SNR, and the average source-destination SNR. Model 2 assumes that the instantaneous SNR of source-relay and relay-destination links are available to the relay while Model 3 assumes that the instantaneous SNR of the source-relay and source-destination links are available to the relay. Finally, Model 4 assumes that the relay knows the instantaneous SNRs of all three links. Expressions for the optimal threshold values[1] and the minimum e2e BER are derived for Rayleigh fading and BPSK modulation.

For all the models considered, although the e2e BERs depend on the average SNR of all three links, the optimal threshold values that minimize the e2e BER are functions of the relay-destination and source-destination link SNRs only. For all the models, it is shown that using the derived threshold values results in significant improvement of the e2e BER compared to simple digital relaying. By analyzing the performance under four different models, we observe that having the instantaneous source-destination SNR information for relaying decisions reduces the e2e BER. The gain from this information is higher when the source-destination link is stronger. However, the gain from instantaneous relay-destination SNR is negligible in most cases. We also compare the performance of the SNR based selective relaying to a performance upper bound that assumes perfect error detection for each symbol, which is not necessarily achievable in practice. The gap between the performance of selective relaying and this upper bound suggests that hybrid schemes that incorporate selective relaying at the relay and smarter detection methods at the destination could provide for further improved e2e BER performance.

The rest of this paper is organized as follows: In Section II, related literature is discussed. The system model is presented in Section III and the optimal threshold and the e2e BER for selective relaying schemes are analyzed in Section IV. In Section V, performance benchmarks are described and numerical examples on the e2e BER performance are presented. The paper concludes with a summary of our findings.

[1]For the first three models we derive approximations to the optimal thresholds and for Model 4 we derive an exact expression for the optimal threshold.

II. Related Work

The trade-off between creating the required diversity branches to the destination and minimizing the risk of error propagation has motivated research on SNR-based threshold relaying [1], [8]–[10]. Some studies considered a system with ideal coding, where no error occurs at the relay as long as source-relay SNR is larger than a target SNR which depends on a specified target rate [1], [11]. This assumption implies that the SNR threshold for relaying must be equal to the target SNR. Herhold et al. studied SNR-based threshold relaying for an uncoded system [8]. In this work, the authors formulate the power allocation and threshold selection jointly. They numerically obtain power allocation fraction and threshold pairs that minimize the e2e BER for a given modulation scheme used by the source and the relay. Based on these numerical results, they also provide empirical rules to approximate the optimal parameters. In [9], the performance of threshold relaying in a multi-antenna multi-relay architecture is studied. It is shown that threshold relaying is essential in uncoded systems when the relay has a small number of receive antennas. In [8], the threshold – if used jointly with the optimal power fraction – is a function of the average SNRs of the source-relay, relay-destination and source-destination links while in [9] the threshold depends on the average SNR of the source-relay link only. Our analytical formulation shows that for arbitrary network configurations and given fixed transmit powers used by the source and the relay, the optimal threshold is independent of the average source-relay SNR.

In [10], the authors derive the BER of threshold-based relaying for an arbitrary threshold value and obtain the optimal threshold and power allocation by minimizing the BER numerically. However, their assumption that the channel coefficients are real Gaussian random variables does not apply to practical wireless scenarios.

Lin et al. [12] study the relay selection problem in the context of coded cooperation. The authors derive the criteria for selecting one of the available relays. If none of the relays is selected, the system reduces to direct transmission from the source. The relay selection is valid for a long period of time, which is longer than the duration of small scale fading. Hence, the relay selection is made based on the average link SNRs.

The idea of selective relaying, or on-off relaying, can be generalized to the adaptation of relay transmit power. In [13] and [14], the authors consider a scheme to control the relay power adaptively based on the link SNRs in order to mitigate error propagation. They propose a scaling factor for relay power that is based on the source-relay and relay-destination SNRs. In these papers, it is reported that if the relay power can be controlled continuously, the proposed scaling factor achieves full diversity. It is also claimed that the proposed scheme has no diversity gain if the relay can only perform on-off power control, i.e., selective relaying. The numerical results of our present paper hints that the latter conclusion is due to the particular choice of the scaling factor rather than the limitation of on-off power control; we observe that, if it is done intelligently, on-off power control can increase the slope of the e2e BER curve, (i.e., the diversity order) compared to simple digital relaying. In [15], it is shown that the SNR-based selective relaying achieves dual diversity.

An alternative approach to mitigate error propagation is to design the destination receiver by taking error propagation into account. In [16], cooperative demodulation techniques for a two-hop parallel relaying system are considered. In this system, the relays always retransmit, which would result in a diversity order of 1 under simple maximal ratio combining (MRC) at the destination. The authors propose maximum-likelihood combining and demodulation at the destination assuming that the destination knows the average bit error probability at each relay during the first hop. They derive ML receivers and piecewise linear approximations to ML receivers for different relaying schemes. They show that in digital relaying systems these receivers can achieve a diversity order of $(M+1)/2 \leq d \leq M/2+1$ for M even and $d = (M+1)/2$ for M odd, where $M-1$ is the number of relays. In addition, it is shown that with a single relay diversity order of 2 can be achieved.

Wang et al. [17] propose a novel combining scheme that can be employed at the destination for digital parallel relaying. This scheme, which is called Cooperative-MRC (C-MRC), exploits the instantaneous BER of source-relay links at the destination. The C-MRC can achieve full diversity in uncoded digital relaying systems. However, it requires the relays to send their instantaneous BER values to the destination.

The models used by [16] and [17] both place the computing burden on the destination while keeping the relays relatively simple. In our model, however, the relay implicitly participates in combining the two branches; the relay assigns weight zero to the relay-destination signal by remaining silent. Then, the destination performs MRC. Avoiding transmissions from branches that make little contribution to the post-processing SNR can reduce interference in the network. Furthermore, in threshold relaying the instantaneous source-relay SNR is exploited at the relay while C-MRC needs the instantaneous source-relay SNR at the destination, which requires additional signaling.

III. SYSTEM MODEL

The network model is shown in Fig. 1. It includes a source node S, a destination node D, and a relay node R that assists the communication between S and D. For clarity of exposition, it is assumed that all the links use BPSK modulation. Appendix C provides a sketch for the extension of some of the analysis to MPSK. S and R work in time division mode in accordance with the half-duplex constraint. This constraint prohibits most practical relay terminals from transmitting and receiving simultaneously on the same channel. The protocol has two phases: In phase 1, S transmits and R and D listen. In phase 2, R detects the signal and either retransmits, in which case S is silent, or declares that it will remain silent and S starts phase 1 with the next data. If R retransmits in phase 2, D combines the signals received in phase 1 and phase 2 using MRC and performs detection based on the combined signal.

Let the signal received at the destination from the source be denoted by y_{sd}.

$$y_{sd} = \alpha_{sd}\sqrt{E_{b,s}}\,x_s + n_{sd}, \quad (1)$$

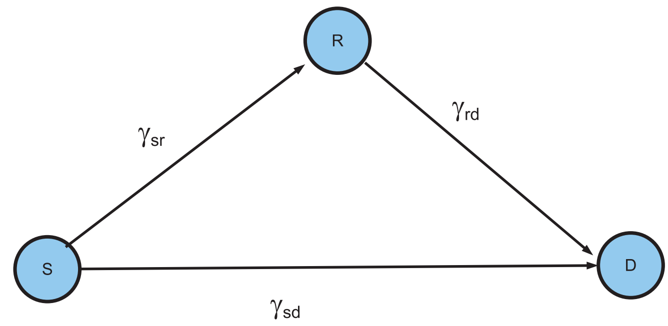

Fig. 1. The system model.

where $x_s \in \{+1,-1\}$, $E_{b,s}$ is the energy per bit spent by the source, α_{sd} is the fading coefficient and n_{sd} is a complex Gaussian random variable with zero mean and a variance of $N_0/2$. Similarly, the signal received at the relay is equal to $y_{sr} = \alpha_{sr}\sqrt{E_{b,s}}\,x_s + n_{sr}$. If the relay transmits, the received signal at the destination as a result of this transmission is given by

$$y_{rd} = \alpha_{rd}\sqrt{E_{b,r}}\,x_r + n_{rd}, \quad (2)$$

where $x_r \in \{+1,-1\}$ is the symbol sent by the relay based on its detection of x_s and $E_{b,r}$ is the energy per bit spent by the relay. The noise components n_{sr}, n_{rd}, and n_{sd} are assumed to be i.i.d. random variables. The instantaneous link SNRs are equal to $\gamma_{sr} = |\alpha_{sr}|^2 E_{b,s}/N_0$, $\gamma_{rd} = |\alpha_{rd}|^2 E_{b,r}/N_0$, and $\gamma_{sd} = |\alpha_{sd}|^2 E_{b,s}/N_0$. All the links are assumed to exhibit flat fading with Rayleigh envelope distribution. However, some of the analysis in the paper is general and not limited to Rayleigh distribution. We assume that both $E_{b,s}$ and $E_{b,r}$ are fixed, predetermined values. Hence, the instantaneous link SNRs can be expressed as $\gamma_{ij} = \sigma_{ij}^2 X_{ij}^2$, where X_{ij}^2 is an exponential random variable and σ_{ij}^2 is the average SNR. All X_{ij}^2's are independently and identically distributed with unit mean. The pdf of γ_{ij} is then given by $p_{\gamma_{ij}}(\gamma_{ij}) = (1/\sigma_{ij}^2)\exp(-\gamma_{ij}/\sigma_{ij}^2)$ for $\gamma_{ij} \geq 0$. The average SNR σ_{ij}^2, incorporates the energy per bit spent by node i and the path loss between node i and node j. Hence, the average SNR of S-R, R-D, and S-D links, denoted by σ_{sr}^2, σ_{rd}^2, and σ_{sd}^2, respectively, are known parameters that are not necessarily identical but constant for at least the duration of the two phases.

The channel states remain constant during phase 1 and phase 2. The two phases constitute one *block*. We assume that the channel states are either independent from block to block or their correlation is not exploited. We assume that the CSI is available at the receiver side for all three links and the signal is demodulated coherently. We consider various models with different levels of adaptation in relaying decisions. In these models, the relay makes use of either the mean or the instantaneous SNR for each link. In Model j, the relay uses the set of parameters denoted by S_j, where $j = 1,2,3,4$, to make relaying decisions. The following sets are considered:

$$S_1 = \{\gamma_{sr}, \sigma_{rd}^2, \sigma_{sd}^2\}, \quad S_2 = \{\gamma_{sr}, \gamma_{rd}, \sigma_{sd}^2\},$$
$$S_3 = \{\gamma_{sr}, \sigma_{rd}^2, \gamma_{sd}\}, \quad \text{and} \quad S_4 = \{\gamma_{sr}, \gamma_{rd}, \gamma_{sd}\}.$$

How well a relaying configuration can adapt to varying channel conditions depends on the information used by the relay. In general, the average SNR values change much more slowly than the instantaneous values. Although a more adaptive scheme is expected to perform better, a system using average channel characteristics is easier to implement since it requires less frequent updates to resource allocations. Another challenge is to acquire the necessary channel state information (CSI) at the relay. Since the relay is the receiver in the S-R link, it can estimate γ_{sr} and additional CSI overhead of Model 1 is minimal. Model 2 requires the relay to make decisions based on the instantaneous SNR of its forward channel γ_{rd}. Thus, a feedback channel from D to R might be necessary. Similarly, Model 3 requires γ_{sd}, which can be estimated in the first phase at D and can be sent to R through the same feedback channel. Model 4 has the highest complexity since it requires that both γ_{rd} and γ_{sd} are sent to R by D. The analysis in this paper focuses on the best possible performance under the different models. Therefore, we assume that the CSI required by each model is available at the relay.

In the rest of the paper we use the following definitions and notation. The error events in the S-R and S-D links are denoted by $\mathcal{E}_{sr}$ and $\mathcal{E}_{sd}$, respectively. The event that an error occurs after the destination combines the source signal and the incorrectly regenerated relay signal is referred to as *error propagation* and is denoted by $\mathcal{E}_{prop}$. We use the term *cooperative error* for the event that an error occurs after the destination combines the source signal and the correctly regenerated relay signal. The cooperative error event is denoted by $\mathcal{E}_{coop}$. The BERs for BPSK in point-to-point links conditioned on the instantaneous link SNR and average link SNR are denoted by $\text{BER}_{awgn}(\gamma_{ij})$ and $\text{BER}_{ray}(\sigma^2_{ij})$, respectively, and are given by [18, pp. 817-818]

$$\mathbb{P}(\mathcal{E}_{ij}|\gamma_{ij}) = \text{BER}_{awgn}(\gamma_{ij}) = Q(\sqrt{2\gamma_{ij}}) \quad \text{and}$$
$$\mathbb{P}(\mathcal{E}_{ij}|\sigma^2_{ij}) = \text{BER}_{ray}(\sigma^2_{ij}) = \frac{1}{2}\left(1-\sqrt{\frac{\sigma^2_{ij}}{1+\sigma^2_{ij}}}\right), \quad (3)$$

where the Q function is defined as $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \mathrm{d}z$.

The function used to calculate the optimal threshold value for Model j is denoted by f_j; the policy used by the relay to make forwarding decisions is denoted by π; and the e2e bit error probability calculated at the relay based on the link SNR observations S_j when the relay follows policy π is denoted by $\mathbb{P}\{\mathcal{E}_{e2e}|S_j, \pi(S_j)\}$. The average e2e BER of the optimal relaying under Model j is denoted by $\text{BER}^{(j)}_{e2e}$.

IV. Analysis of the SNR-Based Selective Relaying

There are two actions that can be taken by the relay node: a_0, which represents remaining silent and a_1, which represents detecting and retransmitting the source signal. In this work, we focus on analyzing the potential of selective relaying to prevent error propagation and to decrease e2e BER. The relay makes decisions to minimize the expected e2e error probability with given SNR observations.[2]

[2]If the relay retransmits in phase 2, the overall transmission uses more bandwidth and more power compared to direct transmission. To keep the analysis tractable these factors are not taken into account in relaying decisions. However, any selective relaying scheme compares favorably to simple relaying in terms multiplexing loss and total average power.

Then, the relaying policy that minimizes the e2e BER is given by

$$\pi^*(S_j) = \arg \min_{a_i \in \{a_0, a_1\}} \mathbb{P}\{\mathcal{E}_{e2e}|S_j, a_i\},$$

which can be expressed as

$$\mathbb{P}\{\mathcal{E}_{e2e}|S_j, a_0\} \underset{a_0}{\overset{a_1}{\gtrless}} \mathbb{P}\{\mathcal{E}_{e2e}|S_j, a_1\}. \quad (4)$$

This policy is optimal for minimizing the e2e BER for memoryless fading channels. The result could be different if link SNRs were correlated in time and the previous instantaneous SNR values were fed back to the relay.

If the relay does not forward the signal received in the first hop, the e2e bit error probability for the block depends only on the S-D channel: $\mathbb{P}\{\mathcal{E}_{e2e}|S_j, a_0\} = \mathbb{P}\{\mathcal{E}_{sd}|S_j\}$. If the relay does forward, we can express the e2e bit error probability as

$$\mathbb{P}\{\mathcal{E}_{e2e}|S_j, a_1\} = \mathbb{P}\{\mathcal{E}_{sr}|S_j\}\,\mathbb{P}\{\mathcal{E}_{prop}|S_j\} + (1-\mathbb{P}\{\mathcal{E}_{sr}|S_j\})\,\mathbb{P}\{\mathcal{E}_{coop}|S_j\}. \quad (5)$$

By substituting (5) into (4), we obtain

$$\mathbb{P}\{\mathcal{E}_{sr}|S_j\} \underset{a_1}{\overset{a_0}{\gtrless}} \frac{\mathbb{P}\{\mathcal{E}_{sd}|S_j\} - \mathbb{P}\{\mathcal{E}_{coop}|S_j\}}{\mathbb{P}\{\mathcal{E}_{prop}|S_j\} - \mathbb{P}\{\mathcal{E}_{coop}|S_j\}}. \quad (6)$$

The derivation up to this point is not specific to Rayleigh channels and is valid under any SNR distribution.

A. Probability of Cooperative Error

Since the destination employs MRC, the SNR after combining the two signals is the sum of the SNRs of the S-D and the R-D channels. If the relay has $S_4 = \{\gamma_{sr}, \gamma_{rd}, \gamma_{sd}\}$, the probability of cooperative error calculated at the relay is equal to

$$\mathbb{P}\{\mathcal{E}_{coop}|S_4\} = \mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd}, \gamma_{sd}\} = \text{BER}_{awgn}(\gamma_{rd}+\gamma_{sd}) = Q\left(\sqrt{2(\gamma_{rd}+\gamma_{sd})}\right). \quad (7)$$

The cooperative error probability given $S_3 = \{\gamma_{sr}, \sigma^2_{rd}, \gamma_{sd}\}$, is equal to

$$\mathbb{P}\{\mathcal{E}_{coop}|S_3\} = \mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd}, \gamma_{sd}\}$$
$$= \mathbb{E}_{\gamma_{rd}}\left[Q\left(\sqrt{2(\gamma_{sd}+\gamma_{rd})}\right)\right] \quad (8)$$
$$= \int_0^\infty \frac{1}{\sigma^2_{rd}} e^{-\gamma_{rd}/\sigma^2_{rd}} Q\left(\sqrt{2(\gamma_{rd}+\gamma_{sd})}\right) \mathrm{d}\gamma_{rd} \quad (9)$$
$$= e^{\gamma_{sd}/\sigma^2_{rd}} \int_{\gamma_{sd}}^\infty \frac{1}{\sigma^2_{rd}} e^{-t/\sigma^2_{rd}} Q\left(\sqrt{2t}\right) \mathrm{d}t$$
$$= e^{\gamma_{sd}/\sigma^2_{rd}} h(\gamma_{sd}, \sigma^2_{rd}), \quad (10)$$

where we use change of variables to obtain (10) from (9) and define $h(.,.)$ as $h(x,y) = \int_x^\infty \frac{1}{y} Q(\sqrt{2t}) e^{-t/y} \mathrm{d}t$. This function can be calculated in terms of Q function (See Appendix A for the derivation.) :

$$h(x,y) = e^{-x/y} Q(\sqrt{2x}) - \sqrt{\frac{y}{1+y}}\, Q\left(\sqrt{2x\left(1+\frac{1}{y}\right)}\right). \quad (11)$$

Similarly, the cooperative error for $S_2 = \{\gamma_{sr}, \gamma_{rd}, \sigma^2_{sd}\}$ is equal to $\mathbb{P}\{\mathcal{E}_{coop}|S_2\} = \mathbb{E}_{\gamma_{sd}}\left[Q\left(\sqrt{2(\gamma_{sd}+\gamma_{rd})}\right)\right]$. Since this expression is the same as (8) with γ_{rd} and γ_{sd} exchanged, $\mathbb{P}\{\mathcal{E}_{coop}|S_2\}$ is given by

$$\mathbb{P}\{\mathcal{E}_{coop}|S_2\} = \mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd}, \sigma_{sd}^2\} = \mathbb{E}_{\gamma_{sd}}\left[Q\left(\sqrt{2(\gamma_{sd}+\gamma_{rd})}\right)\right] = e^{\gamma_{rd}/\sigma_{sd}^2} h(\gamma_{rd}, \sigma_{sd}^2). \quad (12)$$

If the relay utilizes only $S_1 = \{\gamma_{sr}, \sigma_{rd}^2, \sigma_{sd}^2\}$ to make decisions, then the probability of cooperative error is equal to the BER of a 2-branch MRC receiver in Rayleigh fading, which is given as [18, pp. 846-847]

$$\mathbb{P}\{\mathcal{E}_{coop}|S_1\} = \mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2, \sigma_{sd}^2\} = \mathbb{E}_{\gamma_{sd},\gamma_{rd}}\left[Q\left(\sqrt{2(\gamma_{sd}+\gamma_{rd})}\right)\right]$$
$$= \begin{cases} \frac{1}{2}\left(1-\sqrt{\frac{\sigma_{rd}^2}{1+\sigma_{rd}^2}}\right)^2\left(1+\frac{1}{2}\sqrt{\frac{\sigma_{rd}^2}{1+\sigma_{rd}^2}}\right), & \sigma_{rd}^2 = \sigma_{sd}^2; \\ \frac{1}{2}\left[1-\frac{1}{\sigma_{sd}^2-\sigma_{rd}^2}\left(\sigma_{sd}^2\sqrt{\frac{\sigma_{sd}^2}{1+\sigma_{sd}^2}} - \sigma_{rd}^2\sqrt{\frac{\sigma_{rd}^2}{1+\sigma_{rd}^2}}\right)\right] & \sigma_{rd}^2 \neq \sigma_{sd}^2. \end{cases}$$

B. Approximate Expression for the Probability of Error Propagation

Without loss of generality, we assume that the source sends the symbol $x_s = +1$ and the relay sends the symbol $x_r = -1$. The error occurs if the destination decides that -1 was sent by the source. The decision variable after the destination combines the received signals (given in (1) and (2)) using MRC is given by:

$$\begin{aligned} y &= \frac{\alpha_{sd}^*\sqrt{E_{b,s}}}{N_0} y_{sd} + \frac{\alpha_{rd}^*\sqrt{E_{b,r}}}{N_0} y_{rd} \\ &= \left(\frac{|\alpha_{sd}|^2 E_{b,s}}{N_0} - \frac{|\alpha_{rd}|^2 E_{b,r}}{N_0}\right) + \frac{\alpha_{sd}^*\sqrt{E_{b,s}}}{N_0} n_{sd} \\ &\quad + \frac{\alpha_{rd}^*\sqrt{E_{b,r}}}{N_0} n_{rd} \\ &= (\gamma_{sd} - \gamma_{rd}) + \tilde{n}, \end{aligned} \quad (13)$$

where $\tilde{n}$ is the effective noise. The mean and the variance of $\tilde{n}$ are equal to $\mathbb{E}[\tilde{n}] = 0$ and $\mathbb{E}[|\tilde{n}|^2] = \frac{1}{2}(\gamma_{sd} + \gamma_{rd})$. The decision rule at the destination is to declare $+1$ if $y \geq 0$. Then, the probability of error propagation under $S_4 = \{\gamma_{sr}, \gamma_{rd}, \gamma_{sd}\}$ is equal to

$$\begin{aligned} \mathbb{P}\{\mathcal{E}_{prop}|S_4\} &= \mathbb{P}\{\mathcal{E}_{prop}|\gamma_{rd}, \gamma_{sd}\} = \mathbb{P}\{y < 0|\gamma_{rd}, \gamma_{sd}\} \\ &= \mathbb{P}\{\tilde{n} > (\gamma_{sd} - \gamma_{rd})|\gamma_{rd}, \gamma_{sd}\} \\ &= Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{(\gamma_{sd}+\gamma_{rd})/2}}\right). \end{aligned} \quad (14)$$

The probability of error propagation under $S_3 = \{\gamma_{sr}, \sigma_{rd}^2, \gamma_{sd}\}$ can be found by averaging (14) with respect to γ_{rd}

$$\mathbb{P}\{\mathcal{E}_{prop}|S_3\} = \mathbb{P}\{\mathcal{E}_{prop}|\sigma_{rd}^2, \gamma_{sd}\} = \mathbb{E}_{\gamma_{rd}}\left[\mathbb{P}\{\mathcal{E}_{prop}|\gamma_{sd}, \gamma_{rd}\}\right] = \int_0^\infty Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{(\gamma_{sd}+\gamma_{rd})/2}}\right)\frac{1}{\sigma_{rd}^2}e^{-\gamma_{rd}/\sigma_{rd}^2}\mathrm{d}\gamma_{rd}. \quad (15)$$

Similarly

$$\mathbb{P}\{\mathcal{E}_{prop}|S_2\} = \mathbb{P}\{\mathcal{E}_{prop}|\gamma_{rd}, \sigma_{sd}^2\} = \int_0^\infty Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{(\gamma_{sd}+\gamma_{rd})/2}}\right)\frac{1}{\sigma_{sd}^2}e^{-\gamma_{sd}/\sigma_{sd}^2}\mathrm{d}\gamma_{sd}, \quad (16)$$

and

$$\mathbb{P}\{\mathcal{E}_{prop}|S_1\} = \mathbb{P}\{\mathcal{E}_{prop}|\sigma_{rd}^2, \sigma_{sd}^2\} = \int_0^\infty\int_0^\infty Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{(\gamma_{sd}+\gamma_{rd})/2}}\right)\frac{1}{\sigma_{sd}^2\sigma_{rd}^2}e^{-\gamma_{sd}/\sigma_{sd}^2} \times e^{-\gamma_{rd}/\sigma_{rd}^2}\mathrm{d}\gamma_{sd}\mathrm{d}\gamma_{rd}. \quad (17)$$

Due to the complexity of the exact expressions given in (15)-(17), we provide approximate expressions for calculating the probability of error propagation for these models. Equation (13) shows that, if relay forwards an incorrect signal, this has a strong impact on the decision variable y. For instance, for $\gamma_{rd} \approx \gamma_{sd}$, the post-combining SNR is close to zero even if both γ_{rd} and γ_{sd} are large. Assuming that the incorrect relay signal - not the noise term - is the dominant factor that causes the decision variable y to be negative, we approximate the probability of error by the probability of $\{\gamma_{sd} - \gamma_{rd} < 0\}$.

For S_3, using the fact that γ_{rd} is an exponential random variable with mean σ_{rd}^2, we obtain the approximate probability of error as

$$\mathbb{P}\{\mathcal{E}_{prop}|S_3\} \approx \mathbb{P}\{\gamma_{sd} - \gamma_{rd} < 0|\sigma_{rd}^2, \gamma_{sd}\} = \int_{\gamma_{sd}}^\infty \frac{1}{\sigma_{rd}^2}e^{-\gamma_{rd}/\sigma_{rd}^2}\mathrm{d}\gamma_{rd} = e^{-\gamma_{sd}/\sigma_{rd}^2}. \quad (18)$$

Similarly for S_2

$$\mathbb{P}\{\mathcal{E}_{prop}|S_2\} \approx \mathbb{P}\{\gamma_{sd} - \gamma_{rd} < 0|\gamma_{rd}, \sigma_{sd}^2\} = \int_0^{\gamma_{rd}} \frac{1}{\sigma_{sd}^2}e^{-\gamma_{sd}/\sigma_{sd}^2}\mathrm{d}\gamma_{sd} = 1 - e^{-\gamma_{rd}/\sigma_{sd}^2}. \quad (19)$$

For S_1, since γ_{sd} and γ_{rd} are independent, we obtain

$$\begin{aligned} \mathbb{P}\{\mathcal{E}_{prop}|S_1\} &\approx \mathbb{P}\{\gamma_{sd} - \gamma_{rd} < 0|\sigma_{rd}^2, \sigma_{sd}^2\} \\ &= \int_0^\infty\int_0^{\gamma_{rd}} \frac{1}{\sigma_{sd}^2\sigma_{rd}^2}e^{-\gamma_{sd}/\sigma_{sd}^2}e^{-\gamma_{rd}/\sigma_{rd}^2}\mathrm{d}\gamma_{sd}\,\mathrm{d}\gamma_{rd} \\ &= \frac{\sigma_{rd}^2}{\sigma_{sd}^2 + \sigma_{rd}^2}. \end{aligned} \quad (20)$$

To check the accuracy of these approximations at practical SNR values, we compare them with the exact values obtained through the numerical integration of (15)-(17). Fig.s 2-4 show that all three approximations are reasonably accurate for a large range of SNR values.

C. Optimal Threshold Functions and Average e2e BER for SNR-based Selective Relaying

In this section, the optimal decision rule given in (6) is evaluated for all the models using the probability of error propagation and cooperative error expressions derived in Section IV-A and Section IV-B. All the rules simplify to a threshold on the instantaneous SNR of the S-R link.

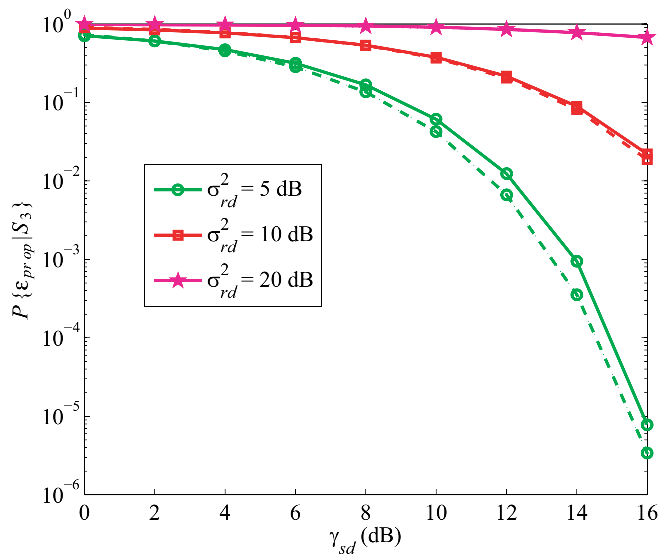

Fig. 2. Comparison of $\mathbb{P}\{\mathcal{E}_{prop}|S_3\}$ values obtained from the approximation in (18) and from the numerical integration of (15) as a function of γ_{sd} for different σ^2_{rd} values. Exact values are shown in solid lines and approximate values are shown in dashed lines.

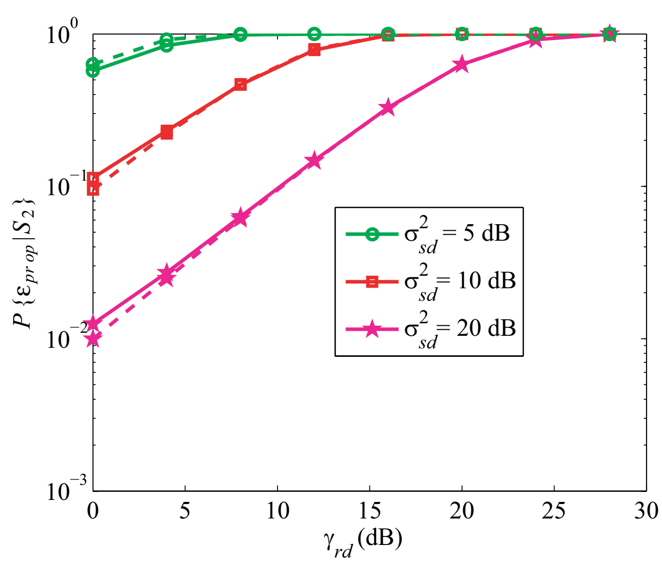

Fig. 3. Comparison of $\mathbb{P}\{\mathcal{E}_{prop}|S_2\}$ values obtained from the approximation in (19) and from the numerical integration of (16) as a function of γ_{rd} for different σ^2_{sd} values. Exact values are shown in solid lines and approximate values are shown in dashed lines.

1) Relaying based on Model 1*:* From (6) we obtain the relaying policy for Model 1 as $\mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\} \underset{a_1}{\overset{a_0}{\gtrless}} \delta_1(\sigma^2_{rd},\sigma^2_{sd})$, where δ_1 is defined as

$$\delta_1(\sigma^2_{rd},\sigma^2_{sd}) = \frac{\mathbb{P}\{\mathcal{E}_{sd}|S_1\} - \mathbb{P}\{\mathcal{E}_{coop}|S_1\}}{\mathbb{P}\{\mathcal{E}_{prop}|S_1\} - \mathbb{P}\{\mathcal{E}_{coop}|S_1\}}$$

$$\approx \frac{\frac{1}{\sigma^2_{sd}-\sigma^2_{rd}}\left(\sigma^2_{sd}\sqrt{\frac{\sigma^2_{sd}}{1+\sigma^2_{sd}}} - \sigma^2_{rd}\sqrt{\frac{\sigma^2_{rd}}{1+\sigma^2_{rd}}}\right) - \sqrt{\frac{\sigma^2_{sd}}{1+\sigma^2_{sd}}}}{\frac{2\sigma^2_{rd}}{\sigma^2_{rd}+\sigma^2_{sd}} - \left[1 - \frac{1}{\sigma^2_{sd}-\sigma^2_{rd}}\left(\sigma^2_{sd}\sqrt{\frac{\sigma^2_{sd}}{1+\sigma^2_{sd}}} - \sigma^2_{rd}\sqrt{\frac{\sigma^2_{rd}}{1+\sigma^2_{rd}}}\right)\right]}, \tag{21}$$

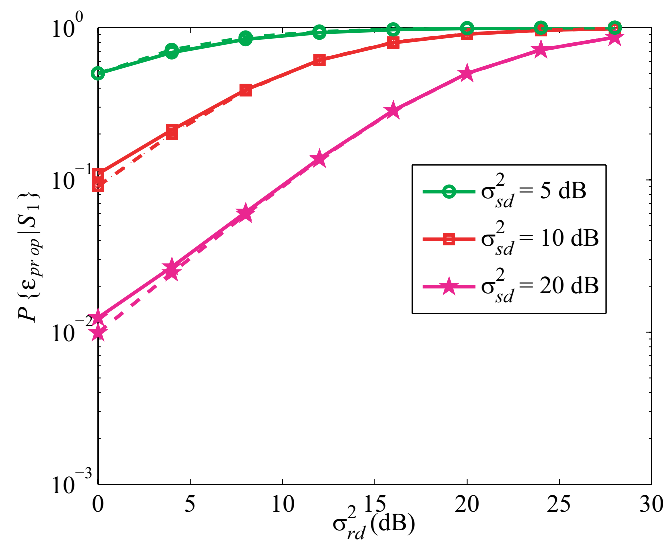

Fig. 4. Comparison of $\mathbb{P}\{\mathcal{E}_{prop}|S_1\}$ values obtained from the approximation in (20) and from the numerical integration of (17) as a function of σ^2_{rd} for different σ^2_{sd} values. Exact values are shown in solid lines and approximate values are shown in dashed lines.

where (3), (13) and (20) have been used to arrive at (21). If $\delta_1(\sigma^2_{rd},\sigma^2_{sd}) > 1/2$, the relay should always transmit since $\mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\}$ is always less than $1/2$. On the other hand, if $\delta_1(\sigma^2_{rd},\sigma^2_{sd}) \leq 1/2$, the relaying policy can be further simplified to

$$\gamma_{sr} \underset{a_0}{\overset{a_1}{\gtrless}} f_1(\sigma^2_{rd},\sigma^2_{sd}), \tag{22}$$

where

$$f_1(\sigma^2_{rd},\sigma^2_{sd}) = \begin{cases} \frac{1}{2}\left(Q^{-1}(\delta_1(\sigma^2_{rd},\sigma^2_{sd}))\right)^2, & \delta_1(\sigma^2_{rd},\sigma^2_{sd}) \leq 1/2; \\ 0, & \text{otherwise,} \end{cases}$$

and $Q^{-1}(z)$ denotes the inverse of the Q function, which is defined for $0 \leq z \leq 1$.

The average e2e BER for Model 1 is derived in (26) (See Appendix B for all average e2e BER derivations.):

$$\begin{aligned} &\mathrm{BER}^{(1)}_{e2e}(\sigma^2_{sr},\sigma^2_{rd},\sigma^2_{sd}) \\ &= \mathbb{P}\{\mathcal{E}_{sd}|\sigma^2_{sd}\}\left(1 - \exp(-f_1(\sigma^2_{rd},\sigma^2_{sd})/\sigma^2_{sr})\right) \\ &+ \mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}\exp(-f_1(\sigma^2_{rd},\sigma^2_{sd})/\sigma^2_{sr}) \\ &+ (\mathbb{P}\{\mathcal{E}_{prop}|\sigma^2_{rd},\sigma^2_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\})h\left(f_1(\sigma^2_{rd},\sigma^2_{sd}),\sigma^2_{sr}\right). \end{aligned} \tag{23}$$

An approximate closed-form expression for $\mathrm{BER}^{(1)}_{e2e}(\sigma^2_{sr},\sigma^2_{rd},\sigma^2_{sd})$ can be found by substituting (11), (13), and (20) into (23).

2) Relaying based on Model 2*:* The optimal decision rule for the case of S_2 is equal to $\mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\} \underset{a_1}{\overset{a_0}{\gtrless}} \delta_2(\gamma_{rd},\sigma^2_{sd})$, where δ_2 is found as

$$\delta_2(\gamma_{rd},\sigma^2_{sd}) = \frac{\mathbb{P}\{\mathcal{E}_{sd}|\sigma^2_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd},\sigma^2_{sd}\}}{\mathbb{P}\{\mathcal{E}_{prop}|\gamma_{rd},\sigma^2_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd},\sigma^2_{sd}\}}$$

$$\approx \frac{\frac{1}{2}\left(1 - \sqrt{\frac{\sigma^2_{sd}}{1+\sigma^2_{sd}}}\right) - e^{\gamma_{rd}/\sigma^2_{sd}}h(\gamma_{rd},\sigma^2_{sd})}{1 - e^{-\gamma_{rd}/\sigma^2_{sd}} - e^{\gamma_{rd}/\sigma^2_{sd}}h(\gamma_{rd},\sigma^2_{sd})}$$

by using (12) and (19). This rule can be expressed as

$$\gamma_{sr} \underset{a_0}{\overset{a_1}{\gtrless}} f_2(\gamma_{rd}, \sigma_{sd}^2),$$

where

$$f_2(\gamma_{rd}, \sigma_{sd}^2) = \begin{cases} \frac{1}{2}\left(Q^{-1}(\delta_2(\gamma_{rd}, \sigma_{sd}^2))\right)^2, & \delta_2(\gamma_{rd}, \sigma_{sd}^2) \le 1/2; \\ 0, & \text{otherwise.} \end{cases}$$

The average e2e BER for Model 2 is given by (28):

$$\begin{aligned} &\mathrm{BER}_{e2e}^{(2)}(\sigma_{sr}^2, \sigma_{rd}^2, \sigma_{sd}^2) \\ &\approx \int_0^\infty \left[\frac{1}{2}\left(1 - \sqrt{\frac{\sigma_{sd}^2}{1+\sigma_{sd}^2}}\right)(1 - \exp(-f_2(\gamma_{rd}, \sigma_{sd}^2)/\sigma_{sr}^2)) \right. \\ &+ \left((1 - e^{-\gamma_{rd}/\sigma_{sd}^2}) - e^{\gamma_{rd}/\sigma_{sd}^2} h(\gamma_{rd}, \sigma_{sd}^2)\right) \\ &\times h(f_2(\gamma_{rd}, \sigma_{sd}^2), \sigma_{sr}^2) \\ &\left. + (1 - e^{-\gamma_{rd}/\sigma_{sd}^2}) \exp(-f_2(\gamma_{rd}, \sigma_{sd}^2)/\sigma_{sr}^2)\right] \frac{1}{\sigma_{rd}^2} e^{\gamma_{rd}/\sigma_{rd}^2} \mathrm{d}\gamma_{rd}. \end{aligned}$$

Since the integrals to calculate the average e2e BERs of Model 2, Model 3, and Model 4 are intractable analytically, we use numerical integration to evaluate them in Section V.

3) Relaying based on Model 3: For Model 3 the optimal decision rule is given by $\mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\} \underset{a_1}{\overset{a_0}{\gtrless}} \delta_3(\sigma_{rd}^2, \gamma_{sd})$, where δ_3 is equal to

$$\begin{aligned} \delta_3(\sigma_{rd}^2, \gamma_{sd}) &= \frac{\mathbb{P}\{\mathcal{E}_{sd}|\gamma_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2, \gamma_{sd}\}}{\mathbb{P}\{\mathcal{E}_{prop}|\sigma_{rd}^2, \gamma_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2, \gamma_{sd}\}} \\ &\approx \frac{Q(\sqrt{2\gamma_{sd}}) - e^{\gamma_{sd}/\sigma_{rd}^2} h(\gamma_{sd}, \sigma_{rd}^2)}{e^{-\gamma_{sd}/\sigma_{rd}^2} - e^{\gamma_{sd}/\sigma_{rd}^2} h(\gamma_{sd}, \sigma_{rd}^2)}. \end{aligned}$$

This rule is equivalent to

$$\gamma_{sr} \underset{a_0}{\overset{a_1}{\gtrless}} f_3(\sigma_{rd}^2, \gamma_{sd}),$$

where

$$f_3(\sigma_{rd}^2, \gamma_{sd}) = \begin{cases} \frac{1}{2}\left(Q^{-1}(\delta_3(\sigma_{rd}^2, \gamma_{sd}))\right)^2, & \delta_3(\sigma_{rd}^2, \gamma_{sd}) \le 1/2; \\ 0, & \text{otherwise.} \end{cases}$$

The average e2e BER is given by (29):

$$\begin{aligned} &\mathrm{BER}_{e2e}^{(3)}(\sigma_{sr}^2, \sigma_{rd}^2, \sigma_{sd}^2) \\ &\approx \int_0^\infty \left[Q(\sqrt{2\gamma_{sd}})(1 - \exp(-f_3(\sigma_{rd}^2, \gamma_{sd})/\sigma_{sr}^2)) \right. \\ &+ \left(e^{-\gamma_{sd}/\sigma_{rd}^2} - e^{\gamma_{sd}/\sigma_{rd}^2} h(\gamma_{sd}, \sigma_{rd}^2)\right) h(f_3(\sigma_{rd}^2, \gamma_{sd}), \sigma_{sr}^2) \\ &\left. + e^{\gamma_{sd}/\sigma_{rd}^2} h(\gamma_{sd}, \sigma_{rd}^2) \exp(-f_3(\sigma_{rd}^2, \gamma_{sd})/\sigma_{sr}^2)\right] \\ &\times \frac{1}{\sigma_{sd}^2} e^{-\gamma_{sd}/\sigma_{sd}^2} \mathrm{d}\gamma_{sd}. \end{aligned}$$

4) Relaying based on Model 4: The optimal decision rule in the case of Model 4 is $\mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\} \underset{a_1}{\overset{a_0}{\gtrless}} \delta_4(\gamma_{rd}, \gamma_{sd})$, where δ_4 is equal to

$$\begin{aligned} \delta_4(\gamma_{rd}, \gamma_{sd}) &= \frac{\mathbb{P}\{\mathcal{E}_{sd}|\gamma_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd}, \gamma_{sd}\}}{\mathbb{P}\{\mathcal{E}_{prop}|\gamma_{rd}, \gamma_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd}, \gamma_{sd}\}} \\ &= \frac{Q(\sqrt{2\gamma_{sd}}) - Q\left(\sqrt{2(\gamma_{sd}+\gamma_{rd})}\right)}{Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{(\gamma_{sd}+\gamma_{rd})/2}}\right) - Q\left(\sqrt{2(\gamma_{sd}+\gamma_{rd})}\right)}, \end{aligned}$$

and this rule can be expressed as $\gamma_{sr} \underset{a_0}{\overset{a_1}{\gtrless}} f_4(\gamma_{rd}, \gamma_{sd})$, where

$$f_4(\gamma_{rd}, \gamma_{sd}) = \begin{cases} \frac{1}{2}\left(Q^{-1}(\delta_4(\gamma_{rd}, \gamma_{sd}))\right)^2, & \delta_4(\gamma_{rd}, \gamma_{sd}) \le 1/2; \\ 0, & \text{otherwise.} \end{cases}$$

The average e2e BER is derived in (30) and is equal to:

$$\begin{aligned} &\mathrm{BER}_{e2e}^{(4)}(\sigma_{sr}^2, \sigma_{rd}^2, \sigma_{sd}^2) \\ &= \int_0^\infty \int_0^\infty \left[Q(\sqrt{2\gamma_{sd}})(1 - \exp(-f_4(\gamma_{rd}, \gamma_{sd})/\sigma_{sr}^2)) \right. \\ &+ \left(Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{(\gamma_{sd}+\gamma_{rd})/2}}\right) - Q(\sqrt{\gamma_{rd}+\gamma_{sd}})\right) \\ &\times h(f_4(\gamma_{rd}, \gamma_{sd}), \sigma_{sr}^2) \\ &\left. + Q(\sqrt{\gamma_{rd}+\gamma_{sd}}) \exp(-f_4(\gamma_{rd}, \gamma_{sd})/\sigma_{sr}^2)\right] \\ &\times \frac{e^{-\gamma_{sd}/\sigma_{sd}^2} e^{-\gamma_{rd}/\sigma_{rd}^2}}{\sigma_{sd}^2 \sigma_{rd}^2} \mathrm{d}\gamma_{rd}\, \mathrm{d}\gamma_{sd}. \end{aligned}$$

V. Results

In this section, we first describe two benchmark schemes: simple digital relaying and genie-aided digital relaying. We then present numerical examples comparing the e2e BER of SNR-based selective relaying under the different models presented in this paper to these benchmark schemes. All the results are obtained from the analytical formulae derived in the paper. We resort to numerical integration where it is required.

A. Benchmark Schemes

The descriptions and e2e BERs of the benchmark schemes are given below.

1) Genie-aided digital relaying: Genie-aided digital relaying is a protocol designed under the hypothetical assumption that the relay has perfect error detection for each symbol. In phase 2, the relay retransmits only those symbols received correctly in phase 1. Since retransmitting a correctly detected symbol decreases e2e BER while transmitting an incorrectly detected symbol increases it, genie-aided protocol constitutes a performance upper bound for any selective digital relaying scheme. The e2e BER of genie-aided digital relaying is equal to

$$\begin{aligned} \mathrm{BER}_{e2e}^{genie}(\sigma_{sr}^2, \sigma_{rd}^2, \sigma_{sd}^2) &= \mathbb{P}\{\mathcal{E}_{sr}|\sigma_{sr}^2\}\mathbb{P}\{\mathcal{E}_{sd}|\sigma_{sd}^2\} \\ &+ (1 - \mathbb{P}\{\mathcal{E}_{sr}|\sigma_{sr}^2\})\mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2, \sigma_{sd}^2\}, \end{aligned}$$

which can be calculated by using (3) and (12).

2) Simple digital relaying: In simple digital relaying, the relay always transmits in phase 2. The e2e BER of simple digital relaying is equal to:

$$\begin{aligned} \mathrm{BER}_{e2e}^{simple}(\sigma_{sr}^2, \sigma_{rd}^2, \sigma_{sd}^2) &= \mathbb{P}\{\mathcal{E}_{sr}|\sigma_{sr}^2\}\mathbb{P}\{\mathcal{E}_{prop}|\sigma_{rd}^2, \sigma_{sd}^2\} \\ &+ (1 - \mathbb{P}\{\mathcal{E}_{sr}|\sigma_{sr}^2\})\mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2, \sigma_{sd}^2\}, \end{aligned}$$

which can be calculated by using (3), (12), and (19).

B. Numerical Results

In Fig. 5, we fix $\sigma_{rd}^2 = 15$ dB and $\sigma_{sd}^2 = 0$ dB and plot the e2e BER as a function of σ_{sr}^2. In this case, f_1 (the optimal threshold for Model 1) remains fixed as seen in (23). The threshold is very low ($f_1 = 0.545$), which can be attributed to the poor quality of the direct link. We observe that when the S-R link is favorable, selective relaying schemes have a small SNR gain (only 1 to 2 dB) compared to simple relaying.

Fig. 6(a) shows the BER performance at $\sigma_{sr}^2 = 15$ dB and $\sigma_{sd}^2 = 5$ dB as a function of σ_{rd}^2. For simple digital relaying as σ_{rd}^2 increases, on one hand the probability of error propagation increases, on the other hand the probability of cooperative error decreases. In Fig. 6(a), the decrease in the probability of cooperative error is the dominant factor. In Fig. 7(a), we plot the e2e BER at $\sigma_{sr}^2 = 15$ dB and $\sigma_{sd}^2 = 15$ dB as a function of σ_{rd}^2. We observe that in Fig. 7(a) the e2e BER of simple digital relaying increases as the R-D channel becomes stronger. This is because in this case the increase in error propagation dominates over the decrease in the cooperative error. In Fig.s 6(a) and 7(a) for large σ_{rd}^2, $\mathbb{P}\{\mathcal{E}_{prop}|\sigma_{rd}^2, \sigma_{sd}^2\} \approx 1$ and $\mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2, \sigma_{sd}^2\} \approx 0$. Thus, the performance of simple digital relaying is limited by the S-R link and can be approximated as $\text{BER}_{e2e}^{(simple)} \approx \text{BER}_{ray}(\sigma_{sr}^2)$ for large σ_{rd}^2. Similarly, $\text{BER}_{e2e}^{(genie)} \approx \text{BER}_{ray}(\sigma_{sr}^2) \times \text{BER}_{ray}(\sigma_{sd}^2)$ for large σ_{rd}^2. Model 1 has a significant performance gain over simple digital relaying in both Fig. 6(a) and Fig. 7(a), since, as shown in Fig. 6(b) and Fig. 7(b), it adaptively increases threshold f_1 as σ_{rd}^2 increases.

Finally, we study a scenario where all the average link SNRs are varied simultaneously. In this scenario, the S-R and R-D links have the same average SNR, while the S-D link has a lower average SNR, which is a typical scenario when R is located around the midpoint of S and D. Specifically, we assume $\sigma_{sr}^2 = \sigma_{rd}^2 = 16\,\sigma_{sd}^2 = \sigma^2$. In Fig. 8(a), we plot e2e BER as a function of σ^2. It is observed that simple digital relaying and direct transmission (i.e., no relay) have the same slope that is equal to 1, while the rest of relaying schemes have a common slope larger than 1, indicating cooperative diversity gains. The asymptotic diversity gains achieved by SNR-based selective relaying is studied in [15]. Fig. 8(b) depicts the behavior of the optimal threshold for Model 1. It is observed that the threshold must be increased as the link SNRs increases.

In all the numerical results, we observe that the performance of SNR-based selective relaying under Model 2 is very close to Model 1 and the performance under Model 4 is very close to Model 3. These observations show that the benefit from exploiting γ_{rd} at the relay is marginal. However, there is a gain both from Model 1 to Model 3 and from Model 2 to Model 4. Hence, it is useful to make use of γ_{sd} in relaying decisions. The gain from adapting according to γ_{sd} increases as the average SNR σ_{sd}^2 increases.

Although SNR-based selection relaying improves the e2e BER compared to simple digital relaying, it still has a significant performance gap compared to genie-aided digital relaying. Therefore, there might be room for improvement through hybrid methods combining SNR-based selection relaying with other methods proposed in the literature such as power control at the relay and better detection methods at the destination.

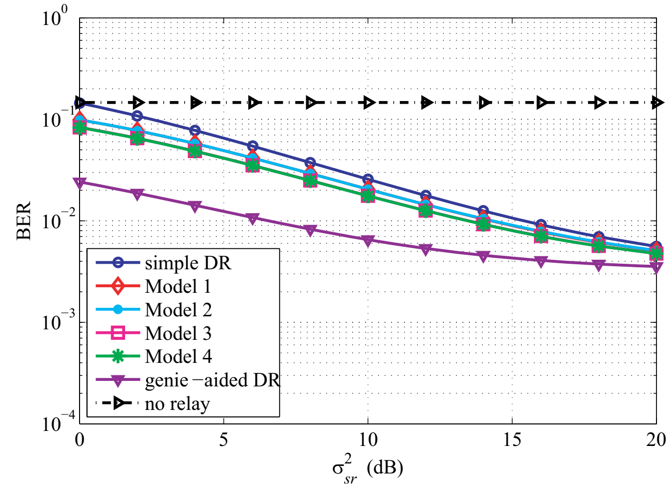

Fig. 5. The e2e BER for different relaying schemes as a function of σ_{sr}^2 for $\sigma_{rd}^2 = 15$ dB, $\sigma_{sd}^2 = 0$ dB.

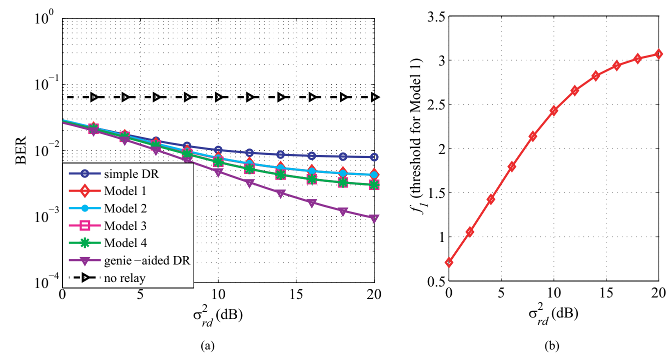

Fig. 6. The e2e BER for different relaying schemes and the threshold for Model 1 (obtained from (23)) as a function of σ_{rd}^2 for $\sigma_{sr}^2 = 15$ dB, $\sigma_{sd}^2 = 5$ dB.

VI. Conclusions

In this paper, we proposed and analyzed SNR-based selective relaying schemes to minimize the end-to-end bit error rate in cooperative digital relaying systems. We considered various models for the knowledge of the relay on the link SNRs in the network. For all the models, the optimal threshold for the source-relay SNR below which the relay must remain silent depends on the SNRs (average or instantaneous) of the relay-destination and source-destination links. In contrast to the assumption in the literature, the optimal threshold is independent of the average source-relay SNR. For BPSK modulation, we derived exact expressions and in some cases approximations for these optimal thresholds and their corresponding average BER. The average BER of the SNR-based selective relaying with these thresholds is compared to the performance of simple digital relaying. Studying the performance of different models, it has been observed that the instantaneous source-destination SNR information can be exploited while making relaying decisions. However, the benefit from knowledge of the instantaneous relay-destination SNR is marginal.

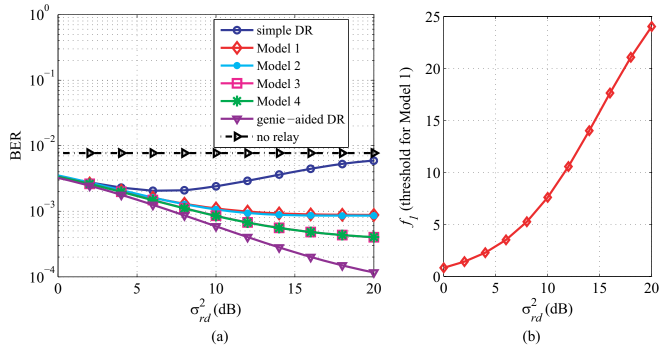

Fig. 7. The e2e BER for different relaying schemes and the threshold for Model 1 (obtained from (23)) as a function of σ^2_{rd} for $\sigma^2_{sr} = 15$ dB, $\sigma^2_{sd} = 15$ dB.

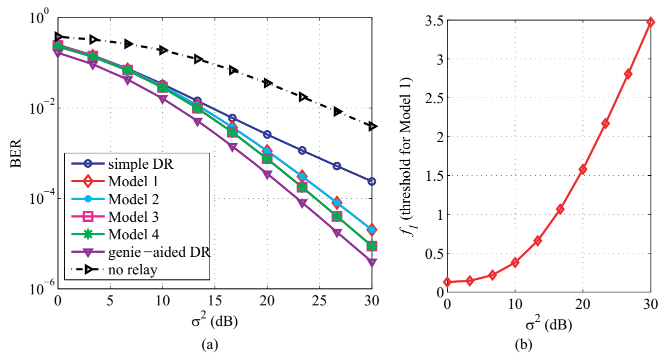

Fig. 8. The e2e BER for different relaying schemes and the threshold for Model 1 (obtained from (23)) as a function of σ^2, where $\sigma^2_{sr} = \sigma^2$, $\sigma^2_{rd} = \sigma^2$ and $\sigma^2_{sd} = (1/16)\,\sigma^2$.

Appendix A
Derivation of $h(x,y)$ Given in (11)

Using integration by parts, the function $h(x,y)$, which is defined as $h(x,y) = \int_x^\infty \frac{1}{y} e^{-t/y} Q(\sqrt{2t})\mathrm{d}t$, can be expressed as

$$h(x,y) = Q(\sqrt{2t})(-e^{-t/y})\Big]_x^\infty - \int_x^\infty (-e^{-t/y})\frac{\mathrm{d}}{\mathrm{d}t}Q(\sqrt{2t})\,\mathrm{d}t.$$

From the definition of Q function ($Q(t) = \int_t^\infty \frac{1}{\sqrt{2\pi}}e^{-z^2/2}\mathrm{d}z$), we have $\frac{\mathrm{d}}{\mathrm{d}t}Q(\sqrt{at}) = -\frac{1}{2\sqrt{2\pi}}\sqrt{\frac{a}{t}}e^{-at/2}$. Hence,

$$\begin{aligned} h(x,y) =& Q(\sqrt{2x})e^{-x/y} \\ &- \int_x^\infty \frac{1}{\sqrt{4\pi}}\frac{1}{\sqrt{t}}\exp\left(-t(1+1/y)\right)\mathrm{d}t. \end{aligned}$$

Rewriting the integral term in terms of the new integration variable $u = \sqrt{2t(1+1/y)}$, we obtain

$$\begin{aligned} h(x,y) =& Q(\sqrt{2x})e^{-x/y} \\ &- \sqrt{\frac{y}{1+y}}\int_{\sqrt{2x(1+1/y)}}^\infty \frac{1}{\sqrt{2\pi}}e^{-u^2/2}\mathrm{d}u \\ =& Q(\sqrt{2x})e^{-x/y} - \sqrt{\frac{y}{1+y}}\,Q\left(\sqrt{2x\left(1+\frac{1}{y}\right)}\right), \end{aligned}$$

which is the same as the expression given in (11). In [8], the authors used a similar derivation to obtain Eqn. (15) of their paper.

Appendix B
Average E2E BER Calculation

Conditioned on γ_{sr}, the e2e BER for Model 1 is equal to

$$\mathrm{BER}^{(1)}_{e2e}(\gamma_{sr},\sigma^2_{rd},\sigma^2_{sd}) = \mathbb{P}\{\mathcal{E}_{sd}|\sigma^2_{sd}\}, \qquad \gamma_{sr} < f_1(\sigma^2_{rd},\sigma^2_{sd});$$

and

$$\begin{aligned} \mathrm{BER}^{(1)}_{e2e}(\gamma_{sr},\sigma^2_{rd},\sigma^2_{sd}) =& \mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\}\Big(\mathbb{P}\{\mathcal{E}_{prop}|\sigma^2_{rd},\sigma^2_{sd}\} \\ &-\mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}\Big) + \mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}, \text{ otherwise.} \end{aligned}$$

The average e2e BER can be obtained by averaging the conditional BER over γ_{sr}:

$$\begin{aligned} &\mathrm{BER}^{(1)}_{e2e}(\sigma^2_{sr},\sigma^2_{rd},\sigma^2_{sd}) = \mathbb{E}_{\gamma_{sr}}\left[\mathrm{BER}^{(1)}_{e2e}(\gamma_{sr},\sigma^2_{rd},\sigma^2_{sd})\right] \\ &= \mathbb{P}\{\mathcal{E}_{sd}|\sigma^2_{sd}\}\int_0^{f_1(\sigma^2_{rd},\sigma^2_{sd})} p_{\gamma_{sr}}(\gamma_{sr})\mathrm{d}\gamma_{sr} \\ &\quad +\left((\mathbb{P}\{\mathcal{E}_{prop}|\sigma^2_{rd},\sigma^2_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}\right) \\ &\quad \times\int_{f_1(\sigma^2_{rd},\sigma^2_{sd})}^\infty \mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\}p_{\gamma_{sr}}(\gamma_{sr})\mathrm{d}\gamma_{sr} \\ &\quad +\mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}\int_{f_1(\sigma^2_{rd},\sigma^2_{sd})}^\infty p_{\gamma_{sr}}(\gamma_{sr})\mathrm{d}\gamma_{sr} \qquad (24) \\ &= \mathbb{P}\{\mathcal{E}_{sd}|\sigma^2_{sd}\}\left(1-\exp(-f_1(\sigma^2_{rd},\sigma^2_{sd})/\sigma^2_{sr})\right) \\ &\quad +\mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}\exp(-f_1(\sigma^2_{rd},\sigma^2_{sd})/\sigma^2_{sr}) \\ &\quad +(\mathbb{P}\{\mathcal{E}_{prop}|\sigma^2_{rd},\sigma^2_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}) \\ &\quad \times\int_{f_1(\sigma^2_{rd},\sigma^2_{sd})}^\infty Q(\sqrt{2\gamma_{sr}})p_{\gamma_{sr}}(\gamma_{sr})\mathrm{d}\gamma_{sr} \qquad (25) \\ &= \mathbb{P}\{\mathcal{E}_{sd}|\sigma^2_{sd}\}\left(1-\exp(-f_1(\sigma^2_{rd},\sigma^2_{sd})/\sigma^2_{sr})\right) \\ &\quad +\mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}\exp(-f_1(\sigma^2_{rd},\sigma^2_{sd})/\sigma^2_{sr}) \\ &\quad +(\mathbb{P}\{\mathcal{E}_{prop}|\sigma^2_{rd},\sigma^2_{sd}\} - \mathbb{P}\{\mathcal{E}_{coop}|\sigma^2_{rd},\sigma^2_{sd}\}) \\ &\quad \times h\left(f_1(\sigma^2_{rd},\sigma^2_{sd}),\sigma^2_{sr}\right). \qquad (26) \end{aligned}$$

For Model 2 the e2e BER conditioned on γ_{sr} and γ_{rd} is equal to

$$\mathrm{BER}^{(2)}_{e2e}(\gamma_{sr},\gamma_{rd},\sigma^2_{sd}) = \mathbb{P}\{\mathcal{E}_{sd}|\sigma^2_{sd}\}, \qquad \gamma_{sr} < f_2(\gamma_{rd},\sigma^2_{sd});$$

and

$$\begin{aligned} \mathrm{BER}^{(2)}_{e2e}(\gamma_{sr},\gamma_{rd},\sigma^2_{sd}) =& \mathbb{P}\{\mathcal{E}_{sr}|\gamma_{sr}\}\Big(\mathbb{P}\{\mathcal{E}_{prop}|\gamma_{rd},\sigma^2_{sd}\} \\ &-\mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd},\sigma^2_{sd}\}\Big) + \mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd},\sigma^2_{sd}\}, \text{otherwise.} \end{aligned}$$

The average e2e BER for Model 2 is given by

$$\begin{aligned} \mathrm{BER}^{(2)}_{e2e}(\sigma^2_{sr},\sigma^2_{rd},\sigma^2_{sd}) =& \mathbb{E}_{\gamma_{sr},\gamma_{rd}}\left[\mathrm{BER}^{(2)}_{e2e}(\gamma_{sr},\gamma_{rd},\sigma^2_{sd})\right] \\ =& \mathbb{E}_{\gamma_{rd}}\left[\mathbb{E}_{\gamma_{sr}}\left[\mathrm{BER}^{(2)}_{e2e}(\gamma_{sr},\gamma_{rd},\sigma^2_{sd})\right]\right]. \end{aligned}$$

Then, $\mathbb{E}_{\gamma_{sr}}\left[\mathrm{BER}^{(2)}_{e2e}(\gamma_{sr},\gamma_{rd},\sigma^2_{sd})\right]$ is calculated following the same steps as in (24), and $\mathrm{BER}^{(2)}_{e2e}(\sigma^2_{sr},\sigma^2_{rd},\sigma^2_{sd})$ is found as

$$\begin{aligned}\mathrm{BER}_{e2e}^{(2)}(\sigma_{sr}^2,\sigma_{rd}^2,\sigma_{sd}^2) = \mathbb{E}_{\gamma_{rd}}\Big[&\mathbb{P}\{\mathcal{E}_{sd}|\sigma_{sd}^2\}\\ &\times(1-\exp(-f_2(\gamma_{rd},\sigma_{sd}^2)/\sigma_{sr}^2))\\ &+\left(\mathbb{P}\{\mathcal{E}_{prop}|\gamma_{rd},\sigma_{sd}^2\}-\mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd},\sigma_{sd}^2\}\right)\\ &\times h(f_2(\gamma_{rd},\sigma_{sd}^2),\sigma_{sr}^2)\\ &+\mathbb{P}\{\mathcal{E}_{coop}|\gamma_{rd},\sigma_{sd}^2\}\exp(-f_2(\gamma_{rd},\sigma_{sd}^2)/\sigma_{sr}^2)\Big],\end{aligned} \quad (27)$$

which is approximately equal to

$$\begin{aligned}&\mathrm{BER}_{e2e}^{(2)}(\sigma_{sr}^2,\sigma_{rd}^2,\sigma_{sd}^2)\\ &\approx\int_0^\infty\Bigg[\frac{1}{2}\left(1-\sqrt{\frac{\sigma_{sd}^2}{1+\sigma_{sd}^2}}\right)(1-\exp(-f_2(\gamma_{rd},\sigma_{sd}^2)/\sigma_{sr}^2))\\ &+\left((1-e^{-\gamma_{rd}/\sigma_{sd}^2})-e^{\gamma_{rd}/\sigma_{sd}^2}h(\gamma_{rd},\sigma_{sd}^2)\right)h(f_2(\gamma_{rd},\sigma_{sd}^2),\sigma_{sr}^2)\\ &+(1-e^{-\gamma_{rd}/\sigma_{sd}^2})\exp(-f_2(\gamma_{rd},\sigma_{sd}^2)/\sigma_{sr}^2)\Bigg]\frac{1}{\sigma_{rd}^2}e^{\gamma_{rd}/\sigma_{rd}^2}\mathrm{d}\gamma_{rd},\end{aligned} \quad (28)$$

after substituting (3), (12) and (19) into (27).

In a similar manner, the e2e BER for Model 3 and Model 4 are calculated as

$$\begin{aligned}&\mathrm{BER}_{e2e}^{(3)}(\sigma_{sr}^2,\sigma_{rd}^2,\sigma_{sd}^2)\\ &\approx\int_0^\infty\Bigg[Q(\sqrt{2\gamma_{sd}})(1-\exp(-f_3(\sigma_{rd}^2,\gamma_{sd})/\sigma_{sr}^2))\\ &+\left(e^{-\gamma_{sd}/\sigma_{rd}^2}-e^{\gamma_{sd}/\sigma_{rd}^2}h(\gamma_{sd},\sigma_{rd}^2)\right)h(f_3(\sigma_{rd}^2,\gamma_{sd}),\sigma_{sr}^2)\\ &+e^{\gamma_{sd}/\sigma_{rd}^2}h(\gamma_{sd},\sigma_{rd}^2)\exp(-f_3(\sigma_{rd}^2,\gamma_{sd})/\sigma_{sr}^2)\Bigg]\\ &\times\frac{e^{-\gamma_{sd}/\sigma_{sd}^2}}{\sigma_{sd}^2}\mathrm{d}\gamma_{sd},\end{aligned} \quad (29)$$

and

$$\begin{aligned}&\mathrm{BER}_{e2e}^{(4)}(\sigma_{sr}^2,\sigma_{rd}^2,\sigma_{sd}^2)\\ &=\int_0^\infty\int_0^\infty\Bigg[Q(\sqrt{2\gamma_{sd}})(1-\exp(-f_4(\gamma_{rd},\gamma_{sd})/\sigma_{sr}^2))\\ &+\left(Q\left(\frac{\gamma_{sd}-\gamma_{rd}}{\sqrt{(\gamma_{sd}+\gamma_{rd})/2}}\right)-Q(\sqrt{\gamma_{rd}+\gamma_{sd}})\right)\\ &\times h(f_4(\gamma_{rd},\gamma_{sd}),\sigma_{sr}^2)\\ &+Q(\sqrt{\gamma_{rd}+\gamma_{sd}})\exp(-f_4(\gamma_{rd},\gamma_{sd})/\sigma_{sr}^2)\Bigg]\\ &\times\frac{e^{-\gamma_{sd}/\sigma_{sd}^2}e^{-\gamma_{rd}/\sigma_{rd}^2}}{\sigma_{sd}^2\sigma_{rd}^2}\mathrm{d}\gamma_{rd}\,\mathrm{d}\gamma_{sd}.\end{aligned} \quad (30)$$

APPENDIX C
THRESHOLD MINIMIZING E2E SYMBOL ERROR RATE FOR MPSK MODULATION

Consider the case where the source and the relay modulate their signals using MPSK. Let the symbols be denoted by $x_0,\dots,x_{M-1}$, where $x_i=e^{j2\pi i/M}$. The symbol sent by the source and the relay are denoted by x_s and x_r, respectively. The received signals are given by $y_{sr}=\alpha_{sr}\sqrt{E_{s,s}}\,x_s+n_{sr}$, $y_{sd}=\alpha_{sd}\sqrt{E_{s,s}}\,x_s+n_{sd}$, and $y_{rd}=\alpha_{rd}\sqrt{E_{s,r}}\,x_r+n_{rd}$, where $E_{s,s}$ is the energy per symbol spent by the source and $E_{s,r}$ is the energy per symbol spent by the relay. Let γ_{ij} and σ_{ij}^2 denote the instantaneous and average SNR per symbol.

Consider Model 1, where the relay makes decisions based on $S_1=\{\gamma_{sr},\sigma_{rd}^2,\sigma_{sd}\}$. The decision rule to minimize e2e symbol error rate (SER) is $\mathbb{P}\{\mathcal{E}_{e2e}|S_1,a_0\}\underset{a_0}{\overset{a_1}{\gtrless}}\mathbb{P}\{\mathcal{E}_{e2e}|S_1,a_1\}$, where $\mathcal{E}_{e2e}$ represents the e2e symbol error event. $\mathbb{P}\{\mathcal{E}_{e2e}|S_1,a_0\}=\mathbb{P}\{\mathcal{E}_{sd}|\sigma_{sd}^2\}$ and is given in [19, Eqn. (8.112)]. Without loss of generality, let us assume that the source transmits x_0. Then the term $\mathbb{P}\{\mathcal{E}_{e2e}|S_1,a_1\}$ can be decomposed as

$$\begin{aligned}\mathbb{P}\{\mathcal{E}_{e2e}|S_1,a_1\}=&\mathbb{P}\{x_r=x_0|x_s=x_0\}\mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2,\sigma_{sd}^2\}\\ &+\sum_{i=1}^{M-1}\mathbb{P}\{x_r=x_i|x_s=x_0\}\\ &\times\mathbb{P}\{\mathcal{E}_{prop}|x_s=x_0,x_r=x_i,\sigma_{rd}^2,\sigma_{sd}^2\}\end{aligned} \quad (31)$$

The term $\mathbb{P}\{\mathcal{E}_{coop}|\sigma_{rd}^2,\sigma_{sd}^2\}$ is given in [19, Eqn. (9.14)]. The term $\mathbb{P}\{x_r=x_i|x_s=x_0\}$ is obtained in integral form in [20, Eqn. (4.198.b)], and [19, Eqn. (8.29)]. We observe that in M-ary modulation, unlike in BPSK, there are $M-1$ ways of making an incorrect decision and their impacts on detection at the destination are not necessarily the same. After MRC the decision variable is given by $y=\gamma_{sd}+\gamma_{rd}e^{j2\pi i/M}+\tilde{n}$ (derivation is given in Section IV-B). As in Section IV-B, we assume that an incorrectly detected symbol sent by the relay constitutes the dominant cause of detection errors at the destination. That is, the term $\mathbb{P}\{\mathcal{E}_{prop}|x_s=x_0,x_r=x_i,\sigma_{rd}^2,\sigma_{sd}^2\}$ can be approximated by the probability that $\gamma_{sd}+\gamma_{rd}e^{j2\pi i/M}$ falls in the decision region of symbol x_i, denoted as $\mathcal{R}_i$. Exploiting the geometry of the MPSK constellation, one can easily derive that $\gamma_{sd}+\gamma_{rd}e^{j2\pi i/M}\in\mathcal{R}_i$ if and only if $\gamma_{sd}-c_{i,M}\,\gamma_{rd}<0$, where

$$c_{i,M}\triangleq\begin{cases}\frac{\sin(\pi(2i-1)/M)}{\sin(\pi/M)}, & i=1,2,\dots,\lfloor M/2\rfloor;\\ -\frac{\sin(\pi(2i+1)/M)}{\sin(\pi/M)}, & i=\lfloor M/2\rfloor+1,\dots,M-1.\end{cases}$$

Then, as in (20), we can calculate an approximate expression for $\mathbb{P}\{\mathcal{E}_{prop}|x_0,x_i,\sigma_{rd}^2,\sigma_{sd}^2\}$:

$$\begin{aligned}\mathbb{P}\{\mathcal{E}_{prop}|x_0,x_i,\sigma_{rd}^2,\sigma_{sd}^2\}&\approx\mathbb{P}\{\gamma_{sd}-c_{i,M}\,\gamma_{rd}<0|\sigma_{rd}^2,\sigma_{sd}^2\}\\ &=\int_0^\infty\int_0^{c_{i,M}\gamma_{rd}}\frac{1}{\sigma_{sd}^2\sigma_{rd}^2}e^{-\gamma_{sd}/\sigma_{sd}^2}e^{-\gamma_{rd}/\sigma_{rd}^2}\mathrm{d}\gamma_{sd}\,\mathrm{d}\gamma_{rd}\\ &=\frac{\sigma_{rd}^2c_{i,M}}{\sigma_{sd}^2+\sigma_{rd}^2c_{i,M}}.\end{aligned} \quad (32)$$

By substituting (32) and the other terms into (31), the decision rule can be determined. Since $\mathbb{P}\{\mathcal{E}_{e2e}|S_1,a_1\}$ decreases with γ_{sr}, the decision rule is a threshold rule on γ_{sr}, where the optimal threshold is a function of σ_{rd}^2 and σ_{sd}^2. Obtaining a closed-form expression for the optimal threshold is quite difficult. However, bounds such as union bound, can be used to derive approximations for $\mathbb{P}\{\mathcal{E}_{e2e}|S_1,a_1\}$, thereby leading to approximate closed-form expressions for the optimal threshold.

REFERENCES

[1] N. Laneman, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–part I: system description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.
[3] S. Valentin, T. Volkhausen, F. A. Onat, H. Yanikomeroglu, and H. Karl, "Enabling partial forwarding by decoding-based one and two-stage selective cooperation," in *Proc. IEEE Cognitive and Cooperative Wireless Networks (CoCoNET) Workshop*, collocated with *IEEE ICC*, May 2008.
[4] Texas Instruments, CC2420 data sheet. 2.4 GHz IEEE 802.15.4 / ZigBee-Ready RF Transceiver. Available at: http://focus.ti.com/lit/ds/symlink/cc2420.pdf.
[5] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Commun. Mag.*, vol. 40, pp. 102–114, 2002.
[6] M. Zorzi and R. R. Rao, "Coding tradeoffs for reduced energy consumption in sensor networks," in *Proc. IEEE Personal, Indoor and Mobile Radio Communications (PIMRC)*, 2004.
[7] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, pp. 1820–1830, Oct. 2004.
[8] P. Herhold, E. Zimmermann, and G. Fettweis, "A simple cooperative extension to wireless relaying," in *Proc. International Zurich Seminar on Communications*, pp. 36–39, 2004. Long version, available at http://wwwmns.ifn.et.tu-dresden.de/publications/2004/Herhold_P_IZS_04.pdf.
[9] A. Adinoyi and H. Yanikomeroglu, "Cooperative relaying in multi-antenna fixed relay networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 533-544, Feb. 2007.
[10] W. P. Siriwongpairat, T. Himsoon, W. Su, and K. J. R. Liu, "Optimum threshold-selection relaying for decode-and-forward cooperation protocol," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, Apr. 2006.
[11] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2415–2425, Oct. 2003.
[12] Z. Lin, E. Erkip, and A. Stefanov, "Cooperative regions and partner choice in coded cooperative systems," *IEEE Trans. Commun.*, vol. 54, pp. 1323–1334, July 2006.
[13] T. Wang, R. Wang, and G. Giannakis, "Smart regenerative relays for link-adaptive cooperative communications," in *Proc. 40th Annual Conference on Information Sciences and Systems (CISS)*, Mar. 2006.
[14] T. Wang, A. Cano, and G. Giannakis, "Link-adaptive cooperative communications without channel state information," in *Proc. Military Communications Conference (MILCOM)*, pp. 1–7, Oct. 2006.
[15] F. A. Onat, Y. Fan, H. Yanikomeroglu, and J. S. Thompson, "Asymptotic BER analysis of threshold digital relaying in cooperative wireless systems." *IEEE Trans. Wireless Commun.*, to appear, 2008.
[16] D. Chen and J. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 1785–1794, July 2006.
[17] T. Wang, A. Cano, G. B. Giannakis, and J. N. Laneman, "High-performance cooperative demodulation with decode-and-forward relays," *IEEE Trans. Commun.*, vol. 55, no. 7, pp. 1427–1438, 2007.
[18] J. G. Proakis, *Digital Communications*. McGraw-Hill, 2000.
[19] M. K. Simon and M. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. Wiley& Sons, Inc., 2000.
[20] M. K. Simon, S. M. Hinedi, and W. C. Lindsey, *Digital Communication Techniques: Signal Design and Detection*. Upper Saddle River, NJ: Prentice Hall, 1995.

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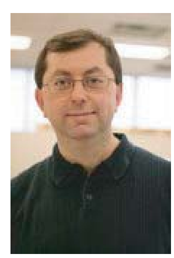

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