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Performance Analysis of Selection Combining of Signals With Different Modulation Levels in Cooperative Communications

Akram Bin Sediq and Halim Yanikomeroglu

***Abstract*—Cooperative relaying introduces spatial diversity through the creation of a virtual antenna array. The vast majority of research in bit-error-rate (BER) performance analysis of selection-combining (SC) schemes used in digital cooperative relaying assumes the modulation level used by both the source and the relay to be the same. This assumption does not necessarily hold when adaptive modulation is implemented. In conventional SC, the branch with the highest signal-to-noise ratio (SNR) is chosen; we refer to this scheme as SNR-based SC (SNR-SC). However, when different modulation levels are employed, the branch that has the maximum SNR may not necessarily be the most reliable branch due to different error-resistance capabilities of the modulation levels. Consequently, the BER-based SC (BER-SC) is a better SC scheme. In BER-SC, the receiver calculates the BER for each branch (using the SNR and the modulation level) and then decodes the signal from the branch that has the minimum BER. In this paper, we provide BER performance analysis for both BER-SC and SNR-SC and show that BER-SC outperforms SNR-SC, with very comparable complexity. Moreover, we analytically quantify the gain achieved by using BER-SC over SNR-SC through asymptotic approximation. We note that BER-SC and SNR-SC schemes are identical when the received signals belong to the same modulation level.**

***Index Terms*—Bit error rate (BER) selection combining, cooperative diversity, diversity analysis, relay networks, selection combining (SC).**

I. Introduction

Cooperative relaying has received tremendous interest in both industry and academia in the recent years. In cooperative relaying, the signals from the source–relay and relay–destination links are properly combined to achieve spatial diversity [2], [3]. Relays can be classified into digital and analog relays. Analog relays amplify and forward the received signal, whereas digital relays decode and forward a regenerated version of the received signal; in this work, digital relaying is considered.

The vast majority of research in bit error rate (BER) performance analysis of selection-combining (SC) schemes used in digital cooperative relaying assumes the modulation level used by both the source and the relay to be the same. This assumption does not necessarily hold in wireless networks where adaptive modulation is implemented.

In [4] and [5], it is shown that the average throughput of a wireless network can be significantly increased if adaptive modulation and coding are implemented in cooperative relaying. To achieve spatial diversity for signals with different modulation levels, SC can be used since this is the least complex diversity combining scheme [4], [5]. In [6], BER performance analysis of soft-bit maximal ratio combining (SBMRC) is presented. SBMRC is introduced as a low-complexity receiver structure that achieves near-optimal performance when used in combining signals with different modulation levels in a way similar to maximal ratio combining (MRC). However, SC is still more attractive than SBMRC from the complexity point of view.

Although the literature is rich in the BER performance analysis of conventional SC [7], as far as we know, it is limited to the case of combining signals with the same modulation level. To this end, this paper addresses performance analysis of SC schemes when they are used for combining signals with different modulation levels.

Notation: For a random variable X, $\bar{X} = E\{X\}$ denotes its mean; $Q(x)$ is the area under the right tail of normalized Gaussian probability density function (pdf) given by $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-(t^2/2)} dt$.

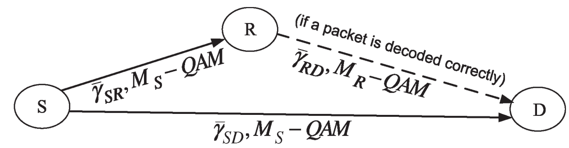

Fig. 1. System model.

II. System Model

We consider a three-node network consisting of a source (S), a relay (R), and a destination (D), all having one antenna, as shown in Fig. 1.

The transmitting nodes S and R transmit on two orthogonal channels, i.e., they do not interfere with each other. For convenience, we consider time-division duplexing to ensure orthogonal transmission from S and R. In the first time slot, S transmits a packet of N bits to R using M_S quadrature amplitude modulation (QAM). This packet is overheard by D because of the broadcast nature of the wireless channel. R fully decodes the packet and forwards it to D in the second time slot using M_R-QAM. We focus on these modulation schemes as they are among the most popular schemes in wireless networks [8]. Deciding on which modulation to be used is beyond the scope of this paper and can be found in [4], [5], and [9, pp. 54–60].

To mitigate the severe effects of detection errors made by R, the following selective-relaying protocol is assumed. The R fully decodes the packet and checks to determine if the packet is correctly decoded with the help of a cyclic redundancy check code. If the packet is decoded successfully, R retransmits the packet to D in the second time slot, and D chooses either to decode the signal from S or R. Otherwise, R refrains from retransmission and sends a one-bit negative acknowledgment to D, indicating that it failed to correctly decode the packet, and D will only utilize the transmission from S. We note that the purpose of this paper is not to design new error-propagation mitigation schemes but rather to revisit the BER performance analysis for SC schemes in this new context.

Due to multipath fading, the channel variations in the links S-R, S-D, and R-D are modeled as independent Rayleigh random variables. The instantaneous SNRs (which are defined as the ratio between the received energy per symbol and the noise power spectral density) in the links S-R, S-D, and R-D are denoted by $\gamma_{\rm SR}$, $\gamma_{\rm SD}$, and $\gamma_{\rm RD}$, respectively, and they are independent exponential random variables. The average SNRs in the links S-R, S-D, and R-D are denoted by $\bar{\gamma}_{\rm SR}$, $\bar{\gamma}_{\rm SD}$, and $\bar{\gamma}_{\rm RD}$, respectively.

Manuscript received July 16, 2010; revised February 1, 2011; accepted February 4, 2011. Date of publication March 17, 2011; date of current version May 16, 2011. This work was supported by an Ontario Graduate Scholarship. This paper was presented in part at the Fall IEEE Vehicular Technology Conference, Anchorage, AK, September 20–23, 2009. The review of this paper was coordinated by Prof. L.-L. Yang.

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Digital Object Identifier 10.1109/TVT.2011.2128357

III. Bit-Error Rate-Selection Combining and Signal-to-Noise Ratio-Selection Combining

In this paper, we investigate the conventional SNR-based SC (SNR-SC) and the BER-based SC (BER-SC). In SNR-SC, the receiver decodes the signal only from the branch that has the maximum SNR. When different modulation levels are employed, the branch that has the maximum SNR may not necessarily be the most reliable branch due to different error-resistance capabilities of the modulation levels. Consequently, BER-SC is a better SC scheme. In BER-SC, the receiver calculates the BER for each branch (using the SNR and the modulation level) and then decodes the signal from the branch that has the minimum BER.[1]

Using the approximate BER expression for Gray-coded square M-QAM given in [10], the selection criterion can be written as

$$\text{select node } i, \text{where } i = \arg\min_{i\in\{\mathrm{S},\mathrm{R}\}} \mathrm{BER}_{M_i}(\gamma_{i\mathrm{D}}) \tag{1}$$

where[2]

$$\mathrm{BER}_{M_i}(\gamma_{i\mathrm{D}}) \approx c_{M_i} Q\left(\sqrt{2d_{M_i}^2\gamma_{i\mathrm{D}}}\right) \tag{2}$$

$$c_{M_i} = \begin{cases} 1, & M_i = 2 \\ 2\frac{1-1/\sqrt{M_i}}{\log_2\sqrt{M_i}}, & M_i \geq 4 \end{cases} \tag{3}$$

$$d_{M_i} = \begin{cases} 1, & M_i = 2 \\ \sqrt{\frac{3}{2(M_i-1)}}, & M_i \geq 4. \end{cases} \tag{4}$$

Note that BER-SC reduces to SNR-SC in the special case where all the signals belong to the same modulation level. We remark that, even though our focus in this paper is on square M-QAMs, all the derived equations in this paper are applicable to any modulation scheme that has instantaneous BER in the form $c_{M_i}Q(\sqrt{2d_{M_i}^2\gamma_{i\mathrm{D}}})$.

IV. Performance Analysis of Bit-Error Rate-Selection Combining and Signal-to-Noise Ratio-Selection Combining

In this section, we derive BER expression for BER-SC. For the sake of completeness and comparison, we also derive BER expression for SNR-SC.

The average BER for BER-SC and SNR-SC can be written as[3]

$$\mathrm{BER}^{\mathrm{BER-SC}} = \mathrm{PER}_{\mathrm{SR}}\mathrm{BER}_{\mathrm{SD}} + (1-\mathrm{PER}_{\mathrm{SR}})\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}} \tag{5}$$

$$\mathrm{BER}^{\mathrm{SNR-SC}} = \mathrm{PER}_{\mathrm{SR}}\mathrm{BER}_{\mathrm{SD}} + (1-\mathrm{PER}_{\mathrm{SR}})\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}} \tag{6}$$

respectively, where

$\mathrm{PER}_{\mathrm{SR}}$	average packet error ratio in S-R;
$\mathrm{BER}_{\mathrm{SD}}$	average BER in S-D;
$\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}}$	average BER at D after SC using BER-SC;
$\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}}$	average BER at D after SC using SNR-SC.

[1]BER-SC can also be viewed as an SNR-based scheme since it requires the knowledge of SNR; nevertheless, we use the term BER-SC to distinguish this scheme from the SNR-SC type.

[2]The expression is exact for $M_i = 2$ [binary phase-shift keying (BPSK)] and $M_i = 4$ (quadrature phase-shift keying). In this paper, BPSK is referred to as 2-QAM.

[3]The BER expressions we derived in [1] and [9] are only applicable to the case where $\mathrm{PER}_{\mathrm{SR}} \approx 0$. In other words, we only provided expressions for $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}}$ and $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}}$ in [1] and [9].

The average BER in the link from nodes i to j, where $ij \in \{\mathrm{SR}, \mathrm{RD}, \mathrm{SD}\}$, for M-QAMs in a point-to-point Rayleigh fading channel can be well approximated as

$$\begin{aligned}\mathrm{BER}_{ij} &\approx \int_0^\infty c_{M_i} Q\left(\sqrt{2d_{M_i}^2\gamma_{ij}}\right)\frac{1}{\bar{\gamma}_{ij}}e^{-\frac{\gamma_{ij}}{\bar{\gamma}_{ij}}}d\gamma_{ij} \\ &= \frac{1}{2}c_{M_i}\left(1-\sqrt{\frac{d_{M_i}^2\bar{\gamma}_{ij}}{1+d_{M_i}^2\bar{\gamma}_{ij}}}\right)\end{aligned} \tag{7}$$

where the previous integration is evaluated in [11, pp. 817–818]. If symbol errors independently occur in an N-bit packet, then $\mathrm{PER}_{\mathrm{SR}}$ can be expressed as

$$\begin{aligned}\mathrm{PER}_{\mathrm{SR}} &= 1-(1-\mathrm{SER}_{\mathrm{SR}})^{\frac{N}{\log_2 M_S}} \\ &\approx 1-\left(1-\frac{1}{2}c_{M_S}\log_2(M_S)\right. \\ &\qquad \left.\times\left(1-\sqrt{\frac{d_{M_S}^2\bar{\gamma}_{\mathrm{SR}}}{1+d_{M_S}^2\bar{\gamma}_{\mathrm{SR}}}}\right)\right)^{\frac{N}{\log_2 M_S}}\end{aligned} \tag{8}$$

where we used the fact that $\mathrm{SER} \approx \mathrm{BER}\log_2 M_S$ for Gray-coded constellations [10].

The derivations of $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}}$ and $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}}$ are given in Sections IV-A and B, respectively. In Section IV-C, we give the final expressions for the average BER for BER-SC and SNR-SC.

A. Derivations of $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}}$

The instantaneous BER at the output of BER-SC, given γ_{SD} and γ_{RD}, can be written as

$$\mathrm{BER}_{\mathrm{comp,inst}}^{\mathrm{BER-SC}} \approx \begin{cases} c_{M_R}Q\left(\sqrt{2d_{M_R}^2\gamma_{\mathrm{RD}}}\right), & \mathrm{BER}_{M_R} \leq \mathrm{BER}_{M_S} \\ c_{M_S}Q\left(\sqrt{2d_{M_S}^2\gamma_{\mathrm{SD}}}\right), & \mathrm{BER}_{M_R} > \mathrm{BER}_{M_S} \end{cases} \tag{9}$$

where BER_{M_S} and BER_{M_R} are given by (2). The common approach in deriving the average BER is to average the instantaneous BER over the pdf of the output SNR [7]. This approach works when the signals belong to the same modulation level since, in this case, the instantaneous BER is a function only of the output SNR. However, it is not straightforward to use this approach in our problem since the instantaneous BER is a piecewise function with intervals that are dependent on the instantaneous SNRs, and it cannot be expressed as a function of the output SNR only. Therefore, to get the average BER, we average (9) over the joint pdf of γ_{SD} and γ_{RD}. The joint pdf of γ_{SD} and γ_{RD} is the multiplication of the individual pdfs and can be expressed as

$$f(\gamma_{\mathrm{SD}},\gamma_{\mathrm{RD}}) = \begin{cases} \frac{1}{\bar{\gamma}_{\mathrm{SD}}\bar{\gamma}_{\mathrm{RD}}}e^{-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}}e^{-\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}}, & \gamma_{\mathrm{SD}},\gamma_{\mathrm{RD}} \geq 0 \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

Using (9) and (10), the average BER can be written as

$$\begin{aligned}\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}} &\approx \iint_{\Psi_1} c_{M_R}Q\left(\sqrt{2d_{M_R}^2\gamma_{\mathrm{RD}}}\right) \\ &\quad\times\frac{1}{\bar{\gamma}_{\mathrm{SD}}}\frac{1}{\bar{\gamma}_{\mathrm{RD}}}e^{-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}}e^{-\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}}d\gamma_{\mathrm{SD}}d\gamma_{\mathrm{RD}} \\ &\quad+\iint_{\Psi_2} c_{M_S}Q\left(\sqrt{2d_{M_S}^2\gamma_{\mathrm{SD}}}\right) \\ &\quad\times\frac{1}{\bar{\gamma}_{\mathrm{SD}}}\frac{1}{\bar{\gamma}_{\mathrm{RD}}}e^{-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}}e^{-\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}}d\gamma_{\mathrm{SD}}d\gamma_{\mathrm{RD}}\end{aligned} \tag{11}$$

where

$$\Psi_1 = \{(\gamma_{\rm SD}, \gamma_{\rm RD}) : {\rm BER}_{M_R} \le {\rm BER}_{M_S}\}$$
$$\Psi_2 = \{(\gamma_{\rm SD}, \gamma_{\rm RD}) : {\rm BER}_{M_R} > {\rm BER}_{M_S}\}. \quad (12)$$

The regions Ψ_1 and Ψ_2 are defined by nonlinear functions [given by (2)], which makes it difficult to perform the integration given by (11). Consequently, we resort to approximating the regions such that the resulted approximated regions are defined by linear functions as follows:

$$\begin{aligned}\Psi_1 &\approx \left\{(\gamma_{\rm SD}, \gamma_{\rm RD}) : c_{M_R} e^{-d^2_{M_R}\gamma_{\rm RD}} \le c_{M_S} e^{-d^2_{M_S}\gamma_{\rm SD}}\right\}\\ &= \left\{(\gamma_{\rm SD}, \gamma_{\rm RD}) : d^2_{M_R}\gamma_{\rm RD} \ge d^2_{M_S}\gamma_{\rm SD} + \log\frac{c_{M_R}}{c_{M_S}}\right\}\\ &\approx \left\{(\gamma_{\rm SD}, \gamma_{\rm RD}) : d^2_{M_R}\gamma_{\rm RD} \ge d^2_{M_S}\gamma_{\rm SD}\right\} \end{aligned} \quad (13)$$

where the first approximation is made using the well-known Chernoff bound on the Q function, and the second approximation is made by dropping the constant $\log(c_{M_R}/c_{M_S})$. This approximation is tight for $\gamma_{\rm SD}, \gamma_{\rm RD} \gg 1$. Similarly

$$\Psi_2 \approx \left\{(\gamma_{\rm SD}, \gamma_{\rm RD}) : d^2_{M_R}\gamma_{\rm RD} < d^2_{M_S}\gamma_{\rm SD}\right\}. \quad (14)$$

The approximations given by (13) and (14) can be interpreted as comparing the normalized SNRs defined by Forney and Ungerboeck [12]. As it will be shown later, such an approximation significantly simplifies the analysis while sustaining high accuracy.

By substituting (13) and (14) in (11), the average BER can be tightly approximated as

$$\begin{aligned}{\rm BER}^{\rm BER-SC}_{\rm comp} \approx \frac{1}{\bar\gamma_{\rm RD}}\frac{1}{\bar\gamma_{\rm SD}} &\times \Bigg(\int_0^\infty \int_0^{\frac{d^2_{M_R}}{d^2_{M_S}}\gamma_{\rm RD}} c_{M_R} Q\left(\sqrt{2d^2_{M_R}\gamma_{\rm RD}}\right)\\ &\times e^{-\frac{\gamma_{\rm SD}}{\bar\gamma_{\rm SD}}} e^{-\frac{\gamma_{\rm RD}}{\bar\gamma_{\rm RD}}}\, d\gamma_{\rm SD}\, d\gamma_{\rm RD}\\ &+ \int_0^\infty \int_{\frac{d^2_{M_R}}{d^2_{M_S}}\gamma_{\rm RD}}^\infty c_{M_S} Q\left(\sqrt{2d^2_{M_S}\gamma_{\rm SD}}\right)\\ &\times e^{-\frac{\gamma_{\rm SD}}{\bar\gamma_{\rm SD}}} e^{-\frac{\gamma_{\rm RD}}{\bar\gamma_{\rm RD}}}\, d\gamma_{\rm SD}\, d\gamma_{\rm RD}\Bigg). \end{aligned} \quad (15)$$

To evaluate the preceding expression, we define the following function:

$$\begin{aligned}H(x; a, b, c) &= \int_0^x aQ(\sqrt{2bt})\frac{1}{c}e^{-\frac{t}{c}}\,dt\\ &= \int_0^\infty aQ(\sqrt{2bt})\frac{1}{c}e^{-\frac{t}{c}}\,dt\\ &\quad - \int_x^\infty aQ(\sqrt{2bt})\frac{1}{c}e^{-\frac{t}{c}}\,dt\\ &= 0.5a\left(1 - \sqrt{\frac{bc}{1+bc}}\right) - aQ(\sqrt{2bx})e^{-\frac{x}{c}}\\ &\quad + a\sqrt{\frac{bc}{1+bc}}\,Q\left(\sqrt{\frac{1+bc}{c}2x}\right) \end{aligned} \quad (16)$$

where the first integration is evaluated in [11, pp. 817–818], and the second integration is evaluated using the same procedure presented in [13, App. A]. We also define $H(\infty; a, b, c)$ as

$$H(\infty; a, b, c) = \lim_{x\to\infty} H(x; a, b, c) = 0.5a\left(1 - \sqrt{\frac{bc}{1+bc}}\right). \quad (17)$$

Moreover, we define the following function:[4]

$$\begin{aligned}J(x; a, b, c, d) &= \int_0^\infty H(xt; a, b, c)\frac{1}{d}e^{-\frac{t}{d}}\,dt\\ &= 0.5a\left(1 - \sqrt{\frac{bc}{1+bc}}\right)\int_0^\infty \frac{1}{d}e^{-\frac{t}{d}}\,dt\\ &\quad - a\int_0^\infty Q(\sqrt{2bxt})e^{-\frac{xt}{c}}\frac{1}{d}e^{-\frac{t}{d}}\,dt\\ &\quad + a\sqrt{\frac{bc}{1+bc}}\int_0^\infty Q\left(\sqrt{\frac{1+bc}{c}2xt}\right)\frac{1}{d}e^{-\frac{t}{d}}\,dt\\ &= 0.5a\left(1 - \sqrt{\frac{bc}{1+bc}}\right)\\ &\quad - H\left(\infty; a\frac{c}{c+xd}, xb, \frac{cd}{c+xd}\right)\\ &\quad + H\left(\infty; a\sqrt{\frac{bc}{1+bc}}, \frac{1+bc}{c}x, d\right)\\ &= 0.5a\frac{xd}{c+xd}\left(1 - \sqrt{\frac{xbcd}{c+xd+xbcd}}\right). \end{aligned} \quad (18)$$

The average BER can be expressed in terms of the function $H(x; a, b, c, d)$ as

$$\begin{aligned}{\rm BER}^{\rm BER-SC}_{\rm comp} \approx &\int_0^\infty c_{M_R} Q\left(\sqrt{2d^2_{M_R}\gamma_{\rm RD}}\right)\\ &\times \left(1 - e^{-\frac{d^2_{M_R}}{d^2_{M_S}}\frac{\gamma_{\rm RD}}{\bar\gamma_{\rm SD}}}\right)\frac{1}{\bar\gamma_{\rm RD}}e^{-\frac{\gamma_{\rm RD}}{\bar\gamma_{\rm RD}}}\,d\gamma_{\rm RD}\\ &+ \int_0^\infty \Bigg(H\left(\infty; c_{M_S}, d^2_{M_S}, \bar\gamma_{\rm SD}\right)\\ &\qquad - H\left(\frac{d^2_{M_R}}{d^2_{M_S}}\gamma_{\rm RD}; c_{M_S}, d^2_{M_S}, \bar\gamma_{\rm SD}\right)\Bigg)\\ &\times \frac{1}{\bar\gamma_{\rm RD}}e^{-\frac{\gamma_{\rm RD}}{\bar\gamma_{\rm RD}}}\,d\gamma_{\rm RD}. \end{aligned} \quad (19)$$

[4]In this paper, the functions $H(x; a, b, c)$ and $J(x; a, b, c, d)$ given in (16) and (18), respectively, are defined slightly differently than those we used in [1, eqs. (7) and (8)] and [9, eqs. (3.7) and (3.8)].

By evaluating the definite integral given by (19), the average BER can be expressed in terms of the functions $H(\infty; a, b, c, d)$ and $J(x; a, b, c, d)$ as

$$\begin{aligned}\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}} \approx\, & H\left(\infty; c_{M_R}, d_{M_R}^2, \bar{\gamma}_{\mathrm{RD}}\right) \\ & - H\left(\infty; \frac{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}}}{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} + d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}} c_{M_R},\right. \\ & \left. d_{M_R}^2, \frac{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}}}{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} + d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}}\right) \\ & + H\left(\infty; c_{M_S}, d_{M_S}^2, \bar{\gamma}_{\mathrm{SD}}\right) \\ & - J\left(\frac{d_{M_R}^2}{d_{M_S}^2}; c_{M_S}, d_{M_S}^2, \bar{\gamma}_{\mathrm{SD}}, \bar{\gamma}_{\mathrm{RD}}\right). \end{aligned} \tag{20}$$

Finally, by evaluating the previous expression using (17) and (18), the average BER can be written as

$$\begin{aligned}\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}} \approx\, & \frac{1}{2} c_{M_S} \left(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}}}{1 + d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}}}}\right) \\ & + \frac{1}{2} c_{M_R} \left(1 - \sqrt{\frac{d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}}{1 + d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}}}\right) \\ & - \frac{1}{2} \frac{c_{M_S} d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}} + c_{M_R} d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}}}{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} + d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}} \\ & \times \left(1 - \sqrt{\frac{\bar{\gamma}_2}{1 + \bar{\gamma}_2}}\right) \end{aligned} \tag{21}$$

where $\bar{\gamma}_2 \triangleq d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}} / (d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} + d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}})$.

As a sanity check, we evaluate the previous expression for the special case when the signals belong to the same modulation level, i.e., $M_S = M_R = M$, as

$$\begin{aligned}&\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}} \\ &\approx \frac{1}{2} c_M \left(1 - \sqrt{\frac{d_M^2 \bar{\gamma}_{\mathrm{SD}}}{1 + d_M^2 \bar{\gamma}_{\mathrm{SD}}}} - \sqrt{\frac{d_M^2 \bar{\gamma}_{\mathrm{RD}}}{1 + d_M^2 \bar{\gamma}_{\mathrm{RD}}}} + \sqrt{\frac{d_M^2 \bar{\gamma}_3}{1 + d_M^2 \bar{\gamma}_3}}\right)\end{aligned} \tag{22}$$

where $\bar{\gamma}_3 \triangleq \bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}} / (\bar{\gamma}_{\mathrm{SD}} + \bar{\gamma}_{\mathrm{RD}})$. Note that (22) is identical to [7, Eq. (9.210)], even though they were derived in very different ways. This is expected since BER-SC reduces to the conventional SNR-SC for this special case. This suggests that [7, Eq. (9.210)] can be viewed as a special case of our derived BER expression for BER-SC.

B. Derivations of $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}}$

The instantaneous BER at the output of SNR-SC, given γ_{SD} and γ_{RD}, can be written as

$$\mathrm{BER}_{\mathrm{comp,inst}}^{\mathrm{SNR-SC}} \approx \begin{cases} c_{M_S} Q\left(\sqrt{2 d_{M_S}^2 \gamma_{\mathrm{SD}}}\right), & \gamma_{\mathrm{SD}} \geq \gamma_{\mathrm{RD}} \\ c_{M_R} Q\left(\sqrt{2 d_{M_R}^2 \gamma_{\mathrm{RD}}}\right), & \gamma_{\mathrm{SD}} < \gamma_{\mathrm{RD}} \end{cases} \tag{23}$$

and the average BER can be written as

$$\begin{aligned}\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}} \approx \frac{1}{\bar{\gamma}_{\mathrm{SD}}} \frac{1}{\bar{\gamma}_{\mathrm{RD}}} \times \Bigg(& \int_0^\infty \int_0^{\gamma_{\mathrm{RD}}} c_{M_R} Q\left(\sqrt{2 d_{M_R}^2 \gamma_{\mathrm{RD}}}\right) e^{-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}} \\ & \times e^{-\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}} d\gamma_{\mathrm{SD}} d\gamma_{\mathrm{RD}} \\ & + \int_0^\infty \int_{\gamma_{\mathrm{RD}}}^\infty c_{M_S} Q\left(\sqrt{2 d_{M_S}^2 \gamma_{\mathrm{SD}}}\right) \\ & \times e^{-\frac{\gamma_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}}} e^{-\frac{\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{RD}}}} d\gamma_{\mathrm{SD}} d\gamma_{\mathrm{RD}} \Bigg). \end{aligned} \tag{24}$$

The integrations in (24) are evaluated using the procedure explained in the previous section; the average BER can be explicitly written as

$$\begin{aligned}\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}} \approx\, & \frac{1}{2} c_{M_S} \left(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}}}{1 + d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}}}}\right) \\ & + \frac{1}{2} c_{M_R} \left(1 - \sqrt{\frac{d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}}{1 + d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}}}\right) \\ & - \frac{1}{2} c_{M_S} \frac{\bar{\gamma}_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{SD}} + \bar{\gamma}_{\mathrm{RD}}} \left(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_3}{1 + d_{M_S}^2 \bar{\gamma}_3}}\right) \\ & - \frac{1}{2} c_{M_R} \frac{\bar{\gamma}_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}} + \bar{\gamma}_{\mathrm{RD}}} \left(1 - \sqrt{\frac{d_{M_R}^2 \bar{\gamma}_3}{1 + d_{M_R}^2 \bar{\gamma}_3}}\right) \end{aligned} \tag{25}$$

where $\bar{\gamma}_3 \triangleq \bar{\gamma}_{\mathrm{SD}} \bar{\gamma}_{\mathrm{RD}} / (\bar{\gamma}_{\mathrm{SD}} + \bar{\gamma}_{\mathrm{RD}})$.

Once again, as a sanity check, one can easily verify that (25) reduces to both (22) and [7, Eq. (9.210)] for the special case when the signals to be combined belong to the same modulation level.

C. Final Expressions for $\mathrm{BER}^{\mathrm{BER-SC}}$ *and* $\mathrm{BER}^{\mathrm{SNR-SC}}$

By substituting (7), (8), and (20) in (5), we get

$$\begin{aligned}&\mathrm{BER}^{\mathrm{BER-SC}} \\ &\approx \left(1 - \left(1 - \log_2(M_S) S_{M_S}(\bar{\gamma}_{\mathrm{SR}})\right)^{\frac{N}{\log_2 M_S}}\right) S_{M_S}(\bar{\gamma}_{\mathrm{SD}}) \\ &\quad + \left(1 - \log_2(M_S) S_{M_S}(\bar{\gamma}_{\mathrm{SR}})\right)^{\frac{N}{\log_2 M_S}} \\ &\quad \times \Bigg(S_{M_S}(\bar{\gamma}_{\mathrm{SD}}) + S_{M_R}(\bar{\gamma}_{\mathrm{RD}}) \\ &\qquad - \frac{1}{2} \frac{c_{M_R} d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} + c_{M_S} d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}}{d_{M_S}^2 \bar{\gamma}_{\mathrm{SD}} + d_{M_R}^2 \bar{\gamma}_{\mathrm{RD}}} \left(1 - \sqrt{\frac{\bar{\gamma}_2}{1 + \bar{\gamma}_2}}\right)\Bigg). \end{aligned} \tag{26}$$

Similarly, by substituting (7), (8), and (25) in (6), we get

$$\begin{aligned}\mathrm{BER}^{\mathrm{SNR-SC}} &\approx \left(1-(1-\log_2(M_S)S_{M_S}(\bar{\gamma}_{\mathrm{SR}}))^{\frac{N}{\log_2 M_S}}\right)S_{M_S}(\bar{\gamma}_{\mathrm{SD}}) \\ &+ (1-\log_2(M_S)S_{M_S}(\bar{\gamma}_{\mathrm{SR}}))^{\frac{N}{\log_2 M_S}} \\ &\times \left(S_{M_S}(\bar{\gamma}_{\mathrm{SD}})+S_{M_R}(\bar{\gamma}_{\mathrm{RD}})-\frac{\bar{\gamma}_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{SD}}+\bar{\gamma}_{\mathrm{RD}}}S_{M_S}(\bar{\gamma}_3)\right. \\ &\left. -\frac{\bar{\gamma}_{\mathrm{SD}}}{\bar{\gamma}_{\mathrm{SD}}+\bar{\gamma}_{\mathrm{RD}}}S_{M_R}(\bar{\gamma}_3)\right) \qquad (27)\end{aligned}$$

where

$$\begin{aligned}S_M(\bar{\gamma}) &= \frac{1}{2}c_M\left(1-\sqrt{\frac{d_M^2\bar{\gamma}}{1+d_M^2\bar{\gamma}}}\right) \\ \bar{\gamma}_2 &\triangleq \frac{d_{M_S}^2\bar{\gamma}_{\mathrm{SD}}d_{M_R}^2\bar{\gamma}_{\mathrm{RD}}}{d_{M_S}^2\bar{\gamma}_{\mathrm{SD}}+d_{M_R}^2\bar{\gamma}_{\mathrm{RD}}} \quad \bar{\gamma}_3 \triangleq \frac{\bar{\gamma}_{\mathrm{SD}}\bar{\gamma}_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{SD}}+\bar{\gamma}_{\mathrm{RD}}}. \qquad (28)\end{aligned}$$

V. Comparison Between Bit-Error Rate-Selection Combining and Signal-to-Noise Ratio-Selection Combining

Although the derived BER for BER-SC and SNR-SC, given by (26) and (27), respectively, are very useful in estimating the BER performance, it is not straightforward to use them to quantify the gain achieved by using BER-SC over SNR-SC. Consequently, we derive simple asymptotic BER expressions for both schemes. Then, we use the asymptotic expressions to quantify the asymptotic gains (AGs) achieved by BER-SC over SNR-SC.

By expressing the average SNRs as $\bar{\gamma}_{\mathrm{SR}} = \sigma_{\mathrm{SR}}^2\mathrm{SNR}$, $\bar{\gamma}_{\mathrm{SD}} = \sigma_{\mathrm{SD}}^2\mathrm{SNR}$, and $\bar{\gamma}_{\mathrm{RD}} = \sigma_{\mathrm{RD}}^2\mathrm{SNR}$, we derive simple expressions for the BER of both BER-SC and SNR-SC as SNR goes to infinity. In Appendix A, we show that the asymptotic BER expression for BER-SC and SNR-SC can be written as

$$\mathrm{BER}^{\mathrm{BER-SC}} \overset{\mathrm{SNR}\to\infty}{\approx} (G^{\mathrm{BER-SC}}\mathrm{SNR})^{-2} \qquad (29)$$

$$\mathrm{BER}^{\mathrm{SNR-SC}} \overset{\mathrm{SNR}\to\infty}{\approx} (G^{\mathrm{SNR-SC}}\mathrm{SNR})^{-2} \qquad (30)$$

where

$$\begin{aligned}G^{\mathrm{BER-SC}} &= 4d_{M_S}^2d_{M_R}^2\sigma_{\mathrm{SD}}\sigma_{\mathrm{RD}} \\ &\times \left(c_{M_S}^2d_{M_R}^4\eta^{-1}+3d_{M_S}^2d_{M_R}^2(c_{M_S}+c_{M_R})\right)^{-\frac{1}{2}} \qquad (31)\end{aligned}$$

$$\begin{aligned}G^{\mathrm{SNR-SC}} &= 4d_{M_S}^2d_{M_R}^2\sigma_{\mathrm{SD}}\sigma_{\mathrm{RD}} \\ &\times \left(c_{M_S}^2d_{M_R}^4\eta^{-1}+3(c_{M_S}d_{M_R}^4+c_{M_R}d_{M_S}^4)\right)^{-\frac{1}{2}} \qquad (32)\end{aligned}$$

and $\eta = (1/N)(\bar{\gamma}_{\mathrm{SR}}/\bar{\gamma}_{\mathrm{RD}}) = (1/N)(\sigma_{\mathrm{SR}}^2/\sigma_{\mathrm{RD}}^2)$. The constants $G^{\mathrm{BER-SC}}$ and $G^{\mathrm{SNR-SC}}$ represent the SNR gain achieved by BER-SC and SNR-SC, respectively. We define η as the cooperation ratio, which is an indicator for the gain achieved by cooperation. The cooperation ratio has determinable effect on BER, as will be shown shortly.

Using (31) and (32), we can make the following proposition that asymptotically holds true for both BER-SC and SNR-SC.

Proposition 1: Asymptotically (sufficiently high SNR for each of the S-R, R-D, and S-D links), it is always better to assign S a modulation level that is lower than or equal to the modulation level used by R.[5] (See Appendix B for the proof.)

By comparing (29) and (30), we observe that both schemes achieve a diversity order of 2, i.e., full diversity (as expected). However, the SNR gain achieved by BER-SC is higher than that by SNR-SC. We define the AG, in decibels, achieved by BER-SC over SNR-SC as

$$\begin{aligned}AG &= 10\log_{10}\left(\frac{G^{\mathrm{BER-SC}}}{G^{\mathrm{SNR-SC}}}\right) \\ &= 10\log_{10}\left(\frac{c_{M_S}^2d_{M_R}^4\eta^{-1}+3\left(c_{M_S}d_{M_R}^4+c_{M_R}d_{M_S}^4\right)}{c_{M_s}^2d_{M_R}^4\eta^{-1}+3d_{M_S}^2d_{M_R}^2(c_{M_S}+c_{M_R})}\right)^{\frac{1}{2}}. \qquad (33)\end{aligned}$$

The AG depends only on the cooperation ratio (η) and the modulation levels of the signals to be combined. By substituting (3) and (4) in (33), one can evaluate AG for different scenarios. Note that AG $= 0$ dB when $M_S = M_R$ since BER-SC and SNR-SC are equivalent in this scenario.

To understand the effect of the cooperation ratio on the BER performance, we investigate the following special cases:

1) $(\bar{\gamma}_{\mathrm{SR}}/\bar{\gamma}_{\mathrm{RD}}) \gg N$ (High Cooperation Ratio): In this case, the S-R link has much better average channel conditions than the R-D link, and $\eta \to \infty$. Therefore, (31)–(33) are well approximated by

$$G^{\mathrm{BER-SC}} \overset{\eta\to\infty}{\approx} \frac{4}{\sqrt{3}}d_{M_S}d_{M_R}\sigma_{\mathrm{SD}}\sigma_{\mathrm{RD}}(c_{M_S}+c_{M_R})^{-\frac{1}{2}}, \qquad (34)$$

$$\begin{aligned}G^{\mathrm{SNR-SC}} &\overset{\eta\to\infty}{\approx} \frac{4}{\sqrt{3}}d_{M_S}d_{M_R}\sigma_{\mathrm{SD}}\sigma_{\mathrm{RD}} \\ &\times \left(c_{M_S}d_{M_R}^4+c_{M_R}d_{M_S}^4\right)^{-\frac{1}{2}} \qquad (35)\end{aligned}$$

$$AG \overset{\eta\to\infty}{\approx} 10\log_{10}\left(\frac{c_{M_S}d_{M_R}^4+c_{M_R}d_{M_S}^4}{d_{M_S}^2d_{M_R}^2(c_{M_S}+c_{M_R})}\right)^{\frac{1}{2}}. \qquad (36)$$

This special case includes the case of perfect cooperation where errors in the S-R link extremely rarely occurs.

2) $(\bar{\gamma}_{\mathrm{SR}}/\bar{\gamma}_{\mathrm{RD}}) \ll N$ (Low Cooperation Ratio): In this case, the R-D link has much better average channel conditions than the S-R link, and $\eta \to 0$. Therefore, (31)–(33) are well approximated by

$$G^{\mathrm{BER-SC}} \overset{\eta\to 0}{\approx} G^{\mathrm{SNR-SC}} \overset{\eta\to 0}{\approx} \frac{4d_{M_S}^2}{c_{M_S}\sqrt{N}}\sigma_{\mathrm{SD}}\sigma_{\mathrm{SR}} \qquad (37)$$

$$AG \overset{\eta\to 0}{\approx} 0. \qquad (38)$$

This special case includes the case of minimal cooperation where R rarely successfully decodes the packet and retransmits it to D. It is interesting to note that the SNR gains for both BER-SC and SNR-SC are independent of the modulation level used by R. Since both BER-SC and SNR-SC achieve the same SNR gain, AG $= 0$ dB.

VI. Analytical and Simulation Results

In Figs. 2 and 3, we plot the BER performance using numerical simulation and the BER expressions for BER-SC [given by (26)] and SNR-SC [given by (27)], respectively. It is clear from the figures that

[5]If the channel conditions allow, R may use a modulation level that is higher than the one used by S to increase the overall spectral efficiency, which, in turn, reduces the number of radio resources required.

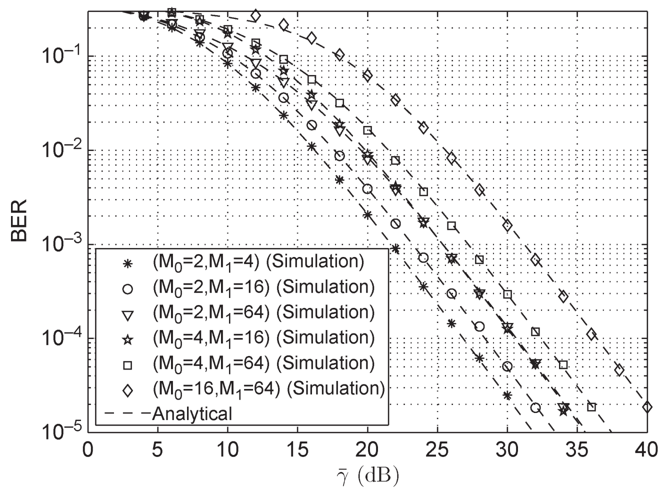

Fig. 2. BER performance of BER-SC, assuming $\bar{\gamma}_{\mathrm{SR}} = \bar{\gamma} + 10$ dB, $\bar{\gamma}_{\mathrm{SD}} = \bar{\gamma} - 10$ dB, $\bar{\gamma}_{\mathrm{RD}} = \bar{\gamma}$ dB, and $N = 264$ bits. It is clear from the figures that the derived BER expressions given by (26) and the simulation results are in excellent agreement.

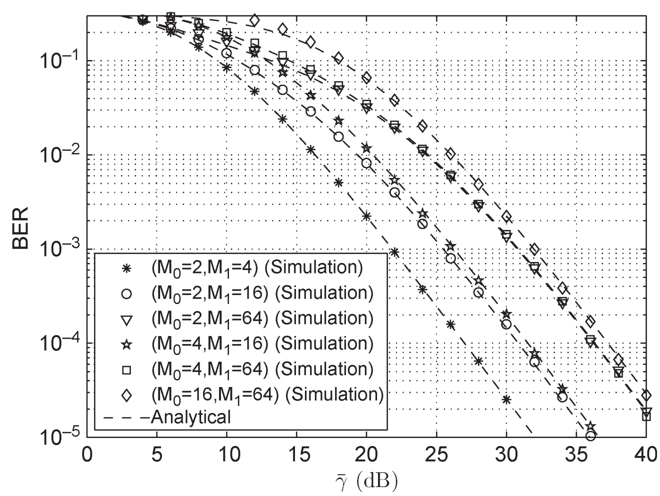

Fig. 3. BER performance of SNR-SC, assuming $\bar{\gamma}_{\mathrm{SR}} = \bar{\gamma} + 10$ dB, $\bar{\gamma}_{\mathrm{SD}} = \bar{\gamma} - 10$ dB, $\bar{\gamma}_{\mathrm{RD}} = \bar{\gamma}$ dB, and $N = 264$ bits. It is clear from the figures that the derived BER expressions given by (27) and the simulation results are in excellent agreement.

the derived BER expressions and the simulation results are in excellent agreement, which validates the mathematical derivations and justifies the approximations made.

To confirm the accuracy of the asymptotic approximation given by (29) and (30), we plot the BER performance using numerical simulation and the asymptotic BER expressions for BER-SC and SNR-SC for different scenarios in Fig. 4. It is clear that the asymptotic expression is tight for high SNRs, which also confirms the accuracy of the derived AGs. It is worth repeating that such an asymptotic approximation is used merely for quantifying the gain of BER-SC over SNR-SC (as will be shown in Fig. 5 and Table I), and it should not be used as an approximate BER since such an approximation is loose in the low-SNR regime, as shown in Fig. 4.

In Fig. 5, we plot the AGs achieved by BER-SC over SNR-SC as a function of the cooperation ratio for different modulation levels. For all scenarios, the AG increases as the cooperation ratio increases, and it saturates at a maximum value that depends only on the modulation levels used. The behaviors of the curves at low and high cooperation ratios are in excellent agreement with (36) and (38), respectively.

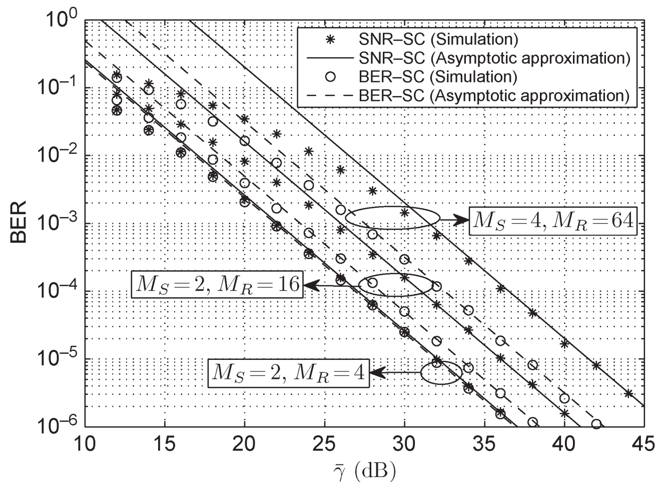

Fig. 4. Asymptotic BER performance of BER-SC and SNR-SC.

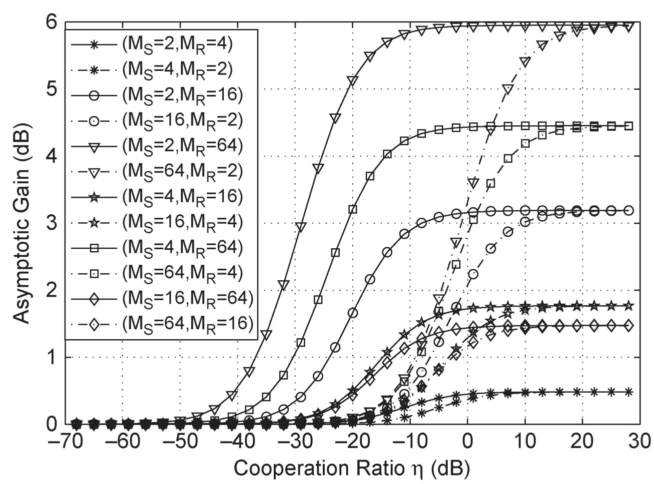

Fig. 5. Asymptotic gain (in decibels) achieved by BER-SC over SNR-SC as a function of the cooperation ratio.

TABLE I
ASYMPTOTIC GAIN ACHIEVED BY BER-SC OVER SNR-SC FOR DIFFERENT SCENARIOS ($\eta_{95\%}$ IS THE REQUIRED COOPERATION RATIO TO ACHIEVE 95% OF THE MAXIMUM ASYMPTOTIC GAIN)

Scenario	Max. Asymptotic Gain (dB), as $\eta \rightarrow \infty$ dB	$\eta_{95\%}$ (dB)
$M_S = 2,\ M_R = 4$	0.48	1.52
$M_S = 4,\ M_R = 2$	0.48	7.54
$M_S = 2,\ M_R = 16$	3.19	-7.25
$M_S = 16,\ M_R = 2$	3.19	10.25
$M_S = 2,\ M_R = 64$	5.95	-15.00
$M_S = 64,\ M_R = 2$	5.95	12.79
$M_S = 4,\ M_R = 16$	1.77	-3.06
$M_S = 16,\ M_R = 4$	1.77	8.42
$M_S = 4,\ M_R = 64$	4.45	-10.98
$M_S = 64,\ M_R = 4$	4.45	10.78
$M_S = 16,\ M_R = 64$	1.47	-3.36
$M_S = 64,\ M_R = 16$	1.47	6.92

Table I shows the maximum AGs that can be achieved for different scenarios [using (36)] and the minimum cooperation required to achieve 95% of this maximum ($\eta_{95\%}$), which is calculated numerically. Among all the scenarios considered, the maximum AG is achieved when $M_S = 2$ and $M_R = 64$ (AG $= 5.95$ dB), and the minimum AG is achieved when $M_S = 2$ and $M_R = 4$. In general, the gain increases as the difference increases between the modulation levels of the signals to be combined. The gain also increases as the

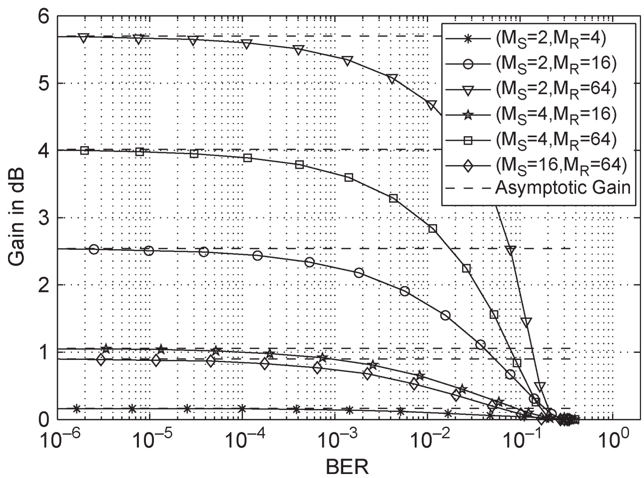

Fig. 6. Gain (in decibels) achieved by BER-SC over SNR-SC, assuming $\bar{\gamma}_{\mathrm{SR}} = \bar{\gamma} + 10$ dB, $\bar{\gamma}_{\mathrm{SD}} = \bar{\gamma} - 10$ dB, $\bar{\gamma}_{\mathrm{RD}} = \bar{\gamma}$ dB, and $N = 264$ bits.

cooperation ratio increases. Moreover, it is clear that, for the same set of modulation levels, the required $\eta_{95\%}$ is much higher when $M_S > M_R$ than the case when $M_S < M_R$.

Since the gains calculated in Table I and plotted in Fig. 5 are only asymptotically valid, we plot the gains achieved by BER-SC over SNR-SC for different finite BERs in Fig. 6. The gains are obtained by numerically inverting the BER formulas for BER-SC and SNR-SC given by (27) and (26), respectively. For all scenarios, the gains increase as the BER decreases (i.e., SNR increases), and they saturate at the AG values calculated by (33). This again validates the asymptotic analysis given in Section VII. Moreover, most of the gains are attained for BER values of 10^{-3} or less, which are reasonable BER values for uncoded schemes.

VII. Conclusion

In this paper, we have derived closed-form BER expressions for both BER-SC and SNR-SC when they are used to combine signals with different modulation levels. The derived expressions for BER-SC and SNR-SC are more general than the existing expression in literature that applies only to combining signals with the same modulation level. In addition, we have analytically quantified the significant asymptotic gain achieved by using BER-SC over SNR-SC.

Appendix A
Derivation of Asymptotic Approximations for $\mathrm{BER}^{\mathrm{BER-SC}}$ and $\mathrm{BER}^{\mathrm{SNR-SC}}$

In this Appendix, we present the derivation of (29) and (30), which give asymptotic approximations for $\mathrm{BER}^{\mathrm{BER-SC}}$ and $\mathrm{BER}^{\mathrm{SNR-SC}}$, respectively. According to (5) and (6), to derive asymptotic approximations for $\mathrm{BER}^{\mathrm{BER-SC}}$ and $\mathrm{BER}^{\mathrm{SNR-SC}}$, we need to derive asymptotic approximations for $\mathrm{PER}_{\mathrm{SR}}$, $\mathrm{BER}_{\mathrm{SD}}$, $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}}$, and $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}}$.

According to [11, pp. 818–819]

$$\mathrm{BER}_{ij} \approx \frac{1}{2} c_{M_i} \left(1 - \sqrt{\frac{d_{M_i}^2 \sigma_{ij}^2 \mathrm{SNR}}{1 + d_{M_i}^2 \sigma_{ij}^2 \mathrm{SNR}}}\right) \overset{\mathrm{SNR}\to\infty}{\approx} \frac{c_{M_i}}{4 d_{M_i}^2 \sigma_{ij}^2 \mathrm{SNR}} \tag{39}$$

where $ij \in \{\mathrm{SR}, \mathrm{RD}, \mathrm{SD}\}$.

Using the previous approximation in the S-R link and the fact that $\mathrm{SER} \approx \log_2 M_S \mathrm{BER}$ for Gray-coded constellations [10], $\mathrm{PER}_{\mathrm{SR}}$ can be approximated as

$$\begin{aligned} \mathrm{PER}_{\mathrm{SR}} &\overset{\mathrm{SNR}\to\infty}{\approx} 1 - \left(1 - \frac{c_{M_S} \log_2 M_S}{4 d_{M_S}^2 \sigma_{\mathrm{SR}}^2 \mathrm{SNR}}\right)^{\frac{N}{\log_2 M_S}} \\ &= 1 - \sum_{k=0}^{\infty} \binom{\frac{N}{\log_2 M_S}}{k} \left(-\frac{c_{M_S} \log_2 M_S}{4 d_{M_S}^2 \sigma_{\mathrm{SR}}^2 \mathrm{SNR}}\right)^k \\ &\overset{\mathrm{SNR}\to\infty}{\approx} \frac{N c_{M_S}}{4 d_{M_S}^2 \sigma_{\mathrm{SR}}^2 \mathrm{SNR}} \end{aligned} \tag{40}$$

where the last asymptotic approximation is obtained by keeping the first two terms and truncating the higher order terms in the binomial series. Note that the preceding binomial series converges when $(c_{M_S} \log_2 M_S/(4 d_{M_S}^2 \sigma_{\mathrm{SR}}^2 \mathrm{SNR})) < 1$, i.e., when $\mathrm{SNR} > (c_{M_S} \log_2 M_S/(4 d_{M_S}^2 \sigma_{\mathrm{SR}}^2))$. This condition is satisfied for the desired asymptotic approximation since $\mathrm{SNR} \to \infty$, and $(c_{M_S} \log_2 M_S/(4 d_{M_S}^2 \sigma_{\mathrm{SR}}^2)) < \infty$.

To derive asymptotic expressions for $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}}$ and $\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}}$, the following asymptotic approximation is used:

$$1 - \sqrt{\frac{x}{x+1}} \overset{x\to\infty}{\approx} \frac{1}{2x} - \frac{3}{8x^2} \tag{41}$$

where the previous approximation is derived by using Taylor series expansion and truncating the higher order terms. Applying the previous approximation in (26) and (27), and going through considerable manipulations and simplifications, we get the following asymptotic expressions:

$$\mathrm{BER}_{\mathrm{comp}}^{\mathrm{BER-SC}} \overset{\mathrm{SNR}\to\infty}{\approx} \frac{3}{16} \frac{c_{M_S} + c_{M_R}}{d_{M_S}^2 d_{M_R}^2 \sigma_{\mathrm{SD}}^2 \sigma_{\mathrm{RD}}^2} \frac{1}{\mathrm{SNR}^2}. \tag{42}$$

$$\mathrm{BER}_{\mathrm{comp}}^{\mathrm{SNR-SC}} \overset{\mathrm{SNR}\to\infty}{\approx} \frac{3}{16} \frac{c_{M_S} d_{M_R}^4 + c_{M_R} d_{M_S}^4}{d_{M_S}^4 d_{M_R}^4 \sigma_{\mathrm{SD}}^2 \sigma_{\mathrm{RD}}^2} \frac{1}{\mathrm{SNR}^2}. \tag{43}$$

By substituting (39), (40), and (42) in (5), we get (29). Similarly, by substituting (39), (40), and (43) in (6), we get (30).

Appendix B
Proof of Proposition 1

We give the proof for the case of BER-SC; the proof for the case of SNR-SC can be constructed similarly. For any M_0 and M_1, where $M_0 < M_1$, we show that $G^{\mathrm{BER-SC}(1)} > G^{\mathrm{BER-SC}(2)}$; $G^{\mathrm{BER-SC}(1)}$ corresponds to the SNR gain for the case where $M_S^{(1)} = M_0$ and $M_R^{(1)} = M_1$, and $G^{\mathrm{BER-SC}(2)}$ corresponds to the SNR gain for the case where $M_S^{(2)} = M_1$ and $M_R^{(2)} = M_0$. Note that both cases have the same end-to-end spectral efficiency.

Using (31), we can write the following:

$$\frac{G^{\mathrm{BER-SC}(1)}}{G^{\mathrm{BER-SC}(2)}} = \frac{\left(c_{M_0}^2 d_{M_1}^4 \eta^{-1} + 3 d_{M_0}^2 d_{M_1}^2 (c_{M_0} + c_{M_1})\right)^{-\frac{1}{2}}}{\left(c_{M_1}^2 d_{M_0}^4 \eta^{-1} + 3 d_{M_1}^2 d_{M_0}^2 (c_{M_1} + c_{M_0})\right)^{-\frac{1}{2}}}. \tag{44}$$

It is clear from (44) that, to prove that $G^{\mathrm{BER-SC}(1)} > G^{\mathrm{BER-SC}(2)}$, it is sufficient to show that $c_{M_0}^2 d_{M_1}^4 < c_{M_1}^2 d_{M_0}^4$, which is equivalent to showing that $f(M_0) < f(M_1)$, where $f(M_i) = c_{M_i}/d_{M_i}^2$. Since $M_0 < M_1$, we can prove that $f(M_0) < f(M_1)$, by showing that $f(M_i)$ is strictly increasing on its domain $\mathcal{D} = \{2\} \bigcup \{M_i : M_i \geq 4\}$. Using (3) and (4), $f(M_i)$ can be written as

$$f(M_i) = \begin{cases} 1, & M_i = 2 \\ \frac{4(M_i - 1)\left(1 - 1/\sqrt{M_i}\right)}{3 \log_2 \sqrt{M_i}}, & M_i \geq 4. \end{cases} \tag{45}$$

The function $f(M_i)$ is differentiable on $\{M_i : M_i > 4\}$, and the first derivative can be expressed as

$$\frac{d}{dM_i} f(M_i) = \frac{4}{3}\left(\sqrt{M_i} - 1\right) M_i^{-3/2} (\log_2 M_i)^{-2} \times \left(\left(1 + \sqrt{M_i}\right) \log_2 M_i + 2M_i \left(\log_2 M_i - \frac{1}{\ln 2}\right) + \frac{2}{\ln 2}\right). \quad (46)$$

Since $\frac{d}{dM_i} f(M_i) > 0$ on $\{M_i : M_i > 4\}$, $f(M_i)$ is strictly increasing on $\{M_i : M_i > 4\}$. Moreover, since $f(2) = 1 < f(4) = 2$, $f(M_i)$ is strictly increasing on its domain $\mathcal{D}$. ■

ACKNOWLEDGMENT

The authors would like to thank the Editor and the anonymous reviewers for their helpful comments and suggestions.

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Performance Analysis of Systematic Upper Layer FEC Codes and Interleaving in Land Mobile Satellite Channels

Nedo Celandroni and Alberto Gotta

***Abstract*—This paper provides an analytical method for evaluating the performance of upper layer forward error correction (FEC), when used together with interleaving on land mobile satellite channels, in terms of residual (after decoding) packet loss rate (PLR). In addition, the analysis highlights the overheads introduced by this technique, which depend on the channel statistics, in terms of additional packet delivery delay, bandwidth consumption, and computation power required. We show why the interleaver is mandatory in some environments, due to the severe disruptive characteristics of the channel, according to measurement campaigns promoted by the European Space Agency. This analysis provides the expression of the residual PLR when systematic FEC coding blocks are interleaved at the packet level (i.e., when a cyclic redundancy check and a sequence number identify a correct data unit). This method can be employed for the performance evaluation of recent and future mobile communication systems that take advantage of upper layer FEC and interleaving techniques.**

***Index Terms*—Interleaving, land mobile satellite channel, upper layer forward error correction (ULFEC).**

I. INTRODUCTION

New digital video broadcasting (DVB) satellite standards are moving the focus toward land mobile communications to provide both global coverage and bandwidth supply. The aim is the integration of satellite links with terrestrial wireless networks, which cannot support high traffic demand. Digital Video Broadcasting–Satellite Services to Handhelds (DVB-SH) [1] and some amendments to Digital Video Broadcasting–Satellite2/Return Channel via Satellite (DVB-S2/RCS) for land mobile satellite communications [2] are examples of this trend. In fact, DVB-SH is designed to almost transparently integrate itself with Digital Video Broadcasting–Handhelds (DVB-H) [3], which is one of the current standards used in Third-Generation cellular networks for multimedia broadcasting.

Automatic repeat request (ARQ), forward error correction (FEC), interleaving, or hybrid techniques are the key solution that can foster reliable information broadcasting [4] to a group of users (identified by a type of service or by a geographical localization) in land mobile communications.

This paper provides an analytical method for the performance evaluation of an upper layer forward error correction (ULFEC) for land mobile satellite channels in terms of residual packet loss when interleaving is applied. We will show why the interleaver is mandatory in environments that exhibit severe disruptive channel characteristics, which are demonstrated by measurement campaigns promoted by the European Space Agency (ESA) [5]. This paper extends the study in [6] by computing the exact residual packet loss rate (PLR) after the decoding process when systematic codes are employed. In disruptive

Manuscript received July 20, 2010; revised November 15, 2010 and January 27, 2011; accepted February 16, 2011. Date of publication March 3, 2011; date of current version May 16, 2011. This work was supported in part by the European Space Agency in the framework of the SatNEx-III research activities. The review of this paper was coordinated by Prof. H.-F. Lu.

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Digital Object Identifier 10.1109/TVT.2011.2122253