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On the Spectral Efficiency of Selective Decode-and-Forward Relaying

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***Abstract*—Multi-relay cooperative relaying enables spatial diversity often at the expense of spectral efficiency. To alleviate the loss in spectral efficiency due to half-duplex relaying and transmission over orthogonal channels, we propose a novel transmission scheme for selective decode-and-forward (DF) networks. In this scheme, we assume that destination may receive signals from transmitting nodes with different modulation levels. Particularly, we obtain a closed-form expression for both end-to-end (E2E) average error probability and spectral efficiency in such scheme. Subsequently, using these closed-form expressions and average channel statistics, we perform joint optimization of power allocation and modulation level selection to maximize the E2E spectral efficiency while maintaining a target E2E average error probability, and a set of transmit power constraints. Simulation results demonstrate that the transmission scheme proposed herein improves the E2E spectral efficiency significantly in comparison with the conventional adaptive DF transmission scheme.**

***Index Terms*—Cooperative diversity, adaptive modulation, selective relaying.**

I. Introduction

Cooperative communication has been recognized as an important enabling technology in wireless networks. This technology has been already deployed in the 3GPP LTE-Advanced standards, and more sophisticated cooperative communication techniques are also expected to be adopted in 5G standards [1].

The use of relay-based cooperative transmission brings important benefits such as effectively extended coverage, and improved link reliability. Despite those benefits, cooperative communication suffers from the loss in spectral efficiency due to half-duplex transmission constraints on relays, and the need of orthogonal time/frequency slots to transmit messages. To mitigate the loss in spectral efficiency, an effective technique, so-called best-relay selection, is proposed, in which only one relay is selected to retransmit the source message [2]. To further improve spectral efficiency, adaptive modulation technique can be used along with the best-relay selection, see [3]–[6] and references therein. In this technique, spectral efficiency is enhanced by changing modulation level adaptively according to channel conditions. It is worth to note that because in amplify-and-forward (AF) relaying, the relays simply amplify the received signal, and then retransmit to the destination, adaptive modulation technique can be only applied to the source [7].

In this paper, we present a novel transmission scheme for a multi-relay selective decode-and-forward (DF) relaying system, in which the destination selects the best relay among a set of candidate relays with different modulation levels. In comparison with the available schemes,[1] the one presented herein is the first to attempt designing modulation level selection and power allocation jointly by using average SNRs when the transmitting nodes have the flexibility to employ different modulation levels. Specifically, in the considered system, we first derive closed-form expressions for the end-to-end (E2E) average error probability and spectral efficiency. Then, using the derived close-form expressions and average channel statistics, we jointly optimize power allocation and modulation level selection to maximize the E2E spectral efficiency while satisfying a predetermined E2E average error probability, and total and individual transmission power constraints. Since the formulated optimization problem is non-linear non-convex and cannot be solved analytically, to implement this optimization problem, we utilize Matlab command *fmincon* with interior-point method. Finally, numerical results are presented to illustrate the achievable performance gains using the proposed scheme.

II. System and Channel Models

We consider a dual-hop network transmitting information from a source (S) to a destination (D) through L relays ($R_1, \ldots, R_l, \ldots, R_L$), where a direct link does not exist between S and D owing to heavy blockage and long distance transmission.[2] Each node is equipped with a single antenna, and operates in the half-duplex mode in which they can either receive or transmit.[3] The circularly symmetric complex Gaussian channel gains between S and R_i, and between R_i and D are $h_i \sim \mathcal{N}(0, \Omega_{h_i})$ and $f_i \sim \mathcal{N}(0, \Omega_{f_i})$, respectively. These gains are constant over a transmission block, and yet they are independent from one block to another.

In the first time slot, S transmits a data packet using M_S-QAM. Since the packet consists of N-bits, the number of transmitted symbols is $N_S = N/k_S$, where $k_S = \log_2(M_S)$ is the number of bits per symbol (bps). Thus, the packet received at the relay R_l can be written as

$$y_{R_l,i} = \sqrt{P_S} h_l x_i + n_{R_l,i}, \; i = 1, \ldots, N_S, \; l = 1, \ldots, L, \quad (1)$$

[1] The vast majority of adaptive-modulation schemes use instantaneous signal-to-noise ratios (SNRs), and assign the same modulation level to all nodes. However, in a realistic scenario, transmitting nodes may employ different modulation levels due to having different channel conditions. In addition, using instantaneous SNR can give rise to an excessive signalling overhead. Furthermore, in some cases, instantaneous SNR estimation accuracy can be practically limited.

[2] We consider the use of the relay technology for coverage limited scenarios. Since in such scenarios, the channel quality of direct link is assumed to be weak, and the discrepancy between the direct and the indirect links is high, there will be a low or no diversity gain utilization.

[3] There is a mounting interest in full-duplex systems in the literature. However, full-duplex systems result in a substantially increased level of complexity. Hence, in this paper, we focus on half-duplex systems.

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where P_S is the transmit power from S and n_{R_i} is the circularly symmetric complex additive white Gaussian noise (AWGN) with zero mean and variance N_0. Then, all relays listen to the transmitted packet by the source, and only the relays which decode the packet correctly can contribute in the packet relaying in the second time slot.[4] We denote by $\mathcal{C}$ and $|\mathcal{C}|$, the set of relays that correctly detect the source signal and cardinality of $\mathcal{C}$, respectively. In the second time slot, the best relay R_j is selected to retransmit the packet based on a given relay selection policy, using M_{R_j}-QAM. The number of transmitted symbols in the second time slot is $N_{R_j} = N/k_{R_j}$, where $k_{R_j} = \log_2\left(M_{R_j}\right)$ bps. The received packet at D from the best relay R_j can be given as

$$y_{D,i} = \sqrt{P_R} f_j x_i + n_{D,i},\ i = 1, \ldots, N_{R_j}, \tag{2}$$

where P_R is the transmit power used in the second time slot, and n_D is the AWGN at D. The resultant instantaneous and average SNRs at the relay R_j in the first time slot can be given as $\gamma_{SR_j} = P_S|h_j|^2/N_0$, and $\overline{\gamma}_{SR_j} = P_S\Omega_{h_j}/N_0$, respectively. The instantaneous and average SNRs at D in the second time slot are $\gamma_{R_jD} = P_S|f_j|^2/N_0$, and $\overline{\gamma}_{R_jD} = P_R\Omega_{f_j}/N_0$, respectively.

Even though the best-relay selection policy based on maximizing the received SNRs works well with the conventional assumption of the same modulation levels at all nodes, it falls short to account for scenarios when the transmitting nodes have different modulation levels. This is due to the fact that different modulation levels have different error resilience properties. Hence, here we choose the best relay in a way to minimize the received bit-error-rate (BER):[5]

select j-th relay, where $j = \arg\min_{j\in\mathcal{C}} \mathsf{P}_{\text{inst}}^{R_j}(\mathsf{e})$,

where for Gray-coded square coherent M_R-QAM over a Rayleigh fading channel, the instantaneous BER at destination can be written as [8]

$$\mathsf{P}_{\text{inst}}^{\text{coop}}(\mathsf{e}|\mathcal{C}) = \mathsf{P}_{\text{inst}}^{\mathsf{R}^*}(\mathsf{e}) \approx \alpha_{R^*} Q\left(\sqrt{2\beta_{R^*}\gamma_{R^*D}}\right),$$
$$\text{with } (\alpha_j, \beta_j) = \begin{cases} (1,1), & M_j = 2, \\ \left(\frac{2-2/\sqrt{M_j}}{\log_2\sqrt{M_j}}, \frac{3}{2(M_j-1)}\right), & M_j \geq 4, \end{cases} \tag{3}$$

where $\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C})$ denotes the instantaneous BER in the cooperative case when the relays in the set of $\mathcal{C}$ decode correctly, $\mathsf{P}_{\text{inst}}^{\mathsf{R}^*}(\mathsf{e})$ is the instantaneous BER between the selected relay R^*, and the destination. It is worth to mention that $\mathsf{P}_{\text{inst}}^{\text{coop}}(\mathsf{e}|\mathcal{C})$ is a piecewise function with intervals that are dependent on the instantaneous SNRs, and it cannot be expressed solely as a function of the output SNR, i.e., (α, β) may also change depending on the selection. Therefore, there is no straightforward expression for the probability density function (PDF) of the output SNR [9].

III. Performance Analysis

For the discussed system model, here we derive the E2E average error probability and spectral efficiency.

A. Special Case: Two Relays ($L = 2$)

1) End-to-End Average Error Probability: The E2E average error probability is equal to the average of the error probabilities over two cases; cooperative and non-cooperative ones, and it can be written as

$$\begin{aligned}\mathsf{P}(\mathsf{e}) &= \Pr\{|\mathcal{C}| = 0\}\,\mathsf{P}^{\text{non-coop}}(\mathsf{e}) + \sum_{\ell=1}^{2}\Pr\{|\mathcal{C}| = \ell\}\mathsf{P}^{\text{coop}}(\mathsf{e}|\ell) \\ &= \mathsf{P}_{\mathsf{SR}_1}^{\mathsf{PEP}}(\mathsf{e})\mathsf{P}_{\mathsf{SR}_2}^{\mathsf{PEP}}(\mathsf{e})\mathsf{P}^{\text{non-coop}}(\mathsf{e}) \\ &+ \left(1 - \mathsf{P}_{\mathsf{SR}_1}^{\mathsf{PEP}}(\mathsf{e})\right)\mathsf{P}_{\mathsf{SR}_2}^{\mathsf{PEP}}(\mathsf{e})\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_1\}) \\ &+ \left(1 - \mathsf{P}_{\mathsf{SR}_2}^{\mathsf{PEP}}(\mathsf{e})\right)\mathsf{P}_{\mathsf{SR}_1}^{\mathsf{PEP}}(\mathsf{e})\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_2\}) \\ &+ \left(1 - \mathsf{P}_{\mathsf{SR}_1}^{\mathsf{PEP}}(\mathsf{e})\right)\left(1 - \mathsf{P}_{\mathsf{SR}_2}^{\mathsf{PEP}}(\mathsf{e})\right)\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_1, R_2\}),\end{aligned} \tag{4}$$

where $\mathsf{P}_{\mathsf{SR}_i}^{\mathsf{PEP}}(\mathsf{e})$ is the average packet error probability (PEP) at the i-th relay, $i \in \{1, 2\}$, when the whole packet is received incorrectly, $\mathsf{P}^{\text{non-coop}}(\mathsf{e})$ and $\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C})$ denote the average BER in the non-cooperative and cooperative cases, respectively.

To find the expressions of $\mathsf{P}_{\mathsf{SR}_i}^{\mathsf{PEP}}(\mathsf{e})$, we follow a reasoning similar to the one given in [10]. Then $\mathsf{P}_{\mathsf{SR}_i}^{\mathsf{PEP}}(\mathsf{e})$ can be obtained as follows:

$$\mathsf{P}_{\mathsf{SR}_i}^{\mathsf{PEP}}(\mathsf{e}) \approx \sum_{w=1}^{N_S}\sum_{z=0}^{w}\binom{N_S}{w}\binom{w}{z}\frac{(-1)^{(w+1)}(k_S\alpha_S)^w A_1^{w-z}A_2^z}{1 + 2\beta_S(a_1(w-z) + a_2 z)\overline{\gamma}_{SR_i}}, \tag{5}$$

where $N_S = N/k_S$, $A_1 = 0.204$, $A_2 = 0.147$, $a_1 = 0.971$, and $a_2 = 0.525$.

When only one relay is active, i.e., $|\mathcal{C}| = 1$, the average BER in the cooperative case, $\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_i\}), i \in \{1, 2\}$, in a Rayleigh fading channel can be derived as

$$\begin{aligned}\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_i\}) &= \int_0^\infty \alpha_i Q(\sqrt{2\beta_i\gamma_{R_iD}})\frac{1}{\overline{\gamma}_{R_iD}}e^{-\frac{\gamma_{R_iD}}{\overline{\gamma}_{R_iD}}}d_{\gamma_{R_iD}} \\ &= I(\alpha_{R_i}, \beta_{R_i}, \overline{\gamma}_{R_iD}),\end{aligned} \tag{6}$$

where $I(a, b, c) = 0.5a\left(1 - \sqrt{\frac{bc}{1+bc}}\right)$.

When the number of active relays is two, i.e., $|\mathcal{C}| = 2$, the instantaneous BER in the cooperative case can be given as

$$\mathsf{P}_{\text{inst}}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_1, R_2\}) = \begin{cases}\alpha_1 Q(\sqrt{2\beta_1\gamma_{R_1D}}), & \mathsf{P}_{\text{inst}}^{\mathsf{R}_1}(\mathsf{e}) \leq \mathsf{P}_{\text{inst}}^{\mathsf{R}_2}(\mathsf{e}), \\ \alpha_2 Q(\sqrt{2\beta_2\gamma_{R_2D}}), & \mathsf{P}_{\text{inst}}^{\mathsf{R}_2}(\mathsf{e}) < \mathsf{P}_{\text{inst}}^{\mathsf{R}_1}(\mathsf{e}).\end{cases} \tag{7}$$

We use the approach given in [9] towards developing the average BER in the cooperative case. The average BER results in [9] are obtained for a scenario where the source communicates to the destination via both a direct and indirect links using a single relay, and then, the destination uses selection combining technique to extract spatial diversity. However, herein we consider a multi-relay scenario without a direct link, where the source communicates to the destination through only the best relay. Hence, the average BER, $\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_1, R_2\})$, when $|\mathcal{C}| = 2$ can be obtained as

$$\begin{aligned}&\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C} = \{R_1, R_2\}) \\ &= \iint_{\rho_1}\alpha_1 Q(\sqrt{2\beta_1\gamma_{R_1D}})\frac{e^{-\left(\frac{\gamma_{R_1D}}{\overline{\gamma}_{R_1D}} + \frac{\gamma_{R_2D}}{\overline{\gamma}_{R_2D}}\right)}}{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}d_{\gamma_{R_1D}}d_{\gamma_{R_2D}} \\ &+ \iint_{\rho_2}\alpha_2 Q(\sqrt{2\beta_2\gamma_{R_2D}})\frac{e^{-\left(\frac{\gamma_{R_1D}}{\overline{\gamma}_{R_1D}} + \frac{\gamma_{R_2D}}{\overline{\gamma}_{R_2D}}\right)}}{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}d_{\gamma_{R_1D}}d_{\gamma_{R_2D}},\end{aligned} \tag{8}$$

where $\rho_1 = \left\{(\gamma_{R_1D}, \gamma_{R_2D}) : \mathsf{P}_{\text{inst}}^{\mathsf{R}_1}(\mathsf{e}) \leq \mathsf{P}_{\text{inst}}^{\mathsf{R}_2}(\mathsf{e})\right\} \approx \left\{(\gamma_{R_1D}, \gamma_{R_2D}) : \beta_{R_1}\gamma_{R_1D} \geq \beta_{R_2}\gamma_{R_2D}\right\}$, and $\rho_2 = \left\{(\gamma_{R_1D}, \gamma_{R_2D}) : \mathsf{P}_{\text{inst}}^{\mathsf{R}_2}(\mathsf{e}) < \mathsf{P}_{\text{inst}}^{\mathsf{R}_1}(\mathsf{e})\right\} \approx \left\{(\gamma_{R_1D}, \gamma_{R_2D}) : \beta_{R_2}\gamma_{R_2D} > \beta_{R_1}\gamma_{R_1D}\right\}$. Note that for the approximate values of ρ_1 and ρ_2, Chernoff bound on the Q-function is first used, and

[4] In practice, a large cyclic redundancy check (CRC) code can be used for error detection in order to guarantee that the probability of occurrence of undetectable errors is sufficiently small.

[5] There is an inversely proportional relation between SNR and BER. However, the type of modulation has also a direct impact on the performance, and hence on the variation between SNR and BER. Therefore, when the different modulation levels are employed, the link that has the maximum SNR might not be the most reliable link.

then the constant terms are dropped. After some mathematical manipulations, $\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C}=\{R_1,R_2\})$ can be rewritten as

$$\begin{aligned}
&\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C}=\{R_1,R_2\})\\
&=\int_{\gamma_{R_1D}=0}^{\infty}\int_{\gamma_{R_2D}=0}^{w_{12}\gamma_{R_1D}}\alpha_1 Q(\sqrt{2\beta_1\gamma_{R_1D}})\frac{e^{-(\frac{\gamma_{R_1D}}{\overline{\gamma}_{R_1D}}+\frac{\gamma_{R_2D}}{\overline{\gamma}_{R_2D}})}}{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}d_{\gamma_{R_1D}}d_{\gamma_{R_2D}}\\
&+\int_{\gamma_{R_2D}=0}^{\infty}\int_{\gamma_{R_1D}=0}^{w_{21}\gamma_{R_2D}}\alpha_2 Q(\sqrt{2\beta_2\gamma_{R_2D}})\frac{e^{-(\frac{\gamma_{R_1D}}{\overline{\gamma}_{R_1D}}+\frac{\gamma_{R_2D}}{\overline{\gamma}_{R_2D}})}}{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}d_{\gamma_{R_1D}}d_{\gamma_{R_2D}}\\
&=I(\alpha_{R_1},\beta_{R_1},\overline{\gamma}_{R_1D})+I(\alpha_{R_2},\beta_{R_2},\overline{\gamma}_{R_2D})\\
&-\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_2D}+w_{12}\overline{\gamma}_{R_1D}}I\Big(\frac{\alpha_{R_1}}{\overline{\gamma}_{R_1D}},\beta_{R_1},\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_2D}+w_{12}\overline{\gamma}_{R_1D}}\Big)\\
&-\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_1D}+w_{21}\overline{\gamma}_{R_2D}}I\Big(\frac{\alpha_{R_2}}{\overline{\gamma}_{R_2D}},\beta_{R_2},\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_1D}+w_{21}\overline{\gamma}_{R_2D}}\Big),
\end{aligned}\tag{9}$$

where $w_{ij}=\beta_{R_i}/\beta_{R_j},\ i,j=1,2$.

By substituting (6), and (9) into (4), the E2E average BER can be rewritten as

$$\begin{aligned}
\mathsf{P}(\mathsf{e})\approx\ &\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_1}(\mathsf{e})\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_2}(\mathsf{e})(1/2)\\
&+(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_1}(\mathsf{e}))\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_2}(\mathsf{e})I(\alpha_{R_1},\beta_{R_1},\overline{\gamma}_{R_1D})\\
&+(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_2}(\mathsf{e}))\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_1}(\mathsf{e})I(\alpha_{R_2},\beta_{R_2},\overline{\gamma}_{R_2D})\\
&+(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_2}(\mathsf{e}))(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_1}(\mathsf{e}))\Big(I(\alpha_{R_1},\beta_{R_1},\overline{\gamma}_{R_1D})\\
&-\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_2D}+w_{12}\overline{\gamma}_{R_1D}}I\Big(\frac{\alpha_{R_1}}{\overline{\gamma}_{R_1D}},\beta_{R_1},\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_2D}+w_{12}\overline{\gamma}_{R_1D}}\Big)\\
&+I(\alpha_{R_2},\beta_{R_2},\overline{\gamma}_{R_2D})\\
&-\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_1D}+w_{21}\overline{\gamma}_{R_2D}}I\Big(\frac{\alpha_{R_2}}{\overline{\gamma}_{R_2D}},\beta_{R_2},\frac{\overline{\gamma}_{R_1D}\overline{\gamma}_{R_2D}}{\overline{\gamma}_{R_1D}+w_{21}\overline{\gamma}_{R_2D}}\Big)\Big),
\end{aligned}\tag{10}$$

where the average BER in the non-cooperative case is considered to be 0.5 [11], viz., $\mathsf{P}^{\text{non-coop}}(\mathsf{e})=0.5$, since we assume that there is no direct link between S and D.[6]

2) End-to-End Spectral Efficiency: For deriving the E2E spectral efficiency, the common approach is to add data rates in each partitioning region multiplied by the occurrence probability of each region; the occurrence probability of each region is calculated using the PDF of output SNR [3]–[7]. However, this approach does not work when the transmitting nodes have different modulation levels, since we do not have the PDF of the output SNR.

We consider a M-QAM system with fixed packet size and fixed symbol duration which are denoted as N-bits and T_s-secs, respectively. Hence, the length of the duration to transmit an N-bits packet using M-QAM is $T_{\text{total}}=T_sN_M$, where $N_M=N/k_M$ symbols and $k_M=\log_2(M)$ bps. If we assume that bandwidth is $B\approx 1/T_s$, then the spectral efficiency in a link-to-link transmission can be defined as $\eta^{\text{non-coop}}=N/(BT_{\text{total}})=k_M$.

Let us first discuss a simple cooperative system with a single relay, where the source and the relay transmit the packet using M_S-QAM and M_R-QAM, respectively. For such a system, the E2E spectral efficiency can be found as $\eta_R^{\text{coop}}=N/(BT_{\text{total}})=k_Sk_R/(k_S+k_R)$, where $T_{\text{total}}=T^{\text{slot-1}}_{\text{total}}+T^{\text{slot-2}}_{\text{total}}$ is total duration, $T^{\text{slot-1}}_{\text{total}}=T_sN/k_S$ and $T^{\text{slot-2}}_{\text{total}}=T_sN/k_R$ are the duration of packet transmission in the first and second slots, respectively.

Let us next discuss a more general scenario with two relays, assuming that relays decode the received packet correctly and transmit the packet using M_{R_1}-QAM and M_{R_2}-QAM while the source employs M_S-QAM. In this case, since only the best relay is chosen to participate in forwarding the received packet, we need to reflect the impact of selection policy. Then, the E2E spectral efficiency can be expressed as follows:

$$\begin{aligned}
\eta^{\text{coop}}_{R_{1,2}}&=N/\big(B(T^{\text{slot-1}}_{\text{total}}+T^{\text{slot-2}}_{\text{total}})\big)\\
&=\left(\frac{1}{k_S}+\frac{\Pr\big(\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\big)}{k_{R_1}}+\frac{\Pr\big(\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})<\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\big)}{k_{R_2}}\right)^{-1},
\end{aligned}\tag{11}$$

where the probabilities of choosing the relay R_1 and R_2 are $\Pr\Big(\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\Big)$ and $\Pr\Big(\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})<\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\Big)$, respectively, the duration of packet transmission in the first time slot is $T^{\text{slot-1}}_{\text{total}}=T_sN/k_S$, the average duration of packet transmission in the second time slot over two cases is $T^{\text{slot-2}}_{\text{total}}=\Pr\Big(\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\Big)T^{\text{slot-2}}_{R_1}+\Pr\Big(\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})<\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\Big)T^{\text{slot-2}}_{R_2}$, and $T^{\text{slot-2}}_{R_\ell}=T_sN/k_{R_\ell},\ \ell=1,2$, denotes the duration of packet transmission in the second slot if ℓ-th relay is chosen.

We provide an approximation, which is tight at high SNRs, for $\Pr\Big(\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\Big)$ expressions as

$$\begin{aligned}
&\Pr\Big(\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\Big)\approx\Pr\Big(\beta_{R_1}\gamma_{R_1D}\ge\beta_{R_2}\gamma_{R_2D}\Big)\\
&=\int_{\gamma_{R_1D}=0}^{\infty}\int_{\gamma_{R_2D}=0}^{w_{12}\gamma_{R_1D}}\frac{1}{\overline{\gamma}_{R_1D}}\frac{1}{\overline{\gamma}_{R_2D}}e^{-\frac{\gamma_{R_1D}}{\overline{\gamma}_{R_1D}}}e^{-\frac{\gamma_{R_2D}}{\overline{\gamma}_{R_2D}}}d_{\gamma_{R_1D}}d_{\gamma_{R_2D}}\\
&=\frac{w_{12}\bar{\gamma}_{R_1D}}{w_{12}\bar{\gamma}_{R_1D}+\bar{\gamma}_{R_2D}},
\end{aligned}\tag{12}$$

In addition, $\Pr\Big(\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\Big)$ can be also given as

$$\begin{aligned}
\Pr\Big(\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\Big)&=\Big(1-\Pr\Big(\mathsf{P}^{\mathsf{R}_1}_{\text{inst}}(\mathsf{e})\le\mathsf{P}^{\mathsf{R}_2}_{\text{inst}}(\mathsf{e})\Big)\Big)\\
&\approx\frac{\bar{\gamma}_{R_2D}}{w_{12}\bar{\gamma}_{R_1D}+\bar{\gamma}_{R_2D}}.
\end{aligned}\tag{13}$$

So far, the link between node S and the relays are assumed to be error-free. To make the analyses more general, we remove this assumption, and then the E2E spectral efficiency can be found as

$$\begin{aligned}
\eta^{\text{E2E}}&=\Pr\{|\mathcal{C}|=0\}\,\eta^{\text{non-coop}}+\sum_{\ell=1}^{2}\Pr\{|\mathcal{C}|=\ell\}\eta^{\text{coop}}_{\ell}\\
&=(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_1}(\mathsf{e}))\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_2}(\mathsf{e})\frac{k_Sk_{R_1}}{(k_S+k_{R_1})}\\
&+(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_2}(\mathsf{e}))\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_1}(\mathsf{e})\frac{k_Sk_{R_2}}{(k_S+k_{R_2})}+(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_1}(\mathsf{e}))(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_2}(\mathsf{e}))\\
&\times\frac{k_Sk_{R_1}k_{R_2}(\overline{\gamma}_{R_1D}w_{12}+\overline{\gamma}_{R_2D})}{k_{R_1}\overline{\gamma}_{R_2D}(k_S+k_{R_2})+k_{R_2}\overline{\gamma}_{R_1D}w_{12}(k_S+k_{R_1})},
\end{aligned}\tag{14}$$

where $\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_i}(\mathsf{e})$ is given in (5) and $\eta^{\text{non-coop}}$ is assumed to be 0 since no communication occurs between S and D when all relays are inactive, i.e., $|\mathcal{C}|=0$.

B. General L Relays Case

1) End-to-End Average Error Probability: For an arbitrary number of relays, the E2E average error probability can be obtained as in (10):

$$\begin{aligned}
\mathsf{P}(\mathsf{e})=&\left(\prod_{r=1}^{L}\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_r}(\mathsf{e})\right)\mathsf{P}^{\text{non-coop}}(\mathsf{e})\\
&+\sum_{r=1}^{L}\sum_{m=1}^{|P_r(\mathcal{S}_{\text{all}})|}\Bigg[\Bigg(\prod_{\kappa_i\in P_{r,m}(\mathcal{S}_{\text{all}})}\Big(1-\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_{\kappa_i}}(\mathsf{e})\Big)\Bigg)\\
&\times\Bigg(\prod_{\kappa_o\notin P_{r,m}(\mathcal{S}_{\text{all}})}\mathsf{P}^{\text{PEP}}_{\mathsf{SR}_{\kappa_o}}(\mathsf{e})\Bigg)\mathsf{P}^{\text{coop}}(\mathsf{e}|\mathcal{C}=P_{r,m}(\mathcal{S}_{\text{all}}))\Bigg],
\end{aligned}\tag{15}$$

[6] The outcome of transmission over such channel acts as a matter of pure chance, similar to tossing a coin.

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where S_{all} is the set of all relays' indexes, i.e., $S_{\text{all}} = \{1, ..., L\}$, $P_r(S_{\text{all}})$ is the r-th element power set of S_{all}, $|P_r(S_{\text{all}})|$ represents the cardinality of $P_r(S_{\text{all}})$, $P_{r,m}(S_{\text{all}})$ is the m-th element of $P_r(S_{\text{all}})$, i.e., $P_r(\mathcal{S}_{\text{all}}) = \{P_{r,1}(\mathcal{S}_{\text{all}}), P_{r,2}(\mathcal{S}_{\text{all}}), ..., P_{r,|P_r(\mathcal{S}_{\text{all}})|}(\mathcal{S}_{\text{all}})\}$.

2) End-to-End Spectral Efficiency: The framework given for $L = 2$ case can be extended to scenarios with any number of relays for the computation of the E2E spectral efficiency:[7]

$$\eta^{\text{E2E}} = \sum_{r=1}^{L} \sum_{m=1}^{|P_r(\mathcal{S}_{\text{all}})|} \left(\prod_{\kappa_i \in P_{r,m}(\mathcal{S}_{\text{all}})} \left(1 - \mathsf{P}^{\text{PEP}}_{\text{SR}_{\kappa_i}}(\mathsf{e})\right) \right) \times \left(\prod_{\kappa_o \notin P_{r,m}(\mathcal{S}_{\text{all}})} \mathsf{P}^{\text{PEP}}_{\text{SR}_{\kappa_o}}(\mathsf{e}) \right) \eta^{\text{coop}}_{P_{r,m}(\mathcal{S}_{\text{all}})}. \tag{16}$$

IV. Joint Optimization of Power Allocation and Modulation Level Selection

We develop a framework to jointly optimize the transmission powers and the modulation levels to maximize the E2E spectral efficiency while meeting the given requirements on the transmission powers and the E2E average BER. In a somewhat similar context, the modulation level selection problem is studied in [12] using a different approach. Particularly, we consider maximizing the E2E spectral efficiency using the average SNRs under power constraints, whereas in [12], the design objective is to find the best transmission route with the highest spectral efficiency using the instantaneous SNR irrespective of the transmission powers. It is worth to note that our proposed scheme requires less signalling overhead, since it relies on average channel statistics.[8]

System constraints: To guarantee a certain level of transmission reliability in the system, the E2E average BER is constrained by a predefined threshold, $\mathsf{P}_{\text{th}}(\mathsf{e})$. The nodes have their own individual power constraints, i.e., $P_S \le P^i_{\text{max}}, i \in \{S, R\}$. Since the interference effect is ignored, the transmitting nodes are inclined to transmit data packets with the maximum power available not only to reduce the E2E average BER but also to improve the spectral efficiency. Hence, to make the scenario more practical, the total power consumption over two time slots is constrained as $P_S + P_R \le P_T$.

Problem formulation: The problem is formulated based on the described system constraints, as shown in the following:[9]

$$\max_{M_S, M_{R_i}, P_S, P_R} \quad \eta^{\text{E2E}} \tag{17a}$$

$$\text{subject to} \quad \mathsf{P}(\mathsf{e}) \le \mathsf{P}_{\text{th}}(\mathsf{e}), \tag{17b}$$

$$P_S + P_R \le P_T, \tag{17c}$$

$$0 < P_j \le P^j_{\text{max}}, \; j \in \{S, R\}, \tag{17d}$$

$$M_S, M_{R_i} \in \{2, 4, 16, 64, 256\}, \; i = 1, \dots, L. \tag{17e}$$

[7]Note that as $P_{r,m}(\mathcal{S}_{all})$ changes, $\eta^{\text{coop}}_{P_{r,m}(\mathcal{S}_{all})}$ changes as well. For instance, when $|\mathcal{C}| = 3$, $\eta^{\text{coop}}_{R_{1,2,3}} = \left(\frac{1}{k_S} + \frac{\Pr\left(1 = \arg\min_{i \in \{1,2,3\}} \mathsf{P}^{\text{R}_i}_{\text{inst}}(\mathsf{e})\right)}{k_{R_1}} + \frac{\Pr\left(2 = \arg\min_{i \in \{1,2,3\}} \mathsf{P}^{\text{R}_i}_{\text{inst}}(\mathsf{e})\right)}{k_{R_2}} + \frac{\Pr\left(3 = \arg\min_{i \in \{1,2,3\}} \mathsf{P}^{\text{R}_i}_{\text{inst}}(\mathsf{e})\right)}{k_{R_3}} \right)^{-1}$.

[8]In our scheme, the modulation level decisions remain the same despite small-scale channel variations; these decisions will only change as the link path loss values change (i.e., due to large-scale channel variations). Making the modulation level decisions based on the average SNR values make the protocol more practical and robust. Hence, in such scheme, signalling overhead is introduced due to the following factors: 1) To acquire the average SNRs of all the links; and 2) To inform the transmitting nodes regarding the outcome of the optimization. This overhead is not expected to be excessive.

[9]Since in our formulation, average error probability is constrained to be less than a predefined threshold, and also, optimization of power allocation is considered, the relatively rare outage events are accounted for.

The optimization problem in (17) is an instance of a non-convex mixed-integer non-linear program. Since finding an analytical solution for (17) is not possible, we resort to numerical optimization. To that end, we employ Matlab command *fmincon* with interior-point method.

Proposed algorithm: We propose an algorithm to find the best combination of transmit powers and modulation levels; a pseudocode of the algorithm is given in Algorithm 1. A centralized design is considered to practically implement the algorithm. In such a design, the destination may collect the average SNR values on all links, carry out the optimization task, and spread the obtained solutions to the source and the relays.

The set of all possible combinations of the modulation levels is denoted by $\mathcal{S}_{\mathcal{M}}$, $\mathcal{M} = (M_S, M_{R_1}, \dots, M_{R_L})$. The cardinality of $\mathcal{S}_{\mathcal{M}}$ is represented as $|\mathcal{S}_{\mathcal{M}}|$, e.g., for $L = 2$, $|\mathcal{S}_{\mathcal{M}}| = 5^3$. We note that the number of such combinations can grow prohibitively high. To lower the number of combinations, we impose a condition; the modulation level assigned to a relay should be higher than the modulation levels of the relays with the worst channel power, $\mathbb{E}(|f_i|^2) = \Omega_{f_i}$. Thereby, for $L = 2$, the number of combinations can be reduced to $|\tilde{\mathcal{S}}_{\mathcal{M}}| = 75$, where $\tilde{\mathcal{S}}_{\mathcal{M}}$ is the set of reduced combinations. For L relays case, the cardinality of the set of the reduced combinations is equal to $|\tilde{\mathcal{S}}_{\mathcal{M}}| = (5L^4 + 50L^3 + 175L^2 + 250L + 120)/24$. Note that the reduction is more pronounced for higher number of relays, and the number of combinations with the proposed simplification does not grow exponentially by the number of relays. On the other hand, from deployment point of view, since the optimal number of relays in a cell will not be extremely high due to cost issues, the proposed centralized algorithm can be viewed as a starting point for the development of more practical ones.

Algorithm 1: Joint Optimization of Power Allocation and Modulation Level Selection

Input: Ω_{h_i}, Ω_{f_i}, P_T, P^S_{max}, P^R_{max}, $\mathsf{P}_{\text{th}}(\mathsf{e})$, $i = 1, \dots, L$.
Output: M^*_S, $M^*_{R_i}$, P^*_S, P^*_R, $i = 1, \dots, L$.

1 **Define the set of combinations:** Reduce the number of combination of modulation levels, and find $\tilde{\mathcal{S}}_{\mathcal{M}}$.
2 **for** $j = 1 : |\tilde{\mathcal{S}}_{\mathcal{M}}|$ **do**
3   **Power allocation optimization:** Find $P^{(j)}_S$, and $P^{(j)}_R$ for the j-th combination $(M^{(j)}_S, M^{(j)}_{R_i}) \in \tilde{\mathcal{S}}_{\mathcal{M}}$.
4   **Find the E2E spectral efficiency:** Using $P^{(j)}_S$, $P^{(j)}_R$, $M^{(j)}_S$ and $M^{(j)}_{R_i}$, obtain $\eta^{\text{E2E}(j)}$ and record all values.
5 **Determine the best combination:** Find the best combination with the highest $\eta^{\text{E2E}(j)}$ and assign it to $(M^*_S, M^*_{R_i}, P^*_S, P^*_R)$.

V. Numerical Results

We evaluate the performance of the proposed method (PM) through numerical comparisons with conventional adaptive modulation method (CM) which serves as a baseline. Even though CM performs a joint optimization similar to PM, in CM, the nodes are constrained to use the same modulation levels.

Simulation setup: We consider a simple scenario in a single-carrier isolated-cell network setup. We assume that a data packet consists of 96-bits, the threshold on the E2E average BER is $\mathsf{P}_{\text{th}}(\mathsf{e}) = 10^{-3}$, the total power available over two time slots is $P_T = 1$, and the individual power limits on source and relay nodes are $P^S_{\text{max}} = 0.8P_T$, and $P^R_{\text{max}} = 0.8P_T$, respectively.

Performance of proposed algorithm: We start by illustrating the E2E average BER performance in two-relay and three-relay scenarios for given modulation levels at the transmitting nodes. The obtained analytical results using (15) are compared with

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Monte Carlo simulations. From Figure 1, it can be observed that the derived analytical results are in good agreement with the simulation results. Figure 2 investigates the gain realized by PM compared to CM in a two-relay scenario, considering the set of the modulation levels of $\mathcal{M} = \{2, 4, 16, 64, 256\}$, e.g., available modulation formats in IEEE 802.11ac standard. At some SNR values, we explicitly mention the amount of achieved gain and the set of the modulation levels employed at nodes, (M_S, M_{R_1}, M_{R_2}). As observed that PM outperforms CM over the entire range of SNR values, since PM achieves full utilization of the degrees of freedom in adaptive modulation. Lastly, in Figure 3, the performance of the algorithms are depicted, considering different number of relays in a variety of scenarios, and a smaller set of the modulation levels,[10] $\mathcal{M} = \{2, 4, 16, 64\}$, e.g., available modulation formats in IEEE 802.11a standard. PM shows superior performance over CM in all cases, yet the improvement gained with PM can change according to the considered scenario. It is worth to note that an increase in the number of relays helps to improve the E2E spectral efficiency, especially at low SNR values. This is because the E2E average BER gets better as the number of relays increases, and in this way, the threshold on the E2E average BER can be satisfied.

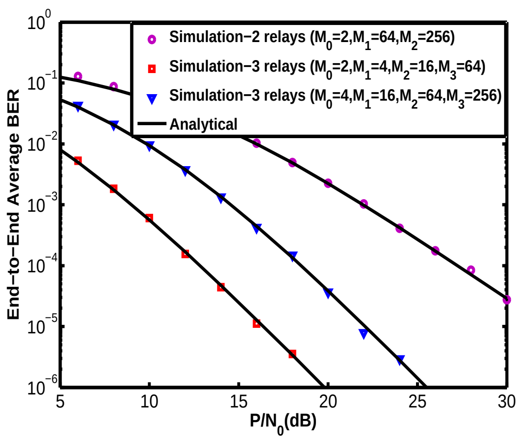

Fig. 1. The E2E average BER performance for $L = 2$ in Scenario I ($\Omega_{h_1} = 2\Omega_{h_2} = 2\Omega_{f_1} = \Omega_{f_2}$), and $L = 3$ in Scenario II ($\Omega_{h_1} = 2\Omega_{h_2} = 4\Omega_{h_3} = 4\Omega_{f_1} = 2\Omega_{f_2} = \Omega_{f_3}$), assuming $P_S = P_{R_1} = P_{R_2} = P_{R_3} = P$.

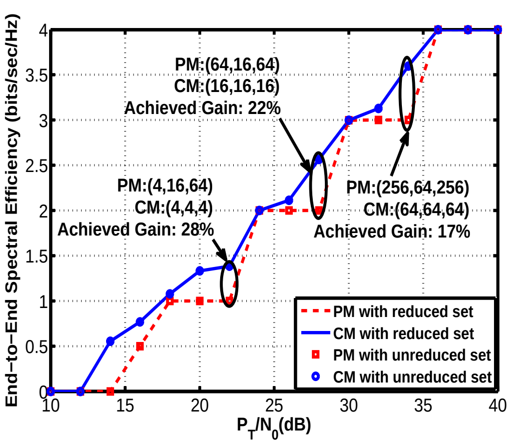

Fig. 2. The gain achieved using PM compared with CM in Scenario I.

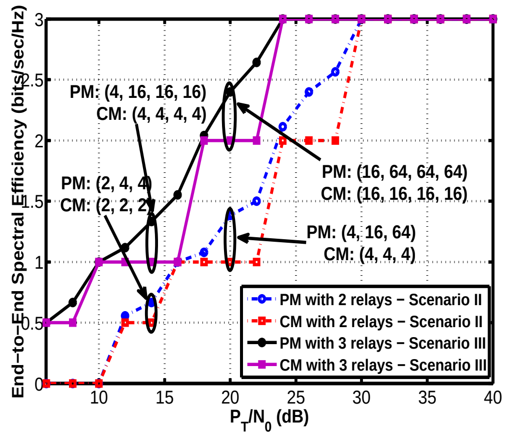

Fig. 3. The E2E spectral efficiency for different number of relays in Scenario II, and in Scenario III ($2\Omega_{h_1} = \Omega_{h_2} = \Omega_{f_1} = 2\Omega_{f_2}$).

VI. Conclusion

We have discussed a new transmission scheme for selective DF relaying networks, considering the employment of different modulation levels at the transmitting nodes. For this scheme, we have derived both the E2E average error probability and spectral efficiency. Using the derived expressions, we have jointly optimized power allocation and modulation level selection to achieve higher E2E spectral efficiency while meeting a predefined E2E average error probability, and total and individual transmit power constraints. Finally, we have showed the performance of the proposed method in comparison with the conventional adaptive modulation method.

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[10] Note that in this case, for L relays, the cardinality of the set of the reduced combinations is equal to $|\tilde{\mathcal{S}}_{\mathcal{M}}| = \frac{2}{3}L^3 + 4L^2 + \frac{22}{3}L + 4$.