A Simple Distributed Antenna Processing Scheme for Cooperative Diversity

Yijia Fan, Abdulkareem Adinoyi, John S. Thompson, Halim Yanikomeroglu, and H. Vincent Poor

***Abstract*—In this letter the performance of multiple relay channels is analyzed for the situation in which multiple antennas are deployed only at the relays. The simple repetition-coded decode-and-forward protocol with two different antenna processing techniques at the relays is investigated. The antenna combining techniques are maximum ratio combining (MRC) for reception and transmit beamforming (TB) for transmission. It is shown that these distributed antenna combining techniques can exploit the full spatial diversity of the relay channels *regardless of* the number of relays and antennas at each relay, and offer *significant* power gain over distributed space-time coding techniques.**

***Index Terms*—Cooperative diversity, MIMO, relay.**

I. INTRODUCTION

THE performance limits of distributed space-time codes, which can exploit *cooperative diversity*, have been investigated in [1] and [2] for single-antenna relay networks using random coding techniques. However, the design and implementation of practical codes that approach these limits are challenging open research areas. One approach to these problems might be to use known space-time codes for the point-to-point multiple-input multiple-output (MIMO) link (e.g. [13]) in relay networks. However, the processing complexity at each relay node for such an approach can increase significantly, as antennas in relay networks are distributed rather than centralized. For example, each relay may need to know all of the uncoded data, before sending only one part of the codeword to the destination. Similarly, the decoding process at the destination might also be very complex when the number of relays are large. Moreover, a more complex protocol is required in order to assign different relays to transmit different parts of the codeword. These points lead to additional time delay and energy cost, while they also present fundamental issues especially for large ad-hoc or sensor networks [4], [10], [12]. Simpler codes such as space-time block codes [14] will result in a rate loss when the number of relays is more than two. Another recent scheme exploits the selection diversity of the network by selecting the best relay among all the available relays [3]. However, the power gain for this scheme is limited due to the limited power at a single relay node; especially in a sensor network environment.

In this letter we exploit the spatial diversity of relay channels in an alternative way to the space-time codes-based approach. We apply two kinds of antenna processing techniques at the relay, namely maximum ratio combining (MRC) [5] for reception and transmit beamforming (TB)[6] for transmission. These techniques are often used in point-to-point single-input multiple-output (SIMO) or multiple-input single-output (MISO) wireless links and have been shown to achieve the optimal diversity multiplexing tradeoff in these cases[7]. More specifically, for a MISO channel, beamforming is often considered as a better approach than space-time coding due to its higher *power gain*, provided that the channel state information (CSI) can be fed back to the transmitter. In our model, we move the multiple antennas to the relays, while the source and the destination are equipped with only a single antenna. Unlike the point-to-point link, the antennas are deployed in a distributed fashion, and MRC and beamforming can only be performed in a distributed rather than a centralized fashion in this scenario. One of the contributions of this letter is to investigate the diversity and power performance tradeoff between the number of relays and the number of antennas at each relay. We will also compare distributed MRC-TB with space-time coding in a multi-antenna multi-relay environment. Some related work on single antenna relay networks has also considered beamforming approach, although this earlier work focuses primarily on the energy efficiency or capacity scaling behavior of such networks [10], [11].

II. SYSTEM DESCRIPTION

We consider a two hop network model with one source, one destination and K relays. For simplicity we ignore the direct link between the source and the destination. The extension of our results to include the direct link is straightforward. We assume that the source and destination are deployed with a single antenna, while relay k is deployed with m_k antennas; the total number of antennas at all relays is fixed at N. We restrict attention to the case in which the channels exhibit slow and frequency-flat fading. We assume a coherent relay channel configuration context in which the kth relay can obtain full knowledge of both the backward channel vector $\mathbf{h}_k$ and the forward channel vector $\mathbf{g}_k$. Note that forward channel knowledge can be obtained easily if the relay-destination link operates in a Time-Division-Duplex (TDD) mode. One example where the relays obtain the required channel information can

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be found in [11], but this might require additional signalling overhead. In a slow fading channel, which is the focus of this letter, this overhead is negligible. For fair comparison, we also assume that for each channel realization, all the backward and forward channel coefficients for all N antennas remain the same regardless of the number of relays K. Fig. 1 shows the system model.

Data is transmitted over two time slots using two hops. In the first transmission time slot, the source broadcasts its signal to all relay terminals. The input/output relation for the source to the kth relay is given by

$$\mathbf{r}_k = \sqrt{\eta}\mathbf{h}_k s + \mathbf{n}_k, \quad (1)$$

where $\mathbf{r}_k$ is the $m_k \times 1$ received signal vector, η is the transmit power at the source, s is the unit mean power transmitted signal, and $\mathbf{n}_k$ is $m_k \times 1$ complex circular additive white Gaussian noise at relay k with zero mean and identity covariance matrix $\mathbf{I}_{m_k}$. The entries of the channel vector $\mathbf{h}_k$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero means and unit variances. We assume that each relay performs MRC of the received signals, by multiplying the received signal vector by the vector $\mathbf{h}_k^H / \|\mathbf{h}_k\|_F$, where $\|\bullet\|_F$ denotes the Frobenius norm. The SNR at the output of the receiver in this scenario can be written as

$$\rho_k^{(m_k)} = \eta \sum_{i=1}^{m_k} |h_{i,k}|^2, \quad (2)$$

where $h_{i,k}$ denotes the channel coefficient from the source to the ith antenna at relay k. Note that for space-time coding, the same MRC scheme is used at the relays when comparing with distributed MRC-TB later in this letter.

After the relays decode the signals, each relay re-encodes the signal using the same codebook as used at the source, then performs TB of the decoded waveform. If we denote the unit variance re-encoded signals as t_k, the transmitted signal vector $\mathbf{d}_k$ for relay k can be written as

$$\mathbf{d}_k = \sqrt{\frac{\eta m_k}{N}} \frac{\mathbf{g}_k^H}{\|\mathbf{g}_k\|_F} t_k, \quad (3)$$

where the vector $\mathbf{g}_k$ is the $m_k \times 1$ channel vector from the kth relay to the destination, where components are i.i.d. complex Gaussian random variables with zero means and unit variances. The vector $\mathbf{d}_k$ in (3) is designed to meet the total transmit power constraint:

$$\mathrm{E}\left[\|\mathbf{d}_k\|_F^2\right] \le \frac{\eta m_k}{N}. \quad (4)$$

Here we assume that the total transmit power from all relays is fixed to be η, i.e., the same as the source transmit power. However, all the conclusions in the paper also hold when the total power from all relays is fixed to *an arbitrary constant*. We note that this power assumption has a meaningful practical implication: in reality a transmitter having a larger number of antennas can often transmit with a higher power (in proportion to the number of transmit antennas in this paper). The destination receiver simply decodes the combined signals from all K relays. If the signals are correctly decoded at all the relays (i.e., $t_k = s$ for all k), the output SNR at the destination receiver can be written as

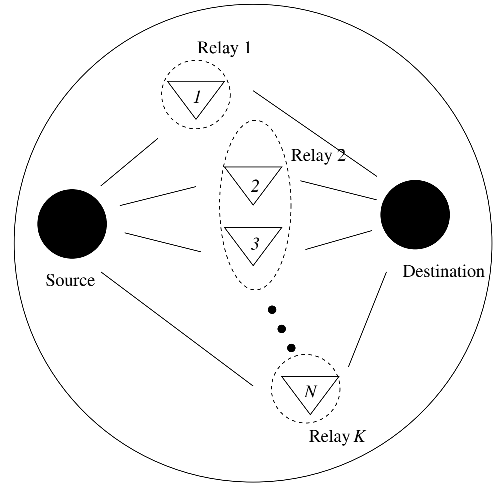

Fig. 1. System model for a two hop network: The source and destination are each deployed with one antenna. Totally N antennas are deployed at K relays. For each channel realization, all the backward or forward channel coefficients for all N antennas remain the same regardless of the number of relays K.

$$\rho_d^{\{m_k\}} = \left(\sum_{k=1}^{K} \sqrt{\frac{\eta m_k}{N} \sum_{i=1}^{m_k} |g_{i,k}|^2} \right)^2. \quad (5)$$

When each of the relays is deployed with a single antenna, there is no MRC gain at the relays, nor is there any beamforming gain at the destination. However, the destination still observes a set of equal-gain-combined [8] amplitude signals from all relays. Since we assume that the backward and forward channel coefficients for each antenna are kept the same for different values of K and m_i, the output SNR at the destination can be rewritten as $\rho_d^{(1)} = \frac{\eta}{N}\left(\sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|\right)^2$; when all the antennas are deployed on one relay (i.e., $K = 1$ and $m_1 = N$), full diversity gain is achieved among all the N antennas at the relay and also at the destination. The SNR for this case can be rewritten as $\rho_d^{(N)} = \eta \sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|^2$.

III. Performance Analysis

A. SNR Gain

We first compare $\rho_d^{(1)}$ with the output SNR at the destination when space-time coding [1] is used, which can be written as

$$\rho_{std} = \frac{\eta}{N}\sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|^2 = \frac{\rho_d^{(N)}}{N}. \quad (6)$$

Clearly we can see that $\rho_d^{(1)} \ge \rho_{std}$. We now introduce the bounds on the value of $\rho_d^{\{m_k\}}$, for $m_k = 1 \ldots N$.

Lemma 1: For any $\{m_k\}$, $\rho_d^{(N)} \ge \rho_d^{\{m_k\}} \ge \rho_d^{(1)}$.

The proof is omitted due to space limitations; please refer to [15] for details. This lemma implies that, generally, the increased "equal gain combining" gain at the destination cannot compensate for the loss of MRC gain at the relay and TB gain at the destination when K is increased and each m_k is reduced, given the power constraint (4). Because $\rho_d^{(1)} \geq \rho_{std}$, we can thus conclude that MRC-TB leads to a higher instantaneous receive SNR than space-time coding at the destination, again given the power constraint (4) and the assumption that all relays can decode the source message correctly.

B. Outage Analysis

To examine the outage properties, we begin with the following result.

Lemma 2: Assuming that all the relays correctly decode the message, the outage probability $P_{out}^{\{m_k\}}$ for the relay network is approximately bounded by

$$\frac{1}{N!}\left(\frac{N\left(2^{2R}-1\right)}{\eta}\right)^N \geq P_{out}^{\{m_k\}} \geq \frac{1}{N!}\left(\frac{2^{2R}-1}{\eta}\right)^N. \quad (7)$$

The right-hand-side (RHS) of (7) is the outage probability for MRC-TB when $K = 1$, while the left-hand-side (LHS) expression is the outage probability for space-time coding for any K.

Proof: The proof can be completed by using the inequality $\rho_d^{(N)} \geq \rho_d^{\{m_k\}} \geq \rho_{std}$, and the following approximation [7]:

$$P\left(\sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|^2 \leq \varepsilon\right) \approx \frac{1}{N!}\varepsilon^N. \quad (8)$$

Further details are omitted due to space limitations. ∎

Lemma 2 indicates that the full diversity of N can be achieved regardless of the number of relays K, provided that the signals are correctly decoded at the relays. However, the diversity of the network might decrease if decoding outages occur at the relays. To avoid this event, we need to select only the relays that can decode the signal correctly. In fact, we can extend the antenna selection protocol proposed by [1], which exploits further the selection diversity of the source to relay channels, to the multi-antenna multi-relay scenario discussed in this letter, as follows.

Protocol 1: **(Selection Decoding)** In order to decode and forward the messages, select $\tilde{K}$ relays with a total number of $\tilde{N}$ antennas, denoted as a set $\Re\left(\tilde{N},\tilde{K}\right)$, that can successfully decode the source message at a transmission rate R.

We can obtain the outage probability for selection decoding in the following theorem:

Theorem 1: For large η, the outage probability for the selection decoding scheme for *any* K and $\{m_k\}$ is bounded approximately by:

$$\left(\frac{2^{2R}-1}{\eta}\right)^N \sum_{\Re\left(\tilde{N},\tilde{K}\right)}\left(\frac{\tilde{N}^{\tilde{N}}}{\tilde{N}!}\prod_{r\notin\Re\left(\tilde{N},\tilde{K}\right)}\frac{1}{m_r!}\right) \geq P_{out}^{\{m_k\}} \geq$$
$$\left(\frac{2^{2R}-1}{\eta}\right)^N \sum_{\Re\left(\tilde{N},\tilde{K}\right)}\left(\frac{1}{\tilde{N}!}\prod_{r\notin\Re\left(\tilde{N},\tilde{K}\right)}\frac{1}{m_r!}\right), \quad (9)$$

where the RHS is achieved for MRC-TB when $\tilde{N}$ antennas are co-located (i.e., $\tilde{K}$ relays cooperate like one relay[1]), the LHS represents the outage probability for the space-time coding scheme.

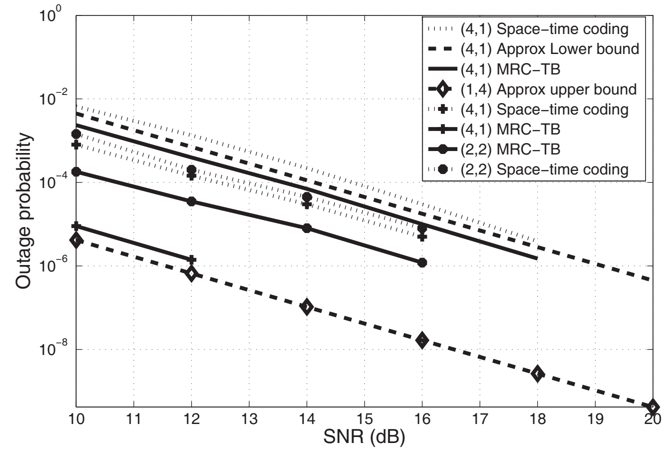

Fig. 2. Outage probability for different pairs of (K, M), where K is the number of relays and M is the number of antennas at each relay. Dashed lines are approximations for high SNR using (9). Dotted curves are simulations for space-time coding. Solid curves are simulations for MRC-TB.

Proof: See Appendix. ∎

It can be seen from *Theorem 1* that for selection decoding full diversity can always be achieved regardless of the choices of K and $\{m_k\}$, and the performance is lower bounded by that of the space-time coding scheme. However, it can also be seen that different choices of K and $\{m_k\}$ might result in different performance, due to different power gains. Comparing the RHS and LHS of (9), we can see a factor of $\tilde{N}^{\tilde{N}}$, where $\tilde{N}$ can be any value from 0 to N. This implies that the performance gap between MRC-TB and space-time coding can be extremely large when N is large. Note that in practice a large N (i.e, the number of transmit antennas) might not be realistic for point-to-point MISO links, and therefore the performance advantage for TB is always limited. However, in a large ad-hoc or sensor network, it is quite possible to have large values of N, and thus the benefits of distributed MRC-TB can be significant. Furthermore, the benefits of deploying multiple antennas at the relays (i.e., applying MRC at the relays) is small when distributed space-time coding is used, as the performance is mainly constrained by the limited power gain of using space-time coding for the relay to destination link. TB in this scenario can offer significant performance advantages. Fig. 2 shows a simulation example for $N = 4$. It can be observed that the performance gap between MRC-TB and space-time coding becomes largest when all the antennas are deployed at a single relay (a $6dB$ difference in this example).

We further note that, in practice, in order to achieve full diversity gain, the relay selection protocol is easier to implement for distributed MRC-TB than for space-time coding. The reason is that for space-time coding, the codes (e.g., block length or code pattern) must be changed whenever the number of selected relays are changed, in order to obtain the full

[1]This implies that the selected relays can always jointly decode and jointly transmit as if they were one relay. Therefore it is an idealistic case and thus can be considered only as a performance upper bound

diversity. This will involves much more channel feedback and signaling overhead.

IV. Conclusions

The performance of the distributed MRC-TB scheme has been studied in a multi-antenna multi-relay environment. We have seen that this technique achieves full diversity regardless of the number of relays and antennas at each relay, and offers a significant power gain over space-time coding.

Note that two important issues about the MRC-Beamforming approach are synchronization and frequency offset among all the relays ([16]–[18]). The impact of these two issues on the relay network is an interesting topic for future research.

Appendix
Proof of Theorem 3

Since $\Re\left(\tilde{N},\tilde{K}\right)$ is a random set, we use the law of total probability and write

$$P_{out}=\sum_{\Re\left(\tilde{N},\tilde{K}\right)} P\left[\Re\left(\tilde{N},\tilde{K}\right)\right] P_{out}^{m_k|\Re\left(\tilde{N},\tilde{K}\right)}, \qquad (10)$$

where $P_{out}^{m_k|\Re\left(\tilde{N},\tilde{K}\right)}$ denotes the outage probability conditioned on the event that $\Re\left(\tilde{N},\tilde{K}\right)$ is chosen, and can be bounded by (7) by replacing N with $\tilde{N}$. The probability that any relay is chosen can be expressed as

$$\begin{aligned} P\left[r\in\Re\left(\tilde{N},\tilde{K}\right)\right]&=P\left[\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2\geq\frac{2^{2R}-1}{\eta}\right]\\ &=1-P\left[\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2\leq\frac{2^{2R}-1}{\eta}\right]. \end{aligned} \qquad (11)$$

Therefore a set $\Re\left(\tilde{N},\tilde{K}\right)$ exists with a probability that can be written as

$$\begin{aligned} P\left[\Re\left(\tilde{N},\tilde{K}\right)\right]=&\prod_{r\in\Re\left(\tilde{N},\tilde{K}\right)}\left(1-P\left[\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2\leq\frac{2^{2R}-1}{\eta}\right]\right)\\ &\times\prod_{r\notin\Re\left(\tilde{N},\tilde{K}\right)} P\left[\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2\leq\frac{2^{2R}-1}{\eta}\right]. \end{aligned} \qquad (12)$$

Based on (8), at high SNR, $P\left[\Re\left(\tilde{N},\tilde{k}\right)\right]$ can be approximated as

$$P\left[\Re\left(\tilde{N},\tilde{K}\right)\right]\approx\left(\frac{2^{2R}-1}{\eta}\right)^{N-\tilde{N}}\prod_{r\notin\Re\left(\tilde{N},\tilde{K}\right)}\frac{1}{m_k!}. \qquad (13)$$

Putting (13) and (7) into (10), we obtain the bound in (9) and thus the proof is complete.

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