Optimization of a Class of Non-Convex Objectives on the Gaussian MIMO Multiple Access Channel: Algorithm Development and Convergence Analysis

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Introduction and Available Results

Introduction

- The capacity region of the GMAC is known [1]. Any rate vector can be achieved using
  - Gaussian signaling,
  - successive interference cancelation (SIC), and
  - time-sharing.
- The optimization of the transmission parameters is, in general, difficult. For the case of K transmitters, these parameters are
  - the covariance matrices of the input signals, $\bar{\boldsymbol{Q}} = \boldsymbol{Q}_1 \oplus \cdots \oplus \boldsymbol{Q}_K$,
  - the user decoding orders, among a total of $K!$, and
  - the time-sharing weights, $\boldsymbol{\beta} \in \mathbb{R}^{K!}$.
- The covariance matrices satisfy certain power constraints, i.e., $\bar{\boldsymbol{Q}} \in \mathcal{P}$, where $\mathcal{P}$ is the set of feasible covariances.
  - We assume $\mathcal{P}$ is convex.
  - The convexity of $\mathcal{P}$ simplifies the subsequent formulation [2]; one $\bar{\boldsymbol{Q}}$ suffices.
- The time-sharing weights vector, $\boldsymbol{\beta}$, belongs to the unit simplex $\mathcal{S} \triangleq \{\boldsymbol{\beta} \in \mathbb{R}^{K!} | \Sigma_{i=1}^{K!} \beta_i = 1,\ \beta_i \geq 0,\ \forall i\}$.

Optimization

For some rate objective $f : \mathbb{R}^K \to \mathbb{R}$, we want to solve

$$\min_{\boldsymbol{\beta} \in \mathcal{S}, \bar{\boldsymbol{Q}} \in \mathcal{P}} f(\boldsymbol{\rho}(\boldsymbol{\beta}, \bar{\boldsymbol{Q}})),$$

where the k-th element of the rate vector $\boldsymbol{\rho}(\boldsymbol{\beta}, \bar{\boldsymbol{Q}})$ is given by $\rho_k(\boldsymbol{\beta}, \bar{\boldsymbol{Q}}) = \Sigma_{i=1}^{K!} \beta_i r_{ki}(\bar{\boldsymbol{Q}})$,and $r_{ki}(\bar{\boldsymbol{Q}})$ is the rate of the k-th transmitter in the i-th decoding order.

Available Results

- Let π_i: permutation of the i-th decoding order, and $\boldsymbol{H}_k$: channel of the k-th transmitter, then

$$r_{ki}(\bar{\boldsymbol{Q}}) = \log \frac{|\boldsymbol{I} + \Sigma_{j \geq \pi_i^{-1}(k)} \boldsymbol{H}_{\pi_i(j)} \boldsymbol{Q}_{\pi_i(j)} \boldsymbol{H}_{\pi_i(j)}^\dagger|}{|\boldsymbol{I} + \Sigma_{j > \pi_i^{-1}(k)} \boldsymbol{H}_{\pi_i(j)} \boldsymbol{Q}_{\pi_i(j)} \boldsymbol{H}_{\pi_i(j)}^\dagger|}.$$

- If f is differentiable and convex in the rates (not necessarily in $\boldsymbol{\beta}$ and $\bar{\boldsymbol{Q}}$) then we have [2]:

Lemma (Necessary and sufficient optimality condition)

- *Let $\boldsymbol{w} = -\nabla f(\boldsymbol{\rho}(\boldsymbol{\beta}^*, \bar{\boldsymbol{Q}}^*))$ for some $\boldsymbol{\beta}^*$ and $\bar{\boldsymbol{Q}}^*$, and*
- *let the users be labelled so that $w_1 \leq \cdots \leq w_K$.*
- *Then, $\boldsymbol{\beta}^*$ and $\bar{\boldsymbol{Q}}^*$ are optimum **if and only if** for each strictly positive element of $\boldsymbol{\beta}^*$, say β_i^*,*
  - *the decoding order i is ordered as $w_1, \ldots, w_K$, and*
  - $\bar{\boldsymbol{Q}}^* = \arg\max_{\bar{\boldsymbol{Q}} \in \mathcal{P}} \Sigma_{k=1}^K (w_k - w_{k-1}) \log |\boldsymbol{I} + \Sigma_{j \geq k} \boldsymbol{H}_j \boldsymbol{Q}_j \boldsymbol{H}_j^\dagger|$.
- This lemma will be used to develop an algorithm that converges to the optimum $\boldsymbol{\beta}^*$ and $\bar{\boldsymbol{Q}}^*$.

References

[1] A. D. Wyner, "Recent results in the Shannon theory," *IEEE Trans. Inf. Theory*, vol. 20, pp. 2-10, 1974.

[2] D. Calabuig, R. H. Gohary, and H. Yanikomeroglu "Optimum transmission through the Gaussian multiple access channel," *Proc. IEEE Int. Symp. Inf. Theory*, (Istanbul, Turkey), pp. 201-205, 2013.

Main Contribution

Motivation and Goal

- The necessary and sufficient condition cannot be readily used to obtain the optimum transmission parameters, $\boldsymbol{\beta}^*$ and $\bar{\boldsymbol{Q}}^*$.
  - The optimum $\boldsymbol{w}$ is not known *a priori*.
- Goal: develop an algorithm that converges to the parameters that satisfy this condition.

Algorithm Approach

- We will define two convex optimization problems.
  - One on $\boldsymbol{\beta}$ and one on $\bar{\boldsymbol{Q}}$.
- The algorithm solves these problems alternately.

Algorithm

1: Initialize $\boldsymbol{\beta}^*(0)$ and $\bar{\boldsymbol{Q}}^*(0)$.
2: For $t = 1, 2, \ldots$ do:
3: $\boldsymbol{w}(t) = -\nabla f(\boldsymbol{\rho}(\boldsymbol{\beta}^*(t-1), \bar{\boldsymbol{Q}}^*(t-1)))$.
4: Find $\bar{\boldsymbol{Q}}^o(t)$ that solves the convex problem (cf. Lemma)

$$\max_{\bar{\boldsymbol{Q}} \in \mathcal{P}} \Sigma_{k=1}^K (w_k(t) - w_{k-1}(t)) \log |\boldsymbol{I} + \Sigma_{j \geq k} \boldsymbol{H}_j \boldsymbol{Q}_j \boldsymbol{H}_j^\dagger|.$$

5: Choose $\bar{\boldsymbol{Q}}^*(t)$ as a convex combination of $\bar{\boldsymbol{Q}}^o(t)$ and $\bar{\boldsymbol{Q}}^*(t-1)$. The weights of the convex combination can be computed solving a univariate minimization problem in the domain $[0, 1]$.
6: Choose $\boldsymbol{\beta}^*(t)$ to solve the convex problem

$$\min_{\boldsymbol{\beta} \in \mathcal{S}} f(\boldsymbol{\rho}(\boldsymbol{\beta}, \bar{\boldsymbol{Q}}^*(t))).$$

7: End for.

Algorithm Convergence

This theorem implies that $\boldsymbol{\beta}^*(t)$ and $\bar{\boldsymbol{Q}}^*(t)$ converge to the optimum transmission parameters as $t \to \infty$.

Theorem (Algorithm convergence)

- *Let $\mathcal{P}$ be convex.*
- *Let f be*
  - *second order differentiable,*
  - *monotonically increasing in each component, and*
  - *convex in the rates.*
- *Then, the proposed algorithm converges to the optimum $\boldsymbol{\beta}$ and $\bar{\boldsymbol{Q}}$, that is, if $\boldsymbol{x}^*$ is the optimum rate vector, then*

$$\lim_{t \to \infty} f(\boldsymbol{\rho}(\boldsymbol{\beta}^*(t), \bar{\boldsymbol{Q}}^*(t))) - f(\boldsymbol{x}^*) = 0.$$

Proof

- To prove this theorem, we first show that
  - f is bounded below,
  - f decreases after every iteration, and
  - the error at every iteration is bounded.
- We use the first two facts to show that the proposed algorithm converges to a particular point.
- From convergence, the properties of f and the error bounds, we show that $f(\boldsymbol{\rho}(\boldsymbol{\beta}^*(t), \bar{\boldsymbol{Q}}^*(t))) - f(\boldsymbol{x}^*) \to 0$ as $t \to \infty$.

Numerical Example and Conclusion

Problem Instance

- Use the proposed algorithm to solve a two-transmitter GMAC optimization problem.
  - The objective is to maximize the weighted proportional fairness,
  - provided that the sum rate exceeds a given threshold R.
  - Transmitters have a power budget of $P = 10$ dB.
- This problem can be cast as

$$\max_{\boldsymbol{\beta}, \bar{\boldsymbol{Q}}} \quad w_1 \log(\rho_1(\boldsymbol{\beta}, \bar{\boldsymbol{Q}})) + w_2 \log(\rho_2(\boldsymbol{\beta}, \bar{\boldsymbol{Q}})),$$
$$\text{subject to} \quad \beta_1 + \beta_2 = 1,\ \beta_i \geq 0,\ i = 1, 2,$$
$$\boldsymbol{Q}_k \succeq 0,\ \mathrm{Tr}(\boldsymbol{Q}_k) \leq P,\ k = 1, 2,$$
$$\log |\boldsymbol{I} + \boldsymbol{H}_1 \boldsymbol{Q}_1 \boldsymbol{H}_1^\dagger + \boldsymbol{H}_2 \boldsymbol{Q}_2 \boldsymbol{H}_2^\dagger| \geq R.$$

- This problem is non-convex in $\boldsymbol{\beta}$ and $\bar{\boldsymbol{Q}}$.
- The proposed algorithm converges to the optimum solution of this problem.

Problem Solution

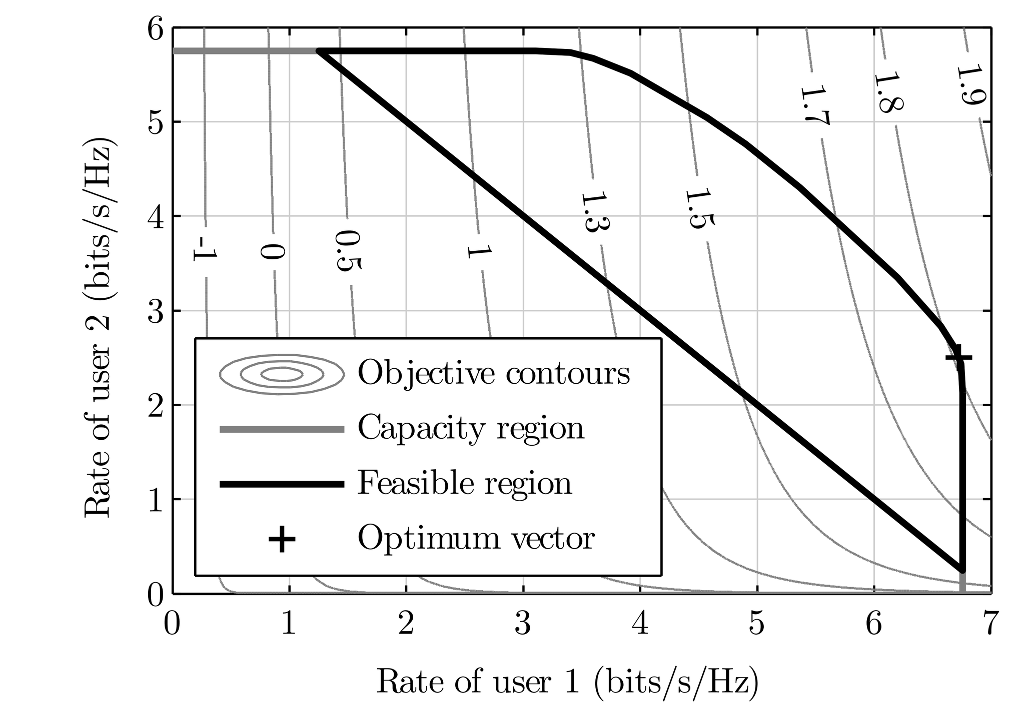

Figure: Feasible rate region and optimum rate vector.

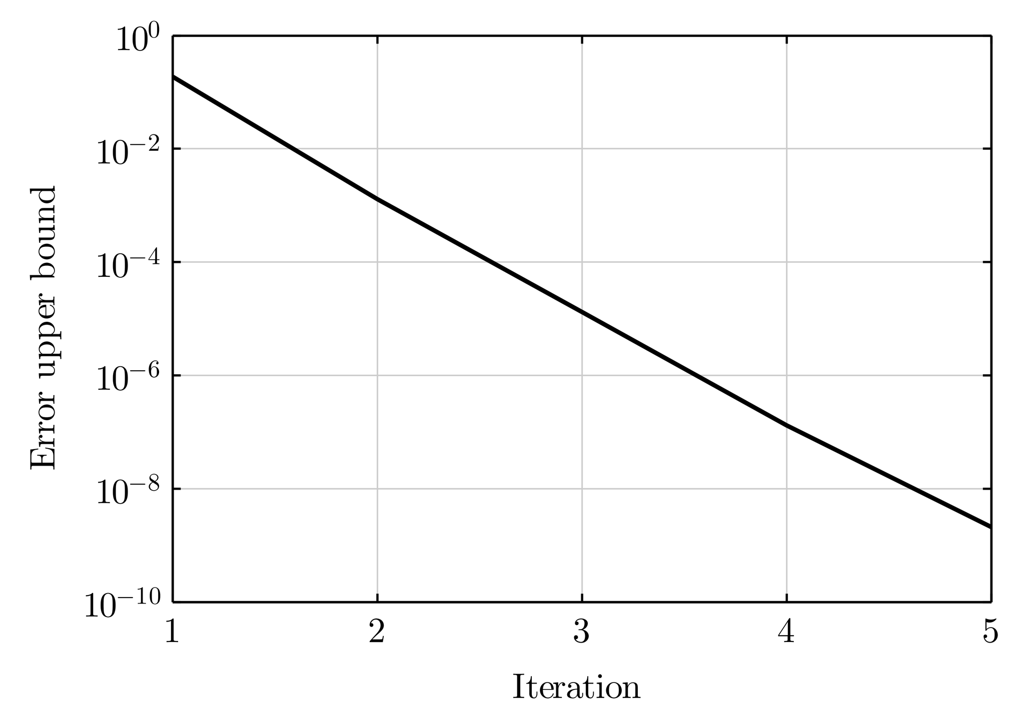

Figure: Error upper bound after each iteration.

Conclusion

- We considered a class of optimization problems with non-linear objectives defined over the capacity region of the MIMO GMAC.
- We developed an efficient iterative algorithm for solving this class of problems.
- The convergence of this algorithm to the optimum solution is guaranteed when the conditions of the Theorem are satisfied.

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