Polar Coded Multi-antenna Multidimensional Constellations in Partially Coherent Channels

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***Abstract*—As one of the multiple-input-multiple-output (MIMO) techniques that work close to capacity, Hochwald and ten Brink proposed to send forward error correction (FEC) coded two-dimensional symbols from multiple antennas in each time slot and decode them using a maximum likelihood decoder. This can be generally considered as the transmission of multidimensional symbols in each time slot and here is referred to as multi-antenna multidimensional constellations (MMCs). Polar codes are a new class of forward error correction codes that benefit from simple rate matching and low complexity decoders, and therefore, facilitate the design of efficient systems. Due to the availability of partial channel state information at the receiver in time varying fading systems, the performance of uncoded MMCs can be improved by employing MMCs designed for partially coherent systems. However, the choice of the constellation in presence of FEC codes is of importance. In this paper, we propose the concatenation of the polar codes and MMC as a high-performance scheme for time varying fading systems. We further study different methods of design of the scheme in partially coherent systems and discuss the choice of the constellation.**

I. Introduction

Multiple-input-multiple-output (MIMO) systems can provide high rates and reliability by using spatial diversity and multiplexing. The linear increase in capacity of MIMO systems with the minimum of the number of transmit and receive antennas at high signal-to-noise ratios (SNRs) as shown in [1], [2] indicates the great potential of these systems as a viable solution to achieve higher spectral efficiency and better performance. To achieve the capacity of MIMO systems, high performance high spectrally efficient schemes are needed. Multilayer MIMO schemes such as V-BLAST, introduced in [3], [4], can provide high spectral efficiencies. Once they are used with forward error correction (FEC) coding across all antennas, they can partially exploit transmit diversity [5]. However, they suffer a performance loss due to the use of successive cancellation based decoders (SCD) and the lack of utilization of full receive and transmit fading diversity. To provide high performance with high spectral efficiency, Hochwald and ten Brink proposed to transmit FEC coded 2^B-quadrature amplitude modulation (QAM) symbols from multiple antennas in each time slot[1] and decode them using a maximum likelihood decoder [6]. This scheme can in general form be described as a multidimensional symbol transmitted on a MIMO vector channel, i.e., the transmission of each two dimensions of a multidimensional constellation using one transmit antenna each. Here, we refer to this scheme as a multi-antenna multidimensional constellation (MMC). As explained in [6], MMC might be one of the most efficient schemes used with FECs to nearly achieve the capacity of MIMO channels.

Later, Lampe et al. in [7] and Lamarca and Lou in [8] evaluated different methods of FEC coding for this scheme, including bit-interleaved coded modulation (BICM), multilevel coding (MLC) and hybrid coded modulation and found that among them MLC shows better performance. For constructing MLC in [7], a B level code is designed for the signal of each transmit antenna independently that results in an MLC with N_tB levels for N_t transmit antennas. However, as the main shortcoming of the aforementioned schemes, the bit-to-symbol mapping was only designed for the signal of each antenna independently. Later, Martin et al. proposed to use MLC with a $2N_t$ dimensional ($2N_t$D) multidimensional bit-to-symbol mapping designed jointly for the signal of all antennas [9].

When designing MLC with multistage decoding (MSD), by employing an appropriate labelling algorithm, the variability between bit-channel capacities to be used with binary codes of different rates should be increased. Forney in [10], [11] proposed a formal set partitioning algorithm that has been only suitable for regular multidimensional constellations that lie on a lattice and cannot provide the best performance for any number of levels. To design the bit-to-symbol mapping, Martin et al. employed Forney's set-partitioning method and they used only a special case of MLC in which only three different component codes were used for a 16-point 8-dimensional (16-8D) constellation. However, a better rate matching, resulting from increasing the number of levels of MLC, can improve the performance substantially. In [12], a general set-partitioning algorithm

[1]This can be seen as the multidimensional Cartesian product of PAM constellations known as cubic constellations.

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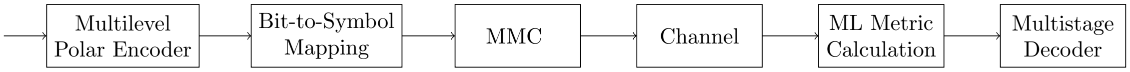

Figure 1. The system block diagram.

for designing multilevel coded-modulation for irregular multidimensional constellations has been proposed. This algorithm can be used for designing multilevel codes with the maximum number of levels, i.e., B.

Polar codes, introduced by Erdal Arikan in [13], are a new class of FEC codes, that benefit from simple rate matching and the availability of a variety of decoders with different complexity-performance trade-offs. Due to the conceptual similarity with MLC/MSD, the multilevel polar code works better than polar coded-modulation constructed using BICM [14]. Therefore, evaluating the performance of polar coded MMCs as a flexible candidate for a variety of applications, e.g., in time-varying channels, is of importance.

In most time varying fading systems, perfect channel state information at the receiver (CSIR) is not available. In addition, by increasing the number of transmit antennas, the spectral loss and the complexity of CSIR estimation increases [15]. Therefore, the performance evaluation and the design of MMC under imperfect CSIR is of importance.

For coherent MMC system models, typically a QAM constellation is used for each two dimensions of the multidimensional signal [6], [7], [9]. Although this constellation works well for the coherent model, it shows an early error floor in partially coherent systems. Therefore, the signal has to be designed given the partially coherent system statistics. Among the notable works for designing partially coherent multidimensional constellations, Borran et al. in [16], [17] employed the Kullback-Leibler (KL) distance and optimized spherical constellations for partially coherent systems. Baccarelli and Biagi in [18] presented flexible space-time constellations that are matched to the channel estimation error and improve the performance. Giese and Skoglund in [19] designed space-time codes based on the asymptotic pairwise error probability of the partially coherent channel. Yadav et al. in [20] employed the cut-off rate to design constellations for partially coherent systems which show higher mutual information in comparison to KL distance based constellation design and the constellation designed in [18]. The cut-off rate can be employed to design constellations for MMCs as well [20]. However, once the FEC code is used, the best choice of the constellation may not be the one with the highest mutual information and may depend on the structure of the FEC code.

In this paper, we design the polar coded MMC scheme using MLC/MSD and BICM; we propose to use B-level set-partitioning-based bit-to-symbol mapping to improve the performance in comparison to MLC with a lower number of levels; and we evaluate the performance of the scheme in the presence of the channel estimation error with a variety of constellations. The rest of the paper is organized as follows: The system model is described in Section II; the constellation design method, the labelling algorithm and the polar code construction method for MLC/MSD and BICM are explained in Section III; the simulation results are reported in Section IV; and the conclusions are presented in Section V.

II. System Model

The communication system is equipped with N_t transmit and N_r receive antennas, respectively. The data are divided into groups of K_{tot} bits each coded with a FEC code of total length N_{tot} and total rate $R_{tot} = K_{tot}/N_{tot}$. Then, each B coded bits are mapped to one of the multidimensional symbols of MMC scheme using a multidimensional bit-to-symbol mapping and each two dimensions of the multidimensional symbol are transmitted using one of the transmit antennas in the same time slot. Therefore, the entire codeword is transmitted in N_{tot}/B channel uses. At the receiver side, the channel output log-likelihood-ratios (LLRs) are calculated and the LLRs are passed to the decoder. The system model is sketched in Fig. 1. The system can be represented as

$$\mathbf{r} = \mathbf{s}\mathbf{H} + \mathbf{w}, \tag{1}$$

where $\mathbf{r}$ is the $1 \times N_r$ received vector, $\mathbf{s}$ is the $1 \times N_t$ transmitted vector, $\mathbf{H}$ is the $N_t \times N_r$ flat fading channel, and $\mathbf{w}$ is zero-mean complex additive white Gaussian noise (AWGN) with variance $N_0/2$ per dimension. The structure of a symbol of MMC can be described as

$$\mathbf{s} = [s^{(1)}, s^{(2)}, \ldots, s^{(N_t)}], \tag{2}$$

where $s^{(1)}, s^{(2)}, ..., s^{(N_t)}$ are different 2D dimensions of a multidimensional constellation point $\mathbf{s}$. The channel $\mathbf{H}$ is independent and identically distributed (i.i.d.) and remains constant during the transmission of signal $\mathbf{s}$. The channel $\mathbf{H}$ can be separated into two parts as

$$\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}, \tag{3}$$

where $\hat{\mathbf{H}}$ is the Rayleigh fading channel estimate with distribution $\mathcal{CN}(\mathbf{0}, (1 - \sigma_E^{\mathbf{2}})\mathbf{I})$, and $\tilde{\mathbf{H}}$ is the additive channel estimation error with distribution $\mathcal{CN}(\mathbf{0}, \sigma_E^{\mathbf{2}}\mathbf{I})$ achieved when the linear minimum mean square error (LMMSE) channel estimator is employed [17]. Here, σ_E^2 is the channel estimation variance varying between 0 and 1, corresponding to coherent and non-coherent system models, respectively. The probability of receiving $\mathbf{r}$, given the transmitted vector $\mathbf{s}$ and the channel estimation $\hat{\mathbf{H}}$, can be written as [17]

$$\Pr(\mathbf{r} \mid \mathbf{s}, \hat{\mathbf{H}}) = \frac{\exp\{(1 + \frac{\sigma_E^2}{N_0}\|\mathbf{s}\|^2)^{-1}\|\mathbf{r} - \mathbf{s}\hat{\mathbf{H}}\|^2\}}{\pi^{N_r}(1 + \frac{\sigma_E^2}{N_0}\|\mathbf{s}\|^2)^{N_r}}. \quad (4)$$

In this system, σ_E^2 is estimated at the receiver and fed back to the transmitter to select the corresponding constellation.

A. MLC/MSD

The MLC scheme consists of independent encoders for each binary channel of a constellation. Each level of MLC uses a code of length N and rate R_i and therefore, the total length and the rate of the code are $N_{tot} = BN$ and $R_{tot} = \frac{1}{B}\sum_{i=1}^{B} R_i$, respectively. After encoding of all levels, each set of code bits $\{c_n^1, c_n^2, ..., c_n^B\}$, for $n = 1, ..., N$, are mapped to an MMC and are transmitted over the channel.

In MSD, the code-bits of each level are deduced with the aid of the received symbol and previously deduced code-bits of upper levels [21]. At each level after decoding, the received message word is fed to an encoder and the generated codeword $\hat{\boldsymbol{c}}$ reduces part of the ambiguity for the demapper of the next level to compute more reliable LLRs. Therefore, the LLR estimation at each level can be given by

$$\lambda_b = \ln \frac{\sum_{\mathbf{s}\in\mathcal{X}_{b,0}} \Pr(\mathbf{r} \mid \mathbf{s}, \hat{\mathbf{H}})}{\sum_{\mathbf{s}\in\mathcal{X}_{b,1}} \Pr(\mathbf{r} \mid \mathbf{s}, \hat{\mathbf{H}})}, \quad (5)$$

where $b = 1, 2, ..., B$, $\mathcal{X}_{b,0}$ and $\mathcal{X}_{b,1}$ are the set of all MMC points containing zero or one in their b^{th} position given the upper level code-bits, respectively.

B. BICM

In BICM, one binary code of length N_{tot} and R_{tot} is interleaved, divided into chunks of B bits, and mapped to the MMC symbols. In the decoder side, the LLRs of binary channels of a constellation are computed interdependently. The LLR estimation of BICM, for the b^{th} binary channel, can be given by

$$\lambda_b = \ln \frac{\sum_{\mathbf{s}\in\bar{\mathcal{X}}_{b,0}} \Pr(\mathbf{r} \mid \mathbf{s}, \hat{\mathbf{H}})}{\sum_{\mathbf{s}\in\bar{\mathcal{X}}_{b,1}} \Pr(\mathbf{r} \mid \mathbf{s}, \hat{\mathbf{H}})}, \quad (6)$$

where $\bar{\mathcal{X}}_{b,0}$ and $\bar{\mathcal{X}}_{b,1}$ are the set of all MMC points with zero or one in their b^{th} position, respectively. In BICM, an additional interleaving is applied on the code-bits prior to bit-to-symbol mapping and the corresponding deinterleaving is applied on estimated LLRs at the receiver.

III. Design Elements

In this section, we briefly review the design elements and explain a method of designing practical polar coded MMC. To this end, we explain the constellation design method, bit-to-symbol mapping and polar code construction method for both MLC/MSD and BICM.

A. Constellation Design Method

As explained in Section I, multidimensional constellations for MMC can be obtained based on KL distance and the cut-off rate. In [20], it is shown that maximizing the cut-off rate generates constellations with equal or higher mutual information than the KL distance. Here, we employ the cut-off rate to generate the constellations and we compare them with the spherical KL constellations optimized in [17]. For generating the cut-off rate, the pairwise Bhattacharyya coefficient for MMCs, $\rho(\mathbf{s}_i, \mathbf{s}_j)$, can be written as [20]

$$\left(\frac{\sqrt{1 + \frac{\sigma_E^2}{N_0}\|\mathbf{s}_i\|^2}\sqrt{1 + \frac{\sigma_E^2}{N_0}\|\mathbf{s}_j\|^2}}{1 + \frac{\sigma_E^2}{2N_0}(\|\mathbf{s}_i\|^2 + \|\mathbf{s}_j\|^2) + \frac{(1-\sigma_E^2)}{4N_0}\|\mathbf{s}_i - \mathbf{s}_j\|^2}\right)^{N_r}. \quad (7)$$

Using (7), the cut-off rate R_0 can be written as

$$R_0 = -\log\left\{\frac{1}{2^{2B}}\sum_{i=1}^{2^B}\sum_{j=1}^{2^B}\rho(\mathbf{s}_i, \mathbf{s}_j)\right\}. \quad (8)$$

To find the constellation, the optimization problem can be given as

$$\begin{aligned} &\text{maximize} \quad R_0 \\ &\text{subject to} \quad \frac{1}{2^B}\sum_{v=1}^{2^B}\|s_v\|^2 \leq 1, \end{aligned} \quad (9)$$

B. Labelling Algorithm for MLC/MSD

A set-partitioning algorithm for multidimensional constellations is proposed in [12] that can be employed to design the bit-to-symbol mapping for partially coherent constellations. Here, we employ this method to generate the bit-to symbol mapping for MMCs. As noted in [16], for a partially coherent constellation, a more relevant metric than the Euclidean distance for the distance of points should be used. Here, for set-partitioning of KL distance based optimized constellations, KL distance and for the cut-off rate based optimized constellations, $-\rho(\mathbf{s}_i, \mathbf{s}_j)$ are used as the pairwise distance to generate the bit-to-symbol mapping.

C. Polar Code Construction for MLC/MSD

In [12], a simulation-based design of multilevel polar codes is proposed. Here, we employ this method to design our codes. In this method, a multilevel polar code with rate one for all levels is constructed and after evaluation of the bit-channel first error events through simulation with a SCD, the best K_{tot} bit-channels are selected as the information set. This includes a different number of information bit-channels $\{k_1, k_2, ..., k_B\}$ for each level of the MLC. We use this method for constructing polar codes used with BICM as well. In this case, only one binary FEC encoder is used.

D. Labelling Algorithm for BICM

For BICM, the union bound on the probability of bit error rate can be used to find a Gray-like mapping [22]. This bound can be written as

$$P_e = \frac{1}{B2^B} \sum_{i=1}^{2^B} \sum_{j=1}^{2^B} \mathrm{H}(i-1, j-1)\rho(\mathbf{s}_i, \mathbf{s}_j). \tag{10}$$

where $\mathrm{H}(i,j)$ is the Hamming distance between index i and j. For generating the bit-to-symbol mapping, the binary switching algorithm (BSA) can be employed [22], [23]. From the first label to the last, BSA swaps the position of each two labels until a mapping that provides a lower P_e is generated. This process is repeated iteratively until P_e remains constant for a large number of iterations. Although this algorithm works well for small size constellations, generating good bit-to-symbol mappings for large size constellations is difficult.

E. The Design Procedure

For designing the polar coded MMC with either MLC or BICM, first we optimize the constellation, we then find a good bit-to-symbol mapping using the BICM or MLC labeling algorithm, and we finally design the polar code given the constellation and the bit-to-symbol mapping. A random interleaver is used to construct the BICM with a length of N_{tot}. For designing the polar code given a FER, the bisection design SNR search algorithm in [12] can be employed.

IV. Numerical Results and Discussions

The performance of the polar coded scheme concatenated with MMC with two and four transmit antennas is evaluated in this section. For all cases, a code of rate $R_{tot} = 1/2$ is employed with SCD and successive cancellation list decoder (SCLD) [24]. For decoding of polar codes, the SC and SCL decoders are used for each level independently. For the SCLD+CRC decoder, the CRC sequence is CRC-16-CCITT for each encoder. All polar codes are designed using the method described in Section III at the SNR corresponding to a FER of 10^{-2}. For the sake of comparison, a turbo coded MMC based on BICM is constructed. The turbo code structure is explained in the LTE standard [25]. The turbo code is decoded using the iterative BCJR with 5, 10 and 12 iterations. The channel is modeled as a Rayleigh fading with AWGN and a coherence time of one channel-use. In the case of MLC, for 4 bits-per-channel-use (bpcu), N_{tot} is 2048 and 2056 bits for polar and turbo codes, respectively. For 6 bpcu schemes, N_{tot} is 1536 and 1608 bits for polar and turbo codes, respectively unless otherwise stated. For BICM, all code lengths are 2048 bits unless otherwise stated. The MMC has 256 and 4096 points corresponding to spectral efficiencies of 4 bpcu and 6 bpcu, respectively. All gains are reported at a FER of 10^{-2}.

A. Coherent Model

In this subsection, the performance of the system with the perfect CSIR ($\sigma^2 = 0$) is evaluated. For constructing polar codes with BICM, a multidimensional Gray bit-to-symbol mapping is used[2].

Fig. 2 shows the performance of the multilevel polar coded MMC (MLPCM), polar coded MMC with BICM (BIPCM) and turbo coded MMC with BICM (BITCM) in 2×2 antenna configuration for 4 bpcu. The MMC is constructed by the Cartesian product of 4-PAM on 4 dimensions. The component code rates of MLPCM are {0.03,0.15,0.24,0.52,0.53,0.75,0.84,0.95}. The results indicate that MLPCM outperforms BIPCM for SCD, SCLD and SCLD+CRC by 1.2 dB, 0.6 dB and 0.8 dB, respectively. Results also show that MLPCM with SCLD and SCLD+CRC outperform BITCM with 12 iterations by 0.9 dB and 0.3 dB and BITCM with 5 iterations by 1.2 dB and 0.6 dB, respectively. BIPCM with SCLD+CRC performs close to BITCM with 10 iterations. Furthermore, MLPCM with SCD and BIPCM with SCLD have the same performance as BITCM with 5 iterations. Clearly, MLPCM with SCLD and SCLD+CRC outperforms BIPCM and BITCM. Note that the complexity of the BCJR decoder with 5 iterations is more than that of SCLD and SCLD+CRC.

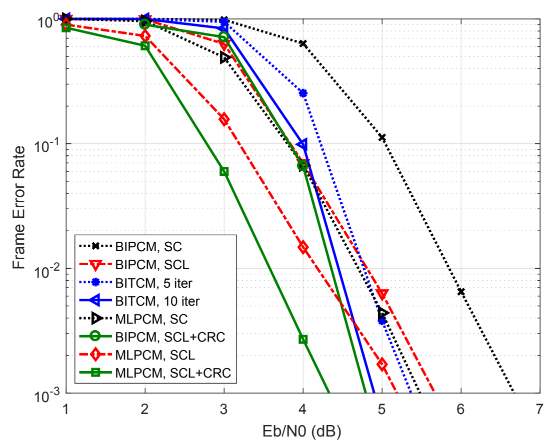

Figure 2. FER comparison of the MLPCM, BIPCM, and BITCM for 4 bpcu and 2×2 antenna configuration.

The comparison of MLPCM and BIPCM and BITCM with 2×2 antenna configuration for 6 bpcu is provided in Fig. 3. The MMC is constructed by the Cartesian product of 8-PAM on 4 dimensions. The component code rates of MLPCM are {0,0.03,0.06,0.23,0.25,0.45,0.59,0.77,0.80,0.91,0.94, 0.98}. The results show that similar to Fig. 2, MLPCM outperforms BIPCM for SCD, SCLD and SCLD+CRC by 2 dB, 1.3 dB and 1.4 dB, respectively. In the case

[2] Multidimensional Gray mapping for cubic constellations is the Cartesian product of PAM Gray mapping.

of 6 bpcu, MLPCM with SCD in addition to SCLD and SCLD+CRC outperforms BITCM with 12 iterations by 0.9 dB, 1.3 dB and 1.9 dB, respectively. It can be seen that BIPCM with SCLD+CRC also outperforms BITCM with 12 iterations by 0.5 dB. It is noticeable that MLPCM even with low complexity SCD shows a substantial performance advantage in comparison to BIPCM and BITCM at high spectral efficiencies.

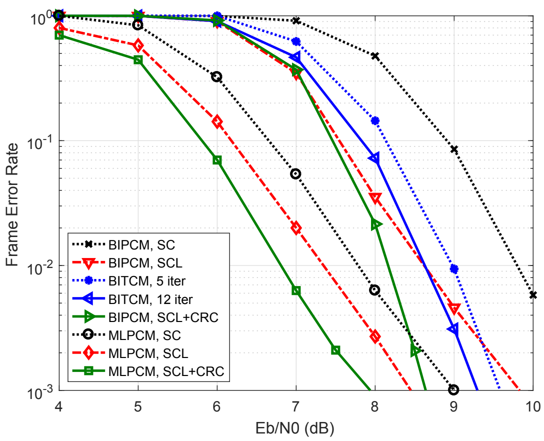

Figure 3. FER comparison of the MLPCM, BIPCM and BITCM, for 6 bpcu and 2×2 antenna configuration.

Fig. 4 shows the MLPCM in comparison to MMC proposed by Martin et al. in [9], and BITCM for 4×4 antenna configuration and 6 bpcu. The MLPCM is constructed using 12 component codes of rates {0.07,0.07,0.12,0.20,0.30,0.41,0.55,0.70,0.77,0.87,0.94, 0.99} by using a circular 8-QAM constellation on each antenna. Multilevel polar codes are optimized for Martin et al. bit-to-symbol mapping, proposed in [9], with 16-QAM constellation for each antenna and the component code rates are {0.09,0.51,0.90}. The MLPCM with SCD and SCLD outperform polar coded scheme constructed using the MMC scheme in [9] by 0.8 dB and 0.4 dB, respectively. Here, MLPCM schemes outperform BITCM with 5 and 12 iterations.

B. Partially Coherent Model

In this subsection, the performance of MMC with 2×2 antenna configuration in the presence of imperfect CSIR is evaluated. In each case, the constellation is optimized using the cut-off rate and the KL distance given a SNR value and a σ^2 value. The comparison of MLPCM and BIPCM with $\sigma^2 = 0.01$ is presented in Fig. 5. The cut-off rate and the KL distance based constellations are optimized at $E_b/N_0 = 5.5$ dB. In this case, MLPCM outperforms BIPCM. Among the constellations used with MLPCM, the cubic constellation outperforms spherical KL distance optimized constellation in [17] by 0.4 dB and the cut-off rate optimized constellation by 0.25 dB. Therefore, the cut-off rate and

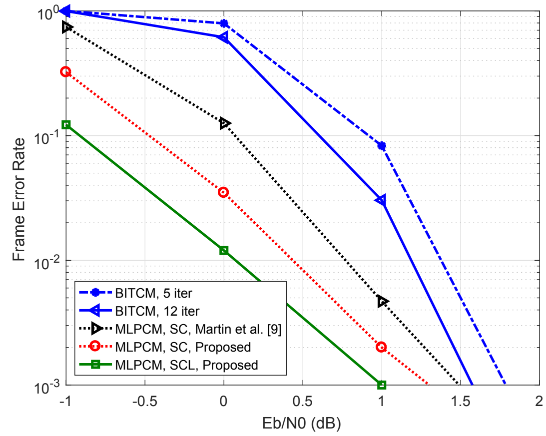

Figure 4. FER comparison of the MLPCM, and BITCM, for 6 bpcu and 4×4 antenna configuration.

KL distance optimized constellations, that have higher mutual information, for low values of σ^2 perform worse than the cubic constellation due to the better match of the cubic constellation with the polar code structure.

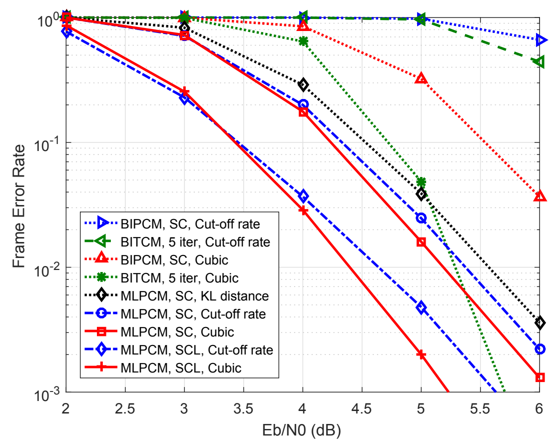

Figure 5. FER comparison of the MLPCM and BIPCM in presence of channel estimation error with $\sigma^2 = 0.01$ for 4 bpcu and 2×2 antenna configuration.

Finally, Fig. 6 provides a comparison of BIPCM and MLPCM when $\sigma^2 = 0.1$ and $N_{tot} = 4096$. The cut-off rate based constellations are optimized at $E_b/N_0 = 12$ dB. The rates of MLPCM with cut-off rate optimized constellation are {0.10,0.17,0.27,0.41,0.54,0.71,0.84,0.95}. The results indicate that BIPCM and BITCM show very poor performance due to the difficulty of finding good bit-to-symbol mapping using a BSA algorithm for large size constellations. Once more, MLPCM presents good performance and the cut-off rate optimized constellation outperforms cubic constellation since cubic constellations are quite suboptimal for $\sigma^2 = 0.1$.

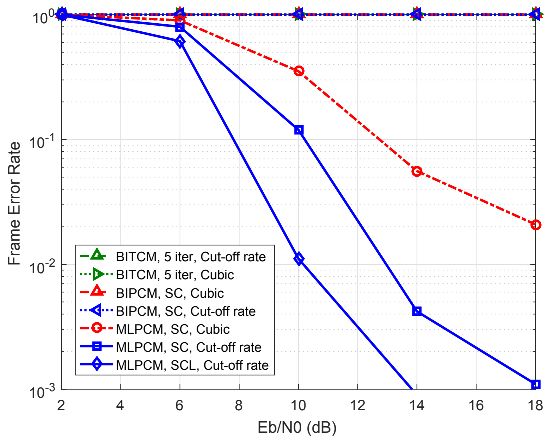

Figure 6. FER comparison of the MLPCM and BIPCM in presence of channel estimation error with $\sigma^2 = 0.1$ for 4 bpcu and 2×2 antenna configuration.

V. Conclusion

In this paper, we proposed the concatenation of polar codes with MMC using MLC and the BICM. We explained the method of construction of the scheme for coherent and partially coherent systems, including the constellation design, the bit-to-symbol mapping generation, and the polar code design and we evaluated the performance of the scheme in different channel conditions. The numerical results showed that MLPCM provides a substantial performance advantage in comparison to BIPCM and BITCM especially at high spectral efficiencies. Furthermore, the constellation with the highest mutual information is not necessarily the best choice when used with polar codes. In partially coherent systems when σ^2 is close to zero, cubic constellations outperform constellations optimized by using the cut-off rate or the KL distance of the partially coherent system. At high values of σ^2, cut-off rate optimized constellations show better performance. In all cases, MLC outperforms the BICM.

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References

[1] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas,” *Bell Labs. Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.

[2] I. E. Telatar, “Capacity of multi-antenna Gaussian channels,” *Europ. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov. 1999.

[3] G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, “Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture,” *Electron. Lett.*, vol. 35, no.1, pp. 14-16, Jan. 1999.

[4] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, “Simplified processing for high spectral efficiency wireless communication employing multi-element arrays,” *J. Select. Areas Commun.*, vol.17, no.11, pp. 1841-1852, Nov. 1999.

[5] X. Li, H. Huang, G. J. Foschini, and R. A. Valenzuela, “Effects of iterative detection and decoding on the performance of BLAST,” in *Proc. IEEE Globecom*, San Francisco, CA, Nov. 2000.

[6] B. M. Hochwald and S. ten Brink, “Achieving near-capacity on a multiple-antenna channel,” *IEEE Trans. Commun.*, vol. 51, no.3, pp. 389-399, Mar. 2003.

[7] L. H.-J. Lampe, R. Schober, and R. F. H. Fischer, “Multilevel coding for multiple-antenna transmission,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 1, pp. 203-208, Jan. 2004.

[8] M. Lamarca and H. Lou, “Spectrally efficient MIMO transmission schemes based on multilevel codes,” in *Proc. IEEE Workshop on Signal Process. Advances in Wireless Commun.*, June. 2005.

[9] P. A. Martin, D. M. Rankin, and D. P. Taylor, “Multi-dimensional space-time multilevel codes,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 11, Nov. 2006.

[10] G. D. Forney, “Coset codes.I: Introduction and geometrical classification,” *IEEE Trans. Inf. Theory*, vol. 34, no. 5, pp. 1123-1151, Sep. 1988.

[11] G. D. Forney, “Coset codes.II: Binary lattices and related codes,” *IEEE Trans. Inf. Theory*, vol. 34, no. 5, pp. 1152–1187, Sep. 1988.

[12] P. R. Balogun, I. Marsland, R. Gohary, and H. Yanikomeroglu, “Polar codes for noncoherent MIMO signalling,” in *Proc. IEEE ICC*, Kuala Lumpur, Malaysia, May 2016.

[13] E. Arikan, “Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels,” *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3051-3073, July 2009.

[14] M. Seidl, A. Schenk, C. Stierstorfer, and J. B. Huber, “Aspects of Polar-Coded Modulation,” in *Proc. 9th Int. ITG Conf. on Syst., Commun. and Coding*, Munich, Germany, Jan. 2013.

[15] B. Hassibi and B. M. Hochwald, “How much training is needed in multiple–antenna wireless links?,” *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp. 951-963, Apr. 2003.

[16] M. J. Borran, “Non-coherent and partially coherent space-time constellations,” Ph.D. thesis, Rice University, Houston, TX, June 2003.

[17] M. J. Borran, A. Sabharwal, and B. Aazhang. “Design criterion and construction methods for partially coherent multiple antenna constellations,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4122-4133, Aug. 2009.

[18] E. Baccarelli and M. Biagi, “Performance and optimized design of space-time codes for MIMO wireless systems with imperfect channel estimates,” *IEEE Trans. Signal Process.*, vol. 52, no. 10, pp. 2911-2923, Oct. 2004.

[19] J. Giese and M. Skoglund, “Space-time constellation design for partial CSI at the receiver,” *IEEE Trans. Inf. Theory*, vol. 53, no. 8, pp. 2715-2731, Aug. 2007.

[20] A. Yadav, M. Juntti, and J. Lilleberg, “Cutoff rate optimized space-time signal design for partially coherent channel,” *IEEE Trans. Commun.*, vol. 60, no. 6, pp. 1563-1574, June 2012.

[21] H. Imai and S. Hirakawa, “A new multilevel coding method using error correcting codes,” *IEEE Trans. Inf. Theory*, vol. 23, no. 3, pp. 371-377, May 1977.

[22] A. Yadav, M. Juntti, and J. Lilleberg, “Partially coherent constellation design and bit-mapping with coding for correlated fading channels,” *IEEE Trans. Commun.*, vol. 61, no. 10, pp. 4243-4255, Oct. 2013.

[23] K. Zeger and A. Gersho, “Pseudo-Gray coding,” *IEEE Trans. Commun.*, vol. 38, no. 12, pp. 2147-2158, Dec. 1990.

[24] I. Tal and A. Vardy, “List decoding of polar codes," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2213-2226, May 2015.

[25] *LTE; Evolved universal terrestrial radio access (E-UTRA); Multiplexing and channel coding*, 3GPP TS 36.212, ver. 12.2.0, release 12, Oct. 2014.