Joint Optimization of Polar Codes and STBCs

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***Abstract*—Space-time block codes (STBCs) have been designed and used to achieve the diversity and multiplexing gains in multiple antenna systems. STBCs have been typically designed based on rank and determinant criteria which can provide good performance at high signal-to-noise ratios (SNRs). Later, STBCs are designed based on mutual information to provide good performance at a specific SNR corresponding to the forward error correction (FEC) code rate. However, once the FEC code and STBC are concatenated, to achieve the best performance, STBC should be designed by considering the structure of the FEC code and the corresponding decoder in addition to the code rate. Polar codes are a new class of FEC codes that benefit from a variety of low complexity decoders and simple rate matching. Polar codes can be efficiently designed for a specific channel and STBC. Therefore, by changing the parameters of a specific STBC and optimizing the polar code for each new STBC, the best match between polar codes and STBCs can be found. Throughout this paper, we introduce a simple method for joint optimization of polar codes and STBCs and show that it can substantially improve the performance of the concatenated scheme.**

I. Introduction

Multiple antenna systems, known as multiple-input multiple-output (MIMO) systems, can provide high-rate and high-reliability to meet the crucial demands of future communication systems. To achieve the high capacity of MIMO, efficient schemes that exploit the spatial diversity are necessary. Orthogonal space-time block codes, introduced by Alamouti in [1] and by Tarokh, Jafarkhani and Calderbank in [2], benefit from low decoding complexity and can use the full spatial diversity. However, they cannot achieve any coding gain and suffer from a rate loss as the number of transmit antennas grows. To overcome these issues, quasi-orthogonal STBCs [3], super-orthogonal space-time trellis codes [4], and algebraic codes [5], [6] were proposed.

Tarokh et al., in [7], introduced the rank and determinant criteria as two useful measures for designing STBCs at high signal-to-noise ratios (SNRs). Even though most STBCs are designed based on these criteria, they do not guarantee good performance at low to moderate SNRs [8]. It has been shown that to achieve good performance with capacity achieving forward error correction (FEC) codes, the modulation (or the STBC) should be designed based on maximizing the mutual information at the SNR corresponding to the FEC code rate [9], [10]. However, due to delay constraints and complexity limitations, typically short to moderate length codes with low complexity decoders are used for most applications. Therefore, designing the modulation (or the STBC) matched with a specific FEC code and decoder structure has been an important open problem.

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Polar codes, introduced by Erdal Arikan, work based on the concept of channel polarization as a method to improve some bit-channels at the expense of others [11]. Arikan also proposed a low complexity successive cancellation decoder (SCD) for polar codes. Polar codes are uniquely designed for a specific channel, modulation and SNR by determining their information set. Given a specific code rate, to design polar codes the information set and the corresponding frame error rate (FER) can be easily determined using Monte Carlo simulation.

For matching binary codes to the modulation and STBC, an efficient coded-modulation scheme should be used. It has been shown that polar coded-modulation, constructed based on multilevel coding (MLC) with multistage decoding (MSD), outperforms polar coded-modulation constructed based on bit interleaved coded modulation (BICM) [12] due to clarity of design and the conceptual similarity of multilevel coding (with set-partitioning-based bit-to-symbol mapping) to channel polarization.

To have the best match between the polar code and STBCs, the parameters of the STBC can be adapted to the structure of the polar code, the corresponding SCD, and the bit-to-symbol mapping. In this paper, we propose to optimize the multilevel polar code and STBC jointly. To this end, we change the parameters of the STBC to create a new space-time code, find a set-partitioning based multidimensional bit-to-symbol mapping and optimize the polar code for the new STBC. Here, given a specific code rate and SNR, the design objective of the concatenated polar code and STBC is to minimize the FER. The design procedure is summarized in the following steps:

- Change the parameters of the STBCs,
- Apply a set-partitioning algorithm to generate the bit-to-symbol mapping for each STBC with the new parameters,
- Optimize the polar code to minimize the FER for the STBC with the new parameters and the bit-to-symbol

mapping, and

- Repeat these steps to find the polar coded STBC with the least FER.

As the main contribution of this paper, we present a method of joint optimization of polar codes and STBCs that can substantially improve the performance of the concatenated scheme. The rest of the paper is organized as follows: The system model is defined in Section II; design elements of multilevel polar coded STBC including MSD and polar code design method are reviewed in Section III; STBC design methods including the joint optimization are explained in Section IV; the numerical results are presented in Section V; and the conclusions are provided in Section VI.

II. System Model

The system consists of a transmitter and a receiver equipped with N_t transmit and N_r receive antennas. Each K bits of data are coded using a multilevel binary polar code of rate $R_{tot} = K_{tot}/N_{tot}$ and length $N_{tot} = BN$ consisting of B levels each with level code length N. Each level encoder of the multilevel code encodes a portion of total K bits corresponding to the component code rates $\{R_1, ..., R_B\}$. After encoding all levels, each set of code bits $\{c_n^1, c_n^2, ..., c_n^B\}$, for $n = 1, ..., N$, are mapped to a space-time symbol by employing a multidimensional bit-to-symbol mapping. The space-time symbol is one of the signal points of STBC, $\mathbf{G}$, distributed on L time slots and N_t transmit antennas. The space-time symbol is then sent through the $N_t \times N_r$ time-varying MIMO Rayleigh flat fading channel $\mathbf{H}$ with normalized Doppler frequency f_d and distribution $\mathcal{CN}(\mathbf{0}, \mathbf{I})$. The $L \times N_r$ received samples are

$$\mathbf{R} = \mathbf{G}\mathbf{H} + \mathbf{W}, \tag{1}$$

where $\mathbf{W}$ is the zero-mean complex additive white Gaussian noise (AWGN) with variance $\sigma_W^2/2 = N_0/2$ per dimension. The probability of detection to derive the soft input for the polar decoder, given perfect channel state information (CSI), for the space-time codeword $\mathbf{G}$ with elements g_p^l being the space-time code symbol transmitted in time slot l from antenna p, is proportional to

$$\Pr(\mathbf{R} \mid \mathbf{G}, \mathbf{H}) \propto \exp\left(\frac{\sum_{l=1}^{L} \sum_{q=1}^{N_r} \left| r_q^l - \sum_{p=1}^{N_t} h_{pq}^l g_p^l \right|^2}{\sigma_w^2} \right), \tag{2}$$

where r_q^l is the signal received from the q^{th} receive antenna during time slot l. Throughout this paper, we use 2×2 STBCs that can carry 2^B bits in every two time slots. The STBC points are chosen using a multidimensional bit-to-symbol mapping that spans all bits of all STBC symbols g_p^l. This method of employing multidimensional bit-to-symbol mapping is elaborated in [13] for the Golden code [5]. The received FEC codewords $\{\hat{\mathbf{c}}^1, \hat{\mathbf{c}}^2, ..., \hat{\mathbf{c}}^B\}$ are decoded using a MSD.

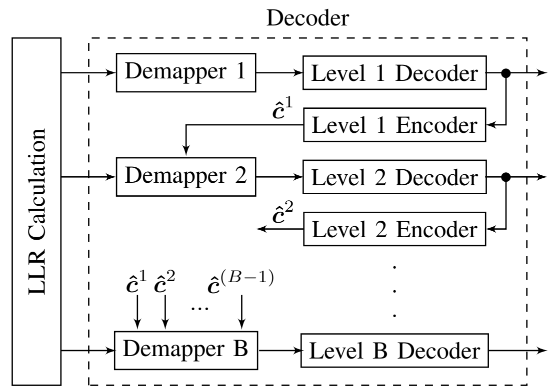

Fig. 1. Multistage decoder.

III. Multilevel Polar Coded STBC Design Elements

In this section, we briefly review different elements and steps of the design of multilevel polar codes and STBC concatenated scheme.

A. Multistage Decoding

In the MSD architecture [14], the code bits of each level are deduced with the aid of the received symbol and previously deduced code bits of the upper levels. As shown in Fig. 1, at each level after decoding, the received message word is fed to an encoder and the generated codeword $\hat{\mathbf{c}}^b$ reduces part of the ambiguity for the demapper of the next level to compute more reliable LLRs. Thus, the LLR estimation at each level can be given by

$$\lambda_b = \ln \frac{\sum_{\mathbf{G} \in \mathcal{X}_{b,0}} \Pr(\mathbf{R} \mid \mathbf{G}, \mathbf{H})}{\sum_{\mathbf{G} \in \mathcal{X}_{b,1}} \Pr(\mathbf{R} \mid \mathbf{G}, \mathbf{H})}, \tag{3}$$

where $b = 1, 2, ..., B$, $\mathcal{X}_{b,0}$ and $\mathcal{X}_{b,1}$ are the set of all constellation points containing zero or one in their b^{th} position given the $\{\hat{\mathbf{c}}^1, ..., \hat{\mathbf{c}}^{b-1}\}$, respectively.

B. Polar Code Construction

For designing a multilevel coding scheme, B binary codes with the set of rates $\{R_1, R_2, ..., R_B\}$ should be constructed. Arikan in [11], proposed the simulation-based design of polar codes for arbitrary channels. Although, polar codes also can be constructed using density evolution and its Gaussian approximation extension, the simulation-based design can be extremely simple and is also slightly more accurate than other numerical methods. Furthermore, the simulation-based method can simply and accurately estimate the component code rates of MLC, while other methods of estimating the component code rates are typically not accurate. As an example, the capacity-based estimation of the component code rates of MLC for the SCD used to decode short to moderate length polar codes generates rates that are more than the achievable rates.

In simulation-based design of polar codes, the position of the first error event of each codeword for a rate one polar code is recorded. After running the simulation with a large number of frames (e.g., 10000) the number of first error events of each bit-channel is recorded. Given a specific total code rate R_{tot}, K_{tot} bit-channels with the lowest number of first error events are chosen as elements of the information set. To avoid running a large number of frames in simulation-based design for each bit-channel, after the first error event occurs, the corresponding position is recorded and corrected and the next bit-channels are examined subsequently. The extension of this method can be used to design multilevel polar codes for space-time signals [13].

In this case, rate one polar codes of length N for each level are simulated and the number of first error events of bit-channels of all levels is estimated and among all $N_{tot} = NB$ different bit-channels, those with the lowest number of first error events construct the information set. In fact, during this step the message word lengths $\{k_1, ..., k_B\}$ can be determined since at each level of the multilevel polar code only some bit channels are in the final total information set and therefore the component code rates $\{R_1, ..., R_B\}$ are determined. This method further facilitates the measurement of FER. In fact, since the first error events for all simulated codewords are recorded, after choosing the information set, the number of codewords containing at least one first error event in the information set is counted to determine the FER. To design the polar coded-modulation for a specific FER, a simple bisection search algorithm is proposed in [13] to find the minimum SNR in which the target FER is feasible.

C. Design Procedure for the Polar Coded STBC Scheme

To design the concatenated scheme of a multilevel polar code and a given STBC, the multidimensional bit-to-symbol mapping is generated using the algorithm proposed in [13]. Then the the multilevel polar code is designed for the constructed set-partitioned STBC scheme by using the code design method mentioned in Section III-B. The polar codes are designed at the minimum SNR such that a target frame error rate of 10^{-2} is achieved.

IV. STBC Design Methodologies

Throughout this paper, we use a variety of STBCs designed based on either the traditional rank and determinant criteria introduced in [7], or on mutual information. Here, we review their structure and the design criteria, and we introduce the joint optimization of polar codes and STBCs as a novel design method.

A. STBC Design based on the Rank and Determinant Criteria

In [7], Tarokh et al. introduced the rank and determinant criteria that are extremely effective on minimizing the union bound of the pairwise error probability of STBCs. They can be employed for designing STBCs at high SNRs. Using the rank of the pairwise difference matrix as a measure of achieving the diversity and the determinant of the pairwise difference matrix multiplied by its Hermitian as a measure of the coding gain for the STBCs have been widely used in the design of the codes. One of the well-known STBCs, designed using this method with full diversity and the highest coding gain for a 2×2 antenna configuration, is the Golden code introduced by Belfiore et al. in [5]. The structure of the Golden code, herein referred to as Matrix A, is given by

$$\mathbf{G}_A = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha(s_1 + s_2\theta) & \alpha(s_3 + s_4\theta) \\ \gamma\bar{\alpha}(s_3 + s_4\bar{\theta}) & \bar{\alpha}(s_1 + s_2\bar{\theta}) \end{bmatrix}, \quad (4)$$

where s_1, s_2, s_3, and s_4 are symbols of the code chosen from a constellation with cardinality $|M|$, $\theta = (1 + \sqrt{5})/2$, $\bar{\theta} = 1 - \theta$, $\alpha = 1 + i(1 - \theta)$, and $\bar{\alpha} = 1 + i(1 - \bar{\theta})$, where $i = \sqrt{-1}$. Later, Sezigner and Sari in [6] introduced a 2×2 STBC to reduce the decoding complexity of the Golden code from $O(|M|^4)$ to $O(|M|^2)$. This code, herein referred to as Matrix B and can be written as

$$\mathbf{G}_B = \begin{bmatrix} as_1 + bs_3 & as_2 + bs_4 \\ -cs_2^* - ds_4^* & cs_1^* + ds_3^* \end{bmatrix}, \quad (5)$$

where $a = c = 1/\sqrt{2}$, $b = 1/\sqrt{2}e^{i\phi}$ and $d = -ib$. By performing a numerical search, $\phi = 114.29°$ was found to maximize the minimum determinant for QPSK [6]. As shown in [6], G_B performs only slightly worse than the Golden code.

B. STBC Design based on Mutual Information

STBCs designed based on the rank and determinant criteria benefit from high mutual information at high SNRs while powerful FEC codes should perform close to capacity at any given SNR corresponding to the code rate and thus, they need to employ high mutual information SNR dependent modulations (or STBCs). In [8], to achieve good performance at low SNRs, the Matrix B code is optimized by maximizing the bit-wise mutual information at low SNRs. As a result, the parameters of the Matrix B code for low SNRs are different than those designed based on the rank and determinant criteria. For example, for a wide range of low to moderate SNRs and by employing QPSK, ϕ is found to be $135°$ for $N_r = 2$.

C. Joint Optimization of FEC Code and STBC

The mutual information is a good criterion to design the modulation (or STBC) for FEC codes that perform close to capacity. However, when there is a gap between the performance of a FEC code and the capacity, the mutual information is not necessarily a good measure. This gap exists for all currently available finite length FEC codes, even in presence of powerful decoders, and motivates the research on design of STBCs that enhance the performance while used with practical FEC codes.

Algorithm 1 Joint Optimization of Polar Code and STBC

Input: Normalized Matrix $\mathbf{G}_X(\rho,\varphi)$ STBC

Output: The information set of the multilevel polar code, rates of different levels, and the optimized parameters of the STBC.

Procedures used in the algorithm:

•*Update_Labelling(): Generates the set-partitioning based bit-to-symbol mapping for the input signal using the algorithm of [13].*

•*Polarcode_Design(): Designs a multilevel polar code for a given constellation and a multidimensional bit-to-symbol mapping using the algorithm of [13] and outputs the information set, code rates of the MLC and the total FER.*

Variables:

$\mathbf{G}_X(\rho,\varphi)$: STBC matrix.

ζ: The set of indices of information bit-channels.

r: Vector of MLC component code rates.

z:Vector of multidimensional bit-to-symbol mapping.

ϵ: FER, ϵ_{Min}: Minimum FER.

$\zeta_{opt}, \mathbf{r}_{opt}, \rho_{opt}, \varphi_{opt}, \mathbf{z}_{opt}$: Optimized values of aforementioned variables.

Initialisation:

$\epsilon_{Min} = 1$
Search vectors **Mag_Vec** and **Phase_Vec**

The body of Algorithm:

for all ρ in **Mag_Vec do**
  for all φ in **Phase_Vec do**
    $\mathbf{G} = \mathbf{G}_X(\rho,\varphi)$
    $\mathbf{z} = Update\_Labelling(\mathbf{G})$
    $[\epsilon, \zeta, \mathbf{r}] = Polarcode\_Design(\mathbf{z}, \mathbf{G})$
    if $(\epsilon < \epsilon_{Min})$ **then**
      $\epsilon_{Min} = \epsilon$
      $[\zeta_{opt}, \mathbf{r}_{opt}, \rho_{opt}, \varphi_{opt}, \mathbf{z}_{opt}] = [\zeta, \mathbf{r}, \rho, \varphi, \mathbf{z}]$
    end if
  end for
end for
return $[\epsilon_{Min}, \zeta_{opt}, \mathbf{r}_{opt}, \rho_{opt}, \varphi_{opt}, \mathbf{z}_{opt}]$

In situations where the use of adaptive modulation and coding (AMC) is not feasible, one of the best strategies is to the design the codes, including FEC codes and STBCs, for known statistics of the fading channel. Since the final codeword constructed by the concatenation of the FEC and the STBC determines the performance of the system, the optimal design of the final concatenated codeword is more effective on the performance of the system rather than the independent optimal design of FECs and STBCs. One way to enhance matching between the FEC code and the STBC is the joint optimization of FEC and STBCs that can improve the system performance substantially.

As described in Section III, multilevel polar codes can easily be designed given a specific STBC and channel statistics. To jointly design the polar code and STBC, the best STBC matched to the polar code structure and the bit-to-symbol mapping should be determined. To present the algorithm for a general structure of STBC, we consider the STBC has two parameters ρ and ϕ to be optimized. To this end, in a linear search, for each combination of the two parameters of the STBC, the set-partitioning algorithm is applied to find a good bit-to-symbol mapping for the generated STBC and polar code design procedure, given the new STBC and the new bit-to-symbol mapping, is repeated. Finally, the best match of information set for the polar codes and parameters of STBC corresponding to the lowest FER is chosen. The design algorithm is formalized in Algorithm 1 and the joint design procedures is shown in Fig. 2.

To jointly optimize polar codes and $\mathbf{G}_B$ in (5), we set $a = c$ and $d = -ib$ and to limit the search space, we set $\measuredangle a = 0°$. Here, unlike [8] that only searches over $\measuredangle b$ to find the highest mutual information, we search over both $\rho = |b|/|a|$ and $\varphi = \measuredangle b$ since this jointly with the set-partitioning bit-to-symbol mapping can polarize the channel more efficiently. Note that there are only two parameters to optimize. For example, each evaluation of the *Polarcode_Design()* for an eight-level $N_{tot} = 512$ bits polar code with 10000 frames only takes around 10 seconds, using Matlab on a computer with 24 GB RAM and a 3.40 GHz i7-3770 CPU, so the linear search only takes a few minutes. Using this design method, by employing QPSK for $N = 64$, the remaining parameters of Matrix B are determined as $|a| = 0.69$, $|b| = 0.73$, and $\measuredangle b = 135°$ in a block fading channel with the coherence time $T_c = 2$ channel-uses at a FER of 10^{-2}. Note that using these new parameters, $\mathbf{G}_B$ has lower mutual information in comparison to the same code optimized in [8]. For all optimizations in this section, we set $N_r = 2$.

In this paper by setting $L = 1$, we also optimize the code $\mathbf{G}_C = [\alpha_1 s_1 + \beta_1 s_2 \quad \alpha_2 s_3 + \beta_2 s_4]$. To limit the emitted power of each antenna, we assume $|\alpha_1|^2 + |\beta_1|^2 = |\alpha_2|^2 + |\beta_2|^2$. To limit the search space, we set $\alpha_2 = \alpha_1$, $\beta_2 = -i\beta_1$ and $\measuredangle \alpha_1 = 0°$. Using Algorithm 1 and by searching on $\rho = |\beta_1|/|\alpha_1|$ and $\varphi = \measuredangle \beta_1$, the remaining parameters of $\mathbf{G}_C$ for $N = 64$ and $N = 128$ are determined as $|\alpha_1| = 0.43$, $|\beta_1| = 0.9$, and $\measuredangle \beta_1 = 136.5°$ in an independent fading channel with $T_c = 1$ channel-use and $|\alpha_1| = 0.5$, $|\beta_1| = 0.86$, and $\measuredangle \beta_1 = 135.5°$ in a time-varying channel with $f_d = 0.01$ at a FER of 10^{-2}. We also optimized β_1 to maximize the mutual information and determined the remaining parameters of $\mathbf{G}_C$ as $|\alpha_1| = 0.48$, $|\beta_1| = 0.88$, and $\measuredangle \beta_1 = 61.5°$ in an independent fading channel. For the sake of comparison, the code proposed in [15], here referred to as Matrix D, is used. The structure of Matrix D is given as $\mathbf{G}_D = [s_1 \quad s_2]$, where s_1 and s_2 are chosen from a QAM constellation.

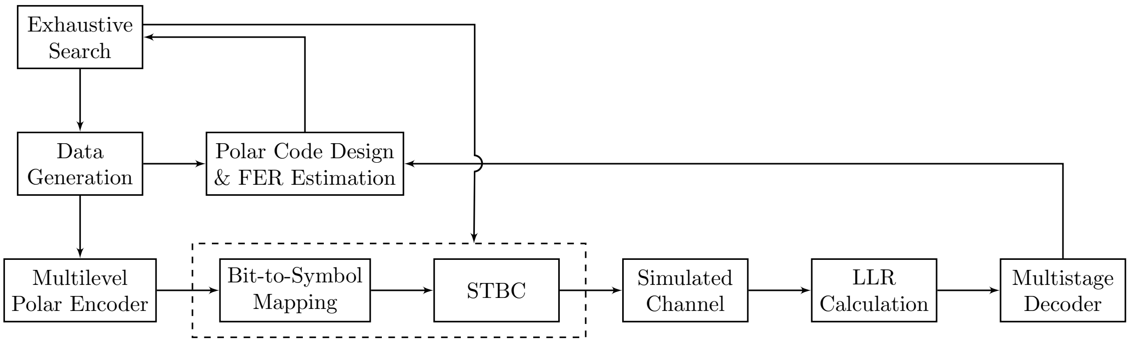

Fig. 2. Joint optimization of polar codes and STBC design method block diagram.

V. Numerical Results and Discussions

In this section, the performance of jointly optimized polar codes and Matrix B STBC is compared with polar codes optimized for Matrix A, B and C designed using the rank and determinant criteria and the mutual information. For designing the concatenated schemes, the construction method explained in Section III-C is employed. We use Method 1 to represent STBCs designed based on the rank and determinant criteria, Method 2 to represent the STBCs designed to maximize the mutual information and Method 3 to represent the joint optimization of FEC and STBCs. The polar decoder for all cases is SCD. To generate the time-varying channel, we employ the Jakes' model and a block fading model. The constellation used for all Matrices A, B and C is QPSK and for Matrix D is 16-QAM. For all curves, we set $N_r = 2$.

The performance of the polar coded Matrices A, B and Alamouti STBCs for 2 bits-per-channel-use (bpcu) over a block fading channel with $T_c = 2$ channel-uses is compared in Fig. 3. Polar coded Alamouti STBC scheme is constructed using a multilevel code with 4 levels of length 128 and $N_{tot} = 512$ for a 16-QAM constellation. For all other curves, the total code length is 512 bits constructed from 8 component codes of length 64. The total code rate is $1/2$ and the component code rates for Matrix B optimized using Method 1 are $\{0.14,0.23,0.31,0.39,0.56,0.69,0.78,0.89\}$, using Method 2 are $\{0.19,0.20,0.31,0.41,0.55,0.67,0.75,0.92\}$ and using Method 3 are $\{0.08,0.19,0.33,0.37,0.64,0.67,0.80,0.92\}$. The results indicate that the polar coded Matrix A designed using Method 1 outperforms the polar coded Matrix B designed using Methods 1 and 2 by 0.15 dB and 0.1 dB, respectively. Furthermore, the polar coded Matrix B designed using Method 3 outperforms Matrix B designed using Method 2 by 0.4 dB at a FER of 10^{-2}.

Fig. 4 shows the performance of the concatenated scheme of polar code and Matrices C and D for 4 bpcu. The total code length is 512 bits constructed from component codes of length 64. The total code rate is $1/2$ and the component code rates for the Matrix C optimized using Method 2 are $\{0.14,0.17,0.25,0.34,0.56,0.72,0.87,0.94\}$ and using Method 3 are $\{0.02,0.14,0.23,0.52,0.56,0.77,0.83,0.94\}$ in an independent fading channel. When $f_d = 0.01$ for a Jake's model, the component code rates for the Matrix C optimized using Method 2 are $\{0.08,0.14,0.25,0.41,0.58,0.75,0.86,0.94\}$ and the rates for Matrix D are $\{0.02,0.11,0.23,0.53,0.55,0.78,0.83,0.95\}$. The results indicate that the polar coded Matrix C designed using Method 3 outperforms the polar coded Matrix D by 0.1 dB in an independent fading, and by 0.7 dB when $f_d = 0.01$ at a FER of 10^{-2}. Furthermore, Matrix C optimized using Method 3 in comparison to Method 2 works 0.5 dB better in an independent fading channel.

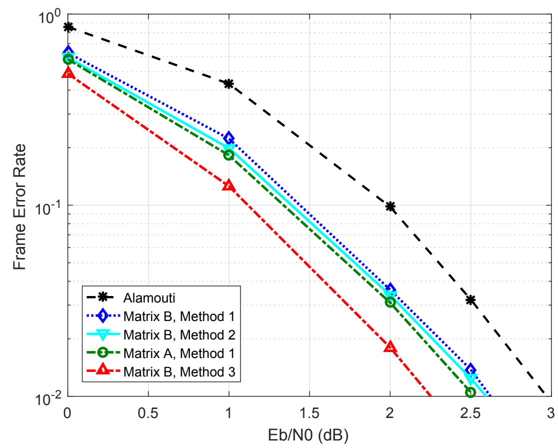

Fig. 3. FER comparison of polar coded Matrix A designed using Method 1, Matrix B designed using Methods 1, 2 and 3, and the Alamouti code for 2 bpcu and $N_{tot} = 512$.

The comparison of polar coded Matrices C and D for 4 bpcu by setting $N = 128$ and $N_{tot} = 1024$ is shown in Fig. 5. The code rates for Matrix C optimized using Method 2 are $\{0.15, 0.16, 0.27, 0.34, 0.56, 0.74, 0.86, 0.93\}$ and using Method 3 are $\{0.04,0.16,0.23,0.52,0.54,0.77,0.81,0.95\}$ in independent fading. For case of $f_d = 0.01$, the code rates for Matrix C designed using Method 2 are $\{0.08,0.15,0.26,0.40,0.57,0.73,0.86,0.95\}$ and for

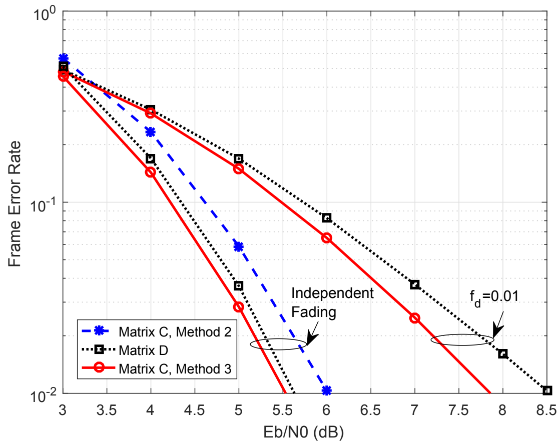

Fig. 4. FER comparison of polar coded Matrix C designed using Methods 2 and 3, and Matrix D, all for 4 bpcu and $N_{tot} = 512$.

Matrix D are $\{0.02,0.14,0.24,0.51,0.54,0.77,0.81,0.96\}$. In an independent fading channel, the improvement of Matrix D designed using Method 2, over the same code designed using mutual information is 0.45 dB and it shows almost the same performance with the Matrix D at a FER of 10^{-2}. It is clear that as the system performance is improved by employing a lengthier FEC code, the gap between the FER of joint design and the mutual information based design decreases. When $f_d = 0.01$, the Matrix C optimized using Method 2 outperforms the Matrix D by 0.7 dB at a FER of 10^{-2}.

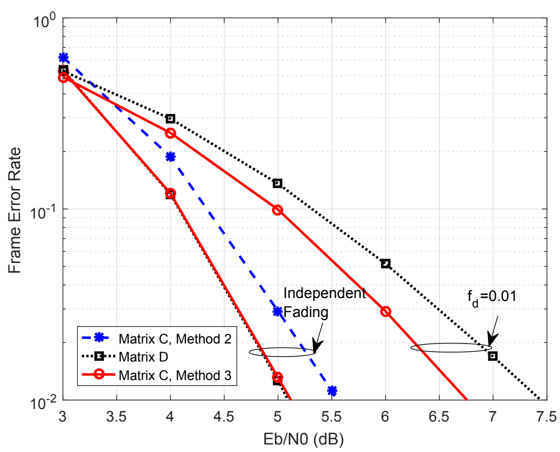

Fig. 5. FER comparison of polar coded Matrix C designed using Methods 2 and 3, and Matrix D, all for 4 bpcu and $N_{tot} = 1024$.

VI. Conclusion

In this paper, we proposed the joint optimization of multilevel polar codes and STBCs. It has been known that the polar code can be designed to minimize FER at a specific SNR given a STBC and a code rate. For joint optimization of polar code and STBCs, here we change the parameters of STBCs to create a new code, find a new multidimensional bit-to-symbol mapping using a set-partitioning algorithm and repeat the code design procedure for the new STBC. After repeating this procedure for a wide range of STBC parameters, we choose an information set and the corresponding parameters for the STBC that minimize the FER jointly. The numerical results show a substantial improvement in comparison to the same STBC designed based on the rank and determinant criteria and the mutual information.

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References

[1] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456-1467, July 1999.

[3] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 1-4, Jan. 2001.

[4] H. Jafarkhani and N. Seshadri, "Super-orthogonal space-time trellis codes," *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp.937-950, Apr. 2003.

[5] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The golden code: A 2×2 full-rate space-time code with nonvanishing determinants," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1432-1436, Apr. 2005.

[6] S. Sezginer and H. Sari, "Full-rate full-diversity 2×2 space-time codes of reduced decoder complexity," *IEEE Commun. Lett.*, vol. 11, no. 12, pp. 973-975, Dec. 2007.

[7] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744-765, Mar. 1998.

[8] A. E. Falou, C. Langlais, C. A. Nour, and C. Douillard, "Low ML-detection complexity, adaptive 2×2 STBC, with powerful FEC codes," in *Proc. 7th International Symposium on Turbo Codes and Iterative Information Processing*, Gothenburg, Sweden, pp. 195-199, 2012.

[9] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans. Inf. Theory*, vol. 48, no. 7, pp. 1804-1824, July 2002.

[10] M. F. Barsoum, C. Jones, and M. Fitz, "Constellation design via capacity maximization," in *Proc. IEEE ISIT*, pp. 1821-1825, Nice, France, June 2007.

[11] E. Arikan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3051-3073, July 2009.

[12] M. Seidl, A. Schenk, C. Stierstorfer, and J. B. Huber, "Aspects of polar-coded modulation," in *Proc. 9th Int. ITG Conf. Systems, Commun. and Coding*, Munich, Germany, Jan. 2013.

[13] P. R. Balogun, I. Marsland, R. Gohary, and H. Yanikomeroglu, "Polar codes for noncoherent MIMO signalling," in *Proc. IEEE ICC*, Kuala Lumpur, Malaysia, May 2016.

[14] H. Imai and S. Hirakawa, "A new multilevel coding method using error correcting codes," *IEEE Trans. Inf. Theory*, vol. 23, no. 3, pp. 371-377, May 1977.

[15] B. M. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. Commun.*, vol. 51, no.3, pp.389-399, Mar. 2003.