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An Upper Bound on BER in a Coded Two-Transmission Scheme with the Same-Size Arbitrary Constellations

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September 4th, 2014

pimrc'14 IEEE WASHINGTON, DC USA

HUAWEI

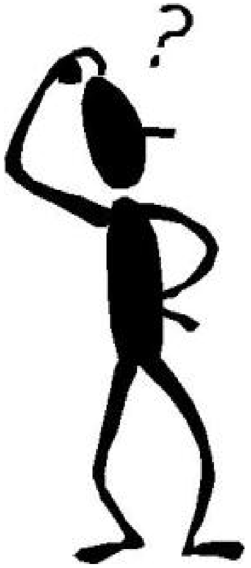
A Fresh Start With These Questions

- Why do we need such an analysis depending on **arbitrary 2D constellation usage** ?

A Fresh Start With These Questions

- Why do we need such an analysis depending on **arbitrary 2D constellation usage** ?

- What does an **upper BER bound expression** bring as a promise to current system scenarios ?



A Fresh Start With These Questions

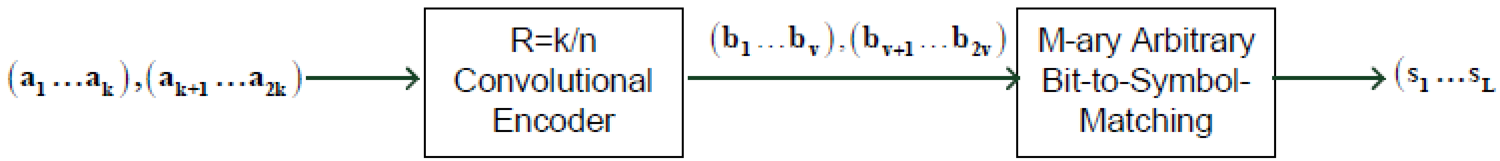
- **What is actually behind in this study ?**

Let's Quick Look System Model

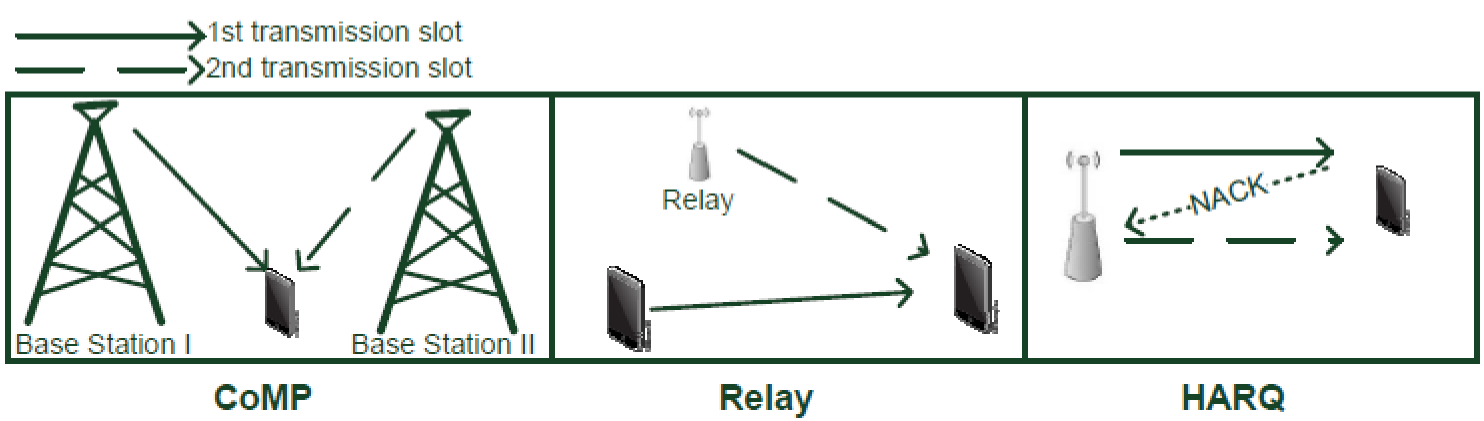
- Convolutional Encoder & Two-Orthogonal Transmission Model

Let's Quick Look System Model

- Convolutional Encoder & Two-Orthogonal Transmission Model
- **Transmitter Scale**



- **Overall System Scale**



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Motivation

- **Finding an good upper bound expression on BER depending on the distances of symbol pairs yields**

  - ✓ **Constellation design opportunity for coded schemes**
    - ➢ ***Optimization variables:* *Location of the signal points***
    - ➢ ***Objective function:* *Error performance expression by utilizing conventional error-state diagram for convolutional, TCM, turbo, etc.***
    - ➢ ***Energy constraint:* *Fair comparison***

  - ✓ **SNR based constellation design framework**
    - ➢ ***There is no M-ary signal set that are optimum for*** $M \geq 7$ ***overall SNR values****

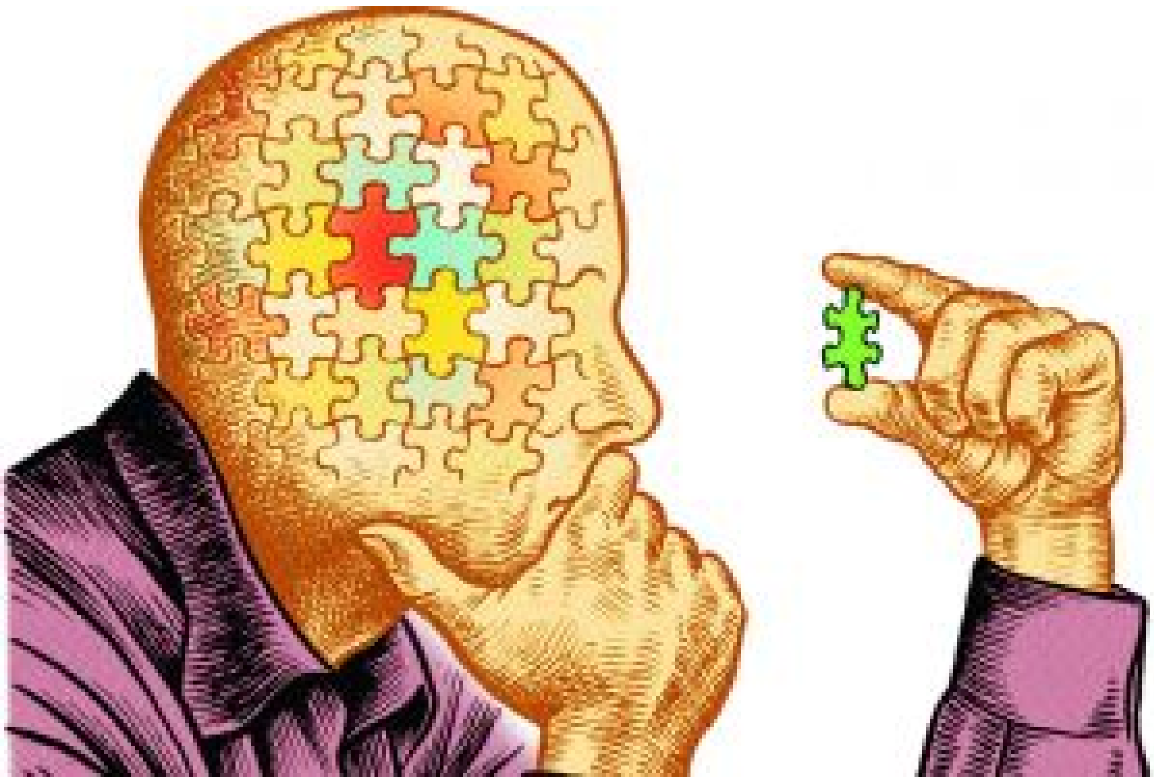
*** M. Steiner,** "The strong simplex conjecture is false," IEEE Transactions on Information Theory, vol. 40, no. 3, pp. 721–731, 1994.

Motivation

"The determination of the optimal signal sets which maximize the probability of detection remains in general unsolved as SNR → 0. Perhaps optimal designs can be found for some partition of the SNR range [0, ∞) as a function of M [number of signal points]"

M. Steiner, "The Strong Simplex Conjecture is False", TIT-40, May'94.

- **Constellation design** has been already used under the divergent topics **since 1970s**.



Rich Literature (1/3)

- Coding and modulation firstly taken into account jointly in

  J. L. Massey, “Coding and modulation in digital communications,” Proc. I974 Int. Zurich Seminar on Digital Comm., Zurich, Switzerland, pp. E2(1)-(4), **1974.** **(231)**

- Trellis coded modulation (TCM):

  Ungerboeck, G., "Channel coding with multilevel/phase signals," IEEE Transactions on Information Theory, vol.28, no.1, pp.55,67, **1982.** **(3314)**

- Constellation design for uncoded systems:
  - 2-D constellation design

    G. Foschini, R. Gitlin, and S. Weinstein, “Optimization of two-dimensional signal constellations in the presence of Gaussian noise,” IEEE Transactions on Communications, vol. 22, no. 1, pp. 28–38, **1974**. **(186)**

    Forney, G.D.; Gallager, R.G.; Lang, G.; Longstaff, F.M.; Qureshi, S.U., "Efficient Modulation for Band-Limited Channels," *IEEE Journal on Selected Areas in Communications,*, vol.2, no.5, pp.632,647, **1984.** **(410)**

  - Multidimensional constellation

    G. D. Forney Jr and L.-F. Wei, “Multidimensional constellations. I. Introduction, figures of merit, and generalized cross constellations,” IEEE Journal on Selected Areas in Communications, vol. 7, no. 6, pp. 877–892, **1989.** **(256)**

    J. Boutros, E. Viterbo, C. Rastello, and J.-C. Belfiore, “Good lattice constellations for both Rayleigh fading and Gaussian channels,” IEEE Transactions on Information Theory, vol. 42, no. 2, pp. 502–518, **1996.** **(227)**

  - Multidimensional constellation + Optimization

    J.-E. Porath and T. Aulin, “Design of multidimensional signal constellations,” IEE Proceedings Communications, vol. 150, no. 5, pp. 317–323, **2003.** **(21)**

    M. Beko and R. Dinis, “Designing good multi-dimensional constellations,” IEEE Wireless Communications Letters, vol. 1, no. 3, pp. 221–224, **2012.** **(4)**

Rich Literature (2/3)

- Constellation design + TCM:
  - Asymmetric modulations

    D. Divsalar, M. Simon, and J. Yuen, "Trellis coding with asymmetric modulations," IEEE Transactions on Communications, vol. 35, no. 2, pp. 130–141, **1987.(69)**

    L. V. Subramaniam, B. S. Rajan, and R. Bahl, "Performance of 4-and 8-state TCM schemes with asymmetric 8-PSK in fading channels," IEEE Transactions on Vehicular Technology, vol. 49, no. 1, pp. 211–219, **2000. (13)**

    X. Zhang, Y. Zhao, and L. Zou, "Optimum asymmetric constellation design for trellis-coded modulation over Gaussian channels," IEEE Communications Letters, vol. 13, no. 7, pp. 528–530, **2009. (1)**
  - Multidimensional constellation (*N*-Dimensional)

    L.-F. Wei, "Trellis-coded modulation with multidimensional constellations," IEEE Transactions on Information Theory, vol. 33, no. 4, pp. 483–501, **1987. (401)**

    C. Dinh and T. Hashimoto, "A systematic approach to the construction of bandwidth-efficient multidimensional trellis codes," IEEE Transactions on Communications, vol. 48, no. 11, pp. 1808–1817, **2000. (8)**
  - Multiple TCM (MTCM)

    Divsalar, D., Simon, Marvin K., "Multiple trellis coded modulation (MTCM)," *Communications, IEEE Transactions on* , vol.36, no.4, pp.410,419, **1988. (121)**

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Rich Literature (3/3)

- Constellation design + Bit-Interleaved Coded Modulation:
  - Bit to Symbol Mapping + Constellation design

    Muhammad, N.S.; Speidel, J., "Joint optimization of signal constellation bit labeling for bit-interleaved coded modulation with iterative decoding," *Communications Letters, IEEE* , vol.9, no.9, pp.775,777, **2005.** **(40)**

    Szczecinski, L.; Diop, F.-K.; Benjillali, M.; Ceron, A.; Feick, R., "BICM in Hybrid ARQ with Mapping Rearrangement: Capacity and Performance of Practical Schemes," *IEEE GLOBECOM*, pp.1410,1415, **2007.** **(1)**

    Kayhan, F., Montorsi, G., "Joint Signal-Labeling Optimization for Pragmatic Capacity under Peak-Power Constraint," *IEEE GLOBECOM*, pp.1,5, **2010.(2)**

- Constellation design + Uncoded + Cooperative Relaying:

  A. Bin Sediq, P. Djukic, H. Yanikomeroglu, and J. Zhang, "Optimized nonuniform constellation rearrangement for cooperative relaying," IEEE Transactions on Vehicular Technology, vol. 60, no. 5, pp. 2340–2347, **2011.** **(3)**

- Constellation design + Physical network coding

  Koike-Akino, T.; Popovski, P.; Tarokh, Vahid, "Optimized constellations for two-way wireless relaying with physical network coding," *IEEE Journal on Selected Areas in Communications*, vol.27, no.5, pp.773,787, June **2009**.**(215)**

- Constellation design + Space time block coding

  Su, W. and Xia X., "Signal Constellations for Quasi-Orthogonal Space-Time Block Codes With Full Diversity," IEEE Transactions on Information Theory, Vol. 50, no. 10, 2331–2347, **2004.** **(434)**

So,...

- What's the novelty part?

So,...

- **Structured imposed constellations**

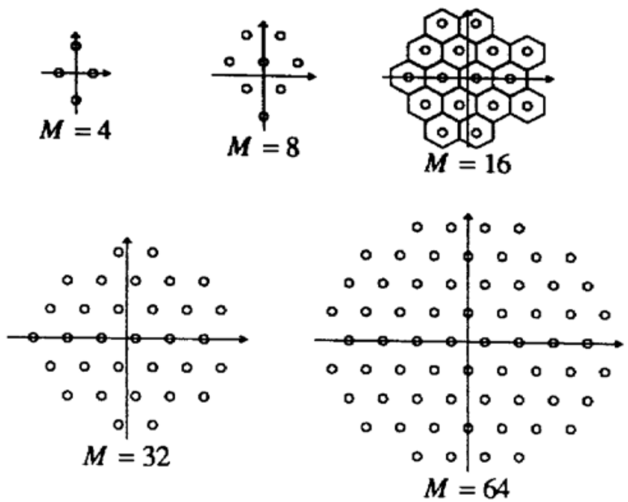

Figure 6-30. Optimal hexagonal constellations. For the $M = 16$ constellation we have shown the hexagonal decision regions. The outer decision regions are approximated as hexagonal for uniformity.

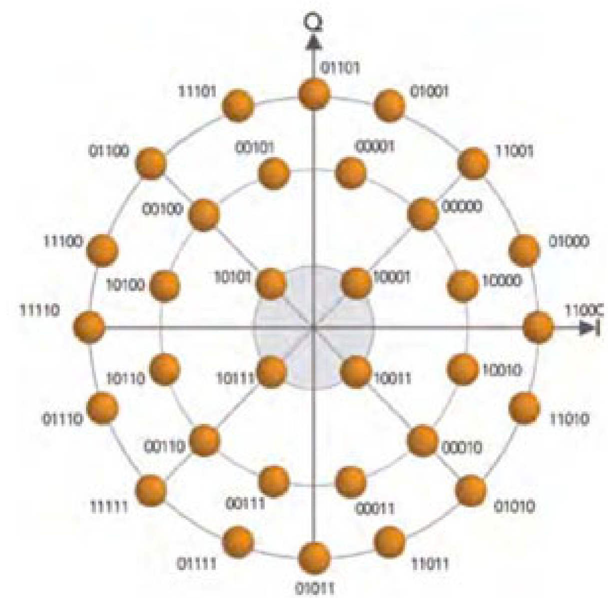

Figure 2: 32-APSK Constellation

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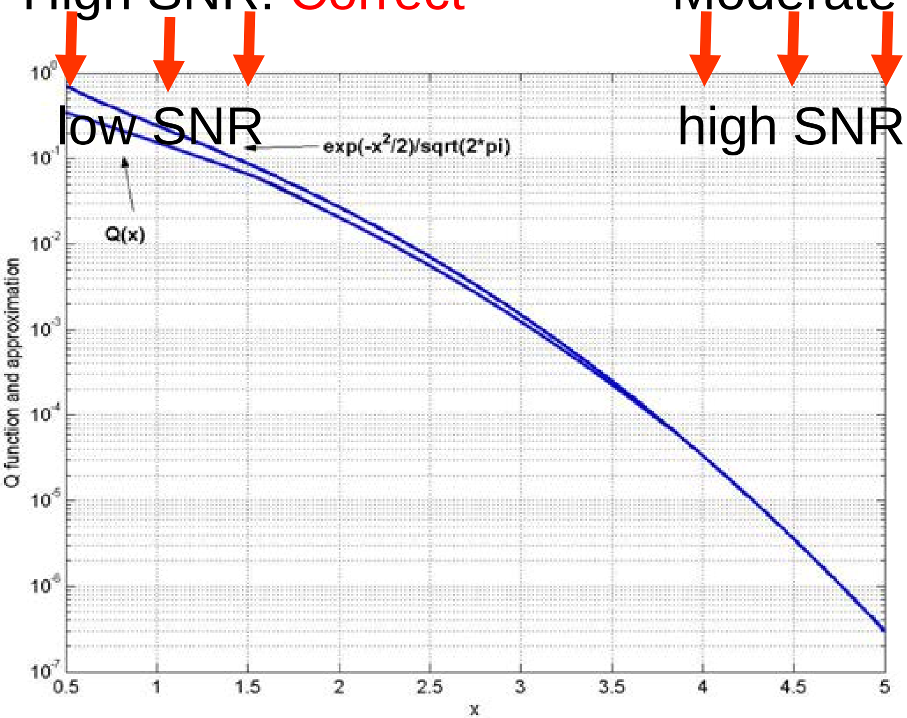
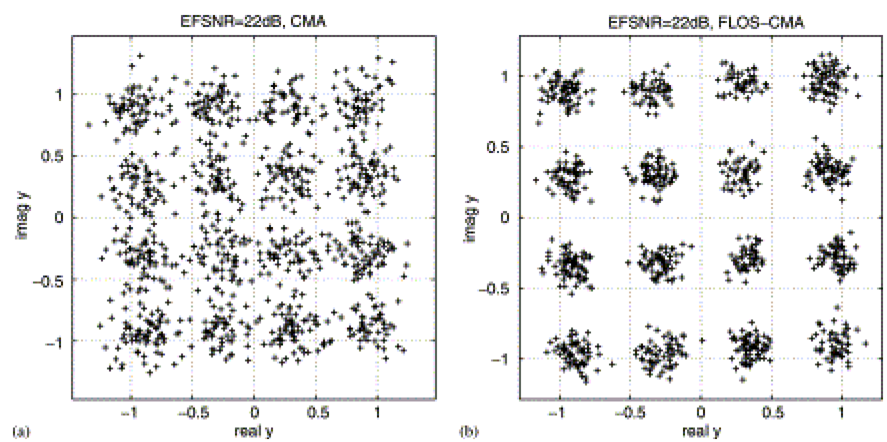
So,...

High SNR Assumptions

- Minimum Euclidean distance between adjacent constellation points determines the performance: Correct or incorrect?

High SNR: Correct   Moderate and especially low SNR: Incorrect

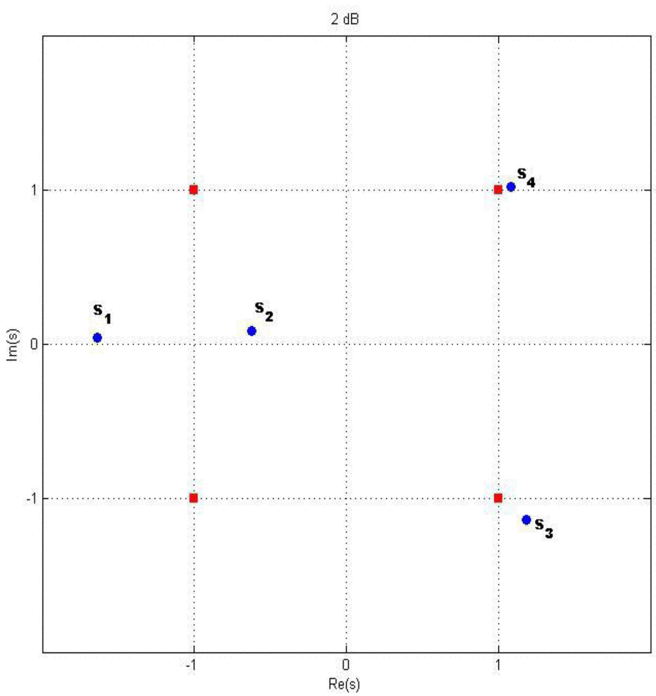


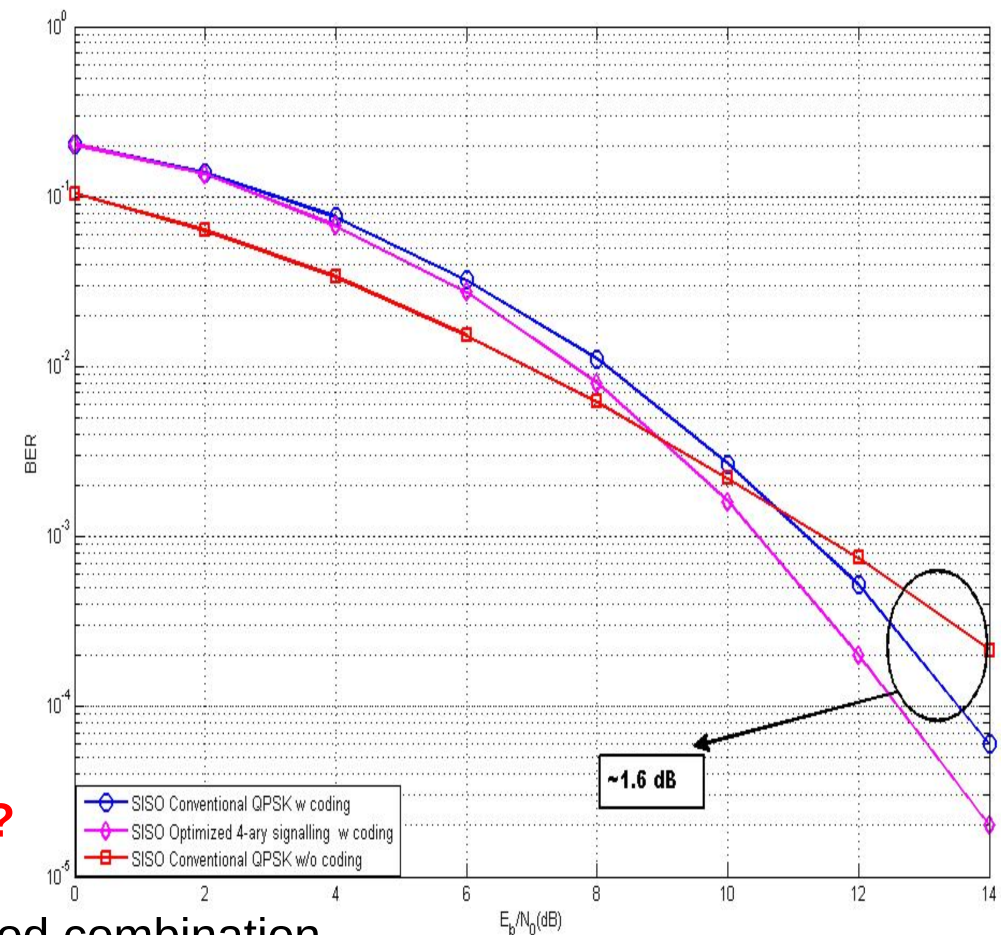


There is no single constellation which is optimal in all SNR levels

SNR Based Constellation Design for SISO

Performance Curves & Constellations for SISO with coding (m=3)





From where the gain is coming?

Generalized set partitioning

Joint modulation and coding: a good combination

BER Bound Expressions

- ✓ BER bound expression for n.i.d. Nakagami-*m case*

$$P_b \leq \frac{1}{k}\left\{\frac{1}{2} - \frac{1}{\sqrt{\pi}}\frac{m_1^{L_\eta m_1} m_2^{L_\eta m_2}}{\Gamma(L_\eta(m_1+m_2))}\sum_{p=0}^{\infty}\frac{(-1)^p \delta_m^{2p+1}}{p!(2p+1)}\right.$$
$$\times\Gamma\left(0.5+p+L_\eta\left(m_1+m_2\right)\right)\left(m_2-\delta_m\right)^{-0.5-p-L_\eta(m_1+m_2)}$$
$$\left.\times{}_2F_1\left(L_\eta m_1,\ p+L_\eta\left(m_1+m_2\right)+0.5;\ L_\eta\left(m_1+m_2\right);\frac{m_1-m_2}{\delta_m-m_2}\right)\right\}$$
$$\times\left.\frac{\partial T(D,I)}{\partial I}\right|_{D=\left(1+\frac{d_{l,1}}{m_1}\right)^{-m_1}\left(1+\frac{d_{l,2}}{m_2}\right)^{-m_2},\, I=1.}$$

- ✓ BER bound expression for i.i.d. Nakagami-*m case*

$$P_b \leq \frac{1}{k}\left(\frac{m^{2L_\eta m}(m-\delta_m)^{-2L_\eta m}\Gamma(2L_\eta m)}{\Gamma(2L_\eta m)} - \frac{2(m-\delta_m)^{-0.5-2L_\eta m}\sqrt{\delta_m}\,\Gamma(0.5(1+4L_\eta m))}{\sqrt{\pi}}\right.$$
$$\left.\times{}_2F_1\left(0.5, 0.5+2L_\eta m; 1.5; -\frac{\delta_m}{m-\delta_m}\right)\right)$$
$$\times\left.\frac{\partial T(D,I)}{\partial I}\right|_{D=\left(1+\frac{d_{l,1}}{m}\right)^{-m}\left(1+\frac{d_{l,2}}{m}\right)^{-m},\, I=1.}$$

$$x = 1-\frac{m}{\delta_m},\ \delta_m = \min\{\delta_l, l\in\eta\}$$

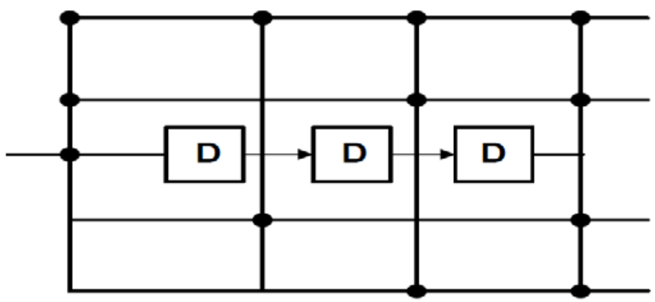
$$\delta_l = \frac{d_l}{1+d_l/m},\ d_l = \frac{\left|s_l-\hat{s}_l\right|^2}{4N_0}$$

L_η : time diversity of the code

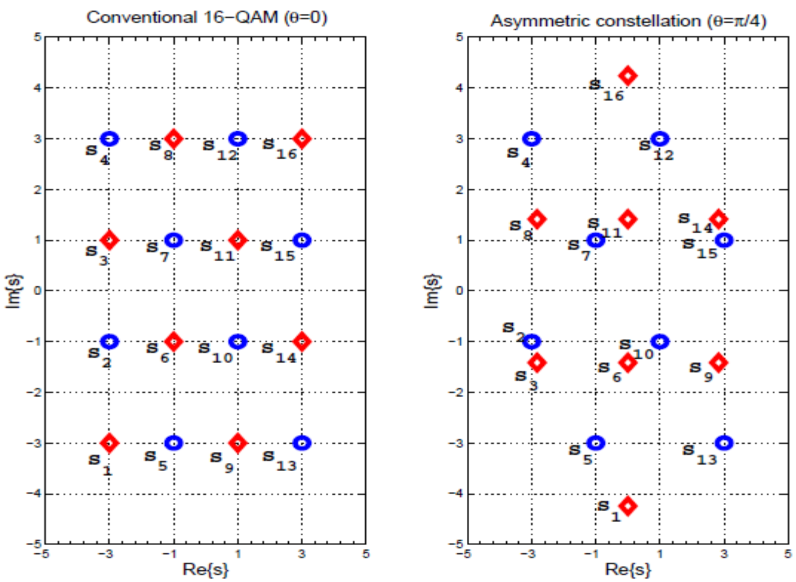
$B_x(.,.)$: incomplete Beta function

It can be used for convolutional, trellis-coded modulation(TCM) and turbo coding scenarios.

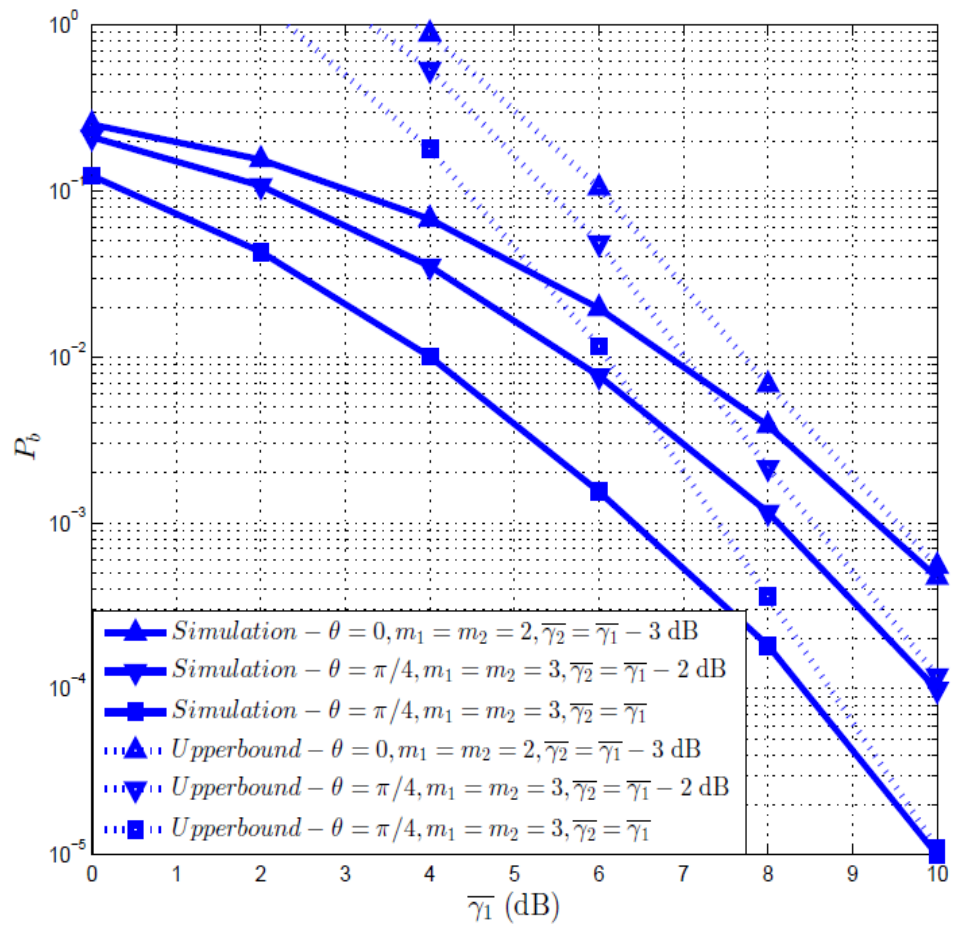
Simulation Results



Convoluational encoder for 16-ary signalling.



Conventional 16-QAM and asymmetric constellation cases.



Future Research Directions

- Correlated channels
- Relevant constraints: PAPR
- Labeling
- Probabilistic signaling
- 2-D → N-D design
- Channel coding
- Source coding
- Non-coherent signaling (optimum signaling unknown even in AWGN)
- Non-coherent MIMO

  Grassmannian signaling, Cayley signaling