Multihop Wireless Communications Channels

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**Multihop Wireless Communications Channels** 

By

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## Abstract

This thesis presents theoretical characterizations and analysis for the physical layer of multihop wireless communications channels. Four channel models are proposed and developed: the decoded relaying multihop channel, the amplified relaying multihop channel, the decoded relaying multihop diversity channel, and the amplified relaying multihop diversity channel. Two classifications are discussed: decoded relaying versus amplified relaying and multihop channels versus multihop diversity channels. The concepts of multihop diversity and multiroute diversity are defined and analyzed. The channel models are compared, through analysis and simulations, with the singlehop reference channel on the basis of signal to noise ratio, probability of outage, probability of error, and optimal power distribution. Each of the four channel models is shown to outperform the singlehop reference channel. Multihop diversity channels are shown to outperform multihop channels. Amplified relaying is shown to outperform decoded relaying despite noise propagation. Finally, a number of implementation considerations are introduced and discussed.

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# List of Acronyms

- AWGN Additive White Gaussian Noise
- BER Bit Error Rate
- BPSK Binary Phase Shift Keying
- CDMA Code-Division Multiple Access
- DSP Digital Signal Processing
- FDMA Frequency-Division Multiple Access
- ISI Inter-Symbol Interference
- LOS Line-of-Sight
- MRC Maximal Ratio Combining
- MWCC Multihop Wireless Communications Channel
- SIR Signal-to-Interference Ratio
- SINR Signal-to-Interference-plus-Noise Ratio
- SNR Signal-to-Noise Ratio
- TDMA Time-Division Multiple Access

# List of Symbols

а	the information symbol at time <i>t</i>
$A_i$	the amplification factor of terminal <i>i</i>
С	the speed of light
С	the processing gain of the wireless system
$C_{Z(i)}$	the approximation of the power sum of chi-square random variables of
	terminal <i>i</i>
$d_{k,i}$	the distance between terminal $i$ and terminal $k$
$G_{k,i}$	the standard optimal gain of branch $k$ of the maximal ratio combiner of
	terminal <i>i</i>
$I_{U(i),i}$	the received feedforward interference of terminal <i>i</i>
$I_{V(i),i}$	the received feedback interference of terminal <i>i</i>
K	the number of branches entering a diversity combiner
$L_{k,i}$	the lognormal random variable associated with the communications channel
	between terminal <i>i</i> and terminal <i>k</i>
$L_{Z(i)}$	the approximation of the sum of lognormal random variables of terminal <i>i</i>
n	the number of hops along the transmission path
$N_0$	the noise power spectral density
$N_{R,i}$	the total noise power applied to the detector of terminal <i>i</i> after maximal ratio
	combining
р	the propagation exponent

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$P_e$	the total probability of error
$P_e(\gamma_{P(i),i})$	the probability of error of terminal <i>i</i>
$P_o$	the total probability of outage
$P_o(\gamma_{P(i),i})$	the probability of outage of terminal <i>i</i>
Q(x)	the Q function of $x$ , related to the error function $erfc(x)$
$r_{k,i}$	the received complex baseband signal amplitude of terminal $i$ from terminal $k$
$R_{k,i}$	the Rayleigh random variable associated with the communications channel
	between terminal <i>i</i> and terminal <i>k</i>
$R_{Z(i)}$	the approximation of the sum of Rayleigh random variables of terminal $i$
s <sub>i</sub>	the transmitted complex baseband signal amplitude of terminal <i>i</i>
$T_i$	the terminal <i>i</i>
$T_A$	the set of all terminals
$T_C$	the set of terminals transmitting correct information
$T_D$	the set of destination terminals
$T_E$	the set of terminals transmitting incorrect information
$T_I$	the set of intermediate terminals
$T_{P(i)}$	the set of terminals that transmit a signal received by terminal <i>i</i>
$T_R$	the set of receiving terminals
$T_S$	the set of source terminals
$T_T$	the set of transmitting terminals

- $T_{U(i)}$  the set of transmitting terminals preceding terminal *i* along the transmission path that do not belong to  $T_{P(i)}$
- $T_{V(i)}$  the set of transmitting terminals following terminal *i* along the transmission path
- $u_1$  the first moment of the approximation using Wilkinson's method of the power sum of lognormal random variables
- $u_2$  the second moment of the approximation using Wilkinson's method of the power sum of lognormal random variables
- $v_{k,i}$  the voltage signal of branch k of the maximal ratio combiner of terminal i
- $v_{R,i}$  the resultant signal envelope applied to the detector of terminal *i* after maximal ratio combining
- *W* the bandwidth of the communication channel
- $\overline{x}$  the expected value of random variable x
- |x| the absolute value of x
- $X_{k,i}$  the multiplicative weight factor of branch *k* of the maximal ratio combiner of terminal *i* for an improved decoded receiver
- $z_{k,i}$  the additive white gaussian noise random variable associated with the communications channel between terminal *i* and terminal *k*
- $\alpha^2$  the free space signal attenuation factor between a transmitting terminal and an arbitrary reference distance
- $\beta_i$  the propagated noise of terminal *i*

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$\chi_{C_{Z(i)}}$	the degrees of freedom of chi-square random variable $C_{Z(i)}$
$\Delta_p$	the processing delay at each intermediate terminal
$\Delta_T$	the total processing delay along the transmission path
$\mathcal{E}_0$	the reference total power constraint
$\mathcal{E}_i$	the transmitted power of terminal <i>i</i>
$\sigma_x^2$	the variance of random variable <i>x</i>
γ	the threshold signal to noise ratio
$\gamma_{P(i),i}$	the received signal to noise ratio of terminal $i$ as a result of the signals from
	all the terminals in set $T_{P(i)}$
$\gamma_{R,i}$	the signal to noise ratio applied to the detector of terminal $i$ after maximal
	ratio combining

- $H_{K,i}$  the received signal to interference plus noise ratio of terminal *i* as a result of the signals from all the terminals in set  $T_{P(i)}$
- $\mu_x$  the mean of random variable *x*
- $\Omega$  the typical standard deviation of lognormal random variables
- $\psi_{P(i),i}$  the received signal to noise ratio  $\gamma_{P(i),i}$  when  $\beta_i = 0$  for all the terminals  $T_i$ in set  $T_{P(i)}$
- $\Psi_{P(i),i}$  the received signal to interference plus noise ratio  $H_{P(i),i}$  when  $\beta_i = 0$  for all the terminals  $T_i$  in set  $T_{P(i)}$

 $\binom{n}{k}$  *n* choose *k* 

## Chapter 1 - Introduction

Traditionally, consumer wireless telecommunications services have been provided using cellular architectures where many mobile wireless terminals communicate directly with a single fixed base station with a wired connection to the public telecommunications infrastructure [24]. As progressive generations of cellular systems have been employed, there has been an evolutionary improvement in the quality and breadth of services offered to the consumer. From first generation analog voice systems, through second generation digital voice systems, and finally into upcoming third generation digital data systems, much of this evolution has been driven by an improved understanding of the nature of the wireless communications channel and supporting developments in digital signal processing hardware.

This thesis is concerned with a proposed wireless system wherein traditional transmission constraints are removed in order to allow direct communication between mobile terminals. This system gives mobile terminals the ability to relay information when they are neither the initial transmitter nor the final receiver. This relaying can be applied to cellular, ad-hoc, and hybrid networks in order to increase coverage, throughput, and capacity. In conjunction with ongoing research in the areas of spatial-temporal processing and smart antennas, among others, this has the potential to revolutionize the field of wireless communication.

## 1.1 Motivation

Relaying systems realize a number of benefits over traditional systems in the areas of deployment, connectivity, adaptability and capacity [14,15]. Minimizing the need for

fixed infrastructure results in networks that are easier, faster and cheaper to deploy. In some scenarios where the installation of fixed infrastructure is difficult due to time and location constraints, relaying techniques enable network connectivity where traditional architectures are impractical. Additionally, these techniques may be used to improve the coverage of existing networks by extending the periphery and closing internal gaps [13].

Mesh connectivity leads to networks that are inherently robust and adaptable to changing environments. Adaptive routing and load balancing techniques can be applied to allow these networks to self-configure in order to scale seamlessly as more terminals are connected [23]. In fact, every additional terminal added to the network increases the overall connectivity and improves fault tolerance. The physical topology of the network emulates the topology and protocols of the Internet itself. Shorter transmission distances and more efficient utilization of transmitted signal energy result in lower power levels throughout the system [18,27]. This implies increased channel capacity, decreased interference levels, reduced terminal radiation, and longer battery life. The application of various spatial diversity techniques enabled by the mesh connectivity of the mobile terminals has the potential to further compound these improvements.

In order to quantify these benefits and gain a better understanding of the associated issues it is first necessary to provide a mathematical foundation for this system through a derived characterization of the multihop wireless communications channel. System and channel models must be developed in order to provide a framework for comparison with the traditional singlehop channel. These models must be considered with the goal of realizable implementation in order to highlight their applicability and constraints with respect to specific design decisions.

#### 1.2 Overview

This thesis proposes four channel models for the case where mobile terminals act as intermediate relays in wireless communications systems. These are referred to as the *decoded relaying multihop channel*, the *amplified relaying multihop channel*, the *decoded relaying multihop diversity channel*, and the *amplified relaying multihop diversity channel*.

The decoded relaying multihop channel corresponds to the case where each intermediate terminal digitally decodes and re-encodes the received signal from the immediately preceding terminal before retransmission. The amplified relaying multihop channel corresponds to the case where each intermediate terminal simply amplifies the received signal from the immediately preceding terminal before retransmission. The decoded relaying multihop diversity channel corresponds to the case where each intermediate terminal combines, digitally decodes, and re-encodes the received signals from all preceding terminals before retransmission. The amplified relaying multihop diversity channel corresponds to the case where each intermediate terminal simply combines and amplifies the received signals from all preceding terminals before retransmission.

As indicated by these descriptions, there are two classifications: decoded relaying versus amplified relaying and multihop versus multihop diversity. When comparing decoded relaying with amplified relaying the primary considerations are noise propagation, error introduction, and delay. Decoded relaying does not propagate noise along the channel, introduces the possibility of decoding error at each intermediate terminal, and experiences delay due to intermediate terminal decoding as well as signal

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propagation. Amplified relaying does propagate noise along the channel, introduces the possibility of decoding error at the destination terminal, and experiences delay due solely to signal propagation. When comparing multihop channels without diversity versus multihop channels with diversity the primary considerations are performance and complexity. Although the performance of the multihop diversity channels is clearly superior, they entail significantly more complex receiver structures and protocols.

These channel models are described via a set of mathematical characterizations that provide comparison with the traditional singlehop channel model used as a base reference. Specific attributes compared include signal to noise ratio, probability of outage, probability of error, and optimal power distribution. Simulations are executed in order to validate the theory and analyze the peak performance and sensitivity to intermediate terminal placement of each proposed channel model. A number of implementation considerations are discussed in order to provide scope for the theoretical results.

### 1.3 Relevant Literature

In order to present this thesis in the proper scope, it is important to distinguish the intended context in relation to relevant publications. The work included in this thesis is in the process of publication in the proceedings of a number of conferences. An overview of Chapters 2, 3, and 4 is presented in [4]. A more detailed discussion of multihop channels without diversity from Chapters 2 and 4 is presented in [5]. A more detailed discussion of multihop channels with diversity from Chapters 3 and 4 is presented in [6].

Recently, a few papers have been published in the area of relaying for wireless communications, although they generally do not deal with physical layer issues. The

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benefits of relaying for power reduction and coverage extension in microcell environments are presented in [13]. The application of relaying for load balancing between cells is presented in [23]. A scheme for increasing the capacity of cellular radio systems through intelligent selection of relaying terminals is presented in [28]. The concept of using cooperation between mobile terminals to generate diversity is presented in [27]. A number of energy efficient receiver designs for relaying channels employing two hops are presented in [18]; this paper is the most similar to the current thesis in intention and content.

There are a number of publications in different areas related to relaying for communications. A set of capacity theorems for generic relaying channels is presented in [9]. The gaussian parallel relay network is presented in [26]. The partial converse for a relay channel is presented in [33]. These three papers are simply examples, and by no means form a representative set of publications in this area. The interested reader is encouraged to investigate the references of the listed papers.

Significant research is ongoing in the area of ad hoc networks, which conceptually employ relaying channels similar to the decoded relaying multihop channel. A good overview, as well as an indication of the expanding interest in this area, is presented in [14] and [15]. Various derivations of the theoretical capacity of ad hoc networks are presented in [12], [29], and [30]. The majority of current research in the area of ad hoc networks is being performed in the derivation and analysis of ad hoc routing protocols. Specific ad hoc routing protocols are presented in [17] and [20]. Analysis and comparison of a number of proposed routing protocols is presented in [7], [10], and [16]. A path availability model for ad hoc networks is presented in [19].

Relaying techniques dramatically change the application of resource allocation and power control. Power control for generic spatial diversity systems is presented in [11]. Power control with the intention of improving performance in packet radio systems in presented in [32]. A large number of publications exist for power control in traditional cellular systems, but are not directly applicable to relaying networks. Derivations for the probability of error performance of diversity systems is presented in [2] and [21]. A set of methods for the approximation of compound lognormal sums is presented in [1], [3], [8], and [25].

## 1.4 Organization

Chapter 2 presents theoretical results for multihop channels where diversity combining is not employed. A mathematical model for multihop channels without diversity is developed and notation is introduced. Two channel models are proposed and developed: the decoded relaying multihop channel and the amplified relaying multihop channel.

Chapter 3 defines the terms *multihop diversity* and *multiroute diversity* and presents theoretical results for multihop channels where diversity combining is employed. A mathematical model for multihop channels with diversity is developed and notation is introduced. Two channel models are proposed and developed: the decoded relaying multihop diversity channel and the amplified relaying multihop diversity channel.

Chapter 4 applies the theoretical results presented in Chapter 2 and 3 in two simulations. The first presents results for fixed terminal positions and provides a comparison with the singlehop reference channel on the basis of probability of outage and probability of error. The second presents results for variable intermediate terminal

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positions and provides a comparison with the singlehop reference channel on the basis of probability of error.

Chapter 5 presents a number of important implementation considerations. The theoretical results are refined and discussed in the light of different system constraints and design decisions.

Chapter 6 provides some concluding remarks. The results of this thesis are summarized and the contributions are described. A number of areas for future research are identified.

## Chapter 2 - Multihop Channels without Diversity

This chapter is concerned with multihop channels where diversity combining is not employed. A mathematical model for multihop channels without diversity is developed and notation is introduced. Two channel models are proposed and developed: one where each intermediate terminal digitally decodes and re-encodes the received signal from the immediately preceding terminal and the other where each intermediate terminal simply amplifies the received signal from the immediately preceding terminal. These are referred to as the decoded relaying multihop channel and the amplified relaying multihop channel. These two channels models for multihop channels without diversity are then characterized with respect to signal to noise ratio, probability of outage, probability of error, and optimal power distribution. A similar analysis for multihop channels with diversity is included in Chapter 3.

## 2.1 System Model

The system model for multihop wireless communications channels without diversity is composed of a source terminal, a receiving terminal, and an indeterminate number of potential intermediate relaying terminals. In Figure 1 the source terminal is identified as  $T_1$ , the destination terminal is identified as  $T_{n+1}$  and the intermediate terminals are identified as  $T_2$  through  $T_n$ , where *n* is the number of hops along the transmission path from the source terminal to the destination terminal.



Figure 1. Multihop Wireless Communications Channel without Diversity

Let  $T_S$  represent the set of source terminals,  $T_I$  represent the set of intermediate terminals, and  $T_D$  represent the set of destination terminals. Therefore  $T_T = T_S \cup T_I$ represents the set of all transmitting terminals and  $T_R = T_I \cup T_D$  represents the set of all receiving terminals. Let  $T_{P(i)}$  represent the set of terminals that transmit a signal received by  $T_i$ . The notation used in this chapter assumes that  $T_{P(i)}$  has cardinality equal to 1 in order to support the characterization of scenarios without diversity. For the communications channel illustrated in Figure 1,  $T_S = \{T_1\}$ ,  $T_I = \{T_2,...,T_n\}$ ,  $T_D = \{T_{n+1}\}$ ,  $T_T = \{T_1,...,T_n\}$ ,  $T_R = \{T_2,...,T_{n+1}\}$  and  $T_{P(i)} = \{T_{i-1}\}$ . These definitions are used in variable subscripts to denote specific terminals or sets of terminals. Notation of the form  $x_{T_i,T_i}$  is abbreviated to  $x_{j,i}$  for simplicity of exposition.

Assuming antipodal signaling, each terminal  $T_i$  transmits a signal with complex baseband amplitude given by

$$s_i = \sqrt{\varepsilon_i} a + \beta_i, \tag{1}$$

where  $\varepsilon_i$  is the transmitted power,  $a \in \{-1,1\}$  is the binary information symbol at time *t*, and  $\beta_i$  is propagated noise. The propagated noise term in (1) is zero for source terminals as well as for intermediate terminals that employ decoded relaying.

Assuming flat fading, each terminal  $T_i$  then receives a signal with complex baseband amplitude given by

$$r_{P(i),i} = \alpha \sqrt{(L_{P(i),i} / d_{P(i),i}^{p})} R_{P(i),i} (\sqrt{\varepsilon_{P(i)}} a + \beta_{P(i)}) + z_{P(i),i},$$
(2)

where  $\alpha^2$  is the free space signal power attenuation factor between the transmitting terminal and an arbitrary reference distance,  $d_{P(i),i}$  is the inter-terminal distance normalized with respect to the reference distance, p is the propagation exponent,  $L_{P(i),i}$ is a lognormal random variable with mean 0 dB and variance  $\sigma^2_{L_{P(i),i}}$ ,  $R_{P(i),i}$  is a complex gaussian (Rayleigh) random variable with variance  $\sigma^2_{R_{P(i),i}} = 0.5$  and  $z_{P(i),i}$  is a zerogaussian random variable with variance  $N_0$ .

Using this model, the received signal to noise ratio at  $T_i$  is given by

$$\gamma_{P(i),i} = \frac{\alpha^2 \varepsilon_{P(i)}}{\left( \frac{d_{P(i),i}}{d_{P(i),i}} \Big| L_{P(i),i} \Big|^2 \right) N_0 + \alpha^2 \Big| \beta_{P(i)} \Big|^2},$$
(3)

where  $|R_{P(i),i}|^2$  is an exponential random variable with mean  $2\sigma_{R_{P(i),i}}^2 = 1$ .

The probability of outage due to lognormal shadowing when  $\beta_{P(i)} = 0$  is given in [24] by

$$\Pr[\gamma_{P(i),i} < \gamma] = Q(\frac{10\log(\overline{\gamma_{P(i),i}}/\gamma)}{\sigma_{L_{P(i),i}}}), \qquad (4)$$

where  $\gamma$  is an arbitrary threshold signal to noise ratio that must be maintained at every

decoder in order to maintain communication and  $Q(x) = \frac{1}{2} \operatorname{erfc}(\frac{x}{\sqrt{2}})$ .

The calculation of probability of error is dependent on the modulation scheme employed. For the special case of BPSK, the probability of error averaged over Rayleigh fading when  $\beta_{P(i)} = 0$  is given in [21] by

$$P_e(\gamma_{P(i),i}) \approx \frac{1}{2\overline{\gamma_{P(i),i}}}, \overline{\gamma_{P(i),i}} >> 1.$$
(5)

The reference channel in Figure 2 is used for comparison purposes and corresponds to the singlehop transmission scheme used in traditional wireless systems. Throughout this thesis, comparative results are expressed in terms of the transmitted power of the reference channel, denoted by  $\varepsilon_0$ .



Figure 2. Singlehop Reference Channel

## 2.2 Decoded Relaying Multihop Channel

As stated in Chapter 1, the decoded relaying multihop channel corresponds to the case where each intermediate terminal digitally decodes and re-encodes the received signal from the immediately preceding terminal before retransmission. This digital relaying channel does not propagate noise along the multihop channel. The possibility of decoding error is introduced at each intermediate terminal.

#### 2.2.1 Channel Model

The channel model for the decoded relaying multihop channel is given by (1) through (3) with  $\beta_{P(i)} = 0$ . The received signal to noise ratio at terminal  $T_i$  as a result of the signal from  $T_{P(i)}$  is given by

$$\gamma_{P(i),i} = \frac{\alpha^2 \varepsilon_{P(i)}}{\left( \frac{d_{P(i),i}}{L_{P(i),i}} \Big| R_{P(i),i} \Big|^2 \right) N_0} \,.$$
(6)

## 2.2.2 Probability of Outage

The total probability of outage for the decoded relaying multihop channel is given by

$$P_{o} = 1 - \prod_{T_{i} \in T_{R}} (1 - \Pr[\gamma_{P(i),i} < \gamma]),$$
(7)

where  $\Pr[\gamma_{P(i),i} < \gamma]$  is the probability of outage at terminal  $T_i$  given a received signal to noise ratio of  $\gamma_{P(i),i}$ . This value can be upper-bounded by

$$P_o \le \sum_{T_i \in T_R} \Pr[\gamma_{P(i),i} < \gamma].$$
(8)

## 2.2.3 Probability of Error

The total probability of decoding error for the decoded relaying multihop channel is approximated by

$$P_{e} \approx 1 - \prod_{T_{i} \in T_{R}} (1 - P_{e}(\gamma_{P(i),i})),$$
(9)

where  $P_e(\gamma_{P(i),i})$  is the probability of decoding error at terminal  $T_i$  given a received signal to noise ratio of  $\gamma_{P(i),i}$ . This value can be upper-bounded by

$$P_e \le \sum_{T_i \in T_R} P_e(\gamma_{P(i),i}) .$$
<sup>(10)</sup>

## 2.2.4 Optimal Power Distribution

In order to provide a fair comparison with the reference channel, the transmit powers at each terminal are constrained so that the sum of the powers at each hop is equal to the reference power  $\varepsilon_0$ , namely  $\varepsilon_0 = \sum_{T_i \in T_T} \varepsilon_i$ . The upper bound of the total probability of

decoding error for the decoded relaying multihop channel is minimized when the sum of

the probabilities of decoding error at each terminal in (10) is minimized. The optimal power distribution based on the upper bound in (10) is then given by

$$\varepsilon_{P(i)} = \frac{\varepsilon_0 \sqrt{d_{P(i),i}^p / L_{P(i),i}}}{\sum_{T_j \in T_R} \sqrt{d_{P(j),j}^p / L_{P(j),j}}},$$
(11)

and substituting (11) into (6) the optimal received signal to noise ratio at terminal  $T_i$  is given by

$$\gamma_{P(i),i} = \frac{\alpha^{2} \varepsilon_{0} / (\sum_{T_{j} \in T_{R}} \sqrt{d_{P(j),j}^{p} / L_{P(j),j}})}{(\sqrt{d_{P(i),i}^{p}} / \sqrt{L_{P(i),i}} |R_{P(i),i}|^{2}) N_{0}}.$$
(12)

For the proof of (11), refer to Appendix A. It is interesting to note from (11) that each optimal power value is proportional to the square root of the signal attenuation on the respective hop. This implies that terminals transmitting across hops with unfavorable conditions (high attenuation) will have optimal power values that are higher than terminals transmitting across hops with favorable conditions (low attenuation). This is intuitively satisfying, in that hops with the least favorable conditions should require the most power.

## 2.3 Amplified Relaying Multihop Channel

As stated in Chapter 1, the amplified relaying multihop channel corresponds to the case where each intermediate terminal simply amplifies the received signal from the immediately preceding terminal before retransmission. This analog relaying channel propagates noise along the multihop channel. The possibility of decoding error is introduced at the destination terminal.

#### 2.3.1 Channel Model

The channel model for the amplified relaying multihop channel is composed of a set of individual transmission channels given by (1) through (3). Assuming that each intermediate terminal can track both lognormal shadowing and Rayleigh fading, the amplification factor at each intermediate terminal  $T_i$  is simply the transmitted power over the received power and is given by

$$A_{i} = \frac{\varepsilon_{i}}{\alpha^{2} (\varepsilon_{P(i)} + \left| \beta_{P(i)} \right|^{2}) L_{P(i),i} \left| R_{P(i),i} \right|^{2} / d_{P(i),i}^{p} + N_{0}}.$$
(13)

Although this amplification factor is exact, the resulting mathematical characterization of the channel model is extremely complex. In order to simplify the amplification factor, an approximation can be made where the noise terms are removed from the denominator of (13). This approximation yields an upper bound on the amplification factor, which is tight provided that the signal to noise ratio at the terminal under consideration is significantly greater than 1.

The amplification factor at each terminal is then given by

$$A_{i} \leq \frac{\varepsilon_{i} d_{P(i),i}^{p}}{\alpha^{2} \varepsilon_{P(i)} L_{P(i),i} \left| R_{P(i),i} \right|^{2}},$$
(14)

and the received signal to noise ratio at the destination terminal  $T_d$  is given by

$$\gamma_{P(d),d} \approx \frac{\alpha^2}{\sum_{T_i \in T_R} \left( d_{P(i),i}^p \left| \varepsilon_{P(i)} L_{P(i),i} \right|^2 \right) N_0} \,. \tag{15}$$

For the proof of (15), refer to Appendix A. The additional terms in the denominator of (15) where  $T_i \in T_I$  are the result of the propagated noise term in (3), given by

$$\left|\beta_{P(d)}\right|^{2} = \sum_{T_{i}\in T_{I}} \left(\varepsilon_{P(d)}d_{P(i),i}^{p} \middle/ \alpha^{2}\varepsilon_{P(i)}L_{P(i),i} \middle| R_{P(i),i} \middle|^{2}\right) N_{0} .$$
(16)

Alternatively, the received signal to noise ratio at the destination terminal (15) can be rearranged and expressed as

$$\gamma_{P(d),d}^{-1} \approx \sum_{T_i \in T_R} \psi_{P(i),i}^{-1},$$
 (17)

where  $\psi_{P(i),i}$  is the received signal to noise ratio  $\gamma_{P(i),i}$  at terminal  $T_i$  with  $\beta_{P(i)} = 0$ .

#### 2.3.1.1 Approximating the Sum of Inverse Exponential Components

The channel model for the amplified relaying multihop channel can be simplified by approximating the sum of inverse exponential components contained in (17) [18]. This approximation does not take into account the lognormal shadowing characteristics of the channel and therefore provides a characterization that is constrained to small-scale channel effects.

It follows from the form of (17) that the total received signal to noise ratio at the destination terminal  $T_n$  can be upper-bounded by

$$\gamma_{P(d),d} \leq \min_{T_i \in T_R} \{ \psi_{P(i),i} \} \,. \tag{18}$$

Since  $\psi_{P(i),i}$  are independent exponential random variables, the minimum is also an exponential random variable with mean  $(\sum_{T_i \in T_R} \overline{\psi_{P(i),i}}^{-1})^{-1}$ , where  $\overline{\psi_{P(i),i}}$  is the expected value of  $\psi_{P(i),i}$  [18]. The total received signal to noise ratio at the destination terminal can now be approximated by

$$\gamma_{P(d),d} \approx \frac{\alpha^2 \left| R_{Z(d)} \right|^2}{\sum\limits_{T_i \in T_R} \left( d_{P(i),i}^p / \varepsilon_{P(i)} L_{P(i),i} \right) N_0},\tag{19}$$

where  $|R_{Z(d)}|^2$  is an exponential random variable with mean  $2\sigma_{R_{Z(d)}}^2 = 1$  created as a result of the compound approximation.

#### 2.3.1.2 Approximating the Power Sum of Lognormal Components

The channel model for the amplified relaying multihop channel can be further simplified by approximating the power sum of lognormal components contained in (17). This approximation does not take into account the Rayleigh fading characteristics of the channel and therefore provides a characterization that is constrained to large-scale channel effects.

It is well known that the power sum of lognormal components can be approximated by another lognormal random variable [1,3,8,25]. There are a number of methods that can be used to calculate the mean and standard deviation of this approximation. The most tractable method, known as Wilkinson's method, yields very good results.

Let the approximation of the power sum of lognormal components be given by

$$L_{Z(d)} = \sum_{T_i \in T_R} L_{P(i),i}$$
  
= 
$$\sum_{T_i \in T_R} e^{Y_{P(i),i}},$$
 (20)

where  $L_{Z(d)}$  is a lognormal random variable with mean  $\mu_{L_{Z(d)}}$  and standard deviation  $\sigma_{L_{Z(d)}}$ . Each lognormal component  $L_{P(i),i}$  is independent with mean  $\mu_{L_{P(i),i}} = \ln(d_{P(i),i}^{p}/\varepsilon_{P(i)})$  and standard deviation  $\sigma_{L_{P(i),i}} = \Omega$ , where  $\Omega$  is typically between 6 and 12 dB or between 1.4 and 2.8 in the natural logarithmic scale.

Following Wilkinson's method, matching the first moment of  $L_{Z(d)}$  gives

$$u_{1} = \sum_{T_{i} \in T_{R}} e^{\ln(d_{P(i),i}^{p}/\varepsilon_{P(i)}) + \Omega^{2}/2} = e^{\Omega^{2}/2} \sum_{T_{i} \in T_{R}} (d_{P(i),i}^{p}/\varepsilon_{P(i)}),$$
(21)

and matching the second moment of  $L_{Z(d)}$  gives

$$u_{2} = \sum_{T_{i} \in T_{R}} e^{2 \ln(d_{P(i),i}^{p}/\varepsilon_{P(i),i}) + 2\Omega^{2}} + 2 \sum_{T_{i} \in T_{I}} \sum_{T_{j} \in T_{R}, j \in T_{R}} e^{\ln(d_{P(i),i}^{p}/\varepsilon_{P(i)}) + \ln(d_{P(j),j}^{p}/\varepsilon_{P(j)}) + \Omega^{2}}$$

$$= e^{2\Omega^{2}} \sum_{T_{i} \in T_{R}} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{I}} \sum_{T_{j} \in T_{R}, j > i} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{I}} \sum_{T_{j} \in T_{R}, j > i} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{I}} \sum_{T_{i} \in T_{R}, j > i} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \in T_{R}} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \in T_{R}} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \in T_{R}} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \in T_{R}} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \in T_{R}} (d_{P(i),i}^{p}/\varepsilon_{P(i)})^{2} + 2e^{\Omega^{2}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \inT_{R}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \in T_{R}} \sum_{T_{i} \inT$$

The total received signal to noise ratio at the destination terminal can now be approximated by

$$\gamma_{P(d),d} \approx \frac{\alpha^2 \left| R_{Z(d)} \right|^2}{L_{Z(d)} N_0},$$
(23)

where  $L_{Z(d)}$  is a lognormal random variable created as a result of the compound approximation and the mean and standard deviation of the corresponding gaussian random variable are given by

$$\mu_{L_{Z(d)}} = \frac{(20\ln(u_1) - 5\ln(u_2))}{\ln(10)}, \text{ and}$$
(24)

$$\sigma_{L_{Z(d)}} = \frac{\sqrt{100 \ln(u_2) - 200 \ln(u_1)}}{\ln(10)}.$$
(25)

## 2.3.2 Probability of Outage

The probability of outage for the amplified relaying multihop channel at the destination terminal is given by

$$P_o = \Pr[\gamma_{P(d),d} < \gamma] = Q(\frac{10\log(\overline{\gamma_{P(d),d}}/\gamma)}{\sigma_{L_{Z(d)}}}), \qquad (26)$$

where  $\Pr[\gamma_{P(d),d} < \gamma]$  is the probability of outage at the destination terminal given a received signal to noise ratio of  $\gamma_{P(d),d}$ .

## 2.3.3 Probability of Error

The total probability of decoding error for the amplified relaying multihop channel is given by

$$P_e = P_e(\gamma_{P(d),d}), \qquad (27)$$

where  $P_e(\gamma_{P(d),d})$  is the probability of decoding error at the destination terminal given a received signal to noise ratio of  $\gamma_{P(d),d}$ .

## 2.3.4 Optimal Power Distribution

As for the decoded relaying multihop channel, in order to provide a fair comparison with the reference channel, the transmit powers at each terminal are constrained so that the sum of the powers at each hop is equal to the reference power  $\varepsilon_0$ , namely  $\varepsilon_0 = \sum_{T_i \in T_T} \varepsilon_i$ . The total probability of decoding error for the amplified relaying multihop

channel is minimized when the received signal to noise ratio at the destination terminal is

maximized. The optimal power distribution based on the approximation in (17) is then given by

$$\varepsilon_{P(i)} = \frac{\varepsilon_0 \sqrt{d_{P(i),i}^p / L_{P(i),i}}}{\sum_{T_j \in T_R} \sqrt{d_{P(j),j}^p / L_{P(j),j}}},$$
(28)

and substituting (28) into (15) the optimal received signal to noise ratio at the destination terminal is given by

$$\gamma_{P(d),d} = \frac{\alpha^{2} \varepsilon_{0} / (\sum_{T_{j} \in T_{R}} \sqrt{d_{P(j),j}^{p} / L_{P(j),j}})}{\sum_{T_{i} \in T_{R}} (\sqrt{d_{P(i),i}^{p}} / \sqrt{L_{P(i),i}} |R_{P(i),i}|^{2}) N_{0}}.$$
(29)

For the proof of (28), refer to Appendix A. It is interesting to note that the form of (28) is identical to the form of (11), and therefore the previous discussion relating to (11) holds for (28) as well. The equality of (11) and (28) is the result of a derived equivalency between the approximations for the total probability of error averaged over Rayleigh fading for the decoded relaying multihop channel and amplified relaying multihop channel. For clarification, consider a channel with *n* hops where the received signal to noise ratio at each terminal is equal and given by  $\gamma_i$ . Then the total probability of error for the decoded relaying multihop channel, according to the upper bound in (10), is given by  $P_e \leq n/2\gamma_i$  and the total probability of error for the amplified relaying multihop channel, according to the approximation in (17), is given by  $P_e \approx 1/2(\sum_{i=1}^n \gamma_i^{-1})^{-1}$ , which can be simplified to result in the same form as that for the decoded relaying multihop channel. Since both the total probability of error and the component signal to noise ratios

of the two multihop channels are identical, the component optimal power values should therefore be identical as well.

## 2.4 Summary

This chapter has presented theoretical characterizations for multihop channels without diversity: the decoded relaying multihop channel where digital relaying is employed and the amplified relaying multihop channel where analog relaying is employed. These characterizations are tractable and enable the quick comparison of the proposed channels with the singlehop reference channel on the basis of signal to noise ratio, probability of outage, probability of error, and optimal power distribution. The approximations and simplifications utilized in the generation of these results will be validated through the use of simulation in Chapter 4. Implementation issues related to these two channel models will be discussed in Chapter 5.
# Chapter 3 - Multihop Channels with Diversity

This chapter is concerned with multihop channels where diversity combining is employed. The terms multihop diversity and multiroute diversity are defined and distinguished. A mathematical model for multihop channels with diversity is developed and notation is introduced. Two channel models are proposed and developed: one where each intermediate terminal digitally combines, decodes, and re-encodes the received signals from all preceding terminals and the other where each intermediate terminal simply combines and amplifies the received signals from all preceding terminals. These are referred to as the decoded relaying multihop diversity channel and the amplified relaying multihop diversity channel. These two channels models for multihop channels with diversity are then characterized with respect to signal to noise ratio, probability of outage, probability of error, and optimal power distribution. A similar analysis for multihop channels without diversity is included in Chapter 2.

## 3.1 Multihop Diversity versus Multiroute Diversity

Before proceeding with the characterization of multihop wireless communications channels employing spatial diversity techniques, it is first necessary to define and distinguish the two relevant classes of techniques: multihop diversity and multiroute diversity. Multiroute diversity results from the concurrent reception of signals that have been transmitted along multiple routes that pass through different intermediate terminals. Multihop diversity results from the concurrent reception of signals that have been transmitted by multiple previous terminals along a single route. Each diversity technique realizes both microdiversity and macrodiversity benefits. Microdiversity benefits are achieved as each received signal experiences independent delay characteristics composed of variable propagation and processing delays. Macrodiversity benefits are achieved as each received signal experiences independent shadowing characteristics.

Figure 3 illustrates an example multihop diversity wireless communications channel. Terminal  $T_1$  transmits a signal  $s_1$  that is received by terminal  $T_2$  as  $r_{1,2}$  and terminal  $T_3$  as  $r_{1,3}$ . Terminal  $T_2$  transmits a signal  $s_2$  and that is received by terminal  $T_3$  as  $r_{2,3}$ . The signals  $r_{2,3}$  and  $r_{1,3}$  are then combined using traditional diversity combining techniques.



Figure 3. Wireless Communications Channel with Multihop Diversity

Figure 4 illustrates an example multiroute diversity wireless communications channel. Terminal  $T_1$  transmits a signal  $s_1$  that is received by terminal  $T_2$  as  $r_{1,2}$  and terminal  $T_4$  as  $r_{1,4}$ . Terminal  $T_2$  transmits a signal  $s_2$  and terminal  $T_4$  transmits a signal  $s_4$  that are received by terminal  $T_3$  as  $r_{2,3}$  and  $r_{4,3}$  respectively. The signals  $r_{2,3}$  and  $r_{4,3}$  are then combined using traditional diversity combining techniques.



Figure 4. Wireless Communications Channel with Multiroute Diversity

When comparing the potential performance and application of these diversity techniques it is important to note that multiroute diversity is caused by the artificial generation of multiple secondary signals whereas multihop diversity is caused by the natural generation of multiple secondary signals. This distinction between artificial and natural generation is based on the requirement for a redistribution of power within the system. Multiroute diversity requires a redistribution of power from terminal  $T_2$  to terminal  $T_4$  whereas multihop diversity requires no such redistribution of power.

#### 3.1.1 Analyzing the Case Supporting Multihop Diversity

The case supporting the multihop diversity channel in comparison to the multihop channel without diversity is straightforward. Since there is no requirement for a redistribution of power within the system, no additional resources must be expended in order to achieve a gain from diversity combining. This is a direct result of the nondirectionality of transmission within wireless systems where omnidirectional antennas are employed. Although a transmission may be intended for a specific terminal, the signal will reach a multitude of other terminals as well. In a multihop channel without diversity combining each terminal receives a signal solely from the immediately preceding terminal along the transmission route. In a multihop diversity channel each terminal receives and combines signals from all the preceding terminals along the transmission route. The only associated cost results from an increase in terminal receiver complexity.

### 3.1.2 Analyzing the Case Supporting Multiroute Diversity

Although it is safe to assume that the multihop diversity channel will outperform the multihop channel without diversity, the same is not the case for the multiroute diversity

channel. The redistribution of power from terminal  $T_2$  to terminal  $T_4$  creates two opposing trends in the resulting bit error rate of the decoder at terminal  $T_3$ . The first trend is a decrease in the bit error rate due to diversity combining. The second trend is an increase in the bit error rate due to a decrease in transmitted power at terminal  $T_2$ . These two opposing trends result in a point of intersection at which the bit error rate performance of a system with multiple routes matches the bit error rate performance of a system with a single route.

In order to support the use of multiroute diversity it is necessary to show an improvement in probability of error and probability of outage performance for reasonable signal to noise ratio values. Consider first the probability of outage due to large-scale lognormal effects. Since the actual average received signal to noise ratio of each route is dependent on the mobility of the respective intermediate terminal relative to the destination terminal, it will change slowly with respect to the period of the signal. The source terminal can therefore determine the best route in terms of actual average received signal to noise ratio and then transmit the signal along that route. The probability of outage when multiroute diversity is applied is therefore greater than or equal to the probability of outage when a single route is used.

Now consider the probability of error due to small-scale Rayleigh effects. Examine the scenario where the statistics of each of the routes are identical. This is in fact the optimal scenario in support of multiroute diversity because in any other scenario there would be more benefit gained from concentrating all the power along the best route. For this optimal scenario, the point of intersection occurs when the expected received signal to noise ratio at  $T_i$  for each branch of the diversity combiner is equal and given by

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$$\overline{\gamma_{k,i}} \approx (\frac{\binom{2K-1}{K}K^{K}}{2^{K-1}})^{\frac{1}{K-1}}, \overline{\gamma_{k,i}} >> 1, K \ge 2, T_{k} \in T_{P(i)},$$
(30)

where K is the number of multihop routes (order of diversity). For the proof of (30), refer to Appendix A. For received signal to noise ratios lower than this value the probability of error of the system with a single route is better. For received signal to noise ratios higher than this value the probability of error of the system with K routes is better.

Figure 5 shows the probability of error with respect to the normalized signal to noise ratio of the singlehop reference channel for channels with K = I, 2, and 4 statistically identical parallel routes, and is valid only for  $\overline{\gamma_{k,i}} \gg 1$  due to approximation of the probability of error. The points of intersection of the singlehop reference channel K = 1 with the other channels K > 1 can be calculated according to (30). Empirical observation of (30) and Figure 5 reveals that for all but large received signal to noise ratios there is a decrease or minimal increase in bit error rate performance versus a single route when multiroute diversity is applied in this optimal scenario in support of multiroute diversity.

This implies that the optimal multiroute diversity scenario, and by extension all suboptimal scenarios, will generally be inferior to the system with a single route. Additionally, since the probability of having a system with statistically identical parallel channels is very low, most systems employing multiroute diversity will significantly underperform this bound. Coupled with the fact that in realistic implementations received signal to noise ratios will be optimized to the minimum value able to support a given quality of service class, this questions the potential benefits of using multiroute diversity. Therefore, from this point forward, all references to diversity within multihop channels will correspond to multihop diversity.



Figure 5. Error for Statistically Identical Parallel Routes with Multiroute Diversity

# 3.2 System Model

The system model for multihop wireless communications channels with diversity is composed of a source terminal, a receiving terminal, and an indeterminate number of potential intermediate relaying terminals. In Figure 6 the source terminal is identified as  $T_1$ , the destination terminal is identified as  $T_{n+1}$  and the intermediate terminals are identified as  $T_2$  through  $T_n$  where *n* is the number of hops along the transmission path. This is of course the same as for the multihop wireless communications channel without diversity. The only difference between the two models is the signal connectivity between non-adjacent terminals.



Figure 6. Multihop Wireless Communications Channel with Diversity

The notation used in this chapter assumes that  $T_{P(i)}$  has a cardinality greater than or equal to 1 in order to support the characterization of scenarios with diversity. For the communications channel illustrated in Figure 6,  $T_{P(i)} = \{T_1, ..., T_{i-1}\}$ . The remainder of the set notation for multihop wireless communications channels without diversity remains applicable. Maximal ratio combining is assumed in all diversity scenarios.

Assuming antipodal signaling, each terminal  $T_i$  transmits a signal with complex baseband amplitude given by

$$s_i = \sqrt{\varepsilon_i} a + \beta_i \,. \tag{31}$$

Assuming flat fading, each terminal  $T_i$  then receives a set of signals with complex baseband amplitudes given by

$$r_{k,i} = \alpha \sqrt{(L_{k,i} / d_{k,i}^p)} R_{k,i} (\sqrt{\varepsilon_k} a + \beta_k) + z_{k,i}, T_k \in T_{P(i)}, \qquad (32)$$

and has a received signal to noise ratio after maximal ratio combining given in [21] by

$$\gamma_{P(i),i} = \sum_{T_k \in T_{P(i)}} \left( \frac{\alpha^2 \varepsilon_k}{\left( \frac{d_{k,i}^p}{L_{k,i} \middle| R_{k,i} \middle|^2} \right) N_0 + \alpha^2 \left| \beta_k \right|^2} \right).$$
(33)

The probability of outage due to lognormal shadowing when  $\beta_k = 0, T_k \in T_{P(i)}$  is given in [24] by

$$\Pr[\gamma_{P(i),i} < \gamma] \approx Q(\frac{10\log(\overline{\gamma_{P(i),i}}/\gamma)}{\sigma_{L_{Z(i)}}}), \qquad (34)$$

where  $\sigma_{L_{Z(i)}}^2$  is the variance of the lognormal approximation of (33) determined using Wilkinson's method [1], and the probability of error for BPSK averaged over Rayleigh fading when  $\beta_k = 0, T_k \in T_{P(i)}$  is given in [21] by

$$P_e(\gamma_{P(i),i}) \approx \binom{2K-1}{K} \prod_{T_k \in T_{P(i)}} \left(\frac{1}{2\overline{\gamma_{k,i}}}\right), \overline{\gamma_{k,i}} \gg 1,$$
(35)

where *K* is the cardinality of  $T_{P(i)}$  and  $\overline{\gamma_{k,i}}$  is the expected received signal to noise ratio at  $T_i$  for branch *k* of the diversity combiner.

#### 3.3 Decoded Relaying Multihop Diversity Channel

As stated in Chapter 1, the decoded relaying multihop diversity channel corresponds to the case where each intermediate terminal combines, digitally decodes, and re-encodes the received signals from all preceding terminals before retransmission. This digital relaying channel does not propagate noise along the multihop channel. The possibility of decoding error is introduced at each intermediate terminal.

### 3.3.1 Channel Model

The channel model for the decoded relaying multihop diversity channel is given by (31) through (33) with  $\beta_k = 0, T_k \in T_{P(i)}$ . The received signal to noise ratio at terminal  $T_i$  is given by

$$\gamma_{P(i),i} = \sum_{T_k \in T_{P(i)}} \left( \frac{\alpha^2 \varepsilon_k}{\left( \frac{d_{k,i}^p}{L_{k,i} | R_{k,i} |^2} \right) N_0} \right).$$
(36)

### 3.3.2 Probability of Outage

The total probability of outage for the decoded relaying multihop diversity channel is given by

$$P_{o} = 1 - \prod_{T_{i} \in T_{R}} (1 - \Pr[\gamma_{P(i),i} < \gamma]), \qquad (37)$$

where  $\Pr[\gamma_{P(i),i} < \gamma]$  is the probability of outage at terminal  $T_i$  given a received signal to noise ratio of  $\gamma_{P(i),i}$ . This value can be upper-bounded by

$$P_o \le \sum_{T_i \in T_R} \Pr[\gamma_{P(i),i} < \gamma].$$
(38)

# 3.3.3 Probability of Error

The total probability of decoding error for the decoded relaying multihop diversity channel is approximated by

$$P_{e} \approx 1 - \prod_{T_{i} \in T_{R}} (1 - P_{e}(\gamma_{P(i),i})),$$
(39)

where  $P_e(\gamma_{P(i),i})$  is the probability of decoding error at terminal  $T_i$  given a received signal to noise ratio of  $\gamma_{P(i),i}$ . This value can be upper-bounded by

$$P_e \le \sum_{T_i \in T_R} P_e(\gamma_{P(i),i}) .$$
(40)

### 3.3.4 Optimal Power Distribution

Once again, in order to provide a fair comparison with the reference channel, the transmit powers at each terminal are constrained so that the sum of the powers at each hop is equal to the reference power  $\varepsilon_0$ , namely  $\varepsilon_0 = \sum_{T_i \in T_T} \varepsilon_i$ . The upper bound of the total probability of decoding error for the decoded relaying multihop diversity channel is minimized when the sum of the probabilities of decoding error at each terminal in (40) is minimized.

In general, the solution to this minimization problem is not tractable. Although (40) is convex with respect to the transmit powers at each terminal and the solution for any finite number of hops can be calculated using Lagrange multipliers, the resulting equations are extremely complex and not easy to generalize. However, the solution for a channel with n = 2 hops is fairly tractable and shall be considered as a representative example.

Consider the channel with source terminal  $T_1$ , intermediate terminal  $T_2$  and destination terminal  $T_3$ . The total probability of decoding error based on the upper bound in (40) is given by

$$P_{e} \leq \frac{1}{2} \left( \frac{\alpha^{2} \varepsilon_{1}}{(d_{1,2}^{p} / L_{1,2}) N_{0}} \right)^{-1} + \frac{3}{4} \left( \frac{\alpha^{2} \varepsilon_{1}}{(d_{1,3}^{p} / L_{1,3}) N_{0}} \right)^{-1} \left( \frac{\alpha^{2} \varepsilon_{2}}{(d_{2,3}^{p} / L_{2,3}) N_{0}} \right)^{-1},$$

$$(41)$$

the optimal power distribution that minimizes (41) is given by

$$\varepsilon_{1} = \varepsilon_{0} + \frac{1}{2(d_{1,2}^{p}/L_{1,2})} (3(d_{1,3}^{p}/L_{1,3})(d_{2,3}^{p}/L_{2,3}) - (6\varepsilon_{0}(d_{1,2}^{p}/L_{1,2})(d_{1,3}^{p}/L_{1,3})(d_{2,3}^{p}/L_{2,3}) , \text{ and } + 9(d_{1,3}^{p}/L_{1,3})^{2}(d_{2,3}^{p}/L_{2,3})^{2})^{1/2})$$
(42)

$$\varepsilon_{2} = \frac{1}{2(d_{1,2}^{p}/L_{1,2})} (-3(d_{1,3}^{p}/L_{1,3})(d_{2,3}^{p}/L_{2,3}) + (6\varepsilon_{0}(d_{1,2}^{p}/L_{1,2})(d_{1,3}^{p}/L_{1,3})(d_{2,3}^{p}/L_{2,3}) , + 9(d_{1,3}^{p}/L_{1,3})^{2}(d_{2,3}^{p}/L_{2,3})^{2})^{1/2})$$

$$(43)$$

and the optimal received signal to noise ratio at each terminal is given by (36) where the transmit powers are given by (42) and (43). For the proof of (42) and (43), refer to Appendix A.

### 3.4 Amplified Relaying Multihop Diversity Channel

As stated in Chapter 1, the amplified relaying multihop diversity channel corresponds to the case where each intermediate terminal simply combines and amplifies the received signals from all preceding terminals before retransmission. This analog relaying channel propagates noise along the multihop channel. The possibility of decoding error is introduced at the destination terminal.

#### 3.4.1 Channel Model

The channel model for the amplified relaying multihop diversity channel is composed of a set of individual transmission channels given by (31) through (33). Assuming that each intermediate terminal can track both lognormal shadowing and Rayleigh fading, the amplification factor at each intermediate terminal  $T_i$  is simply the transmitted power over the received power and is given by

$$A_{i} = \frac{\varepsilon_{i}}{\sum_{T_{k} \in T_{P(i)}} (\alpha^{2} (\varepsilon_{k} + |\beta_{k}|^{2}) L_{k,i} |R_{k,i}|^{2} / d_{k,i}^{p} + N_{0})}.$$
(44)

Although this amplification factor is exact, the resulting mathematical characterization of the channel model is extremely complex. In order to simplify the amplification factor, an approximation can be made where the noise terms are removed from the denominator of (44). This approximation yields an upper bound on the amplification factor, which is tight provided that the signal to noise ratio at the terminal under consideration is significantly greater than 1.

The amplification factor at each terminal is then given by

$$A_{i} \leq \frac{\varepsilon_{i}}{\sum\limits_{T_{k} \in T_{P(i)}} \left( \alpha^{2} \varepsilon_{k} L_{k,i} \middle| R_{k,i} \middle|^{2} \middle/ d_{k,i}^{p} \right)},$$
(45)

and received signal to noise ratio at the destination terminal  $T_d$  can be expressed recursively as

$$\gamma_{P(d),d} \approx \sum_{\substack{T_k \in T_{P(d)} \\ k \neq s}} (\gamma_{P(k),k}^{-1} + \psi_{k,d}^{-1})^{-1} + \psi_{s,d}, \qquad (46)$$

where  $\psi_{k,d}$  is the received signal to noise ratio  $\gamma_{k,d}$  at terminal  $T_d$  for branch k of the diversity combiner with  $\beta_k = 0$  and  $T_s$  is the source terminal. For the proof of (46), refer to Appendix A.

#### 3.4.1.1 Approximating the Sum of Inverse Chi-Square Components

The channel model for the amplified relaying multihop diversity channel can be simplified by approximating the inverse sum of inverse chi-square components contained in (46) [18]. This approximation does not take into account the lognormal shadowing characteristics of the channel and therefore provides a characterization that is constrained to small-scale channel effects.

It follows from (46) that the total received signal to noise ratio at the destination terminal  $T_d$  can be upper-bounded by

$$\gamma_{P(d),d} \le \sum_{\substack{T_k \in T_{P(d)} \\ k \neq s}} (\min\{\gamma_{P(k),k}, \psi_{k,d}\}) + \psi_{s,d}.$$
(47)

Since the two terms in the minimization in (47) are independent chi-square random variables, each minimum is also a chi-square random variable with mean  $(\overline{\gamma_{P(k),k}}^{-1} + \overline{\psi_{k,d}}^{-1})^{-1}$ , where  $\overline{\gamma_{P(k),k}}$  is the expected value of  $\gamma_{P(k),k}$  and  $\overline{\psi_{k,d}}$  is the expected value of  $\psi_{k,d}$  [18]. The degrees of freedom of the resulting chi-square random variable is approximated by  $\chi_{\gamma_{k,d}} = \min{\{\chi_{\gamma_{P(k),k}}, \chi_{\psi_{k,d}}\}}$ , where  $\chi_{\gamma_{P(k),k}}$  is the degrees of freedom of  $\psi_{k,d}$ . Due to the recursive nature of (47), the chi-square random variable at each branch of the combiner has two degrees of freedom. The total received signal to noise ratio at the destination terminal can now be approximated by

$$\gamma_{P(d),d} \approx \left(\sum_{\substack{T_k \in T_{P(d)} \\ k \neq s}} (\overline{\gamma_{P(k),k}}^{-1} + \overline{\psi_{k,d}}^{-1})^{-1} + \overline{\psi_{s,d}}) C_{Z(d)},$$
(48)

where  $C_{Z(d)}$  is a chi-square random variable with  $\chi_{Z(d)} = 2K$  degrees of freedom and mean  $2K\sigma_{R_{Z(d)}}^2 = 1$ .

# 3.4.2 Probability of Outage

The probability of outage for the amplified relaying multihop diversity channel at the destination terminal is given by

$$P_o = \Pr[\gamma_{P(d),d} < \gamma] \approx Q(\frac{10\log(\overline{\gamma_{P(d),d}}/\gamma)}{\sigma_{L_{Z(d)}}}), \qquad (49)$$

where  $\Pr[\gamma_{P(d),d} < \gamma]$  is the probability of outage at the destination terminal given a received signal to noise ratio of  $\gamma_{P(d),d}$  and  $\sigma_{L_{Z(d)}}^2$  is the variance of the lognormal approximation of (46) determined using Wilkinson's method [1].

### 3.4.3 Probability of Error

The total probability of decoding error for the amplified relaying multihop diversity channel is given by

$$P_e = P_e(\gamma_{P(d),d}) \approx {\binom{2K-1}{K}} \prod_{T_k \in T_{P(d)}} (\frac{1}{2\overline{\gamma_{k,d}}}), \overline{\gamma_{k,d}} \gg 1,$$
(50)

where  $P_e(\gamma_{P(d),d})$  is the probability of decoding error at the destination terminal given a received signal to noise ratio of  $\gamma_{P(d),d}$ , K is the cardinality of  $T_{P(d)}$ , and  $\overline{\gamma_{k,d}}$  is the expected received signal to noise ratio at the destination terminal for branch k of the diversity combiner.

### 3.4.4 Optimal Power Distribution

As for the decoded relaying multihop diversity channel, in order to provide a fair comparison with the reference channel, the transmit powers at each terminal are constrained so that the sum of the powers at each hop is equal to the reference power  $\varepsilon_0$ ,

namely  $\varepsilon_0 = \sum_{T_i \in T_T} \varepsilon_i$ . The total probability of decoding error for the amplified relaying multihop diversity channel is minimized when the product of the received signal to noise ratios at each branch of the diversity combiner in (50) is maximized.

In general, the solution to this minimization problem is not tractable. Although (50) is convex with respect to the transmit powers at each terminal and the solution for any finite number of hops can be calculated using Lagrange multipliers, the resulting equations are extremely complex and not easy to generalize. However, the solution for a channel with n = 2 hops is fairly tractable and shall be considered as a representative example.

Consider the channel with source terminal  $T_1$ , intermediate terminal  $T_2$  and destination terminal  $T_3$ . The total probability of decoding error based on the approximation in (50) is given by

$$P_{e} \approx \frac{3}{4} \left( \frac{\alpha^{2} \varepsilon_{1}}{(d_{1,3}^{p} / L_{1,3}) N_{0}} \right)^{-1} \left( \left( \frac{\alpha^{2} \varepsilon_{1}}{(d_{1,2}^{p} / L_{1,2}) N_{0}} \right)^{-1} + \left( \frac{\alpha^{2} \varepsilon_{2}}{(d_{2,3}^{p} / L_{2,3}) N_{0}} \right)^{-1} \right)$$
(51)

the optimal power distribution that minimizes (51) is given by

$$\varepsilon_{1} = \frac{\varepsilon_{0}}{4((d_{1,2}^{p}/L_{1,2}) - (d_{2,3}^{p}/L_{2,3}))} (4(d_{1,2}^{p}/L_{1,2}) - (d_{2,3}^{p}/L_{2,3}) - ((d_{2,3}^{p}/L_{2,3})) (8(d_{1,2}^{p}/L_{1,2})), \text{ and } (52) + (d_{2,3}^{p}/L_{2,3})))^{1/2})$$

$$\varepsilon_{2} = \frac{\varepsilon_{0}}{4((d_{1,2}^{p}/L_{1,2}) - (d_{2,3}^{p}/L_{2,3}))} (-3(d_{2,3}^{p}/L_{2,3})) + ((d_{2,3}^{p}/L_{2,3})(8(d_{1,2}^{p}/L_{1,2}) + (d_{2,3}^{p}/L_{2,3})))^{1/2})$$
(53)

and the optimal received signal to noise ratio at the destination terminal is given by (46) where the transmit powers are given by (52) and (53). For the proof of (52) and (53), refer to Appendix A.

## 3.5 Summary

This chapter has presented theoretical characterizations for multihop channels with diversity: the decoded relaying multihop diversity channel where digital relaying is employed and the amplified relaying multihop diversity channel where analog relaying is employed. These characterizations are tractable and enable the quick comparison of the proposed channels with the singlehop reference channel on the basis of signal to noise ratio, probability of outage, probability of error, and optimal power distribution. Additionally, the terms multihop diversity and multiroute diversity have been defined and distinguished, and a case made against the application of multiroute diversity. The approximations and simplifications utilized in the generation of these results will be validated through the use of simulation in Chapter 4. Implementation issues related to these two channel models will be discussed in Chapter 5.

# Chapter 4 - Simulation Results

The work presented in previous chapters of this thesis has been focused on the derivation of theoretical characterizations for the multihop wireless communications channel. In Chapter 2, results were derived for multihop channels without diversity, namely the decoded relaying multihop channel and the amplified relaying multihop channel. In Chapter 3, results were derived for multihop channels with diversity, namely the decoded relaying multihop diversity channel and the amplified relaying multihop diversity channel and the amplified relaying multihop diversity channel. These derivations lead to the characterization of the four channel models with respect to signal to noise ratio, probability of outage, probability of error, and optimal power distribution.

In this chapter, these results are applied in two simulations that validate the derived theory and provide a comparison with the singlehop reference channel. The first simulation presents theoretical and simulated results for fixed terminal positions and provides a comparison with the singlehop reference channel on the basis of probability of outage and probability of error. This simulation serves to validate the theoretical characterizations and highlights the performance improvement that can occur under optimal intermediate terminal placement. The second simulation presents theoretical results for variable intermediate terminal positions and provides a comparison with the singlehop reference channel on the basis of probability of error. This simulation indicates the sensitivity of performance to intermediate terminal position, where lower sensitivity indicates that a channel achieves a performance gain over the singlehop reference channel for a larger range of intermediate terminal positions. Again, the figures generated from these simulations are only valid for large signal to noise ratios due to the approximations used in the derivation of the probability of error.

A BPSK modulation scheme is used for simplicity of exposition. Maximal ratio combining at receiving terminals is assumed for multihop channels with diversity. The simulated multihop channel is composed of n + 1 terminals: source  $T_1$ , intermediate  $T_2$ through  $T_n$  and destination  $T_{n+1}$ . The coordinates of the channel are normalized with respect to the distance between the source and destination terminals such that  $d_{1,n+1} = 1$ .

The propagation exponent is p = 4. The lognormal shadowing components are independent with zero-mean and variance  $\sigma_{L_{P(i),i}}^2 = 12dB$ . The Rayleigh fading components are independent with variance  $\sigma_{R_{P(i),i}}^2 = 0.5$ . For the purpose of simplifying the comparison, and without loss of generality, the free space signal attenuation factor is  $\alpha^2 = 1$ . The optimal power distribution is assumed for each of the channel models with total power constrained to the reference power  $\varepsilon_0$ . A diagram of the generic channel is illustrated in Figure 3. Each simulation was executed for 100 million iterations.

#### 4.1 Fixed Intermediate Terminal Positions with *n* Hops

For this simulation the intermediate terminals are fixed so that they divide the direct path between the source and destination terminals into equal length segments. This serves to validate the theory presented thus far as well as illustrate the power gain that can be realized under an optimal placement of the intermediate terminals with respect to the source and destination terminals. The graphs provide a comparison in terms of probability of outage and probability of error versus the normalized signal to noise ratio of the singlehop reference channel.

Figures 7 shows the simulated probability of outage performance of the decoded relaying multihop channel. The theoretical characterization (8) is represented by dotted lines and indicates good agreement with the simulated results. Empirical observation suggests that at a probability of outage of  $10^{-5}$  the decoded relaying multihop channel achieves a power gain on the order of  $8(\log n / \log 2)dB$ . The theoretical and simulated results are almost identical since no approximations were used in the derivation of (8). The slight difference at low probability of error for 4 hops is statistically insignificant.



Figure 7. Outage for Decoded Relaying Multihop Channel

Figures 8 shows the simulated probability of outage performance of the amplified relaying multihop channel. The theoretical characterization (26) is represented by dotted lines and indicates good agreement with the simulated results. Empirical observation suggests that at a probability of outage of  $10^{-5}$  the amplified relaying multihop channel achieves a power gain on the order of  $9(\log n / \log 2)dB$ . The theoretical and simulated results cross at moderate probability of error due to competing trends resulting from the approximations used in the derivation of (26).



Figure 8. Outage for Amplified Relaying Multihop Channel

Figures 9 shows the simulated probability of outage performance of the decoded relaying multihop diversity channel. The theoretical characterization (38) is represented

by dotted lines and indicates good agreement with the simulated results. Empirical observation suggests that at a probability of outage of  $10^{-5}$  the decoded relaying multihop diversity channel achieves a power gain on the order of  $9(\log n / \log 2)dB$ .



Figure 9. Outage for Decoded Relaying Multihop Diversity Channel

Figures 10 shows the simulated probability of outage performance of the amplified relaying multihop diversity channel. The theoretical characterization (49) is represented by dotted lines and indicates fair agreement with the simulated results. The difference between the theoretical and simulated results is due to the approximation of the amplification factor in (45). Empirical observation suggests that at a probability of outage

of  $10^{-5}$  the amplified relaying multihop diversity channel achieves a power gain on the order of  $12(\log n / \log 2)dB$ .



Figure 10. Outage for Amplified Relaying Multihop Diversity Channel

Figures 11 shows the simulated probability of error performance of the decoded relaying multihop channel. The theoretical characterization (10) is represented by dotted lines and indicates good agreement with the simulated results. Empirical observation suggests that at a probability of error of  $10^{-5}$  the decoded relaying multihop channel achieves a power gain on the order of  $6(\log n / \log 2)dB$ .



Figure 11. Error for Decoded Relaying Multihop Channel

Figures 12 shows the simulated probability of error performance of the amplified relaying multihop channel. The theoretical characterization (27) is represented by dotted lines and indicates good agreement with the simulated results. Empirical observation suggests that at a probability of error of  $10^{-5}$  the amplified relaying multihop channel achieves a power gain on the order of  $6(\log n / \log 2)dB$ .



Figure 12. Error for Amplified Relaying Multihop Channel

Figures 13 shows the simulated probability of error performance of the decoded relaying multihop diversity channel. The theoretical characterization (40) is represented by dotted lines and indicates good agreement with the simulated results. Empirical observation suggests that at a probability of error of  $10^{-5}$  the decoded relaying multihop diversity channel achieves a power gain on the order of  $9(\log n / \log 2)dB$ .



Figure 13. Error for Decoded Relaying Multihop Diversity Channel

Figures 14 shows the simulated probability of error performance of the amplified relaying multihop diversity channel. The theoretical characterization (50) is represented by dotted lines and indicates fair agreement with the simulated results. Again, the difference between the theoretical and simulation results is due to the approximation of the amplification factor in (45). Empirical observation suggests that at a probability of error of  $10^{-5}$  the amplified relaying multihop diversity channel achieves a power gain on the order of 24(2-2/n)dB.



Figure 14. Error for Amplified Relaying Multihop Diversity Channel

Not surprisingly, the multihop channels with diversity significantly outperform the multihop channels without diversity. This is expected, as discussed previously, due to the fact that the power gain due to multihop diversity combining is achievable without expending additional systems resources. What is somewhat unexpected, however, is that the amplified relaying channels experience performance gains that are equal to or greater than the decoded relaying channels.

The performance gains of the decoded relaying multihop channel and the amplified relaying multihop channel are almost identical. This is simply the result of the derived equivalency discussed in Chapter 2 for the probability of error equations for relaying over

Rayleigh channels. If similar results were derived for relaying over gaussian channels, the probability of error performance of the decoded relaying multihop channel would be significantly better than that of the amplified relaying multihop channel.

The performance gains of the amplified relaying multihop diversity channel are much greater than those of the decoded relaying multihop diversity channel. An explanation for this result lies in the structure of the decoded relaying receiver implied by (37) through (40). Whereas the amplified relaying channel responds gracefully to severe signal degradation on any individual link between two terminals, the decoded relaying channel produces an error at the destination receiver if any individual link between two terminals produces an error. This is a result of the propagation of errors to following terminals along the transmission path, significantly decreasing the error performance.

The relevant question therefore becomes whether the decoded relaying receiver can be modified to improve system performance. In fact, this can be accomplished by transmitting out-of-band information regarding the estimated probability of decoding error for each signal link to every terminal further along the signal path [18]. The incorporation of more complex decoded relaying receiver structures into the framework provided by this thesis, as well as discussion related to the performance impact and implementation of this out-of-band transmission, is included in Chapter 5.

# 4.2 Variable Intermediate Terminal Position with 2 Hops

For this simulation the single intermediate terminal is placed at locations uniformly distributed across a unit square. The source and destination terminals are located at (0,0) and (1,0) respectively. The intermediate terminal ranges from 0 to 1 along the x-axis and  $-\frac{1}{2}$  to  $\frac{1}{2}$  along the y-axis. This serves to illustrate the robustness of the channel models

with respect to distance from the optimal placement of the intermediate terminal. The graphs provide a comparison in terms of probability of error versus intermediate terminal location when the normalized average signal to noise ratio is fixed at 10dB.

Figure 15 shows the variation of the probability of error performance of the decoded relaying multihop channel with respect to the position of the intermediate terminal. The concave hull represents the theoretical characterization presented in (10) and indicates that the performance gain with respect to the reference channel is fairly sensitive to the relative position of the intermediate terminal. A horizontal plain indicates the probability of error performance of the singlehop reference channel.



Figure 15. Robustness of Decoded Relaying Multihop Channel- View 1

Figure 16 shows the same graph as Figure 15 from a vantage point parallel to the yaxis at 0 degrees elevation. The probability of error performance of the decoded relaying multihop channel is symmetrical about both the x-axis and y-axis. This is a result of the symmetrical form of (10), in that the expected performance when transmitting an information signal from the source terminal to the destination terminal is identical to the expected performance when transmitting an information signal from the destination terminal to the source terminal. The error performance is therefore solely a function of the hop distances and is not dependant on the respective distances of the intermediate terminal from the source and destination terminals.



Figure 16. Robustness of Decoded Relaying Multihop Channel – View 2

Figure 17 shows the variation of the probability of error performance of the amplified relaying multihop channel with respect to the position of the intermediate terminal. The concave hull represents the theoretical characterization presented in (27) and also indicates that the performance gain with respect to the reference channel is fairly sensitive to the relative position of the intermediate terminal. A horizontal plain indicates the probability of error performance of the singlehop reference channel.



Amplified Relaying Using BPSK Under Fading Conditions: SNR = 10dB

Figure 17. Robustness of Amplified Relaying Multihop Channel - View 1

Figure 18 shows the same graph as Figure 17 from a vantage point parallel to the yaxis at 0 degrees elevation. As for the decoded relaying multihop channel, the probability of error performance of the amplified relaying multihop channel is symmetrical about both the x-axis and y-axis. Again, this is a result of the symmetrical form of (27). It is interesting to note that the probability of error equivalence of the decoded and amplified relaying multihop channel first indicated by Figures 11 and 12 still holds in Figures 15 through 18. This of course implies that the equivalence is therefore also solely a function of the hop distances and is not dependent on the respective distances of the intermediate terminal from the source and destination terminals.



Figure 18. Robustness of Amplified Relaying Multihop Channel - View 2

Figure 19 shows the variation of the probability of error performance of the decoded relaying multihop diversity channel with respect to the position of the intermediate terminal. The concave hull represents the theoretical characterization presented in (40)

and indicates that the performance gain with respect to the reference channel is fairly sensitive to the relative position of the intermediate terminal, although less so than the multihop channel models without diversity. A horizontal plain indicates the probability of error performance of the singlehop reference channel.



Decoded Relaying With Diversity Using BPSK Under Fading Conditions: SNR = 10dB

Figure 19. Robustness of Decoded Relaying Multihop Diversity Channel - View 1

Figure 20 shows the same graph as Figure 19 from a vantage point parallel to the yaxis at 0 degrees elevation. The probability of error performance of the decoded relaying multihop diversity channel is symmetrical about the y-axis, but not the x-axis. The error performance is generally better when the intermediate terminal is closer to the source terminal than the destination terminal. This is due to the fact that the limiting term in (40) is the probability of error on the hop between the source terminal and the intermediate terminal, where no benefit is gained from diversity combining. Therefore, the total probability of error performance is generally improved when decreasing the distance between the source terminal and intermediate terminal reduces the probability of error on that hop.



Figure 20. Robustness of Decoded Relaying Multihop Diversity Channel – View 2

Figure 21 shows the variation of the probability of error performance of the amplified relaying multihop diversity channel with respect to the position of the intermediate terminal. The concave hull represents the theoretical characterization presented in (50) and indicates that the performance gain with respect to the reference

channel is fairly sensitive to the relative position of the intermediate terminal, although less so than the other three multihop channel models. A horizontal plain indicates the probability of error performance of the singlehop reference channel.



Amplified Relaying With Diversity Using BPSK Under Fading Conditions: SNR = 10dB

Figure 21. Robustness of Amplified Relaying Multihop Diversity Channel - View 1

Figure 22 shows the same graph as Figure 21 from a vantage point parallel to the yaxis at 0 degrees elevation. The probability of error performance of the amplified relaying multihop diversity channel is symmetrical about the y-axis, but not the x-axis. The error performance is generally better when the intermediate terminal is closer to the destination terminal than the source terminal. This is due to the fact that as the distance from the intermediate terminal to the destination terminal decreases, the intermediate terminal requires a smaller percentage of the total allocated channel power, allowing the source terminal to transmit with greater power. Consider the two limiting cases. In the first case the intermediate terminal is asymptotically close to the source terminal and both source and intermediate terminals transmit with equal power. In the second case the intermediate terminal is asymptotically close to the destination terminal and the source terminal transmits with the total allocated power, double that for the first case. Since the noise introduced on the infinitely short hop in both these cases is obviously negligible in comparison to the received power and both cases experience benefit from combining *2* diversity branches, the second case will outperform the first.



Figure 22. Robustness of Amplified Relaying Multihop Diversity Channel - View 2

These results attest to the importance of good decisions when selecting intermediate terminals. Although the performance gain is significant when the intermediate terminal is positioned close to the midpoint between the source and destination terminals, the gain becomes negligible and in some cases negative as the distance with respect to that midpoint position increases. That generality aside, it is evident that the multihop diversity channels, and especially the amplified relaying multihop diversity channel, are less sensitive to intermediate terminal position in that they achieve a performance gain over the singlehop reference channel for a larger range of intermediate terminal positions.

The determination of whether a particular terminal should be used for relaying and which terminals from the global set for a particular wireless system represent the best relaying set is generally referred to as *ad hoc routing* [14]. A number of different ad hoc routing protocols have been proposed [7,10,16,17,26] that use a variety of methods to cope with the wild changes in network topology caused by the combination of terminal mobility and multipath fading. It is assumed that the characterizations derived in this thesis can be used as link metrics in any proposed routing scheme.

#### 4.3 Summary

This chapter has presented two simulations for multihop channels with and without diversity. The first simulation validated the theoretical characterizations presented in Chapters 2 and 3 and indicated that significant performance improvements can be realized through the use of multihop channels, especially those employing multihop diversity combining. The second simulation indicated that these performance improvements are fairly sensitive to the location of the intermediate terminals and highlighted the care that must be taken when selecting intermediate terminals. The
multihop channels without diversity were shown to outperform the singlehop reference channel, with the performance of the decoded relaying multihop channel and the amplified relaying multihop channel being almost identical. The multihop channels with diversity were shown to outperform both the singlehop reference channel and multihop channels without diversity, with the performance of the amplified relaying multihop diversity channel being significantly better than the performance of the decoded relaying multihop diversity channel. This motivates the search for an improved decoded relaying receiver, a possible implementation for which is proposed in Chapter 5.

# **Chapter 5 - Implementation Considerations**

The results presented in previous chapters provide a good comparison in terms of average power, probability of outage, probability of bit error, and optimal power distribution. However, there are a number of other factors that are important to consider as well. These factors include receiver tracking speed in amplified relaying channels, the development of an improved decoded relaying receiver, relaying channel allocation, feedback and feedforward interference, propagation and processing delay characteristics of the channel, interference distribution and power control, multiple access schemes, adaptive modulation, and terminal complexity. This chapter briefly discusses each of these factors, highlighting their impact on the theoretical characterizations and raising interesting points for further research. Additional implementation considerations are discussed in [14] and [15].

### 5.1 Receiver Tracking Speed

One of the assumptions inherent in the characterization of the amplified relaying channels is that each intermediate terminal can track both lognormal shadowing and Rayleigh fading. This assumption requires additional receiver complexity, and may in fact be unrealizable depending upon the speed of the Rayleigh fading. Therefore, it is interesting to consider the performance impact when intermediate terminals can track only the lognormal shadowing, and calculate the amplification factor at each terminal using only the mean (averaged over the Rayleigh fading) received signal power.

Considering the amplified relaying multihop channel, the amplification factor at each intermediate terminal  $T_i$  is given by

$$A_{i} = \frac{\varepsilon_{i}}{\alpha^{2} (\varepsilon_{P(i)} + \left| \beta_{P(i)} \right|^{2}) L_{P(i),i} / d_{P(i),i}^{p} + N_{0}}.$$
(54)

Following a similar process to that used for the simplification of (13), an approximation can be made where the noise terms are removed from the denominator of (54). This approximation yields an upper bound on the amplification factor, which is tight provided that the signal to noise ratio at the terminal under consideration is significantly greater than 1.

The amplification factor at each terminal is then given by

$$A_i \le \frac{\varepsilon_i d_{P(i),i}^p}{\alpha^2 \varepsilon_{P(i)} L_{P(i),i}},\tag{55}$$

and the received signal to noise ratio at the destination terminal  $T_d$  is given by

$$\gamma_{P(d),d} \approx \frac{\alpha^2}{\sum_{T_i \in T_R} \left( \frac{d_{P(i),i}^p}{\varepsilon_{P(i)}} \Big| \frac{\varepsilon_{P(i),i}}{\varepsilon_{P(i)}} \prod_{\substack{T_j \in T_R \\ j \le i}} \left| R_{P(j),j} \right|^2 \right) N_0}.$$
(56)

For the proof of (56), refer to Appendix A. Alternatively, the received signal to noise ratio at the destination terminal can be expressed as

$$\gamma_{P(d),d}^{-1} \approx \sum_{T_i \in T_R} \psi_{P(i),i}^{-1} , \qquad (57)$$

where  $\psi_{P(i),i}$  is the received signal to noise ratio  $\gamma_{P(i),i}$  at terminal  $T_i$  with  $\beta_{P(i)} = 0$  and is given by

$$\psi_{P(i),i} \approx \frac{\alpha^2 \varepsilon_{P(i)}}{\left(\frac{d_{P(i),i}}{2} \middle| \begin{array}{c} L_{P(i),i} \prod_{\substack{T_j \in T_R \\ j \le i}} \left| R_{P(j),j} \right|^2 \right) N_0},$$
(58)

where the notation  $j \le i$  indicates that the terminals over which  $T_j$  ranges are those preceding and including terminal  $T_i$  along the transmission path.

A similar analysis can be performed for the amplified relaying multihop diversity channel. Figures 23-24 show the simulated probability of error performance of the amplified relaying multihop channel and the amplified relaying multihop diversity channel when the relaying receivers are unable to track the Rayleigh fading. The simulated results are compared against the theoretical characterizations (27) and (50) when the relaying receivers are able to track the Rayleigh fading.



Figure 23. Amplified Relaying Multihop Channel with Slow Tracking



Figure 24. Amplified Relaying Multihop Diversity Channel with Slow Tracking

These results indicate a severe degradation in performance for the slow tracking receiver versus the fast tracking receiver. In fact, the probability of error performance of the slow tracking receiver is similar to, if not worse than, the singlehop reference channel. This is the result of the product of Rayleigh random variables in the denominator of (56) and (58), which becomes progressively more destructive as the number of hops increases. This implies that there is minimal or no benefit to using the amplified relaying multihop and amplified relaying multihop diversity channels unless the Rayleigh fading can be tracked in some fashion by the intermediate terminals. An

analysis of the relationship between the probability of error performance and the accuracy of tracking estimates presents an interesting area for further research.

### 5.2 An Improved Decoded Relaying Receiver

As stated previously, the poor performance of the decoded relaying multihop diversity channel can be improved by modifying the decoded relaying receiver to take into account out-of-band information regarding the estimated probability of decoding error for each signal branch entering the diversity combiner [18]. Consider the generalized decoded relaying multihop diversity channel with *n* hops. The  $K^{th}$  receiving terminal  $T_i$  along the path will receive and combine signals from the *K* previous terminals  $T_{P(i)}$ . If this terminal can acquire an estimate of the probability of decoding error at each of these preceding terminals it can apply a multiplicative weight factor  $X_{k,i}$  to each branch of the diversity combiner that decreases the contribution of those branches with a high probability of decoding error. Figure 25 shows a block diagram of this improved diversity combiner.



Figure 25. Improved Diversity Combiner with Out-Of-Band Information

Each weight factor  $X_{k,i}$  is constrained to the range between 0 and 1. When the estimated probability of error of branch k of the combiner is  $\frac{1}{2}$  then the optimal weight factor is  $X_{k,i} = 0$ . When the estimated probability of error of branch k of the combiner is 0 then the optimal weight factor is  $X_{k,i} = 1$  and the results are identical to those presented in [24]. Following a similar derivation to that in [24] for the output signal to noise ratio, the voltage signals  $v_{k,i}$  from each of the K diversity branches are co-phased to provide coherent voltage addition and are individually weighted to produce the optimal output signal to noise ratio. If each branch has gain  $G_{k,i}X_{k,i}$  then the resulting signal envelope applied to the detector is given by

$$v_{R,i} = \sum_{k=1}^{K} G_{k,i} X_{k,i} v_{k,i},$$
(59)

and assuming that each branch has the same average noise power  $N_0$ , the total noise power applied to the detector is given by

$$N_{R,i} = N_0 \sum_{k=1}^{K} (G_{k,i} X_{k,i})^2 .$$
(60)

Given that  $G_{k,i} = v_{k,i}/N_0$  [24], the signal to noise ratio applied to the detector is given by

$$\gamma_{R,i} = \frac{v_{R,i}^2}{2N_{R,i}}$$

$$= \frac{\left(\sum_{k=1}^{K} X_{k,i} v_{k,i}^2 / N_0\right)^2}{2\sum_{k=1}^{K} X_{k,i}^2 v_{k,i}^2 / N_0}$$
(61)

This signal to noise ratio can be approximated by a more tractable form if the factors  $X_{k,i}^2$  are replaced by  $X_{k,i}$ . This approximation can be made as a result of the range of the  $X_{k,i}$ . As  $X_{k,i}$  approaches 1,  $X_{k,i}^2$  also approaches 1. As  $X_{k,i}$  approaches 0 and the other terms become dominant,  $X_{k,i}^2$  approaches 0 even faster and leads to the same result. This approximation produces a lower bound for the combined signal to noise ratio. With this approximation, the signal to noise ratio applied to the detector is then given by

$$\gamma_{R,i} \geq \frac{\left(\sum_{k=1}^{K} X_{k,i} v_{k,i}^{2} / N_{0}\right)^{2}}{2\sum_{k=1}^{K} X_{k,i} v_{k,i}^{2} / N_{0}}$$

$$\geq \frac{\sum_{k=1}^{K} X_{k,i} v_{k,i}^{2} / N_{0}}{2}$$

$$\geq \sum_{k=1}^{K} X_{k,i} \gamma_{k,i}$$
(62)

Given a received signal to noise ratio of  $\gamma_{R,i}$  the probability of error for BPSK under fading conditions at terminal  $T_i$  when  $\beta_k = 0, T_k \in T_{P(i)}$  and none of the diversity branches is receiving incorrect information due to a decoding error at a preceding terminal is given by

$$P_e(\gamma_{P(i),i} \mid \stackrel{correct}{branches}) \approx \binom{2K-1}{K} \prod_{T_k \in T_{P(i)}} (\frac{1}{2X_{k,i}\overline{\gamma_{k,i}}}), X_{k,i}\overline{\gamma_{k,i}} \gg 1.$$
(63)

Let  $T_E$  represent the set of diversity branches where the respective preceding terminals are transmitting incorrect information. Let  $T_C$  represent the set of diversity branches where the respective preceding terminals are transmitting correct information. Then the probability of error for BPSK under fading conditions at terminal  $T_i$  when  $\beta_k = 0, T_k \in T_{P(i)}$  and at least one of the diversity branches is receiving incorrect information due to a decoding error at a preceding terminal is approximated by the probability that the weighted sum of the signal to noise ratios for all incorrect branches is greater than the weighted sum of the signal to noise ratios for all correct branches and is given by

$$P_{e}(\gamma_{P(i),i} \mid \stackrel{incorrect}{branches}) \approx \Pr(\sum_{T_{k} \in T_{E}} X_{k,i} \gamma_{k,i} > \sum_{T_{k} \in T_{C}} X_{k,i} \gamma_{k,i}))$$

$$\approx \sum_{T_{k} \in T_{E}} \sum_{T_{l} \in T_{C}} \frac{\pi_{E,k,i} \pi_{C,l,i} X_{k,i} \overline{\gamma_{k,i}}}{X_{k,i} \overline{\gamma_{k,i}} + X_{l,i} \overline{\gamma_{l,i}}},$$
(64)

where  $\pi_{E,k,i}$  is given by

$$\pi_{E,k,i} = \prod_{\substack{T_j \in T_E \\ j \neq k}} \frac{X_{k,i} \overline{\gamma_{k,i}}}{X_{k,i} \overline{\gamma_{k,i}} - X_{j,i} \overline{\gamma_{j,i}}},$$
(65)

and  $\pi_{C,k,i}$  is given by

$$\pi_{C,k,i} = \prod_{\substack{T_j \in T_C \\ j \neq k}} \frac{X_{k,i} \gamma_{k,i}}{X_{k,i} \overline{\gamma_{k,i}} - X_{j,i} \overline{\gamma_{j,i}}} \,. \tag{66}$$

For the proof of (64), refer to Appendix A.

Substituting (64) for the probability of error for BPSK under fading conditions at terminal  $T_i$  when  $\beta_k = 0, T_k \in T_{P(i)}$  and at least one of the diversity branches is receiving incorrect information due to a decoding error at a preceding terminal, the total probability of error at terminal  $T_i$  can now be approximated by

$$P_{e}(\gamma_{P(i),i}) \approx \prod_{T_{k} \in T_{P(i)}} \Pr(T_{k} \in T_{C}) \binom{2^{K-1}}{K} \prod_{T_{k} \in T_{P(i)}} \left( \frac{1}{2X_{k,i}\overline{\gamma_{k,i}}} \right) + \sum_{T_{E} \in T_{P(i)}} \left( \prod_{T_{k} \in T_{E}} \Pr(T_{k} \in T_{E}) \prod_{T_{k} \in T_{C}} \Pr(T_{k} \in T_{C}) \sum_{T_{k} \in T_{E}} \sum_{T_{l} \in T_{C}} \frac{\pi_{E,k,i}\pi_{C,l,i}X_{k,i}\overline{\gamma_{k,i}}}{X_{k,i}\overline{\gamma_{k,i}} + X_{l,i}\overline{\gamma_{l,i}}} \right).$$
(67)

where  $\sum_{T_E \in T_{P(i)}}$  represents the set of all possible combinations of incorrect branches. Now that the probability of error at terminal  $T_i$  can be expressed in terms of the branch weight factors, these weight factors can be optimized to minimize the probability of error.

In order to illustrate the discussion, consider a multihop channel with source terminal  $T_1$ , intermediate terminal  $T_2$ , and destination terminal  $T_3$ . The probability of error at terminal  $T_3$  from (67) is given by

$$P_{e}(\gamma_{P(3),3}) \approx \Pr(T_{1} \in T_{C}) \Pr(T_{2} \in T_{C}) (\frac{3}{4X_{1,3}\overline{\gamma_{1,3}}X_{2,3}\overline{\gamma_{2,3}}}) + \Pr(T_{1} \in T_{C}) \Pr(T_{2} \in T_{E}) (\frac{X_{2,3}\overline{\gamma_{2,3}}}{X_{2,3}\overline{\gamma_{2,3}} + X_{1,3}\overline{\gamma_{1,3}}}) .$$
(68)  
$$\approx (1 - \frac{1}{2\overline{\gamma_{1,2}}}) (\frac{3}{4\overline{\gamma_{1,3}}X_{2,3}\overline{\gamma_{2,3}}}) + (\frac{1}{2\overline{\gamma_{1,2}}}) (\frac{X_{2,3}\overline{\gamma_{2,3}} + X_{1,3}\overline{\gamma_{1,3}}}{X_{2,3}\overline{\gamma_{2,3}} + X_{1,3}\overline{\gamma_{1,3}}}) .$$

where  $X_{1,i} = 1$  since terminal  $T_1$  is the source terminal and therefore cannot make a decoding error. The only variable is  $X_{2,3}$ , which can be chosen to minimize (68). It is important to note again that (68) only holds for  $X_{k,i}\overline{\gamma_{k,i}} \gg 1$ .

#### 5.3 Relaying Channel Allocation

Current literature [13,18] on the subject of multihop wireless communications channels places the restriction that relaying must be performed in a separate channel due to limitations in the signal processing capabilities of the terminal hardware. The concern is that using the same channel will result in feedback from the transmitter to the receiver, and since the transmitted power is generally a few orders of magnitude greater than the received power, the received signal may be completely obscured. A solution to this problem should become tractable as signal processing and antenna design technologies continue to advance. Possible methods for removing this antenna feedback include digital subtraction of the transmitted signal from the received signal as well as having a separate receive antenna placed in a spatial null of the transmit antenna.

Therefore, it is interesting to consider multihop channels with and without this restriction. With this restriction, relaying channels are forced to allocate a separate channel for relaying and therefore use more system resources per communications link than the singlehop reference channel. Regardless of the number of hops along a particular transmission route, only one additional channel is required since the two channels can be used in alternating fashion at each hop such that each intermediate terminal receives and transmits in separate channels. Further discussion related to the problem of selecting channels for relaying in a cellular network is presented in [28]. Without this restriction, relaying channels require no additional resources and can be compared directly to the singlehop reference channel.

### 5.4 Feedback and Feedforward Interference

It is important to note that the primary metric used thus far to characterize the channel models is the signal to noise ratio (SNR) as opposed to the signal to interference plus noise ratio (SINR). The motivation for this is that the scope of consideration is constrained to a single channel in isolation. Thus, the interference generated by signals propagating along other channels is irrelevant and cochannel interference from other

users is ignored. However, there are two forms of interference that are unique to multihop channels. These are referred to as feedback interference and feedforward interference and are a result of the non-directionality of transmission within wireless systems that employ omnidirectional antennas.

Feedback interference is caused by the reception of relayed signals from following terminals along the transmission route. Feedback interference affects both multihop and multihop diversity channels. Feedforward interference is caused by the reception of relayed signals from preceding terminals along the transmission route other than the immediately preceding terminal. Feedforward interference affects only multihop channels without diversity, since these same transmitted signals, if processed and combined properly, provide the diversity in the multihop diversity channels.

First consider a multihop channel without diversity with source terminal  $T_1$ , intermediate terminals  $T_2$  through  $T_4$ , and destination terminal  $T_5$ . Although terminal  $T_2$ receives the desired signal from terminal  $T_1$ , it also receives feedback interference from terminals  $T_3$  and  $T_4$ . Although terminal  $T_3$  receives the desired signal from terminals  $T_2$ , it also receives feedforward interference from terminal  $T_1$  and feedback interference from terminal  $T_4$ .

Now consider a multihop channel with diversity with source terminal  $T_1$ , intermediate terminals  $T_2$  through  $T_4$ , and destination terminal  $T_5$ . Although terminal  $T_2$ receives the desired signal from terminal  $T_1$ , it also receives feedback interference from terminals  $T_3$  and  $T_4$ . Although terminal  $T_3$  receives the desired signal from terminals  $T_1$ and  $T_2$ , it also receives feedback interference from terminal  $T_4$ . The feedback interference at terminal  $T_i$  is given by

$$I_{V(i),i} = \frac{1}{C} \sum_{T_{j} \in T_{V(i)}} \left( \frac{\alpha^{2} \varepsilon_{j}}{d_{j,i}^{p} / L_{j,i} |R_{j,i}|^{2}} \right),$$
(69)

where *C* is the processing gain of the system, and  $T_{V(i)}$  represents the set of transmitting terminals following terminal  $T_i$  along the transmission route. The definition of the system processing gain *C* for specific multiple access and relaying schemes is discussed later in this section. The feedforward interference at terminal  $T_i$  is given by

$$I_{U(i),i} = \frac{1}{C} \sum_{T_j \in T_{U(i)}} \left( \frac{\alpha^2 \varepsilon_j}{d_{j,i}^p / L_{j,i} |R_{j,i}|^2} \right),$$
(70)

where  $T_{U(i)}$  represents the set of transmitting terminals preceding terminal  $T_i$  along the transmission route that do not belong to  $T_{P(i)}$ . Therefore, the set of all terminals along the transmission route can be represented by  $T_A = T_{U(i)} \cup T_{P(i)} \cup T_i \cup T_{V(i)} \cup T_D$ .

For the decoded relaying multihop channel, it follows from (6) that the received signal to interference plus noise ratio at terminal  $T_i$  is given by

$$\mathbf{H}_{P(i),i} = \frac{\alpha^2 \varepsilon_{P(i)}}{\left( \frac{d_{P(i),i}^p}{L_{P(i),i} \left| R_{P(i),i} \right|^2} \right) \left( N_0 + I_{U(i),i} + I_{V(i),i} \right)}.$$
(71)

For the amplified relaying multihop channel, it follows from (17) that the received signal to interference plus noise ratio at the destination terminal is given by

$$\mathbf{H}_{P(d),d}^{-1} \approx \sum_{T_i \in T_R} \Psi_{P(i),i}^{-1} , \qquad (72)$$

where  $\Psi_{P(i),i}$  is the received signal to interference plus noise ratio  $H_{P(i),i}$  at terminal  $T_i$ with  $\beta_{P(i)} = 0$  and is given by

$$\Psi_{P(i),i} = \frac{\alpha^2 \varepsilon_{P(i)}}{\left(\frac{d_{P(i),i}}{L_{P(i),i}} \left| R_{P(i),i} \right|^2\right) \left(N_0 + I_{U(i),i} + I_{V(i),i}\right)}.$$
(73)

For the proof of (72), refer to Appendix A.

For the decoded relaying multihop diversity channel, it follows from (36) that the received signal to interference plus noise ratio at terminal  $T_i$  is given by

$$\mathbf{H}_{P(i),i} = \sum_{T_k \in T_{P(i)}} \left( \frac{\alpha^2 \varepsilon_k}{\left( \frac{d_{k,i}^p}{L_{k,i}} \middle| R_{k,i} \middle|^2 \right) (N_0 + I_{V(i),i})} \right).$$
(74)

For the amplified relaying multihop diversity channel, it follows from (46) that the received signal to interference plus noise ratio at the destination terminal  $T_d$  can be expressed recursively as

$$\mathbf{H}_{P(d),d} \approx \sum_{\substack{T_k \in T_{P(d)} \\ k \neq s}} (\mathbf{H}_{P(k),k}^{-1} + \Psi_{k,d}^{-1})^{-1} + \Psi_{s,d},$$
(75)

where  $\Psi_{k,d}$  is the received signal to noise ratio  $H_{k,d}$  at terminal  $T_d$  for branch k of the diversity combiner with  $\beta_k = 0$  and  $T_s$  is the source terminal. For the proof of (75), refer to Appendix A.

Of particular importance in the preceding equations is the determination of the processing gain C. The processing gain of the system is dependent on the chosen multiple access and relaying scheme. For code division multiple access (CDMA) systems, C is equal to the number of chips per bit (spreading gain). For time division multiple access (TDMA) and frequency division multiple access (FDMA) systems that

utilize the same channel for relaying as the source transmission, C is equal to unity. For TDMA and FDMA systems that utilize a separate channel for relaying, C approaches infinity and is limited only by adjacent channel interference.

Since C is a controllable parameter, it is useful to gain a qualitative understanding of how the choice of multiple access and relaying scheme can limit the performance of the multihop wireless communications channel. First consider the multihop channel with diversity. Averaging over all intermediate terminals, all intermediate terminal locations, and all shadowing and fading characteristics, the received power of all interference transmissions before processing will be approximately equal to the received power of all signal transmissions. Now consider the multihop channel without diversity. Although the previous equality does not hold, it serves as an upper bound.

Let  $P_{Signal}$  represent the average total received power from all signal transmissions. Let  $P_{Interference}$  represent the average total received power from all interference transmissions before processing. Then the mean signal to noise ratio at terminal  $T_i$  can be approximated by

$$\overline{\gamma_{P(i),i}} \approx \frac{P_{Signal}}{N_0},\tag{76}$$

and the mean signal to interference plus noise ratio at terminal  $T_i$  can be approximated by

$$\overline{\mathbf{H}_{P(i),i}} \approx \frac{P_{Signal}}{N_0 + \frac{1}{C} P_{Interference}} \,. \tag{77}$$

Substituting (76) into (77) and letting  $P_{Signal} = P_{Interference}$ , the mean signal to interference plus noise ratio can be rewritten as

$$\overline{\mathbf{H}_{P(i),i}} \approx \left(\frac{1}{\gamma_{P(i),i}} + \frac{1}{C}\right)^{-1}.$$
(78)

Examining (78), it can be seen that

$$\overline{\gamma_{P(i),i}} \to \infty \Longrightarrow \overline{\mathcal{H}_{P(i),i}} \to C, \qquad (79)$$

revealing that no matter how much power is available to the multihop channel, the mean signal to interference plus noise ratio is limited by the processing gain. This provides some insight into the possible performance of various multiple access and relaying schemes. CDMA systems will have a signal to interference plus noise ratio limited by the spreading gain, TDMA and FDMA systems utilizing the same channel for relaying will have a signal to interference plus noise ratio limited by unity. TDMA and FDMA systems utilizing a separate channel for relaying will have a signal to interference plus noise ratio limited by unity.

#### 5.5 Propagation and Processing Delay

Relaying channels suffer in general from a greater propagation delay in comparison to the singlehop reference channel due to the indirect nature of the route over which the information signal is transmitted. The decoding relaying channel also incorporates an additional processing delay at each terminal in order to digitally decode and re-encode the signal. The processing delay may be anywhere from a bit to a full frame in duration depending on whether error control coding is performed at intermediate terminals. Unlike the propagation delay, this additional processing delay can increase the total delay of the channel by orders of magnitude, and may in fact make the decoded relaying channel unsuitable for delay sensitive transmissions. The delay introduced by the reception, amplification, and retransmission of the signal is considering negligible in comparison to the propagation delay, implying that the processing delay incurred for amplified relaying channels can be approximated by zero.

This additional processing delay will also affect the application of diversity combining techniques with the decoded relaying channel. Although the destination terminal will receive the transmitted signal from multiple intermediate terminals concurrently, the signal received along each path will be separated in time by up to the duration of one frame. This will render infeasible the use of multipath technologies like conventional rake receivers. In order to apply spatial diversity combining techniques, complex receiver structures will have to be developed that have the ability to buffer the received signals and calculate cross-correlation metrics on the different signals across a number of frames.

The total delay for a relaying channel with n hops is given by

$$\Delta_T = \left(\sum_{T_i \in T_R} d_{p(i),i} \middle/ c\right) + (n-1)\Delta_p, \qquad (80)$$

where  $\Delta_p$  is the processing delay at each intermediate terminal, *c* is the speed of light, and  $T_{p(i)}$  is the terminal immediately preceding terminal  $T_i$  along the transmission route. This compares to the singlehop reference channel, where the total delay is given by

$$\Delta_T = d_{s,d} / c , \qquad (81)$$

where  $T_s$  is the source terminal and  $T_d$  is the destination terminal.

### 5.6 Interference Distribution and Power Control

Given that these relaying techniques are applied to a complete wireless communications system, the interference experienced at each terminal due to the transmit power from other users will have a very different distribution than a system using the singlehop reference channel. Although the average power transmitted per terminal per signal will be much lower, terminals that relay a large number of signals will have a total transmit power that in some cases will be greater than the system using the singlehop reference channel. Although the optimal transmit powers for the four channel models are different, the interference distributions should be highly correlated as a result of very similar if not identical network layer topologies and routing paths.

Power control in both the decoded relaying and amplified relaying channels is very different from power control in the singlehop reference channel [11,32]. The reasons for this are analogous to the differences outlined in the discussion of interference distribution. Of specific interest to relaying channels is the problem of how to propagate power control information to individual terminals along a transmission route given a particular power control algorithm. Since the terminals communicate independently and are not always directly connected to a base station, any power control algorithm used must by distributed. However, it is not clear whether there is an optimal distributed algorithm.

A relevant note is that this problem corresponds very closely to the problem of propagating routing information to individual terminals along a transmission route. It should therefore be possible to leverage the routing algorithms proposed for the network layer of multihop wireless communications systems. This parallelism raises an interesting point. Currently, power control is performed at the physical layer and routing is performed at the network layer. However, if both kinds of information are propagated using the same methods, then it makes sense that the same communications layer manages them concurrently. Conversely, it may be that the application of power control at the physical layer is too restrictive and therefore beneficial to view power control as a vertical plane orthogonal to the conventional layered network architecture.

### 5.7 Multiple Access Schemes

One of the most important decisions in wireless systems design is the choice of multiple access scheme. Three multiple access schemes are considered for applicability to multihop channels: FDMA, TDMA, and CDMA. FDMA systems are possible for multihop channels without diversity, but not for multihop channels with diversity. This is due to the fact that the coherence bandwidth of the individual wireless links will generally be greater than the bandwidth of the FDMA channel, implying that the receiving terminals will not be able to distinguish and combine individual copies of the signal from different preceding terminals. If relaying is performed in the same channel as the initial transmission, this will introduce significant ISI due to feedback interference.

Diversity combining at receiving terminals in TDMA systems requires that the signals received from different preceding terminals be separated by at least the duration of one bit in order to ensure their reception and combination as independently fading signals. Combining is carried out through the use of an equalizer (linear or decision-feedback), where the tap coefficients are chosen to minimize the mean square error of the combined signal. For scenarios where relaying is performed in the same channel as the initial transmission, out-of-band information can also be transmitted back to preceding

terminals with instructions to introduce additional processing delay in order to control the inter-symbol interference (ISI) due to feedback interference seen at the receiver. Assuming a favorable ISI pattern, the performance of the equalizer may be upper bounded by the "matched filter bound", comparable to maximal ratio combining.

Diversity combining at receiving terminals in CDMA systems requires that the signals received from different preceding terminals be separated by at least the duration of one chip in order to ensure their reception and combination as independently fading signals. Combining is carried out through the use of a Rake receiver, where each finger of the Rake correlates to the signal from a single preceding terminal. Out-of-band information can also be transmitted back to preceding terminals with instructions to introduce additional processing delay in order to provide better separation between individual signal components. The performance of the Rake receiver is also upper bounded by that of maximal ratio combining.

Assuming that a symbol rate equalizer is used in the case of TDMA, the number of taps required in the equalizer for TDMA or the Rake receiver for CDMA is given by

$$K = \left( \left( \left( \sum_{T_i \in T_R} d_{p(i),i} \right) - d_{s,d} \right) \middle| c + (n-1)\Delta_p \right) W + 1,$$
(82)

where W is the bandwidth of the channel

Another interesting option is possible for TDMA systems. Although diversity combining must be performed at each intermediate terminal for decoded relaying multihop channels, the same is not necessarily the case for amplified relaying multihop channels. Instead of combining the individual signal copies at each intermediate terminal, it is possible to simply amplify the total received signal and only perform combining at the destination terminal. In that case, as opposed to receiving K = n copies of the signal

at the destination terminal, where *n* is the number of hops, the destination terminal would receive and combine  $K = 2^{n-1}$  copies of the signal. The performance of this system should be greater than or equal to the performance of the system where combining is performed at every intermediate terminal, since this is analogous to comparing a global optimization problem to a set of local optimization problems.

### 5.8 Adaptive Modulation

Adaptive modulation techniques allow the transmitter and receiver along a particular communications link to dynamically change the modulation scheme as the characteristics of the communications channel vary. Typically, this involves using modulation schemes with higher signal constellations when channel conditions are favorable and dropping down to modulation schemes with lower signal constellations when channel conditions are unfavorable. This results in the rate of transmission under favorable channel conditions being significantly higher than the rate of transmission under unfavorable channel conditions. The conceptual limit of adaptive modulation uses a store-and-forward approach to transmit all information during peak channel conditions. In [12], this approach is used to show that mobility can significantly increase the capacity of ad-hoc wireless networks. All information is transmitted when the transmitter-receiver pairs are physically close together.

Adaptive modulation techniques are only applicable to the decoded relaying multihop channel. Each transmitting terminal along the transmission route has the capability of selecting a different, dynamically changing modulation scheme depending on the channel conditions of the respective communications link. This is not possible for channels employing amplified relaying as receiving using one modulation scheme and transmitting using another modulation scheme involves an intermediate decoding process. Adaptive modulation techniques are also not applicable to the decoded relaying multihop diversity channel as diversity combiners do not currently exist that can combine branches using different modulation schemes. However, it is possible for each of these channels as a unit to dynamically change the modulation scheme employed as channel conditions vary, provided that every terminal along the transmission route can be synchronized to switch to the new global modulation scheme at the same time.

### 5.9 Terminal Complexity

Given that both relaying channels are significantly more complex than the reference channel, it is not surprising that there is a corresponding increase in terminal complexity. For both relaying channels, this increased complexity includes more complex power control and routing algorithms, the capability of handling multiple signals from different users concurrently, and more complex antenna structures if the same channel is used for relaying. TDMA systems may require equalizers with a large number of taps. CDMA systems may require fractionally spaced Rake receivers with a large number of taps. This complexity will increase linearly with the number of hops, resulting in equalizers and Rake receivers with an increasing number of taps and increasingly parallel receiver structures for CDMA systems. In addition, the decoded relaying channel may also include increased complexity at the receiver in order to apply spatial diversity combining techniques. Although the possible terminal complexity increases without limit as the number of hops goes to infinity, it can be assumed that in any realistic implementation the maximum number of hops will be limited. It should therefore be possible to find a point where the benefits in terms of performance outweigh the required increase in terminal complexity.

### 5.10 Summary

This chapter has presented a number of important implementation considerations for multihop channels. The inability to track multipath fading in multihop channels employing amplified relaying is shown to cause a severe performance degradation. An improved decoded relaying receiver is proposed that uses the error metrics of previous terminals to weight individual combiner branches. Relaying in the same channel as the original transmission is proposed as a possibility for future communications systems. Feedback and feedforward interference are defined and integrated into the theoretical characterizations. The propagation and processing delay characteristics of the decoded relaying channels and the amplified relaying channels are discussed. The redistribution of other-user interference is highlighted and system level power control is equated to the problem of propagating routing information. The realization of each of the channel models is discussed for FDMA, TDMA, and CDMA systems. The applicability of adaptive modulation techniques is considered. Terminal complexity is highlighted as a limiting factor to the maximum number of hops in a realistic implementation.

## Chapter 6 - Conclusions

### 6.1 Summary

The results presented in this thesis provide a firm foundation for the characterization of multihop wireless communications channels. Four models for multihop wireless communications channels are proposed and characterized. The decoded relaying multihop channel employs digital relaying of the signal from the immediately preceding terminal. The amplified relaying multihop channel employs analog relaying of the signal from the immediately preceding terminal. The decoded relaying multihop diversity channel employs diversity combining and digital relaying of the signals from all preceding terminals. The amplified relaying multihop diversity channel employs diversity combining and analog relaying of the signals from all preceding terminals. The mathematical characterizations outlined are tractable and enable the quick comparison of the proposed channels with the singlehop reference channel.

Multihop diversity and multiroute diversity are defined and distinguished. Multihop diversity results from the concurrent reception of signals that have been transmitted by multiple previous terminals along a single route. Multiroute diversity results from the concurrent reception of signals that have been transmitted along multiple routes that pass through different intermediate terminals. Multiroute diversity is caused by the artificial generation of multiple secondary signals whereas multihop diversity is caused by the natural generation of multiple secondary signals. An argument is presented against the application of multiroute diversity, and the rest of the thesis assumes the use of multihop diversity.

Two simulations for each of the channel models are generated and analyzed. The first simulation validates the theoretical characterizations and indicates that significant performance improvements can be realized through the use of multihop channels. The second simulation indicates that these performance improvements are fairly sensitive to the location of the intermediate terminals under a total power constraint and highlights the care that must be taken when selecting intermediate terminals. All four multihop channels are shown to outperform the singlehop reference channel, with the multihop channels with diversity outperforming the multihop channels without diversity. The performance of amplified relaying is shown to be comparable to or better than that of decoded relaying. This highlights the need for an improved decoded relaying receiver.

A number of implementation considerations are presented and discussed. Amplified relaying channels without the capability to track multipath fading exhibit severe performance degradation. Decoded relaying receivers that utilize the error metrics of previous terminals can improve the performance of decoded relaying channels. Relaying in the same channel as the original transmission is presented as a viable future option. Feedback and feedforward interference generally limit the maximum signal to interference plus noise ratio to a value on the order of the processing gain. Decoded relaying channels generally have a higher end-to-end delay than amplified relaying channels and therefore may not be useful for delay sensitive applications. System-level interference distribution and power control are significantly more complex than traditional cellular systems. Different multiple access schemes realize different advantages and disadvantages for each of the channel models. Adaptive modulation techniques are only applicable for the decoded relaying multihop channel. Terminal complexity is greater than the singlehop reference channel, but can be mitigated through careful design decisions including restrictions on the maximum number of hops.

In conclusion, it is shown that although the results presented in this thesis indicate that there are significant performance advantages to be gained, these multihop channels generally suffer from increased complexity at both the system and terminal levels. The issue to consider will therefore be whether the benefits in terms of enabling communication in the absence of existing infrastructure and increased capacity and coverage outweigh these additional costs. Given the traditional exponential evolutionary path taken by new technologies, it is almost certain that these costs will become comparatively insignificant as the demand for wireless services and bandwidth increases.

### 6.2 Contributions

This thesis is concerned with an area of research where very limited work has been performed to date: the physical layer of multihop wireless communications channels. This thesis covers the following contributions:

- 1. The theoretical characterization of four novel mathematical channel models: the decoded relaying multihop channel, the amplified relaying multihop channel, the decoded relaying multihop diversity channel, and the amplified relaying multihop diversity channel.
- 2. The definition and analysis of multihop diversity and multiroute diversity, including an argument against the application of multiroute diversity.
- 3. The generation of simulations that illustrate the significant performance gains that are possible with multihop communications channels and indicate the importance of careful selection of intermediate terminals.

- 4. The analysis of results that indicate the superiority of multihop channels with diversity over multihop channels without diversity and the superiority of amplified relaying channels over decoded relaying channels
- 5. Discussion that indicates the performance of decoded relaying multihop diversity channels is generally better when the intermediate terminals are closer to the source terminal than the destination terminal.
- 6. Discussion that indicates the performance of amplified relaying multihop diversity channels is generally better when the intermediate terminals are closer to the destination terminal than the source terminal.
- 7. The analysis of results that indicate there is minimal or no benefit to using amplified relaying channels unless Rayleigh fading can be tracked in some fashion by intermediate relaying terminals.
- 8. The analysis of results that indicate the mean signal to interference plus noise ratio of multihop channels is limited by the system processing gain.
- 9. The presentation of numerous implementation considerations that highlight a number of interesting areas for further research.

### 6.3 Suggestions for Future Research

This thesis raises a number of interesting areas for future research, including:

- 1. The refinement of the theoretical characterizations to reduce approximations and better match simulation results.
- 2. The formalization of the argument against multiroute diversity.
- 3. The characterization of performance versus tracking error.
- 4. The derivation of the optimal decoded relaying receiver model.

- 5. The extension of the results to system level characterizations including coverage, interference distribution, throughput distribution, and capacity.
- 6. The integration of mobility models.
- 7. The application of these results as the link metrics for ad-hoc routing algorithms.

# Appendix A – Proofs

### A.1 Proof of (11)

The optimal power distribution that minimizes the total probability of error (10) can be calculated in the following fashion using Lagrange multipliers:

1. Let the total probability of error (10) be given by

$$\int_{T_i \in T_R} (\varepsilon_{P(i)}) = \sum_{T_i \in T_R} \frac{(d_{P(i),i}^p / L_{P(i),i}) N_0}{2\alpha^2 \varepsilon_{P(i)}}$$

2. Let the total power constraint be given by

$$g_{T_i \in T_R}(\varepsilon_{P(i)}) = \varepsilon_0 - \sum_{T_i \in T_R} \varepsilon_{P(i)} = 0.$$

3. Using Lagrange multipliers, calculate a set of partial derivatives given by

$$\frac{\partial}{\partial \varepsilon_{P(i)}} \left[ f_{T_i \in T_R}(\varepsilon_{P(i)}) + \lambda g_{T_i \in T_R}(\varepsilon_{P(i)}) \right] = -\frac{(d_{P(i),i}^p / L_{P(i),i}) N_0}{2\alpha^2 \varepsilon_{P(i)}^2} - \lambda, \forall T_i \in T_R$$

4. Set the partial derivatives equal to  $\theta$  and solve the resulting system of equations for  $\lambda$  and  $\varepsilon_{P(i)}, \forall T_i \in T_R$ .

This calculation results in an equation for the optimal power distribution given by (11) and completes the proof.

### A.2 Proof of (15)

Consider first a channel with 1 hop with source terminal  $T_1$  and destination terminal

 $T_2$ . The received signal to noise ratio at terminal  $T_2$  is given by

$$\gamma_{1,2} = \frac{\alpha^2}{(d_{1,2}^p / \varepsilon_1 L_{1,2} | R_{1,2} |^2) N_0}$$

Now consider a channel with *n* hops with source terminal  $T_1$ , intermediate terminals  $T_2$  through  $T_n$ , and destination terminal  $T_{n+1}$ . Selecting an intermediate terminal at random, we assume that the received signal to noise ratio at terminal  $T_k$  is according to (15) and given by

$$\gamma_{k-1,k} = \frac{\alpha^2}{\sum_{i=2}^{k} (d_{i-1,i}^{p} / \varepsilon_{i-1} L_{i-1,i} | R_{i-1,i} |^2) N_0},$$

which obviously holds for k = 2. If this signal is amplified by (14) then the signal transmitted by terminal  $T_k$  is given by

$$s_{k} = \sqrt{\varepsilon_{k}} \left( a_{t} + \sum_{i=2}^{k} \left( \sqrt{d_{i-1,i}^{p} / \varepsilon_{i-1} L_{i-1,i} | R_{i-1,i} |^{2}} z_{i-1,i} \right) \right),$$

the signal received by terminal  $T_{k+1}$  is given by

$$r_{k,k+1} = \alpha \sqrt{L_{k,k+1} \left| R_{k,k+1} \right|^2 / d_{k,k+1}^p} \sqrt{\varepsilon_k} \left( a_t + \sum_{i=2}^k \left( \sqrt{d_{i-1,i}^p / \varepsilon_{i-1} L_{i-1,i} \left| R_{i-1,i} \right|^2} z_{i-1,i} \right) \right) + z_{k,k+1}},$$

and the received signal to noise ratio at terminal  $T_{k+1}$  is given by

$$\gamma_{k,k+1} = \frac{\alpha^2}{\sum_{i=2}^{k+1} (d_{i-1,i}^p / \varepsilon_{i-1} L_{i-1,i} | R_{i-1,i} |^2) N_0}.$$

Since it has been shown that the equation holds for the first receiving terminal and that given that the equations holds for a particular receiving terminal it will also hold for the next receiving terminal, the equation must hold for all receiving terminals. The equation therefore holds for any channel regardless of the number of hops along the transmission path. Using the set notation from Section 2.1 results in the generalization provided in (15) and completes the proof.

## A.3 Proof of (28)

The proof of (28) is identical to the proof of (11) except that the primary equation is the total probability of decoding error (27) instead of (10). Refer to Section A.1 for more information.

## A.4 Proof of (30)

Consider first the channel with a single route between the source and destination terminals. The total probability of error for a single route is given in [21] by

$$P_e = \frac{1}{2\overline{\gamma_{k,i}}}.$$

Now consider the channel with multiple parallel routes with identical channel statistics and equal (optimal) power distribution. The total probability of error for a *K*-diversity system derived from multiple parallel routes is given in [21] by

$$P_e = \frac{\binom{2K-1}{K}}{\left(2\overline{\gamma_{k,i}}/K\right)^K}.$$

Setting these two equations equal and solving for  $\overline{\gamma_{k,i}}$  results in (30) and completes the proof.

### A.5 Proof of (42) and (43)

The optimal power distribution that minimizes the total probability of error (41) can be calculated in the following fashion:

- 1. Substitute  $\varepsilon_2 = \varepsilon_0 \varepsilon_1$  into (41).
- 2. Set the derivative of (41) with respect to  $\varepsilon_1$  equal to  $\theta$ .
- 3. Solve for  $\varepsilon_1$  and then substitute back to solve for  $\varepsilon_2$ .

This calculation results in equations for the optimal power distribution given by (42) and (43) and completes the proof.

### A.6 Proof of (46)

The proof of (46) is straightforward. Given the proof of (15) and its inverse sum of inverses representation (16), as well as the fact that the output signal to noise ratio of a maximal ratio combiner is the sum of the signal to noise ratios of the input branches [24], multihop wireless communications channels can be considered in the light of resistance theory. Signal links in serial are analogous to resistors in parallel. Signal links in parallel are analogous to resistors in serial.

Using classical circuit theory, the complete network of serial and parallel signal links can be expressed in an equivalent representation. The equivalent representation is derived in an iterative fashion, where the equivalent representation of the received signal to noise ratio of a particular terminal is composed of the equivalent representation of the received signal to noise ratios of all the preceding terminals along the transmission path. The equivalent representation of the received signal to noise ratio of every terminal along the transmission path, including the destination terminal, is therefore given by (46), thus completing the proof.

### A.7 Proof of (52) and (53)

The optimal power distribution that minimizes the total probability of error (51) can be calculated in the following fashion:

- 1. Substitute  $\varepsilon_2 = \varepsilon_0 \varepsilon_1$  into (51).
- 2. Set the derivative of (51) with respect to  $\varepsilon_1$  equal to  $\theta$ .
- 3. Solve for  $\varepsilon_1$  and then substitute back to solve for  $\varepsilon_2$ .

This calculation results in equations for the optimal power distribution given by (52) and (53) and completes the proof.

### A.8 Proof of (56)

The proof of (56) is identical to the proof of (15) except that the amplification factor at each intermediate terminal is (55) instead of (14). Refer to Section A.2 for more information.

## A.9 Proof of (64)

The weighted sum of the signal to noise ratios for all incorrect branches at terminal  $T_i$  is given by  $\gamma_{E,i} = \sum_{T_k \in T_E} X_{k,i} \lambda_{k,i}$ . The weighted sum of the signal to noise ratios for all correct branches at terminal  $T_i$  is given by  $\gamma_{C,i} = \sum_{T_k \in T_C} X_{k,i} \lambda_{k,i}$ . The probability equation given in (64),  $\Pr(\gamma_{E,i} > \gamma_{C,i})$ , can be rewritten as  $\Pr(\gamma_{E,i} - \gamma_{C,i} > 0)$  and calculated directly.

The probability distribution function of  $\gamma_{E,i}$  is given in [21] by

$$p(\gamma_{E,i}) = \sum_{T_k \in T_E} \frac{\pi_{E,k,i}}{X_{k,i} \overline{\gamma_{k,i}}} \exp(-\gamma_{E,i} / X_{k,i} \overline{\gamma_{k,i}}),$$

where  $\pi_{E,k,i}$  is given by (65) and the probability distribution function of  $\gamma_{C,i}$  is given in [21] by

$$p(\gamma_{C,i}) = \sum_{T_k \in T_C} \frac{\pi_{C,k,i}}{X_{k,i} \overline{\gamma_{k,i}}} \exp(-\gamma_{C,i} / X_{k,i} \overline{\gamma_{k,i}}),$$

where  $\pi_{C,k,i}$  is given by (66).

Now consider the transformation of variables given by  $Y = \gamma_E - \gamma_C$ ,  $W = \gamma_C$ ,  $\gamma_E = Y + W$ , and  $\gamma_C = W$ . The Jacobian determinant of the transformation is 1. The joint probability density function of (Y, W) is given by

$$f_{YW}(y,w) = \sum_{T_k \in T_E} \frac{\pi_{E,k,i}}{X_{k,i}\overline{\gamma_{k,i}}} \exp(-(y+w) / X_{k,i}\overline{\gamma_{k,i}}) \sum_{T_l \in T_C} \frac{\pi_{C,l,i}}{X_{l,i}\overline{\gamma_{l,i}}} \exp(-w / X_{l,i}\overline{\gamma_{l,i}})$$
$$= \sum_{T_k \in T_E} \sum_{T_l \in T_C} \frac{\pi_{E,k,i}\pi_{C,l,i}}{X_{k,i}\overline{\gamma_{k,i}}} \exp(-(y+w) / X_{k,i}\overline{\gamma_{k,i}} - w / X_{l,i}\overline{\gamma_{l,i}})$$

and the probability density function of Y is given by

$$f_{Y}(y) = \int_{0}^{\infty} \sum_{T_{k} \in T_{E}} \sum_{T_{l} \in T_{C}} \frac{\pi_{E,k,i} \pi_{C,l,i}}{X_{k,i} \overline{\gamma_{k,i}} X_{l,i} \overline{\gamma_{l,i}}} \exp(-(y+w) / X_{k,i} \overline{\gamma_{k,i}} - w / X_{l,i} \overline{\gamma_{l,i}}) dw$$

$$= \sum_{T_{k} \in T_{E}} \sum_{T_{l} \in T_{C}} \frac{\pi_{E,k,i} \pi_{C,l,i}}{X_{k,i} \overline{\gamma_{k,i}} X_{l,i} \overline{\gamma_{l,i}}} \exp(-y / X_{k,i} \overline{\gamma_{k,i}}) \int_{0}^{\infty} \exp(-w(1 / X_{k,i} \overline{\gamma_{k,i}} + 1 / X_{l,i} \overline{\gamma_{l,i}})) dw$$

$$= \sum_{T_{k} \in T_{E}} \sum_{T_{l} \in T_{C}} \frac{\pi_{E,k,i} \pi_{C,l,i}}{X_{k,i} \overline{\gamma_{k,i}} X_{l,i} \overline{\gamma_{l,i}}} \exp(-y / X_{k,i} \overline{\gamma_{k,i}}) (-1 / (1 / X_{k,i} \overline{\gamma_{k,i}} + 1 / X_{l,i} \overline{\gamma_{l,i}}))$$

$$= \sum_{T_{k} \in T_{E}} \sum_{T_{l} \in T_{C}} \frac{\pi_{E,k,i} \pi_{C,l,i}}{X_{k,i} \overline{\gamma_{k,i}} + X_{l,i} \overline{\gamma_{l,i}}}} \exp(-y / X_{k,i} \overline{\gamma_{k,i}})$$

Finally, the probability that Y > 0 is given by

$$Pr(Y > 0) = \int_{0}^{\infty} \sum_{T_k \in T_E} \sum_{T_l \in T_C} \frac{\pi_{E,k,i} \pi_{C,l,i}}{X_{k,i} \overline{\gamma_{k,i}} + X_{l,i} \overline{\gamma_{l,i}}} \exp(-y/X_{k,i} \overline{\gamma_{k,i}}) dy$$
$$= \sum_{T_k \in T_E} \sum_{T_l \in T_C} \frac{\pi_{E,k,i} \pi_{C,l,i}}{X_{k,i} \overline{\gamma_{k,i}} + X_{l,i} \overline{\gamma_{l,i}}} \int_{0}^{\infty} \exp(-y/X_{k,i} \overline{\gamma_{k,i}}) dy,$$
$$= \sum_{T_k \in T_E} \sum_{T_l \in T_C} \frac{\pi_{E,k,i} \pi_{C,l,i} X_{k,i} \overline{\gamma_{k,i}}}{X_{k,i} \overline{\gamma_{k,i}} + X_{l,i} \overline{\gamma_{l,i}}}$$

which of course is equivalent to the probability that  $\gamma_{E,i} - \gamma_{C,i} > 0$ . This equation can be rewritten as the probability that  $\gamma_{E,i} > \gamma_{C,i}$ , the probability equation given in (64), completing the proof.

# A.10 Proof of (72)

The proof of (72) is identical to the proof of (15) except that the signal to noise plus interference ratio (71) is used in place of the signal to noise ratio (3). Refer to Section A.2 for more information.

# A.11 Proof of (75)

The proof of (75) is identical to the proof of (46) except that the signal to noise plus interference ratio (74) is used in place of the signal to noise ratio (33). Refer to Section A.6 for more information.

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