Multi-Resolution Broadcasting Over the Grassmann and Stiefel Manifolds

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***Abstract*—We consider the design of space-time codes for multi-resolution multiple-input multiple-output (MIMO) broadcast communication systems. Two classes of receivers are considered: high-resolution (HR) receivers, which have access to reliable channel state information (CSI) and can perform coherent detection, and low-resolution (LR) receivers which do not have access to CSI and can only perform non-coherent detection. We propose a layered encoding structure, whereby, for the LR receivers, the transmitted codewords are chosen to be points on the Grassmann manifold whereas, for the HR receivers, incremental information is encoded in the particular bases of the transmitted codewords, thereby representing points on the Stiefel manifold. For the HR receivers, we develop a computationally-efficient two-step detector. Using this detector, we show that the proposed structure enables reliable coherent communication of the incremental HR information without compromising the reliability with which the basic LR information is non-coherently communicated. We also show that this structure enables full diversity to be achieved for both LR and HR receivers. Finally, we show that this structure achieves the maximum number of degrees of freedom for non-coherent LR channels and coherent HR channels with unitarily-constrained input signals.**

I. Introduction

Multiple-input multiple-output (MIMO) communication systems offer a spectrally-efficient means for the reliable transmission of high data rates [1], [2]. However, the mode in which these systems operate depends, to a large extent, on the accuracy of the channel state information (CSI) available at the receivers. For instance, when accurate CSI is available at the receiver, the communication channel operates in a coherent mode, whereas when no CSI is available, the channel operates in a non-coherent mode [3], [4]. In comparison with their non-coherent counterparts, coherent channels are amenable to simpler detection mechanisms and can support the reliable communication of higher data rates.

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Similar to point-to-point communications, MIMO systems are expected to be effective in broadcast scenarios wherein one transmitter wishes to send multi-resolution information to multiple receivers. Depending on their mobility and relative locations from the transmitter, a receiver may or may not be able to acquire reliable CSI. In such cases, the transmitter may wish to send basic low-resolution (LR) information that can be detected by all receivers, including those without CSI, and in addition, it may wish to send incremental high-resolution (HR) information to receivers with reliable CSI.

In [5] it was shown that, when the signal-to-noise ratio (SNR) is sufficiently high and the coherence time of the channel is at least as large as the sum of the number of transmit and receive antennas, the capacity of the non-coherent MIMO channel is achieved by a signaling scheme in which the transmitted codewords are isotropically distributed on the so-called Grassmann manifold. Distinct points on this manifold are equivalence classes that represent distinct subspaces. In contrast, the capacity of the coherent MIMO channel is achieved by Gaussian-distributed codewords [6], which, for several considerations are often difficult to use in practice, and unitarily-structured block codes are utilized instead [2].

In this paper, we address the problem of designing space-time codes that allow the simultaneous transmission of information to two classes of receivers: HR receivers, which have access to reliable CSI and can perform coherent detection, and LR receivers which do not have access to reliable CSI and can only perform non-coherent detection. For the LR receivers, the transmitted codewords represent points on the Grassmann manifold, whereas for the HR receivers incremental information is transmitted in the particular bases of the LR codewords, which represent points on the Stiefel manifold. In particular, to encode incremental information, we will exploit the fact that Grassmannian-structured codewords are invariant under the right action of unitary groups. Right multiplication of an element from this group with a Grassmannian-structured codeword will rotate its bases, but will preserve the subspace it spans. Using this layered structure a receiver with no CSI will be able to detect the basic LR information, whereas a receiver with reliable CSI will be able to detect both the LR and the

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incremental HR information. It will be shown that imposing the unitary constraint on the HR information codewords, not only facilitates their detection, but also preserves the reliability with which the basic LR information is communicated. In particular, for the HR receivers, we develop a computationally-efficient two-step detector, which will enable us to show that the proposed layered structure achieves full diversity for both the LR and HR receivers. Furthermore, it will be shown that the number of communications degrees of freedom achieved by the proposed layered structure is maximal for both non-coherent LR channels and coherent HR channels with unitarily-constrained input signals.

Similar ideas were presented in [7], [8] for transmitting data in a mulituser MIMO (MU-MIMO) settings. However, in our work we consider a multilayer (multi-resolution) broadcasting setup. Also, we propose some practical structures for the coherent and non-coherent space-time codes whereas in [7], [8] the authors were interested in codes that can achieve the degrees of freedom (DoF).

II. Preliminaries

In this section we will provide a brief background on the Stiefel and Grassmann manifolds, which will be necessary for subsequent analysis.

For $T \geq M$, the Stiefel manifold $\mathbb{S}_{T,M}(\mathbb{C})$ is defined as the set of all unitary $T \times M$ matrices, that is,

$$\mathbb{S}_{T,M}(\mathbb{C}) = \{\mathbf{Q} \in \mathbb{C}^{T \times M} : \mathbf{Q}^{\mathcal{H}}\mathbf{Q} = \mathbf{I}_M\}. \quad (1)$$

The Stiefel manifold $\mathbb{S}_{T,M}(\mathbb{C})$ is submanifold of $\mathbb{C}^{T \times M}$ of $TM - M^2/2$ complex dimensions.

The Grassmann manifold $\mathbb{G}_{T,M}(\mathbb{C})$ is defined as the quotient space of $\mathbb{S}_{T,M}(\mathbb{C})$ with respect to the equivalence relation that renders two elements $\mathbf{P}, \mathbf{Q} \in \mathbb{S}_{T,M}(\mathbb{C})$ equivalent if their T-dimensional column vectors span the same subspace, i.e.,

$$\mathbf{P} = \mathbf{Q}\mathbf{V} \quad (2)$$

for some matrix $\mathbf{V}$ in the unitary group $\mathbb{U}_M = \mathbb{S}_{M,M}(\mathbb{C})$.

Since each element of $\mathbb{G}_{T,M}(\mathbb{C})$ represents an equivalence class in $\mathbb{S}_{T,M}(\mathbb{C})$, the number of complex dimensions of the Grassmann manifold can be expressed as:

$$\begin{aligned}\dim(\mathbb{G}_{T,M}(\mathbb{C})) &= \dim(\mathbb{S}_{T,M}(\mathbb{C})) - \dim(\mathbb{S}_{M,M}(\mathbb{C})) \\ &= M(T - M). \end{aligned} \quad (3)$$

III. System Model

We consider a broadcast MIMO communication system with M transmit antennas with two classes of receivers operating over the block Rayleigh flat-fading channel in Figure 1. The channel is assumed to be constant over T consecutive time slots. Using N_i to denote the number of receive antennas of the i-th receiver, the communication system can be modeled as

$$\begin{aligned}\mathbf{Y}_i &= \mathbf{X}\mathbf{H}_i + \mathbf{W}_i \\ &= \mathbf{U}\mathbf{A}\mathbf{H}_i + \mathbf{W}_i, \ i = 1, 2, \cdots, \end{aligned} \quad (4)$$

where $\mathbf{Y}_i$ is the $T \times N_i$ received matrix of the i-th receiver, $\mathbf{X} = \mathbf{U}\mathbf{A}$ is the $T \times M$ transmitted matrix which contains both the LR and the HR information. When the SNR is sufficiently high, the capacity of the LR channel observed by, say, receiver 1 can be achieved if $\mathbf{X}$ were isotropically distributed on $\mathbb{G}_{T,M}(\mathbb{C})$, provided that $N_1 \geq M$, $T \geq M + N_1$ and $M \leq \lfloor T/2 \rfloor$ [5]. For ease of exposition, these conditions will be assumed to be satisfied throughout. To realize the isotropic distribution for $\mathbf{X}$ on $\mathbb{G}_{T,M}(\mathbb{C})$ under these conditions, in this paper, the LR information is encoded in the subspace spanned by the matrix $\mathbf{U}$ and the HR information is encoded in the $M \times M$ unitary matrix $\mathbf{A} \in \mathbb{U}_M$, which represents a rotation of the subspace spanned by $\mathbf{U}$. The matrices $\mathbf{H}_i$ and $\mathbf{W}_i$ represent the channel and noise observed by receiver i, and the elements of these matrices are assumed to be statistically independent, identically distributed (i.i.d) circularly-symmetric zero mean complex Gaussian random variables. The entries of $\mathbf{H}_i$ have unit variance and the entries of $\mathbf{W}_i$ have variance N_0. For notational convenience, we will drop the receiver index i and will allude to the coherent and non-coherent operating modes as necessary.

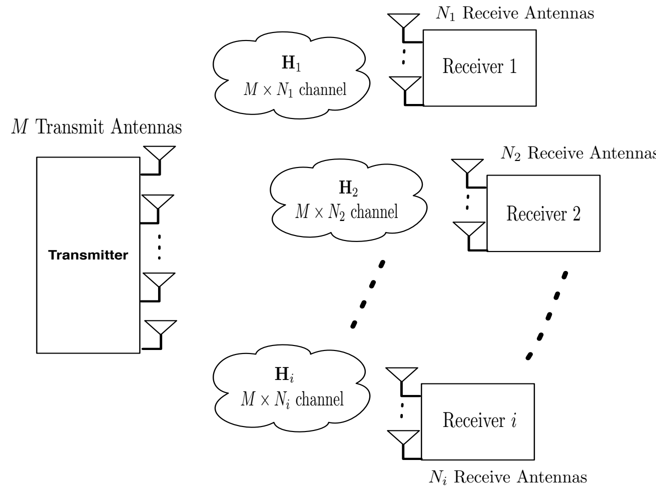

Fig. 1: The MIMO broadcast system model.

In the proposed layered encoding scheme, the LR layer contains the basic information and is represented by a unitary non-coherent code, with codewords that are isotropically distributed on the complex Grassmann manifold $\mathbb{G}_{T,M}(\mathbb{C})$. Since this manifold represents the set of M-dimensional subspaces in $\mathbb{C}^T$, every codeword in this non-coherent code represents a particular M-dimensional subspace, which is spanned by the $T \times M$ matrix $\mathbf{U}$. This LR layer has the advantage that it can be decoded coherently if the receiver has reliable CSI or non-coherently if CSI is not available. In contrast, in our construction, the HR layer contains the incremental information and is represented by a coherent code with particular codewords drawn from the group of $M \times M$ complex unitary matrices $\mathbb{U}_M$. This layer can only be decoded coherently by a receiver that has access to reliable CSI.

We will later show that restricting the matrix $\mathbf{A}$ in (4), which represents the incremental HR information, to possess a unitary structure ensures the preservation of the distance characteristics of the non-coherent code. This implies that the performance of the LR (non-coherent) code will not be affected by the transmission of the HR information. Also, restricting the matrix $\mathbf{A}$ to be unitary does not increase the required transmitted power, i.e., the HR (coherent) layer is completely transparent to the LR (non-coherent) layer and no additional power provisioning is required to maintain the performance of the LR layer.

IV. The Optimum Non-Coherent Detector

In this section, we will discuss the optimum non-coherent detector, which is equivalent to the following generalized likelihood ratio test (GLRT) detector [9].

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \sup_{\mathbf{H}} p(\mathbf{Y}|\mathbf{U}, \mathbf{H}). \tag{5}$$

Using the facts that the matrix $\mathbf{U}$ is unitary and that the fading coefficients are i.i.d Gaussian-distributed random variables, it can be shown that the detector in (5) is equivalent to the following maximum likelihood (ML) detector [9]:

$$\hat{\mathbf{U}} = \arg\max_{\mathbf{U}} \mathrm{Trace}(\mathbf{Y}^{\mathcal{H}}\mathbf{U}\mathbf{U}^{\mathcal{H}}\mathbf{Y}). \tag{6}$$

From (6) it can be readily verified that encoding the HR information in the unitary matrix $\mathbf{A} \in \mathbb{U}_M$ does not compromise the performance of the non-coherent GLRT detector. In particular, we can write $\mathbf{H} \overset{d}{=} \mathbf{AH}$, where $\overset{d}{=}$ denotes equality in distribution. This implies that the encoded HR information will 'see' an equivalent channel matrix $\mathbf{AH}$ with the same statistics as the original channel matrix $\mathbf{H}$. As such, it can be readily seen that the GLRT detector will exhibit the same performance, as if the HR information layer were not present; a Grassmannian-structured codebook will exhibit a particular diversity order regardless of whether incremental HR information is transmitted.

Restricting attention to the current case in which $\mathbf{U} \in \mathbb{G}_{T,M}(\mathbb{C})$, and using the approach in [10], the pairwise error probability (PEP) can be upper bounded by

$$\mathrm{PEP}(\mathbf{U}_1 \to \mathbf{U}_2) \leq \frac{1}{2} \prod_{m=1}^{M} \left[1 + \frac{\left(\frac{\mathrm{SNR}}{M}\right)^2 (1 - s_m^2)}{4(1 + \frac{\mathrm{SNR}}{M})} \right]^{-N}, \tag{7}$$

where $\mathrm{SNR} \triangleq \mathbf{E}(\mathrm{Trace}\,\mathbf{X}\mathbf{X}^{\mathcal{H}})/(N_0 MT)$ and $1 \geq s_1 \geq \cdots \geq s_M \geq 0$ are the singular values of the $M \times M$ matrix $\mathbf{U}_2^{\mathcal{H}}\mathbf{U}_1$. By properly designing the Grassmannian codebook, the greatest singular value of $\mathbf{U}_2^{\mathcal{H}}\mathbf{U}_1$, s_1, can be guaranteed to be strictly less than 1 for any two distinct codewords $\mathbf{U}_1$ and $\mathbf{U}_2$ in $\mathbb{G}_{T,M}(\mathbb{C})$. In that case, it can be readily verified that the asymptotic SNR exponent equals MN, thereby ensuring that the Grassmannian codebook achieves full diversity order, as if the HR information were not transmitted.

V. Coherent Detectors

In this section, we will discuss the optimum coherent detector for the class of receivers with reliable CSI. For such receivers both the HR and LR information can be reliably detected, and the optimal detector in this case is the conventional ML one. Despite its optimality, the ML detector is computationally expensive to implement in practice. To circumvent this difficulty, we will develop a computationally-efficiency suboptimum detector, and we will show that this detector achieves full diversity, as does the optimal ML one.

A. The Optimum One-Step Coherent Detector

Since the noise matrix is Gaussian-distributed, the optimum coherent detector that "jointly" decodes the LR and HR information layers can be expressed as the detector that yields

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{Y} - \mathbf{XH}\|^2. \tag{8}$$

Using S_L and S_H to denote the cardinality of the LR Grassmannian codebook and the HR unitary codebook, respectively, it can be seen that the detector in (8) requires an exhaustive search over $S_L S_H$ codewords, which is computationally expensive to implement in practice. To alleviate this difficulty, in the next section we will develop a sequential two-step detector that is computationally-efficient and that will be shown to achieve full diversity.

B. The Two-Step Coherent Detector

In this section we develop a sequential two-step detector that is less complex than the one presented in Section V-A. In the first step of this detector, the GLRT approach in (6) is used to detect the LR Grassmannian codeword. In the second step of the sequential detector, the GLRT output, $\hat{\mathbf{U}}$, is assumed to be the correct Grassmannian codeword and is subsequently fed to an ML detector for detecting the HR information in $\mathbf{A}$. The output of this ML detector is given by

$$\hat{\mathbf{A}} = \arg\min_{\mathbf{A}} \|\mathbf{Y} - \hat{\mathbf{U}}\mathbf{AH}\|^2. \tag{9}$$

This two-step detector is significantly less complex than the one-step detector in (8), as it requires searching over $S_L + S_H$ codewords, as opposed to the $S_L S_H$ codewords that are searched over in (8). From a performance perspective, the two-step detector does not take advantage of the available CSI when detecting the LR information in $\mathbf{U}$. This results in a performance degradation in comparison with one-step detector. However, as the following theorem shows, both detectors yield the same diversity order (the proof of the theorem is omitted due to space limitation).

Theorem 5.1: Let the LR and HR codebooks, $\{\mathbf{U}\}$ and $\{\mathbf{A}\}$, satisfy the full diversity singular values criterion for non-coherent codes in [10] and the full diversity determinant criterion for coherent codes in [1], respectively. Then, the sequential two-step coherent detector achieves a diversity of order MN, i.e., full diversity.

Remark: Since full diversity is achieved by the suboptimal sequential two-step detector, this diversity order must be also achieved by the optimal one-step coherent detector. □

VI. Degrees of Freedom

Having considered the detection of the LR and HR information layers, we will now focus our attention on the number of degrees of freedom that they achieve. To do so, we will use the fact that the incremental HR information is sent over the Stiefel manifold, whereas the basic LR information is sent over the Grassmann manifold, cf. Section II. The following corollary characterizes the degrees of freedom achieved by the proposed layered encoding structure.

Corollary 6.1: The achievable degrees of freedom, over T time slots, for the conjoined LR and HR layers is $TM-M^2/2$, whereas the achievable degrees of freedom for the LR layer is $M(T-M)$.

Proof: By construction, the LR information is encoded over matrices that are isotropically distributed on the Grassmann manifold, $\mathbb{G}_{T,M}(\mathbb{C})$, and the total LR and HR information is encoded over matrices that are are isotropically distributed on the Stiefel manifold, $\mathbb{S}_{T,M}(\mathbb{C})$. The proof of this lemma follows directly from the dimensionality of these manifolds discussed in Section II. The achievability of these degrees of freedom follows directly from the achievability of the non-coherent degrees of freedom; as mentioned above, with our proposed code structure, the presence of the coherent layer is completely transparent to the non-coherent layer, and therefore the maximum number of degrees of freedom for the non-coherent layer is achievable. Hence, it is straightforward to show that the degrees of freedom of the coherent layer are also achievable. ■

It is also worth mentioning that the proposed construction does not achieve the maximum number of degrees of freedom for the HR receivers. In particular, for these receivers, when no constraints are imposed on $\mathbf{X}$, the maximum number of degrees of freedom is given by $T\min\{M,N\}$, which, under the conditions in Section III, reduces to TM. However, by restricting $\mathbf{X}$ to be in $\mathbb{S}_{T,M}(\mathbb{C})$, this number is reduced by $M^2/2$. This reduction can be regarded as the price paid to ensure that the basic LR information rate, which can be decoded by all receivers, is maximized. Restricting $\mathbf{X}$ to be in $\mathbb{S}_{T,M}(\mathbb{C})$ is equivalent to restricting $\mathbf{A}$ to be in $\mathbb{U}_M$, which offers the advantage of preserving the channel statistics of the LR channel; cf. Section IV. This ensures that code designs that are favorable for standard point-to-point non-coherent MIMO systems can be readily utilized in the current multi-resolution layered scheme. For completeness, in the next section, we will allude to some of these designs.

VII. Coherent and Non-Coherent Code Constructions

In this section, we present coherent and non-coherent code constructions that will be used in Section VIII in evaluating the performance of the proposed layered coding scheme.

A. HR Layer (Coherent) Code Construction

In Section V, we showed that HR coherent codes drawn from the unitary group $\mathbb{U}_M$ ensure that the performance of the LR non-coherent codes is not compromised. A candidate of such coherent codes is the standard 2×2 Alamouti scheme [2], whereby the matrix $\mathbf{A}$ is constructed as follows:

$$\mathbf{A}=\frac{1}{\sqrt{2}}\begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}, \tag{10}$$

where s_1 and s_2 are two complex symbols drawn from any constant modulus constellation, e.g., PSK. Note that Alamouti's scheme can be used for $M=2$ with PSK modulation to construct our coherent orthogonal code. For larger M, square orthogonal coherent code designs that exhaust all the $M^2/2$ degrees of freedom are not readily available, but can be constructed directly on $\mathbb{U}_M$.

B. LR Layer (Non-Coherent) Code Construction

For the LR (non-coherent) code construction, we consider two approaches: the exponential parameterization approach [4] and the direct design approach [3].

1) The exponential parameterization method [4]: In this approach non-coherent codes are obtained from coherent block codes using the exponential map. In particular, the non-coherent code matrices $\{\mathbf{U}\}$ in (4) are constructed using

$$\mathbf{U}=\left[\exp\begin{pmatrix} \mathbf{0}_M & \alpha\mathbf{V} \\ -\alpha\mathbf{V}^{\mathcal{H}} & \mathbf{0}_M \end{pmatrix}\right]\mathbf{I}_{T,M}, \tag{11}$$

where $\mathbf{V}\in\mathbb{C}^{M\times(T-M)}$ is the matrix representing the coherent code, $\mathbf{I}_{T,M}\in\mathbb{C}^{T\times M}$ is the matrix containing the first M columns of the $T\times T$ identity matrix, and α is a homothetic factor which ensures that the singular values of $\mathbf{V}$ are less than $\pi/2$ [4]. Although this approach facilitates the design of non-coherent codes, it does not provide performance guarantees. Another approach that addresses this issue is the direct one, which we present next.

2) The direct design [3]: In this approach, the minimum chordal Frobenius norm between the spaces spanned by any two matrices $\mathbf{U}_i,\mathbf{U}_j\in\mathbb{G}_{T,M}(\mathbb{C})$ is maximized. This norm is given by $\sqrt{2M-2\,\mathrm{Trace}\,(\mathbf{\Sigma}_{ij})}$, where $\mathbf{\Sigma}_{ij}$ is the matrix containing the singular values of $\mathbf{U}_i^{\mathcal{H}}\mathbf{U}_j$, cf. [11].

Using the approach in [3], the S_L Grassmannian constellation points required for the non-coherent code of the LR layer can be cast as the following optimization problem:

$$\begin{aligned}&\min_{\{\mathbf{U}_r\}_{r=1}^{S_L}}\ \max_{1\le i,j\le S_L}\ \mathrm{Trace}(\mathbf{\Sigma}_{ij})\\ &\text{subject to}\quad \mathbf{U}_k\in\mathbb{G}_{T,M}(\mathbb{C}),\quad k=1,\ldots,S_L.\end{aligned} \tag{12}$$

VIII. Simulation Results

In this section, we provide numerical evaluation of the performance of the proposed layered coding scheme. In all cases, we will use $T=4$ and $M=N=2$. For the HR layer, we will use Alamouti's scheme in Section VII-A for sending two 4-QAM symbols, which gives rise to an HR transmission rate of 1 bit per channel use (bpcu). For the LR layer, we will use the exponential parameterization method and the direct design in Sections VII-B1 and VII-B2, respectively.

In Figure 2, we plot the bit error probability when the exponential parameterization method is used for constructing

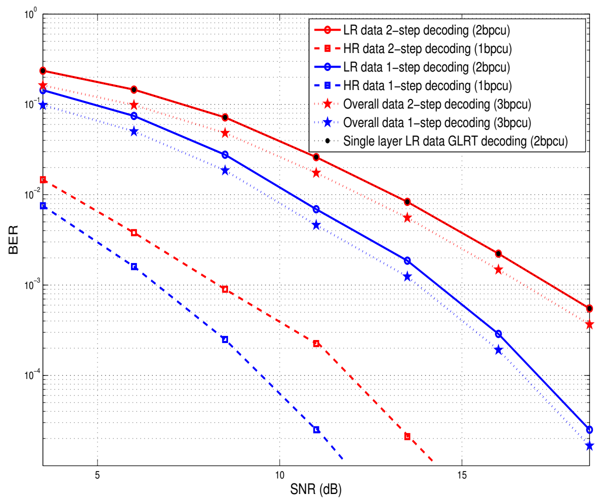

Fig. 2: The LR layer is constructed on $\mathbb{G}_{4,2}(\mathbb{C})$ using the exponential parameterization method and the HR layer is constructed using the 2×2 Alamouti code.

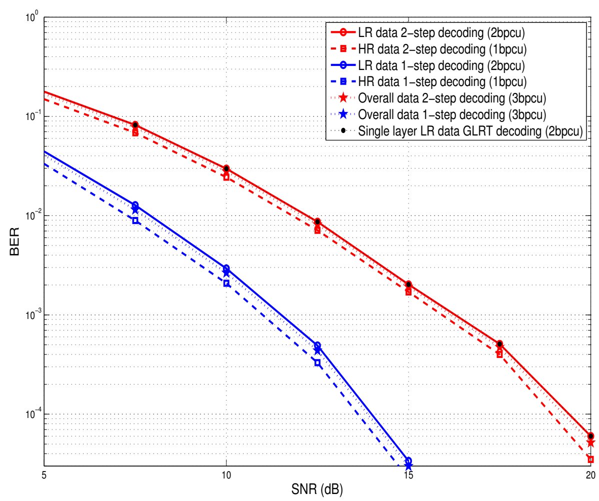

Fig. 3: The LR layer is constructed on $\mathbb{G}_{4,2}(\mathbb{C})$ using the direct design technique and the HR layer is constructed using the 2×2 Alamouti code.

the non-coherent code of the LR layer. In this method we used the space-time code matrix $\mathbf{V}$ given by [12]

$$\mathbf{V} = \begin{pmatrix} s_1 + \theta s_2 & \phi(s_3 + s_4) \\ \phi(s_3 - \theta s_4) & s_1 - \theta s_2 \end{pmatrix} \qquad (13)$$

where $\phi^2 = \theta = e^{i\frac{\pi}{4}}$ and s_i, $i = 1, \cdots, 4$, are the four 4-QAM symbols to be transmitted on the LR layer. In this case, the homothetic factor to maximize the product distance is selected to be $\alpha = 0.3$ [4].

In Figure 3, we plot the bit error probability when the direct technique is used for designing a 256-point Grassmannian constellation for the communication on the LR layer.

From Figures 2 and 3, it can be readily seen that all the bit error probability curves have the same high SNR slopes. This implies that the optimal one-step detector and the sequential two-step detector achieve the same diversity order, which is the same diversity order achieved by the non-coherent receiver. This confirms the result reported in Theorem 5.1 and, in addition, confirms that the performance of the non-coherent layer receiver is not adversely affected by the transmission of the HR layer.

IX. Conclusion

In this paper, we proposed a new layered multi-resolution broadcast space-time coding scheme which allows the simultaneous transmission of LR non-coherent information for all receivers, including those with no CSI, and HR coherent information to those receivers that have reliable CSI. The proposed scheme ensures that the communication of the HR layer is transparent to the underlying LR layer. We showed that both the non-coherent and coherent receivers achieve full diversity, and we showed that the proposed scheme achieves the maximum number of communication degrees of freedom for non-coherent LR channels and coherent HR channels with unitarily-constrained input signals.

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