Optimum Transmission through the Gaussian Multiple Access Channel

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Outline

Introduction

- The capacity region of the GMAC is known [Wyner74]
  - Gaussian signaling
  - Successive Interference Cancelation (SIC)
  - Time-sharing
- The computation of the optimum transmission parameters is, in general, difficult
  - Transmission parameters:
    - Covariance matrices of input signals
    - User decoding orders
    - Time-sharing weights
  - Solution known for linear rate objectives [Tse&Hanly98]

Introduction

- Features of linear rate objectives
  - Complexity reduction
    - The optimum user decoding order is given by the order of the rate weights
    - The optimization only has to find the optimum covariance matrices
  - Convexity of the optimization problem
    - The linear rate objective can be expressed as a concave function of the covariance matrices
- What about non-linear rate objectives?

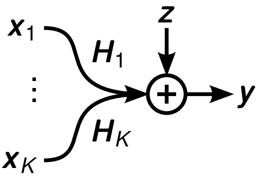
Contributions

- Solution of a class of problems with **non-linear** objectives
  - Objectives convex in rates not necessarily in transmission parameters
  - Aided by variational inequalities
- Complete description of the optimum parameters
  - Necessary and sufficient condition
- An algorithm that finds the optimum parameters
  - We can now solve problems that could not be solved before

Outline

System model

MIMO GMAC



- Number of users: K
- Antennas of user k: N_k
- Signal of user k: $\boldsymbol{x}_k$

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{z}$$

- Let $\mathrm{E}[\boldsymbol{z}\boldsymbol{z}^\dagger] = \boldsymbol{I}$, $\boldsymbol{Q}_k = \mathrm{E}[\boldsymbol{x}_k \boldsymbol{x}_k^\dagger]$ and

$$\bar{\boldsymbol{Q}} = \boldsymbol{Q}_1 \oplus \cdots \oplus \boldsymbol{Q}_K = \begin{bmatrix} \boldsymbol{Q}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{Q}_K \end{bmatrix}$$

- $\bar{\boldsymbol{Q}}$ must satisfy certain power constraints

$$g_\ell\left(\bar{\boldsymbol{Q}}\right) \leq 0, \quad \ell = 1, \ldots, L$$

Problem formulation

- Time-sharing implies a convex combination of (at most) $K+1$ rate vectors (Carathéodory's theorem)
- Achievable rate: $\boldsymbol{\rho}(\boldsymbol{\alpha}, \mathcal{Q})$, with the k-th entry given by

$$\rho_k(\boldsymbol{\alpha}, \mathcal{Q}) = \sum_{m=1}^{K+1} \sum_{i=1}^{K!} \alpha_{mi} r_{ki}\left(\bar{\boldsymbol{Q}}^{(m)}\right), \quad \mathcal{Q} = \{\bar{\boldsymbol{Q}}^{(m)}\}_{m=1}^{K+1}$$

- We define $\boldsymbol{\alpha} \in \mathbb{R}^{(K+1)\times K!}$ as the time-sharing matrix
  - It jointly represents time-sharing and decoding orders
  - The entries must belong to the unit simplex

$$\mathcal{S} \triangleq \left\{\boldsymbol{\alpha} \middle| \sum_{m=1}^{K+1} \sum_{i=1}^{K!} \alpha_{mi} = 1,\ \alpha_{mi} \geq 0,\ \forall m, i\right\}.$$

Outline

Optimization

$$\begin{aligned} \min_{\boldsymbol{\alpha},\mathcal{Q}} \quad & f\left(\boldsymbol{\rho}\left(\boldsymbol{\alpha},\mathcal{Q}\right)\right), \quad \mathcal{Q} = \{\bar{\boldsymbol{Q}}^{(m)}\}_{m=1}^{K+1} \\ \text{subject to} \quad & \boldsymbol{\alpha} \in \mathcal{S} \\ & \bar{\boldsymbol{Q}}^{(m)} \in \mathcal{P},\ m = 1, \ldots, K+1 \end{aligned}$$

Problems with non-linear objectives
Preliminaries

Lemma 1 (Variational inequalities)

- *Let $f : \mathcal{X} \to \mathbb{R}$ be convex and continuously differentiable*
- *If* $\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in\mathcal{X}} \boldsymbol{x}^\dagger \nabla f(\boldsymbol{x}^*)$
- *then* $\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in\mathcal{X}} f(\boldsymbol{x})$

- *Let $\mathcal{X}$ be convex, and let f be continuously differentiable*
- *If* $\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in\mathcal{X}} f(\boldsymbol{x})$
- *then* $\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in\mathcal{X}} \boldsymbol{x}^\dagger \nabla f(\boldsymbol{x}^*)$

Problems with non-linear objectives

Theorem 1

- *Let* $\boldsymbol{w} = -\nabla f(\boldsymbol{\rho}(\boldsymbol{\alpha}^*, \mathcal{Q}^*))$ *for some* $\boldsymbol{\alpha}^*$, $\mathcal{Q}^*$ *and convex* f
- *Let users be labelled so that* $w_1 \leq \cdots \leq w_K$
- *Then,* $\boldsymbol{\alpha}^*$ *and* $\mathcal{Q}^*$ *are optimum* ***if and only if*** *for each strictly positive* $\alpha^*_{mi} \in \boldsymbol{\alpha}^*$
  1. *the decoding order* i *is ordered as* $w_1, \ldots, w_K$, *and*
  2. $\bar{\boldsymbol{Q}}^{*(m)}$ *solves*

$$\max_{\bar{\boldsymbol{Q}}} \sum_{k=1}^{K} (w_k - w_{k-1}) \log \left| \boldsymbol{I} + \sum_{j \geq k} \boldsymbol{H}_j \boldsymbol{Q}_j \boldsymbol{H}_j^\dagger \right|, \quad s.t.\ \bar{\boldsymbol{Q}} \in \mathcal{P}$$

Problems with non-linear objectives
Remarks on Theorem 1

D. Calabuig, R. Gohary, H. Yanikomero.

$$\max_{\bar{\boldsymbol{Q}}} \sum_{k=1}^{K} (w_k - w_{k-1}) \log \left| \boldsymbol{I} + \sum_{j \geq k} \boldsymbol{H}_j \boldsymbol{Q}_j \boldsymbol{H}_j^\dagger \right|, \quad \text{s.t. } \bar{\boldsymbol{Q}} \in \mathcal{P}$$

- Theorem 1 gives a complete description of the optimum transmission parameters for convex rate objectives
- The above optimization problem
  - is independent of the decoding order, and
  - is convex if the power constraints are convex
  - Theorem 1 is easily testable
- Computation of the optimum pair $(\boldsymbol{\alpha}^*, \mathcal{Q}^*)$ is still difficult
  - Need to find all the solutions and try all combinations
  - Next theorem solves this problem

Problems with convex and monotonic power constraints

Theorem 2

- *Let the power constraint functions, g_ℓ, $\ell = 1, \ldots, L$, be convex and monotonic*
- *Then, **any achievable rate vector** in the corresponding GMAC can be achieved with one collection of covariance matrices*

Problems with convex and monotonic power constraints

Remarks on Theorem 2

$$\max_{\bar{\boldsymbol{Q}}} \sum_{k=1}^{K} (w_k - w_{k-1}) \log \left| \boldsymbol{I} + \sum_{j \geq k} \boldsymbol{H}_j \boldsymbol{Q}_j \boldsymbol{H}_j^{\dagger} \right|, \quad \text{s.t. } \bar{\boldsymbol{Q}} \in \mathcal{P}$$

- Theorem 2 is true for the entire capacity region
  - It is true for any objective f
- We just need one collection of covariance matrices
  - The search of optimum parameters is simplified
    - All solutions of the above problem are equally optimum

Outline

Algorithm

- We use Theorems 1 and 2 to design an algorithm that converges to the optimum $\boldsymbol{\alpha}^*$ and $\mathcal{Q}^*$
- Each algorithm iteration is divided into two steps
  1. We fix the time-sharing matrix and compute the covariance matrices that satisfy condition 2 of Theorem 1
  2. We fix the covariance matrices and compute the optimum time-sharing matrix
     - This time-sharing matrix necessarily satisfies condition 1 of Theorem 1

Algorithm

Proposition 1

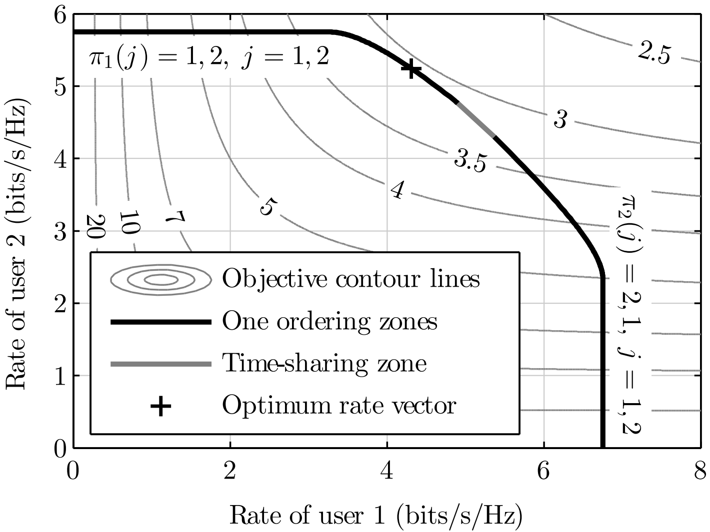
- *If the rate objective f is bounded below for rates inside the capacity region*
- *then, the previous algorithm converges to the optimum pair* $(\boldsymbol{\alpha}^*, \mathcal{Q}^*)$

Application

- We used the previous algorithm to minimize the total completion time of a two-user GMAC
- Completion time [Liu&Erkip2011]: Time required to transmit the data stored in the buffer $\rightarrow$ bits / bit rate
- Let b_k be the number of bits in the buffer of user k
- Let P be the total available power at each transmitter

$$\begin{aligned} \min_{\alpha_1,\alpha_2,\boldsymbol{Q}_1,\boldsymbol{Q}_2} \quad & \frac{b_1}{\rho_1\left(\alpha_1,\alpha_2,\boldsymbol{Q}_1,\boldsymbol{Q}_2\right)} + \frac{b_2}{\rho_2\left(\alpha_1,\alpha_2,\boldsymbol{Q}_1,\boldsymbol{Q}_2\right)} \\ \text{subject to} \quad & \alpha_1+\alpha_2=1,\ \alpha_1\geq 0,\ \alpha_2\geq 0 \\ & \boldsymbol{Q}_1\succeq 0,\ \mathrm{tr}\left(\boldsymbol{Q}_1\right)\leq P,\ \boldsymbol{Q}_2\succeq 0,\ \mathrm{tr}\left(\boldsymbol{Q}_2\right)\leq P \end{aligned}$$

Application



Outline

Conclusions

- GMAC problems with non-linear rate objectives
- Main results
  - Complete description of the optimum pair $(\boldsymbol{\alpha}^*, \mathcal{Q}^*)$ for convex rate objectives
    - Convex in rates, not in $\boldsymbol{\alpha}$ and $\mathcal{Q}$
    - Variational inequalities as a linear-to-convex bridge
  - A simplification of the problem when the power constraints are convex and monotonic
- An algorithm has been proposed
  - It converges to the optimum pair $(\boldsymbol{\alpha}^*, \mathcal{Q}^*)$