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Throughput fairness and efficiency of link adaptation techniques in wireless networks

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Abstract: Link adaptation techniques aim to maximise the quality of service and resource utilisation in wireless networks. However, fairness must be taken into consideration, particularly, in low-mobility environments where the channel dynamic variation is small. The authors propose and analyse three link adaptation techniques [using joint power control (PC) and adaptive coding and modulation (ACM)] for fairness enhancement. In the first technique, called aggregate throughput maximisation with fairness constraint, the authors formulate the fairness problem as a constrained optimisation problem where the authors try to maximise the aggregate throughput subject to the throughput fairness constraint. In order to solve the optimisation problem, the authors convert the constrained optimisation problem to an unconstrained optimisation one using the penalty method. Then, the unconstrained optimisation problem is solved using the steepest descent technique. The second techniques, called individual throughput balancing, tries to equalise the individual throughput by using a higher throughput level for disadvantaged users and using a lower throughput level for advantaged users. Finally, the third technique, called adaptive virtual maximum power constraint, uses virtual maximum power cap, which is lower than the real maximum power cap. The virtual maximum power cap of each user is variable and it adapts based on the user's individual throughput to compensate disadvantaged users. The authors analyse the three proposed techniques in terms of the throughput fairness and the throughput efficiency and compare them with three basic link adaptation techniques (PC, ACM, and joint PC and ACM). The three proposed techniques are shown to be able to enhance the fairness with different degrees and with different levels of aggregate throughput degradation.

1 Introduction

Unlike wire-line links, wireless links experience significant temporal and spatial variation in the link quality. In wireless cellular networks, there are always disadvantaged users who are distant from their serving base stations (BSs) or experience strong shadowing and/or deep fading. On the other hand, there are advantaged users who are close enough to their serving BSs and might be experiencing little/no shadowing. Furthermore, the disadvantaged users usually suffer from high-interference levels as they can be close enough to cochannel interferer BSs, while the advantaged users usually have low-interference levels since they are far-away from cochannel interferer BSs. The problem is more significant with stationary and low-mobility users due to the small temporal channel variation; hence, the disadvantaged users will always have bad channels and high interference while advantaged ones will enjoy good channels and low interference.

Aggregate throughput performance is usually optimised in wireless network by employing opportunistic algorithms. However, opportunistic techniques always favour advantaged users and penalise disadvantaged users, which degrades the fairness among users. Fairness can be measured by the variation of the individual throughput. If there is a big variation in the throughput of different users, this is an indicator of low fairness and vice versa. On the other hand, fairness enhancement schemes tend to penalise advantaged users; and as a result, they might degrade the throughput performance. Hence, there should be a trade-off between the overall system performance and the fairness requirements.

Wireless networks are usually designed based on the worst-case scenario such that all users including the disadvantaged ones are guaranteed minimum quality of service in terms of the signal-to-interference-and-noise-ratio (SINR). However, this approach is not efficient since it wastes part of the resources as the advantaged users often have much higher SINR than the minimum required value. Link adaptation techniques are proposed as a remedy of the variation in the signal quality. Adaptive power control (PC) can be used to equalise the signal quality throughout the whole network by balancing SINR of all users [1]. It should be noted that PC is used for other purposes such as solving near–far problem in CDMA, reducing the interference and reducing the power consumption. Alternatively, adaptive coding and modulation (ACM) is employed to exploit the variation in the signal quality by assigning different coding and modulation levels to each user depending on SINR or any related parameter [2]. Joint PC and ACM schemes can be utilised to maximise the throughput and/or improve the signal quality [3–5]. These three link adaptation techniques vary in their fairness and efficiency performance.

The performance of link adaptation techniques (PC, ACM, and joint PC and ACM) are often analysed in the literature focusing on the aggregate (or average) throughput as a metric of their efficiency without considering fairness among users [4, 5]. In [5], the aggregate throughput maximisation (ATM) is achieved using iterative algorithms without taking the fairness into consideration. Although the proposed algorithms in [5] can maximise the aggregate throughput, this comes at the expense of the disadvantaged users who are shut off completely to allow advantaged users to get the maximum possible throughput.

Maximising the aggregate throughput using joint PC and adaptive modulation subject to some fairness constraints has been addressed in [6]. It has been shown that the problem is hard to be solved because constrained combinatorial optimisation problems cannot be always solved analytically [7]. Therefore the problem is heuristically divided into two sub-problems. The first part deals with the fairness by selecting a limited set of potential modulation levels for each user depending on the achieved time-averaged throughput. Hence, users having low time-averaged throughput are compensated in the next frame by selecting higher modulation levels while those having high time-averaged throughput are assigned in the next frame lower modulation levels. The second part selects the modulation level of each user from the selected set given by part one with an objective of maximising the total network throughput.

The optimal resource allocation with fairness constraint is addressed in [8] for CDMA systems. Fairness is achieved by formulating the resource (transmission power, spreading gain and error correction coding rate) allocation problem as the maximisation of the product of individual throughput of all users in the cell. Then, the problem is solved using the Kuhn–Tucker condition [7]. The same problem is investigated in OFDM systems [9, 10]. In this case, the subcarrier allocation is utilised (in addition to the transmission power and modulation levels) to control the fairness among users.

In addition to the transmission power, adaptive modulation and the frequency subcarriers, time-slots can be used to achieve fair resource allocation. In this case, the scheduling scheme modifies the buffer management policy to give a higher priority to disadvantaged users. Proportional fairness scheduling [11] and max–min fairness scheduling [12] are examples of such schemes. In proportional fairness scheduling, fairness is accomplished by using the ratio of the achievable throughput of the user to its time-average throughput as the scheduling criterion, while in max–min fairness scheduling, fairness is improved by trying to maximise the throughput of the user with the lowest throughput. Although such scheduling schemes can achieve long-term fairness, they do not guarantee fairness at the short-term level [9].

In this paper, we focus on the fairness problem using the transmission power and adaptive modulation as the controlling parameters. We propose and analyse three fairness enhancement techniques and analyse their throughput performance. In the first scheme, called ATM with fairness constraint (ATMFC), we formulate the fairness problem as a constrained optimisation problem where we try to maximise the aggregate throughput subject to fairness constraint. Then, we solve it by converting the optimisation problem to unconstrained one using the penalty method. The other two schemes [individual throughput balancing (ITB) and adaptive virtual maximum power constraint (AVMPC)] are heuristic ones that try to enhance the fairness performance by assisting disadvantaged users. We compare the performance of these three schemes with three reference ones (PC, ACM, and joint PC and ACM) in terms of average throughput and throughput fairness.

The rest of this paper is organised as follows. Section 2 presents the system model. The three reference link adaptation techniques are presented briefly in Section 3. Section 4 discusses the proposed fairness enhancement techniques. The performance results are provided in Section 5. Finally, the conclusions are given in Section 6.

2 System model and link adaptation techniques

A cellular system with omni-directional antennas are employed at the BSs and user stations. We assume that

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orthogonal channels are employed in each cell, so that the interference comes only from the inter-cell interference.

The average throughput that is used here as an indicator of the aggregate throughput is defined as

$$\overline{\mathrm{Thr}} = \frac{1}{N}\sum_{i=1}^{N}\mathrm{Thr}_i \tag{1}$$

where Thr_i is the individual throughput of user i and N is the number of users in the network. In order to quantify the fairness of the individual throughput, the fairness coefficient (FC) is defined as follows

$$\mathrm{FC} = \frac{1}{1+V} \tag{2}$$

where V is the variance of the individual throughput values and it is given by

$$V = \frac{1}{N}\sum_{i=1}^{N}(\mathrm{Thr}_i)^2 - (\overline{\mathrm{Thr}})^2 \tag{3}$$

It is apparent from the definition in (2) that the FC is inversely proportional to the throughput variance. The FC ranges from zero to one and the larger the value of FC, the better the fairness performance will be. The FC can be considered as a modified version of Jain's fairness index given by $(\sum_{i=1}^{N}\mathrm{Thr}_i)^2/(N\sum_{i=1}^{N}(\mathrm{Thr}_i)^2)$ [1], which can be shown to be equal to $1/(1+(V/(\overline{\mathrm{Thr}})^2))$. Jain's fairness index also ranges from zero to one. Both fairness metrics (FC and Jain's fairness index) are inversely proportional to the throughput variance since the variance can be considered as an indicator of the variation or unfairness among users. FC and Jain's fairness index approach zero when a few users have very high-throughput values while many other users are deprived (particularly when N is very large). In this case V will approach infinity and $\overline{\mathrm{Thr}}$ will approach zero. On the other hand, both of them will approach one when all users have the same throughput values. The only difference between FC and Jain's fairness index is that in the latter, the variance is normalised by the average throughput squared. We choose to remove the dependence of FC on $\overline{\mathrm{Thr}}$ so that FC depends on V only.

Adaptive coding–modulation is employed to adjust the coding/modulation based on the achievable SINR. Eleven combinations of coding–modulation levels using bit-interleaved coded modulation [13] are employed. Table 1 lists these coding–modulation combinations associated with their spectral efficiency and SINR requirements at 10^{-6} bit error rate (BER).

Three link adaptation techniques are described here and will be used as references for comparison in the results section. PC tries to achieve the same SINR for all users. Although PC can lead to high fairness, its efficiency in terms of average throughput is not necessarily high. On the other hand, ACM, exploits the variation in the signal quality (in terms of SIR) experienced by each user by allocating different coding and modulation levels to each user depending on the SIR or any related parameter. ACM improves the efficiency but it causes large variation in the throughput of different users. Joint PC and ACM scheme has more degrees of freedom and can be used for different objectives (e.g. to maximise the throughput with or without fairness constraint). In the three schemes, we use an iterative approach where the power or/and throughput (using coding rates and modulation level) are updated every frame.

2.1 SINR-based power control (SINR-BPC)

The transmitted power is adjusted so that all users achieve the same SINR regardless of their location or channel conditions. We use here the distributed constraint PC algorithm [1] because of its practical features such as the distributed nature (no central entity is required) and maximum power constraints. Hence, the power of user i, P_i, is updated as follows

$$P_i(n+1) = \min\left\{P^{\max}, \frac{\delta P_i(n)\gamma}{\mathrm{SINR}_i(n)}\right\} \tag{4}$$

where n is the frame index, γ is the target SINR, δ (>1) is a protection margin constant and $\mathrm{SINR}_i(n)$ is the SINR of user i at the nth frame, which is given by

$$\mathrm{SINR}_i(n) = \frac{G_{ii}P_i(n)}{\sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}P_j(n) + \eta_i} \tag{5}$$

where G_{ij} is the path loss between user i and the BS serving user j and η_i is the thermal noise power of user i. The coding rate and modulation level are kept fixed when PC is used. The employed coding and modulation combination is 1/2 coding rate with 16-QAM, which corresponds to a spectral efficiency of 2 b/s/Hz. The goal of this scheme is to balance SINR such that all users can achieve the same SINR regardless of their location, channel conditions or encountered interference levels. Hence, this scheme tries to impose complete fairness by assigning the same individual throughput to all users.

2.2 Adaptive coding and modulation (ACM)

The transmitted power is fixed and set to the maximum power ($P^{\max}$), while the coding rate and modulation level are adapted (in each frame) according to the achieved SINR such that the BER is fixed for all users. A target BER is chosen (e.g. 10^{-6} and the corresponding SINR threshold values are found for all possible combinations of coding rates and modulation levels as shown in Table 1. Before the beginning of the frame, the SINR is predicted (or achieve SINR in the previous fame is used instead).

Table 1 SINR of different coding–modulation levels

Coding rate and modulation level index (l)	Coding rate and modulation level combination	Spectral efficiency, b/s/Hz	SINR at 10^{-6} BER, (γ^k) dB
1	1/2, QPSK	1.00	4.65
2	2/3, QPSK	1.33	6.49
3	3/4, QPSK	1.50	7.45
4	7/8, QPSK	1.75	9.05
5	1/2, 16-QAM	2.00	10.93
6	2/3, 16-QAM	2.66	12.71
7	3/4, 16-QAM	3.00	14.02
8	7/8, 16-QAM	3.50	15.74
9	2/3, 64-QAM	4.00	18.50
10	3/4, 64-QAM	4.50	19.88
11	7/8, 64-QAM	5.25	21.94

Then, the coding rate and modulation level combination (that give the highest spectral efficiency with predicted SINR > SINR threshold value) is chosen. For instance, if the achieved SINR = 8.5 dB, 3/4-QPSK scheme is chosen according to Table 1. This scheme assigns different throughput levels to different users based on their channel conditions. ACM enables advantaged users to achieve high throughput. Meanwhile, ACM neither boosts disadvantaged users (like SINR-BPC does) nor penalises them (like ATM does as discussed below).

2.3 Aggregate throughput maximisation [5]

This is a joint PC and ACM scheme that updates the power and throughput (coding rate and modulation level combination) every frame as discussed below. This scheme tries to maximise the aggregate throughput (without fairness constraints) by solving the following optimisation problem

$$\begin{aligned} &\max_{\boldsymbol{P}} \overline{\text{Thr}} \\ &\text{s.t.} \\ &P^{\min} \le P_i \le P^{\max}, \quad \forall i \end{aligned} \tag{6}$$

where $\boldsymbol{P}$ is the allocated power vector ($\boldsymbol{P} = \{P_i\}, i = 1, 2, \ldots, N$) and $P^{\min}$ is the minimum power level. Using the logarithmic throughput model, the throughput of user is given

$$\text{Thr}_i = \log_2(1 + k\,\text{SINR}_i) \tag{7}$$

where SINR_i is the SINR of users i as in (5) (but without the time-index n) and k is a constant that depends on the required BER. This optimisation problem is solved using the maximum–minimum theorem [14] and the power update is given by [5]

$$P_i(n+1) = \begin{cases} P^{\min}, & \text{if } X_i(\boldsymbol{P}(n)) < P^{\min} \\ P^{\max}, & \text{if } X_i(\boldsymbol{P}(n)) > P^{\max} \\ X_i(\boldsymbol{P}(n)), & \text{otherwise} \end{cases} \tag{8}$$

where $X_i(\boldsymbol{P}(n))$ is given by

$$X_i(\boldsymbol{P}(n)) = \frac{1}{\sum_{j \ne i} (G_{ji})/(I_j(\boldsymbol{P}(n)))(\text{SINR}_j(\boldsymbol{P}(n)))/(1 + k\,\text{SINR}_j(\boldsymbol{P}(n)))(1 + k\,\text{SINR}_i(\boldsymbol{P}(n)))/(\text{SINR}_i(\boldsymbol{P}(n)))} \tag{9}$$

where $I_j(\boldsymbol{P}(n)) = \sum_{i \ne j} G_{ji} P_i + \eta_j$. Then the coding rate and modulation level combination is determined based on the achieved SINR. This scheme (as will be shown later) maximises the aggregate throughput by de-emphasising disadvantaged users (with bad channel conditions) by assigning them very low power (almost zero) to minimise the interference so that advantaged users (with good channel conditions) get high throughput.

These three schemes represent the main possible approaches in link adaptation techniques. SINR-BPC tries to achieve good fairness but not necessarily very high efficiency. ACM, on the other hand, seeks high efficiency without caring about fairness. Finally, ATM aims to achieve the highest possible efficiency even if this comes at the expense of very low fairness.

3 Fairness enhancement techniques

Fairness enhancement can be achieved by either compensating the disadvantaged users, penalising the advantaged users, or a combination of both. The following three schemes employ joint PC and ACM in different ways to improve fairness.

3.1 Aggregate throughput maximisation with fairness constraint (ATMFC)

The aggregate throughput maximisation with individual throughput fairness constraint can be modelled as a constrained optimisation problem as follows

$$\begin{aligned} &\max_{\boldsymbol{P}} \overline{\text{Thr}} \\ &\text{s.t} \\ &\text{FC} \ge \text{FC}^{\min} \\ &P^{\min} \le P_i \le P^{\max}, \quad \forall i \end{aligned} \tag{10}$$

where $\text{FC}^{\min}$ is the minimum FC. Obviously, the value of $\text{FC}^{\min}$ controls the level of required fairness among users. The fairness constraint given by the first inequality above in

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(10) can be equivalently rewritten as $V < V^{\max}$, where $V^{\max}$ is the maximum value of the throughput variance, which is equal to $(1/\mathrm{FC}^{\min} - 1)$ using (2).

In order to solve (10), the average throughput and the FC (or equivalently the throughput variance) should be expressed as a function of the power vector ($\boldsymbol{P}$). As in [5], we use the logarithmic throughput model given in (7). Using (1)–(3), (7) and (10), the optimisation problem can be rewritten as

$$\max_{\boldsymbol{P}} \overline{\mathrm{Thr}} = \frac{1}{N}\sum_{i=1}^{N} \log_2(1 + k\,\mathrm{SINR}_i)$$
$$\text{s.t.}$$
$$V = \left(\frac{1}{N}\sum_{i=1}^{N}(\log_2(1+k\,\mathrm{SINR}_i))^2 - \left(\frac{1}{N}\sum_{i=1}^{N}(\log_2(1+k\,\mathrm{SINR}_i))\right)^2\right) \le V^{\max} \tag{11}$$
$$P^{\min} \le P_i \le P^{\max}, \quad \forall i$$

Now, we need to find the power vector that maximises the objective function (average throughput) and satisfies the constraint (of the throughput variance). In fact, it is very hard to solve such nonlinear constrained optimisation problem analytically [6, 9]. However, the problem can be converted to an unconstrained problem by adding the fairness constraint to the objective function as a penalty term as follows

$$\max_{\boldsymbol{P}} f(\boldsymbol{P}) = \left(\overline{\mathrm{Thr}} - \alpha C\right) \tag{12}$$

where f is the objective function, α is a large positive constant called the penalty parameter and C is the penalty term defined as

$$C = \begin{cases} (V - V^{\max})^2, & V \ge V^{\max} \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

The reason for using the square of $(V - V^{\max})$ (not just $(V - V^{\max})$) is to ensure the differentiability of the penalty term (C) at $V = V^{\max}$. According to (13), the penalty term (C) will have a positive value only if V is greater than $V^{\max}$ (i.e. when the fairness constraint is violated), otherwise it is equal to zero. The power constraints will be imposed on the power update as shown below by (19).

It is still hard to find a global maximum solution for (12) since the objective function is not concave (as discussed in the next section) [9]. Hence, we opt to local maximum solutions, which can be considered as suboptimal solutions. Since $f(\boldsymbol{P})$ is a continuous differentiable function. The unconstrained optimisation problem given in (12) can be solved using the steepest descent technique, which is a gradient search method that finds the (local) optimal solution iteratively [9]. Therefore the solution of (12) can be found using the following power vector update

$$\boldsymbol{P}(n+1) = \boldsymbol{P}(n) + \beta\nabla f(\boldsymbol{P}(n)) \tag{14}$$

where β is a positive constant called the search step and $\nabla f(\boldsymbol{P}(n))$ is the objective function gradient, which is given by

$$\nabla f(\boldsymbol{P}(n)) = \left[\frac{\partial f(\boldsymbol{P})}{\partial P_1} \quad \frac{\partial f(\boldsymbol{P})}{\partial P_2} \quad \cdots \quad \frac{\partial f(\boldsymbol{P})}{\partial P_N}\right]_{\boldsymbol{P}=\boldsymbol{P}(n)} \tag{15}$$

In the steepest descent method, the best value of β (β^*) is determined by solving the following maximisation problem

$$\beta^* = \arg\max_{\beta>0} f\left[\boldsymbol{P}(n) + \beta\nabla f(\boldsymbol{P}(n))\right] \tag{16}$$

This maximisation problem can be easily solved by finding the root of the first derivative of the function with respect to β, f', iteratively using the Secant method as follows [9]

$$\beta(m+1) = \beta(m) + \frac{\{\beta(m) - \beta(m-1)\}f' \times [\mathbf{P}(n) + \beta(m)\nabla f(\mathbf{P}(n))]}{f'[\mathbf{P}(n) + \beta(m)\nabla f(\mathbf{P}(n))] - f'[\mathbf{P}(n) + \beta(m-1)\nabla f(\mathbf{P}(n))]} \tag{17}$$

where m is the search iteration index for β^*, and f' is the first derivative of f with respect to β and it is given by

$$f' = \frac{\partial f}{\partial \beta} = \nabla f\left[\frac{\partial P_1}{\partial \beta} \quad \frac{\partial P_2}{\partial \beta} \quad \cdots \quad \frac{\partial P_N}{\partial \beta}\right]^{\mathrm{T}} \tag{18}$$

where $[\ ]^{\mathrm{T}}$ is the transpose operator. Since we have to limit the power between $P^{\min}$ and $P^{\max}$, the power update can be expressed as

$$P_i(n+1) = \begin{cases} P^{\min}, & \text{if } P_i(n) + \beta^*\dfrac{\partial f(\boldsymbol{P})}{\partial P_i} < P^{\min} \\ P^{\max}, & \text{if } P_i(n) + \beta^*\dfrac{\partial f(\boldsymbol{P})}{\partial P_i} > P^{\max} \\ P_i(n) + \beta^*\dfrac{\partial f(\boldsymbol{P})}{\partial P_i}, & \text{otherwise} \end{cases} \tag{19}$$

3.2 Individual throughput balancing

This technique tries to balance the discrete-constellation throughput of all users around the discrete-constellation average value. In this scheme, the resource management controller compares the individual throughput of user i (Thr_i) with the average throughput ($\overline{\mathrm{Thr}}$). If $\mathrm{Thr}_i > \overline{\mathrm{Thr}}$, the transmission rate is reduced one step by moving the user to the next lower coding–modulation levels combination, otherwise it is increased one step. Hence, the allocated throughput is updated by [15]

$$\mathrm{Thr}_i(n+1) = \mathrm{Thr}_i(n) + \Delta\mathrm{Thr}\chi(\mathrm{Thr}_i(n) < \overline{\mathrm{Thr}}) \tag{20}$$

where ΔThr is the throughput step size and $\chi(a < b)$ is the indicator function, which is equal to $+1$ if $a < b$, and -1 otherwise. Then, the transmission power of that user is updated to achieve the required SINR. This process is done for all users each frame. The value of ΔThr is equal to the difference between spectral efficiency of the new coding/modulation level and spectral efficiency of the current coding/modulation level. For example, if a user is found to have throughput of 2 b/s/Hz (corresponding to 1/2 coding rate and 16-QAM modulation) while the average throughput is 2.9 b/s/Hz, then the throughput of that user will be increased to 2.66 b/s/Hz by moving to the next higher coding/modulation level (2/3 coding rate and 16-QAM modulation), which corresponds to ΔThr of 0.66 b/s/Hz.

3.3 Adaptive virtual maximum power constraint

In this technique, throughput is equalised using the adaptation of a virtual maximum power limit ($P^{\text{vir\_max}}$). Instead of having a fixed maximum power constraint for all users, the maximum power constraint is considered as a variable that depends on the relationship between user throughput ($\text{Thr}_i(n)$) and average throughput ($\overline{\text{Thr}}$). If $\text{Thr}_i(n) < \overline{\text{Thr}}$, the virtual maximum power of user i in the next update ($\text{Thr}_i(n+1)$) is increased by $\Delta P^{\max}$, otherwise $P_i^{\text{vir\_max}}(n+1)$ is reduced by $\Delta P^{\max}$, where $\Delta P^{\max}$ is a design parameter. Therefore $P_i^{\text{vir\_max}}$ is updated as follows [15]

$$P_i^{\text{vir\_max}}(n+1) = \min[P^{\text{abs\_max}}, P_i^{\text{vir\_max}}(n) + \Delta P^{\max}\chi\{\text{Thr}_i(n) < \overline{\text{Thr}}\}] \quad (21)$$

where $P^{\text{abs\_max}}$ is the absolute (physical) maximum power. By adapting the virtual maximum power, the disadvantaged users are compensated by increasing their virtual maximum power limit that lead to throughput increase. Meanwhile, the advantaged users' virtual maximum power is decreased, which yields throughput reduction. The power of each user is chosen to achieve the highest possible coding/modulation level without violating the virtual maximum power constraint ($P_i < P_i^{\text{vir\_max}}, \forall i$).

4 Results

A network with 16 hexagonal cells with a wraparound structure is simulated to analyse the performance of the proposed techniques as well as the link adaptation schemes. The channel model consists of an exponential path loss model (α) with log-normal shadowing with a standard deviation (σ) and flat Rayleigh fading. Results are obtained for the downlink only but the proposed scheme can easily be extended to the uplink case. Also, we considered one channel (i.e. one set of cochannel users) only in the simulation to minimise the simulation and computational time. This, of course, does not affect the generality of the results because in the multiple channel case each set of cochannel users interact independently from other sets of cochannel users. Since there are 16 cells in the network, the maximum loading value is limited to 16 users per cochannel set. An interference-limited case is assumed such that the noise power is much smaller than the interference power. The maximum and minimum transmit powers ($P^{\max}$ and $P^{\min}$) are set at 50 and -40 dBm. This very dynamic range is chosen so that we can analyse the performance without the effect of power constraint. However, we also examined the impact of the power constraint on the performance as will be at the end of this section.

Throughput is calculated in b/s/Hz using two methods [5]: the first method, continuous-constellation throughput, uses (10) to calculate the throughput as a continuous differentiable function of SINR, while the second method, discrete-constellation throughput, calculates the throughput using the spectral efficiency of the assigned coding–modulation level as in Table 1. Although the latter is more meaningful as it reflects the achievable throughput in practice, the use of continuous-constellation throughput is essential for solving the optimisation problem as explained in the previous section. The path loss exponent (α) is equal to 3 while the log-normal shadowing standard deviation (σ) is equal to 8.

In ATMFC, we set the largest step of the power update (ΔP) to 0.3 dB while in AVMPC we choose the absolute maximum power ($P^{\text{abs\_max}}$) to be equal to 50 dBm and the largest step of $P^{\text{vir\_max}}(\Delta P^{\max}) = 1$ dB. As shown in Fig. 1, the discrete-constellation throughput is approximated by the continuous-constellation throughput [given by (7)]. It has been found that value of constant k that minimises the difference between the continuous-constellation throughput and discrete-constellation throughput is equal to 0.2.

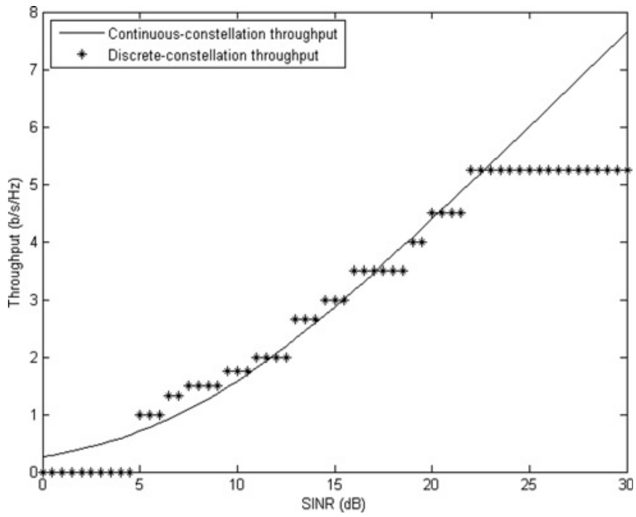

Figure 1 *Approximation of the discrete-constellation throughput using the continuous-constellation throughput*

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4.1 Illustrative example

An example of four cochannel users is given here for illustration. The channel path-loss matrix (path-loss between the four users and their serving BSs) is given by

$$G = \begin{pmatrix} 128.2 & 157.2 & 133.9 & 145.5 \\ 138.4 & 123.1 & 145.3 & 136.0 \\ 151.6 & 135.1 & 106.0 & 155.5 \\ 120.8 & 135.1 & 142.9 & 111.0 \end{pmatrix}$$

Table 2 lists the achieved SINR, individual throughput (continuous-constellation and discrete-constellation), allocated power, average throughput average ($\overline{\text{Thr}}$) and throughput FC of the six investigated schemes.

As expected, when SINR-based power control (SINR-BPC) is employed, high fairness (unity FC) is achieved by equalising the SINR (and the throughput) of all users. However, balancing the SINR of all users is obtained by increasing the SINR of the first two users and reducing

Table 2 Results of the four-users example

	Scheme	Metric	User index				$\overline{\text{Thr}}$	FC
			1	2	3	4		
Reference schemes	SINR-BPC	SINR, dB	12.35	12.35	12.35	12.35		
		Thr (continuous-constellation), b/s/Hz	2.15	2.15	2.15	2.15	2.15	1
		Thr (discrete-constellation), b/s/Hz	2	2	2	2	2	1
		power, dBm	25.89	13.74	10.32	5.68		
	ACM	SINR, dB	−7.46	8.98	27.14	24.44		
		Thr (continuous-constellation), b/s/Hz	0.05	1.36	6.7	5.8	3.47	0.11
		Thr (discrete-constellation), b/s/Hz	0	1.5	5.25	5.25	3.0	0.16
		power, dBm	50	50	50	50		
	ATM [5]	SINR, dB	−70	−63	49.7	31.5		
		Thr (continuous-constellation), b/s/Hz	0	0	14.2	8.1	5.58	0.02
		Thr (discrete-constellation), b/s/Hz	0	0	5.25	5.25	2.62	0.12
		power, dBm	−40	−40	35.76	22.77		
Proposed schemes	ATMFC − $FC_{min} = 0.5$)	SINR, dB	5.9	16.27	14.7	15.11		
		Thr (continuous-constellation), b/s/Hz	0.83	3.24	2.76	2.9	2.44	0.53
		Thr (discrete-constellation), b/s/Hz	1.0	3.5	3	3	2.6	0.52
		power, dBm	26.9	21.5	13.7	13.1		
	ATMFC − $FC_{min} = 0.9$)	SINR, dB	10.7	13.7	12.9	13.2		
		Thr (continuous-constellation), b/s/Hz	1.75	2.51	2.29	2.36	2.23	0.92
		Thr (discrete-constellation), b/s/Hz	1.75	2.66	2.66	2.66	2.43	0.86
		power, dBm	29.3	19.1	14.3	10.7		
	ITB	SINR, dB	−144.5	20.12	18.74	18.84		
		Thr (continuous-constellation), b/s/Hz	0	4.43	3.99	4.02	3.11	0.23
		Thr (discrete-constellation), b/s/Hz	0	4	4	4	3	0.25
		power, dBm	−40	47.8	47.8	45.3		
	AVMPC	SINR, dB	11.96	24.03	13.56	11.84		
		Thr (continuous-constellation), b/s/Hz	2.04	5.70	2.47	2.02	3.05	0.30
		Thr (discrete-constellation), b/s/Hz	2.0	5.25	2.66	2.0	2.97	0.36
		power, dBm	30	20.1	16.7	27.01		

that of the other two users. For instance, the SINR of the first user is increased from −7.46 dB without SINR-BPC to 12.35 dB with SINR-BPC, while the SINR of the third user is reduced from 27.1 dB without SINR-BPC to 12.35 dB with SINR-BPC.

By using ACM, we managed to translate the high SINR of the third and fourth users into high throughput. Since ACM does not boost disadvantaged users, the first user is unable to achieve non-zero throughput. The big difference in the throughput values between the first user in one side and the third and fourth users on the other side causes the FC of ACM to be significantly low.

The unfairness of ATM is manifested by the very low SINR (leading to zero throughput) of the first two users while the other two users enjoy high SINR (leading to high throughput) because ATM maximise the aggregate throughput by giving high power, SINR and throughput to advantaged users and reducing the power, SINR and throughput to disadvantaged users. ATM even turns off disadvantaged users for the benefit of the advantaged one. For instance, ATM turns the first two users off by reducing their allocated power to $P^{\min}$ to increase the SINR of the third and fourth users to 49.7 and 31.5 dB, respectively. However, these very high SINR values are not translated to higher discrete-constellation throughput (e.g. compared with the results of ACM) since the threshold value of SINR for the highest coding/modulation level is equal to 21.94 dB and any increase of SINR above this value cannot be converted to throughput gain in the discrete-constellation throughput.

Like SINR-BPC, ATMFC scheme achieves fairness by increasing the power of disadvantaged users and decreasing the power of the advantaged ones. For instance, ATMFC (with $FC_{\min} = 0.9$) allocates high power to the first user (29.3 dBm) and low power to the fourth user (10.7 dBm), which increases SINR of the first user to 10.7 dB and reduces the SINR of the fourth users to 13.2 dB. Furthermore, it is evident that the efficiency and fairness of ATMFC can be adjusted using the value of the $FC_{\min}$. For example, when $FC_{\min}$ is reduced from 0.9 to 0.5, the average continuous-constellation throughput and discrete-constellation throughput increased from 2.23 to 2.44 and from 2.43 to 2.6, respectively, while the FC of the continuous-constellation throughput and discrete-constellation is reduced from 0.92 to 0.53 and from 0.86 to 0.52, respectively.

Also, it is clear that the ITB and the AVMPC schemes can achieve medium FC and relatively high-average throughput. AVMPC, like SINR-BPC and ATMPC, tries to accommodate all users. But unlike SINR-BPC and ATMPC, AVMPC does not penalise advantaged users. Hence, it allocates high power to the first user to keep it on while the other three users can still have relatively high SINR. On the other hand, ITB removes the first user for a better throughput of the other three users. Also, it is obvious that ITB achieves high fairness but among the un-removed users only as the second, third and fourth users have close SINR values (20.12, 18.74 and 18.84 dB) and the same discrete-constellation throughput values (4 b/s/Hz).

4.2 Effect of loading on aggregate throughput and fairness performance

In this section, we analyse the effect of loading (number of co-channel users in the network) on the performance of the investigated algorithms. Figs. 2*a* and 2*b* show the average throughput against loading of the six schemes discussed above (three link adaptation schemes and three fairness-enhancement algorithms) while Figs. 3*a* and 3*b* show their FC. We show the results of ATMFC at two different levels of fairness requirements. These two values represent medium fairness constraint ($FC = 0.5$) and stringent fairness constraint ($FC = 0.9$).

It is evident that the average throughput (continuous-constellation and discrete-constellation) is decreasing with loading, while FC does not show strong dependence on the loading level. It is also apparent that the efficiency measured by the average throughput and fairness measured by FC are conflicting requirements. For instance, ATM scheme achieves the highest average throughput value but it has the lowest FC values. On the other, SINR-BPC and ATMFC (with $FC = 0.9$) achieve high FC but at the expense of the average throughput. This is because fairness is usually achieved by limiting the throughput of advantaged users and boosting disadvantaged users, which has two negative effects on the average throughput. First, advantaged users do not fully exploit the good channels they have. Second, boosting disadvantaged users might lead to higher interference levels.

Fig. 2 also shows that the continuous-constellation average throughput (especially for ATM scheme) is higher than the discrete-constellation average throughput. For instance, the continuous-constellation average throughput of ATM scheme goes from 7.36 to 1.98 b/s/Hz with increasing the loading from 4 to 16 users while the discrete-constellation average throughput of the same scheme goes from 2.48 to 1.49 b/s/Hz for the same loading values. This is due to the fact that the discrete-constellation average throughput cannot take advantage of the very high SINR values (>21.94 dB) according to the employed coding/modulation levels. However, if higher modulation levels are employed, the discrete-constellation average throughput can be increased considerably, particularly, for ATM.

Among the three proposed fairness enhancement techniques, it is clear that ATMFC is the best in terms of the fairness performance as shown in Fig. 3. However, ITB is the best in terms of the average throughput as depicted in Fig. 2. Furthermore, it is apparent that ATMFC scheme has adjustable fairness and efficiency depending on the value of $FC_{\min}$. High values for $FC_{\min}$ will give high FC

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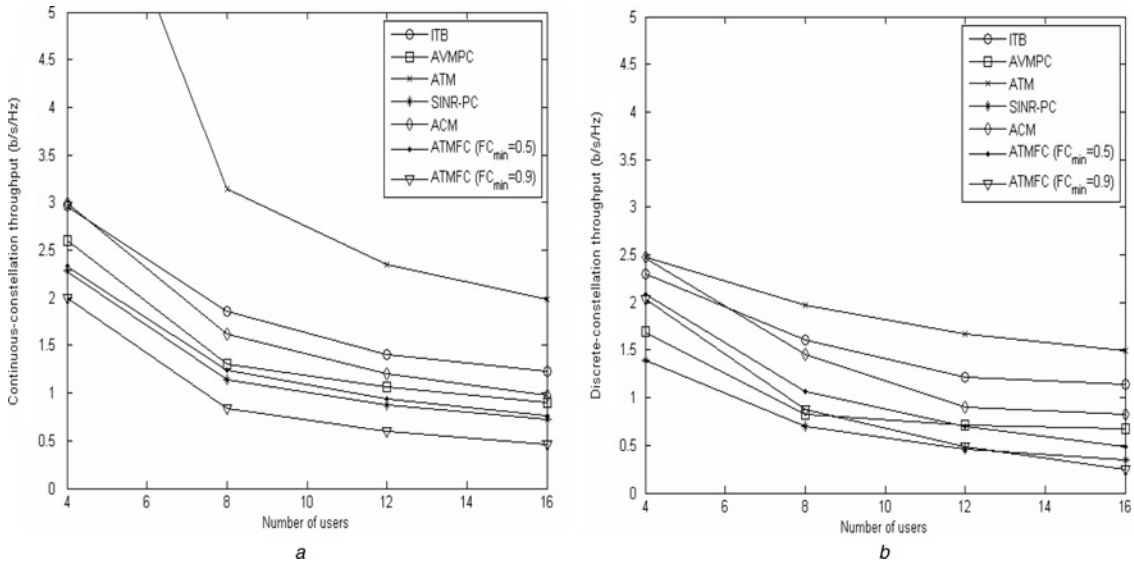

Figure 2 *Average throughput against loading of the six schemes*

a Average continuous-constellation throughput against network loading
b Average discrete-constellation throughput against network loading

but relatively low-average throughput and vice versa. This trade-off is very useful since it gives the network designer/ operator the flexibility to strike a balance between the fairness level and the aggregate throughput efficiency.

From Fig. 3, we can also see that ATMFC is always able to achieve the required fairness constraint ($FC \geq FC_{min}$) of the continuous-constellation throughput. However, it does not guarantee a minimum value for the FC of the discrete-constellation throughput since ATMFC scheme solves the optimisation problem (including the FC constraint) using the continuous-constellation throughput definition. Nevertheless, it is worth noting that the achieved FC of the discrete-constellation throughput is close enough to (and can be even higher than) FC_{min}. For instance, as the loading increases from 4 to 16 users, FC of ATMFC scheme for discrete-constellation throughput goes from 0.75 to 0.81 and from 0.54 to 0.6 for $FC_{min} = 0.9$ and 0.5, respectively.

4.3 Effect of P^{max} on aggregate throughput and fairness performance

So far the results are obtained with unconstrained power in interference-limited system. In the interference-limited

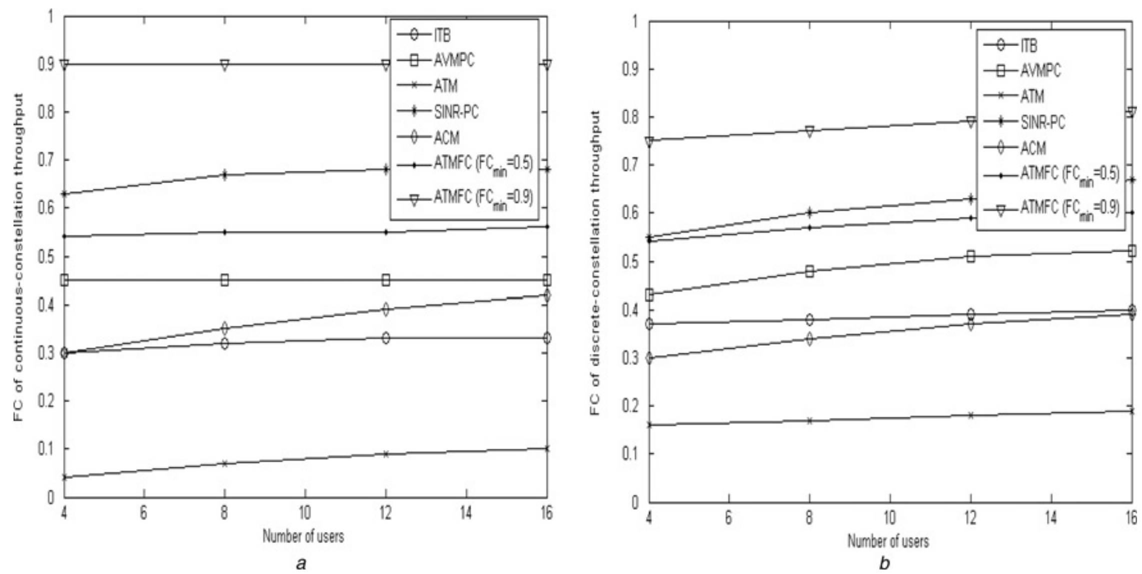

Figure 3 *Fairness performance*

a FC of continuous-constellation throughput against network loading
b FC of discrete-constellation throughput against network loading

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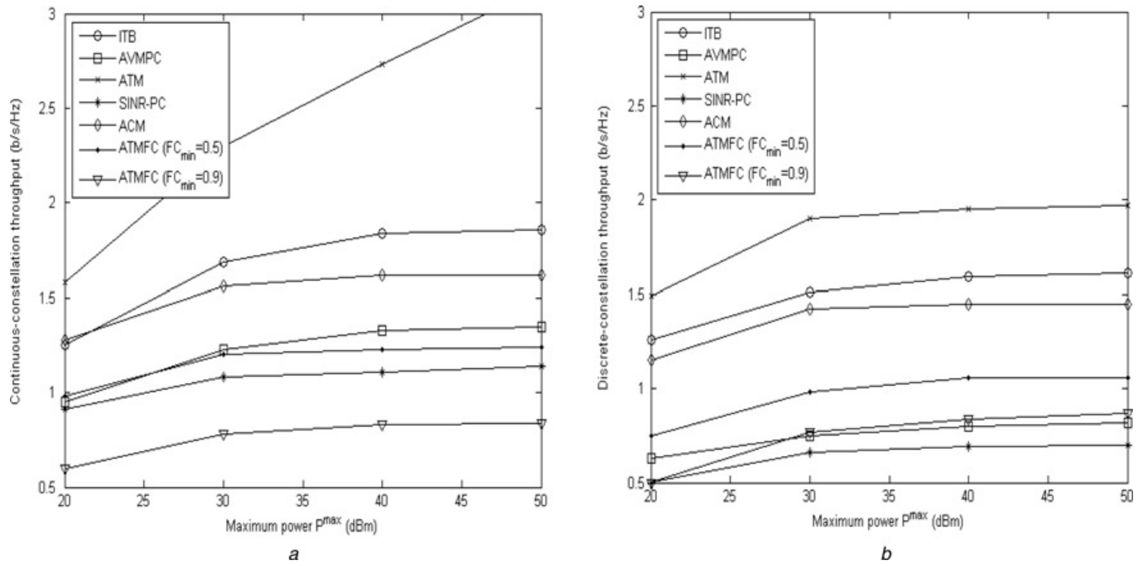

Figure 4 *Average throughput and FC of the six schemes against the maximum power limit*

a Average continuous-constellation throughput against maximum power (P^{max})
b Average discrete-constellation throughput against maximum power (P^{max})

scenario, the change in P^{max} did not make any difference in the performance. When the system is not interference-limited and the noise power is increased to −107 dBm, the effect of P^{max} becomes apparent. Figs. 4 and 5 depict the average throughput and FC of the six schemes against the maximum power limit (P^{max}) with a noise power level of −107 dBm for eight users. First of all, it is clear that the relative performance of the six schemes does not change with P^{max}. Also, it is evident that at low values of P^{max}, the average throughput is reduced while FC is increased, especially for ITB, ATM and ACM. This is due to the fact that at low values of P^{max}, the advantaged users cannot have very high-transmission power compared with that of the disadvantaged ones because of the tight power constraint.

Hence, advantaged users are restricted and cannot achieve very high throughput compared with disadvantaged ones. On the other hand, AVMPC, ATMPC and SINR-BPC show less sensitivity to P^{max} (particularly for FC) since such schemes are always able to achieve their fairness target even by penalising advantaged users. Moreover, both the average throughput and FC show no dependence on P^{max} after high values (>40 dBm) with the exception of the ATM scheme. Also, if we compare average throughput and FC of

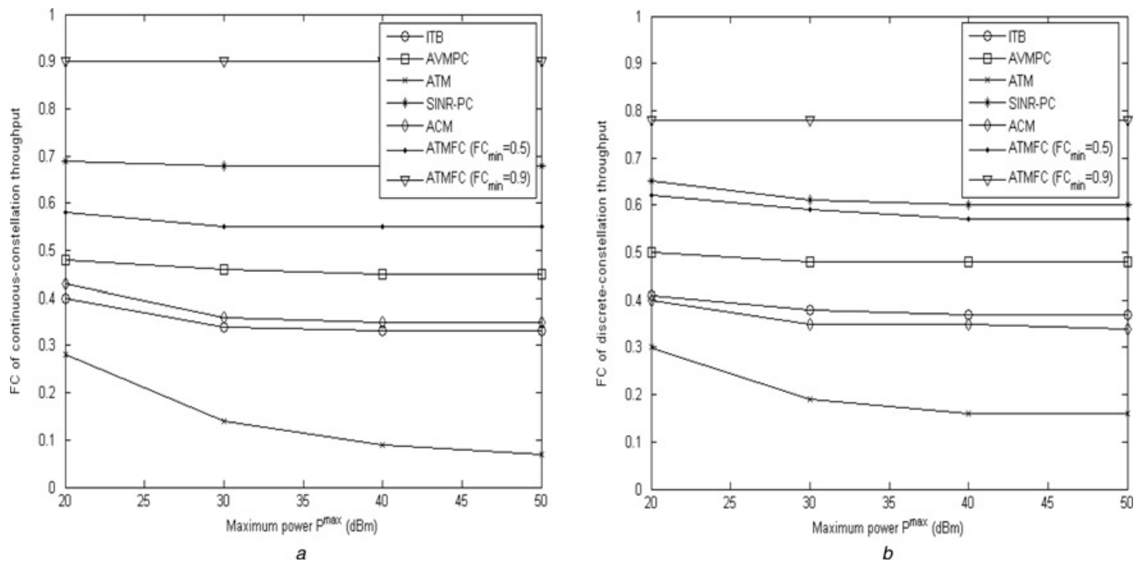

Figure 5 *Average throughput and FC of the six schemes against the maximum power limit*

a FC of continuous-constellation throughput against maximum power (P^{max})
b FC of discrete-constellation throughput against maximum power (P^{max})

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the six schemes at $P^{max} = 50$ dBm with those obtained for the interference-limited case in Figs. 2 and 3, we can see that they are identical. Hence, it can be concluded that the noise has a small impact on the performance as long as the transmission power in unconstrained (or has a high-maximum constraint).

4.4 Convergence of ATMFC

ATMFC is an iterative technique that uses the steepest descent method for solving the constrained optimisation problem given by (12) and (13). The steepest descent method is guaranteed to converge with at least an order of one [9]. However, there is no guarantee that the found maximum is the global one unless the objective function is concave [9]. The objective function $f(\boldsymbol{P})$ in (14) can easily be proven to be not concave by showing that $f(\boldsymbol{P})$ does not satisfy the concavity condition (function $g(\boldsymbol{X})$ is concave if and only if $g(a\boldsymbol{X} + (1-a)\boldsymbol{Y}) > ag(\boldsymbol{X}) + (1-a)g(\boldsymbol{Y})$, for any $\boldsymbol{X}$ and $\boldsymbol{Y}$ [9]).

In order to verify the non-concavity of $f(\boldsymbol{P})$, we plot $f(\boldsymbol{P})$ for a two-user scenario in Fig. 6. It is obvious that $f(\boldsymbol{P})$ is not concave. Therefore we can conclude that ATMFC is able to find local maximum solutions only. It should be noted that even if we are able to find the global maximum continuous-throughput solution, we have to convert it to the discrete-constellation format. This conversion renders the discrete-constellation throughput solution suboptimal. In order to check how far the suboptimal discrete-constellation throughput of ATMFC from the global maximum discrete-constellation throughput, we found the latter using exhaustive search (by trying all possible allocations of coding/modulation levels to the users and choosing the coding/modulation allocation that gives that highest discrete-constellation throughput without violating the fairness or power constraints) for the example of the four users discussed above. The global maximum discrete-constellation throughput was found to be 2.875 and 2.495 b/s/Hz for $FC_{min} = 0.5$ and 0.9, respectively. Hence, the suboptimal discrete-constellation throughput values found by ATMFC (from table II) are equal to 91.3 and 97.4% of the corresponding global maximum discrete-constellation throughput for $FC_{min} = 0.5$ and 0.9, respectively.

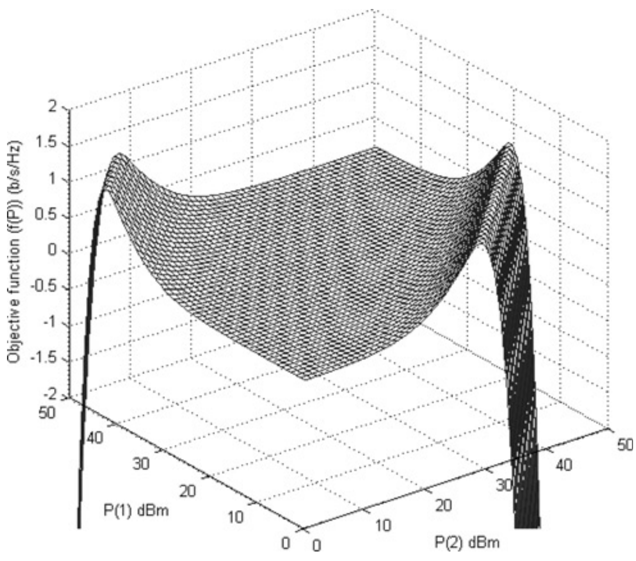

Figure 6 *Illustration of the objective function dependence on the transmission power of different users (N = 2)*

5 Conclusions

The fairness and efficiency of three link adaptation techniques as well as three proposed fairness enhancement schemes have been analysed. It is shown that efficiency (measured by aggregate or average throughput) and fairness (measured by the FC) are conflicting goals. Among the three fairness enhancement schemes, ATMFC is shown to be the best in term of fairness enhancement, while ITB and AVMPC can achieve medium fairness and relatively high efficiency. Furthermore, it is shown that ATMFC is always able to achieve the fairness requirement for the continuous-constellation throughput. Although the proposed scheme is not always able to meet the fairness requirement for the discrete-constellation throughput, it is always close enough or even better in some cases. It is shown that maximum power constraint does not have an impact on the results if the system is interference-limited. However, if noise cannot be neglected, the maximum power cap is shown to have a significant impact on the average throughput and a much less impact on the throughput FC.

The design of distributed fairness enhancement techniques that do not require global information (such as channel gain matrix) and the development of methods to find the global optimum throughput with fairness constraints (instead of the local ones as in ATMFC) are considered for future work.

6 References

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