

# Interference Characterization and Spectrum Sharing in Large Wireless Networks

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## Outline

- ▶ Introduction
- I Cumulant-based Characterization of the Aggregate Interference Power
- II Gaussianity of the Distribution of the Aggregate Interference Power
- III Impact of the Spatial Size of the Secondary Network on Spectrum Sharing
- IV Dominant Regions Dictating Spectrum Sharing Opportunities
- ▶ Conclusions and Future Work

## Introduction

- ▶ There is an exponential growth in the number of wireless systems and devices.
- ▶ Radio spectrum is a scarce resource; however, it is under-utilized.
- ▶ Spectrum management is going through a paradigm shift.
- ▶ Secondary users (SUs) could share the spectrum with primary users (PUs) under the following condition:
  - ▶ SUs don't introduce "harmful interference" towards PUs.
- ▶ Different metrics are proposed to gauge the harmful interference.

## Interference Probability

A harmful interference metric [Ghasemi08] and [Win09]

▶ Non-harmful interference:

$$P(I_A \geq I_{th}) \leq \beta$$

⇒ spectrum sharing allowed

▶ Harmful interference:

$$P(I_A \geq I_{th}) > \beta$$

⇒ spectrum sharing NOT allowed

## System Model

▶ Aggregate Interference:

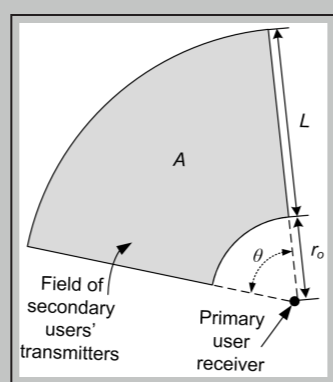
$$I_A = \sum_{i \in \mathcal{N}} I_i = \sum_{i \in \mathcal{N}} g(r_i) X_i$$

▶ Distance-Dependant Attenuation

$$g(r_i) = \begin{cases} kr_i^{-n}, & r_i \geq r_c \\ kr_c^{-n}, \text{ constant}, & r_i < r_c \end{cases}$$

▶ Other system and channel parameters  
 $\lambda$ : Density of active nodes  
 $n$ : Path loss exponent  
 $X_i$ 's are i.i.d.

$$X_i = \prod_l X_{i,l}$$



## I. Cumulant-based Characterization of the Aggregate Interference Power

**Motivations:**

- ▶ Characteristic function is known.
- ▶ No closed-form expressions for PDF/CDF.
- ▶ Numerical inversion is possible, however, cumulants approach is more attractive.

## I. Cumulant-based Characterization of the Aggregate Interference Power

**Lit. Review:**

- ▶ A number of recent papers in literature have dealt with cumulants of the aggregate interference but under specific scenarios.
  - ▶ [Lichte10] considers the first cumulant, i.e., the mean.
  - ▶ [Chan01] provides an integral form to compute the cumulants for out-of-cell interference in a CDMA networks.
  - ▶ [Menon05,06] deal with cumulants for non-fading scenarios.
  - ▶ [Ghasemi08] considers an infinite field with a very small exclusion region.
- ▶ Extending these results and generalizing them for a wide range of scenarios are of great importance and advantage to study the spectrum sharing in large secondary networks.

## I. Cumulant-based Characterization of the Aggregate Interference Power

**Results:**

$$I_A = \sum_{i \in \mathcal{N}} g(r_i) X_i$$

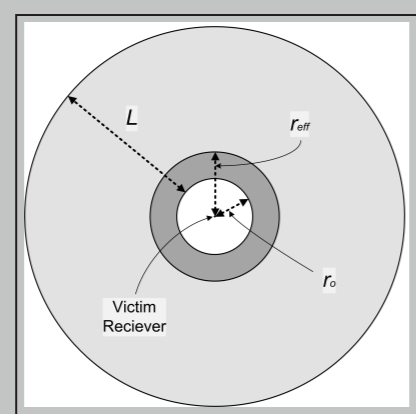
$$\kappa_m(I_A) = N_{\text{eff}}(m) [g(r_o)]^m \tilde{\mu}_m(X)$$

$$N_{\text{eff}}(m) = \lambda A_{\text{eff}}(m)$$

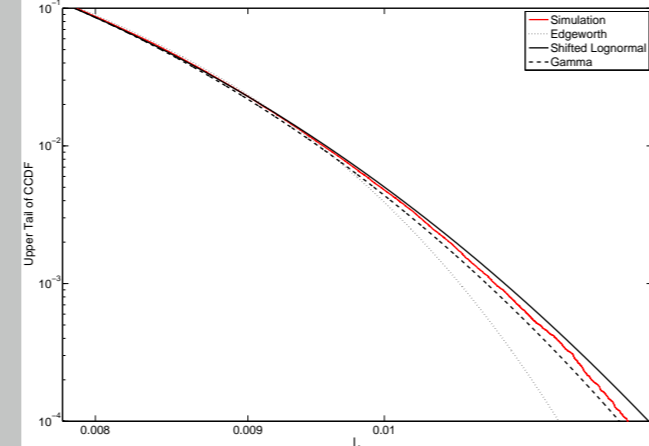
$$A_{\text{eff}}(m) = \frac{1}{2} \theta [r_{\text{eff}}^2(m) - r_o^2]$$

$$r_{\text{eff}}(m) = \hat{r} \sqrt{1 + \frac{2}{mn-2} \left(1 - \left[\frac{\hat{r}}{r_o + L}\right]^{mn-2}\right)}$$

$$\hat{r} = \max(\min(r_c, r_o + L), r_o)$$



Cumulant-based Approximation of the Distribution of  $I_A$



## I. Cumulant-based Characterization of the Aggregate Interference Power

**Contributions:**

- ▶ Introduced a simple yet comprehensive method to determine the cumulants of the aggregate interference power originating from a wireless network.
- ▶ This method is quite general and applicable for finite and infinite network sizes, and it is flexible to encompass different system and propagation parameters such as large-scale fading, small-scale fading, or even composite fading.
- ▶ Investigated the behavior of these cumulants with respect to changes in some system and channel parameters.

## II. Gaussianity of the Distribution of the Aggregate Interference Power

**Motivations and Lit. Review:**

- ▶ As the number of interfering nodes increases, there might be a tendency to approximate the distribution of the aggregate interference power by a Gaussian random variable.
- ▶ Some scattered observations in literature (e.g., [Evans99], [Chan01], [Hasan07], [Ganti08], and [Win09]) suggest that this Gaussian approximation is not valid, except under some specific scenarios.
- ▶ Literature lacks a thorough discussion of the Gaussianity of the aggregate interference power in large wireless networks.

## II. Gaussianity of the Distribution of the Aggregate Interference Power

**Results:**

▶ Exclusion Region ( $r_o \geq r_c$ )

$$|F_Z(y) - F_N(y)| < 2.21 \frac{2(n-1)^{\frac{3}{2}}}{3n-2} \frac{1}{\sqrt{\lambda \pi r_o^2}} \frac{\tilde{\mu}_3(X)}{[\tilde{\mu}_2(X)]^{\frac{3}{2}}}$$

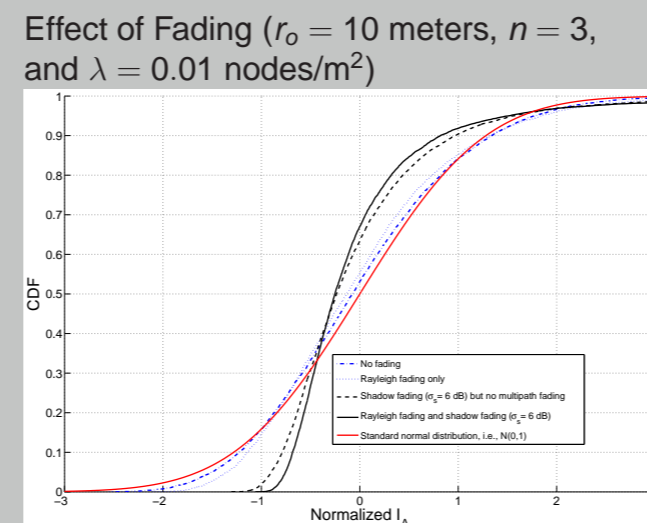
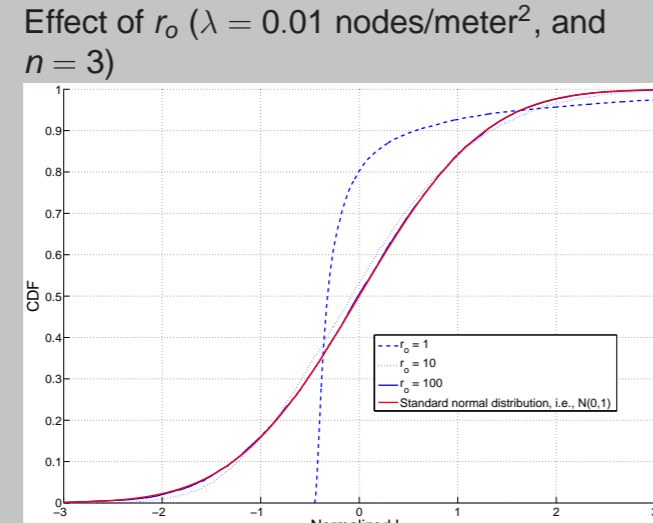
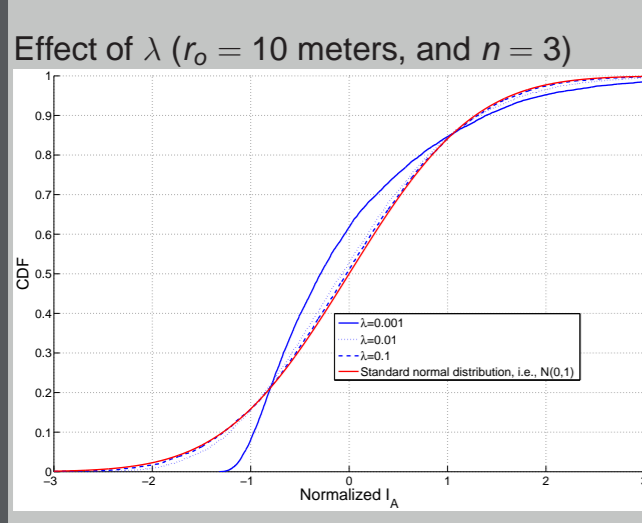
▶ No Exclusion Region ( $r_o = 0$ )

$$|F_Z(y) - F_N(y)| < 2.21 \frac{3(n-1)^{\frac{3}{2}}}{\sqrt{n}(3n-2)} \frac{1}{\sqrt{\lambda \pi r_o^2}} \frac{\tilde{\mu}_3(X)}{[\tilde{\mu}_2(X)]^{\frac{3}{2}}}$$

▶ Gaussian approximation is valid if

$$\sqrt{\lambda \pi r_o^2} \gg 2.21 \frac{2(n-1)^{\frac{3}{2}}}{3n-2} \frac{\tilde{\mu}_3(X)}{[\tilde{\mu}_2(X)]^{\frac{3}{2}}}, \text{ for } r_o \geq r_c$$

## II. Gaussianity of the Distribution of the Aggregate Interference Power



## II. Gaussianity of the Distribution of the Aggregate Interference Power

**Contributions:**

- ▶ Casted scattered observations in a single mathematical framework using Berry-Esseen bound.
- ▶ Expressed the conditions for which the Gaussian approximation will be valid for the aggregate interference power generated by a Poisson field of interferers.
- ▶ Discussed the effect of different system and channel parameters on the convergence of the distribution of the aggregate interference to a Gaussian distribution.

## III. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing

**Motivations and Lit. Review:**

- ▶ Previous works such as [Menon05], [Pinto07], [Ghasemi08] and [Ofcom08] studied the effect of different system parameters on spectrum sharing opportunities.
- ▶ However, a parameter that has received little attention is the spatial size of the secondary network.
- ▶ Usually, the spatial size is assumed to be infinite, e.g., [Menon05], [Menon06], [Ghasemi08] and [Win09].
- ▶ Results developed for infinite networks might be too pessimistic leading to missing spectrum sharing opportunities.
- ▶ Impact of spatial size of the secondary network on spectrum sharing opportunities?

## III. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing

**Impact of the Spatial Size on Cumulants of  $I_A$ :**

▶ Cumulants of  $I_A$ :

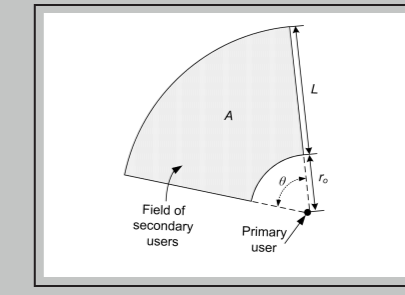
$$\kappa_m(I_A) = \frac{1}{nm-2} \lambda \theta \tilde{\mu}_m(X) r_o^{2-mn} \times \left[ 1 - \left( \frac{r_o}{r_o + L} \right)^{mn-2} \right]$$

▶ For  $L \ll r_o$ :

$$\kappa_m(I_A) \approx \lambda \theta r_o^{1-mn} L \tilde{\mu}_m(X)$$

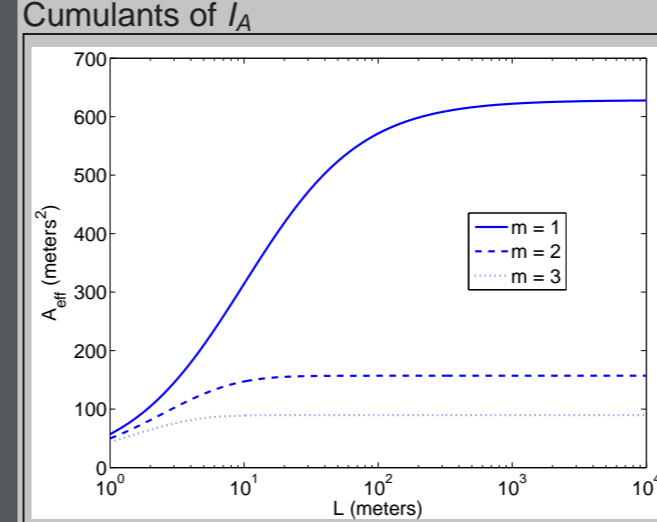
▶ For  $L \gg r_o$ :

$$\kappa_m(I_A) \approx \frac{1}{nm-2} \lambda \theta \tilde{\mu}_m(X) r_o^{2-mn}$$

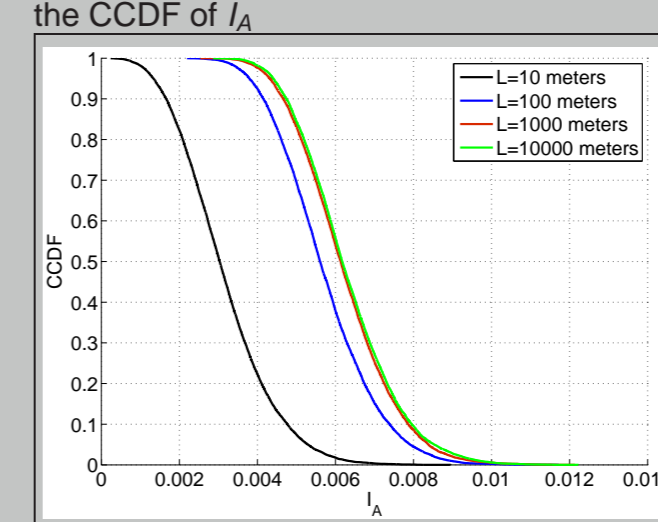


## III. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing

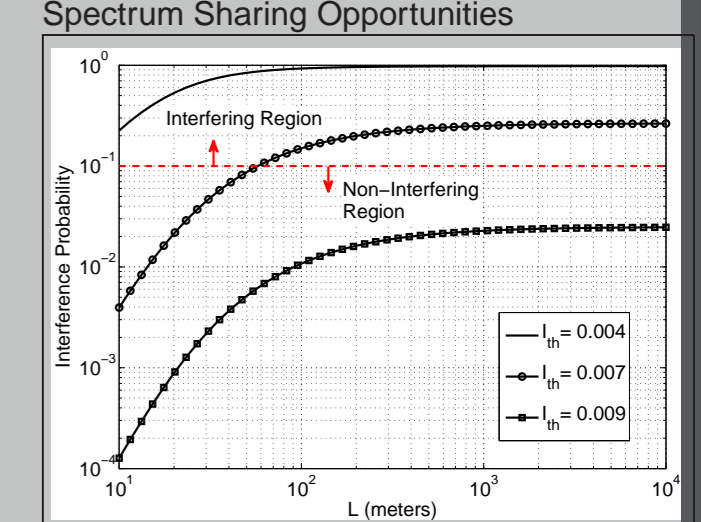
**Impact of the Spatial Size on Cumulants of  $I_A$**



**Impact of the Spatial Size on the CCDF of  $I_A$**



**Impact of the Spatial Size on Spectrum Sharing Opportunities**



## III. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing

**Contributions:**

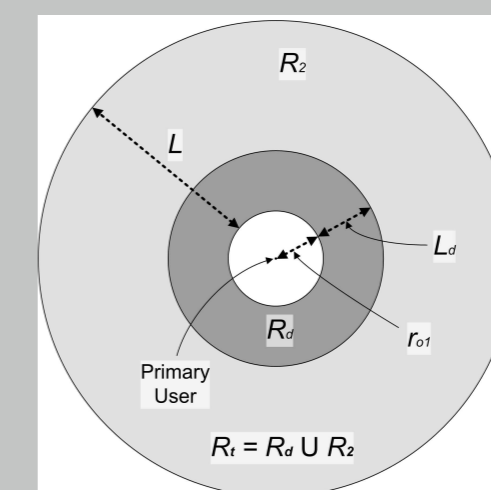
- ▶ Studied the effect of the field size on  $I_A$  and spectrum sharing opportunities.
- ▶ Observations:
  - ▶ Asymptotic results obtained for infinite fields are applicable for finite but relatively large fields (when the radial depth of the field is much greater than the minimum distance to the primary user) as well.
  - ▶ In some cases, however, asymptotic results are too pessimistic hiding some spectrum sharing opportunities.
  - ▶ In certain situations, a small reduction in the field size may create spectrum sharing opportunities while in certain other situations a huge increase in the field size may not eliminate spectrum sharing opportunities.

## IV. Dominant Regions Dictating Spectrum Sharing Opportunities

**Motivations and Lit. Review:**

- ▶ There are some comments in literature (e.g., [Etkin06] and [Weber07]) indicating that the aggregate interference is dominated by the nearby interferers to the victim receiver.
- ▶ There is to the best of our knowledge no work devoted to precisely identifying the boundary of the dominant region.
- ▶ A contribution is required to fill this gap, especially in the context of spectrum sharing.

## IV. Dominant Regions Dictating Spectrum Sharing Opportunities



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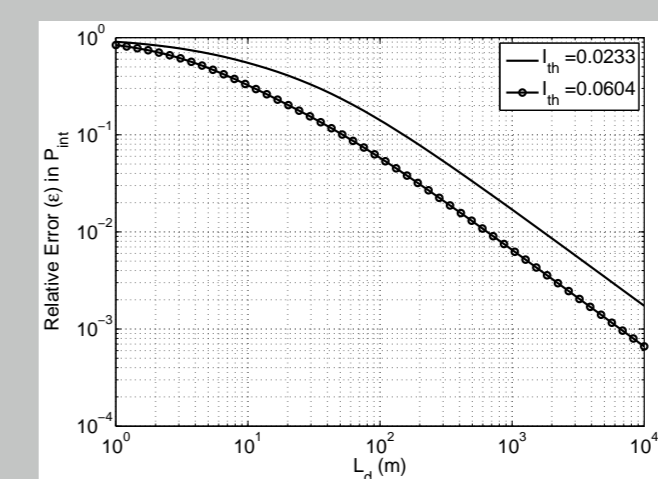
▶ Cumulant-Based Approach:

$$\frac{\kappa_m(I_A, R_d) - \kappa_m(I_A, R_s)}{\kappa_m(I_A, R_d)} \leq \epsilon_k$$

▶ Interference Probability-Based Approach:

$$\frac{P_{\text{int}}(I_{th}, R_d) - P_{\text{int}}(I_{th}, R_s)}{P_{\text{int}}(I_{th}, R_d)} \leq \epsilon$$

## IV. Dominant Regions Dictating Spectrum Sharing Opportunities



## IV. Dominant Regions Dictating Spectrum Sharing Opportunities

**Contributions:**

- ▶ Identified the smallest portion (dominant region) of the secondary network that would impact spectrum sharing opportunities.
- ▶ Results reflect the following:
  - ▶ The dominant region is not necessarily a small region encompassing a few interferers within the proximity of the primary user.
  - ▶ Far interferers may tangibly contribute to spectrum sharing decisions when a higher approximation accuracy is required or when a wide exclusion region is considered.
  - ▶ On the other hand, the dominant region shrinks with the increase in the path-loss exponent or in the level of the interference threshold specified by the primary user or a regulator.
- ▶ Some implications of these results are highlighted.

## Summary of Contributions

- I Cumulant-based Characterization of the Aggregate Interference Power
- II Gaussianity of the Distribution of the Aggregate Interference Power
- III Impact of the Spatial Size of the Secondary Network on Spectrum Sharing
- IV Dominant Regions Dictating Spectrum Sharing Opportunities

## Suggested Future Work

- ▶ Spatially clustered secondary users
- ▶ Spectrum sharing for a secondary network overlapped with spatially distributed many primary users
- ▶ Further accurate approximating distributions for the aggregate interference power
- ▶ Effect of correlation among  $X_i$ 's on spectrum sharing

## Publications

- ▶ M. Aljuaid and H. Yanikomeroglu, "On the asymptotic analysis of average interference power generated by a wireless sensor network," in Proc. IEEE Vehicular Technology Conference (VTC) 2008-Fall, Calgary, AB, Canada, Sep. 2008.
- ▶ M. Aljuaid and H. Yanikomeroglu, "Impact of secondary users field size on spectrum sharing opportunities," in Proc. IEEE Wireless Communications and Networking Conference (WCNC), Sydney, Australia, Apr. 2010.
- ▶ M. Aljuaid and H. Yanikomeroglu, "A cumulant-based characterization of the aggregate interference power in wireless networks," in Proc. IEEE Vehicular Technology Conference (VTC) 2010-Spring, Taipei, Taiwan, May 2010.
- ▶ M. Aljuaid and H. Yanikomeroglu, "A cumulant-based investigation of the impact of secondary users' field size on spectrum sharing opportunities," submitted to IEEE Transactions on Vehicular Technology (submissions: December 2010).
- ▶ M. Aljuaid and H. Yanikomeroglu, "Investigating the validity of a Gaussian approximation for the distribution of the aggregate interference power in large wireless networks," in Proc. 25th Biennial Symp. on Communications (QBSC 2010), Queen's University, Kingston, Canada, May 2010.
- ▶ M. Aljuaid and H. Yanikomeroglu, "Investigating the Gaussian convergence of the distribution of the aggregate interference power in large wireless networks," IEEE Transactions on Vehicular Technology, vol. 59, no. 9, pp. 4418-4424, November 2010.
- ▶ M. Aljuaid and H. Yanikomeroglu, "Identifying boundaries of dominant regions dictating spectrum sharing opportunities for large secondary networks," in Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC) 2010, Istanbul, Turkey, Sep. 2010.
- ▶ M. Aljuaid and H. Yanikomeroglu, "Impact of Secondary Network Partitioning on Aggregate Interference and Spectrum Sharing," IEEE journal manuscript in preparation.