Joint Backhaul and Access Optimization for Service-Segment-Based VN Admission Control

Meisam Mirahsan, Hamid Farmanbar, and Halim Yanikomeroglu

Abstract—**In this paper, we consider the problem of admission control of wireless virtual network (VN) service requests in a multi-service network with service-specific service function chains (SFC). SFC imposes traversal constraints on flows, i.e., each flow must visit certain service-specific nodes in a specific order. We leverage the fact that all flows within a service have common traversal constraints and propose a "service-based" admission control approach. For both of the service-based and flow-based approaches, we propose novel joint optimizations of backhaul and access networks for the admission control of VN service requests. We show that our optimization formulation is convex, hence computationally efficient and tractable. We also show that the proposed method is applicable to general backhaul and access networks. The simulation results show that the service-based approach incurs no loss in optimality while greatly reducing problem complexity compared to the flow-based counterpart.**

Index Terms—**Admission Control, Wireless Virtual Networks, Service Segments, Convex Optimization.**

I. Introduction

The future 5G+ multi-service wireless networks must incorporate not only short-term single-session requests by individual users, but also long-term virtual network (VN) service requests by groups of users. Indeed, user groups with statistical distribution of traffic demands and specific quality-of-service (QoS) requirements submit requests for VN services, and the service provider, considering its limited resources and capabilities, must decide whether to accept or reject the requests. This is performed in the admission control module of the service provider's network operating system.

Admission control is performed based on a large number of parameters including the customer traffic profile (user equipment (UE) locations), available bandwidth at base stations (BSs), and customer QoS requirements. The virtual network admission control in a wireless network is different from (and more complicated than) the single-session admission control. In the single-session admission control, decision is made based on a (deterministic) snapshot of already existing UE locations and the new UE location; while in the VN admission control, the UE locations and traffic demand distribution information are provided statistically by the VN customers. For instance, VN customers might indicate the expected distribution of traffic in various areas of the network. Therefore, the VN requests must be admitted if their QoS can be satisfied statistically. After (long-term) admission of a VN, (short-term) single-sessions of the VN users are admitted only if they follow the restrictions and limitations of the VN admission contract.

In service-based networks, services may require processing at certain nodes in a specific order. Such requirements are referred to as service function chain (SFC) requirements. Traffic aggregation, caching, encryption, and video trans-coding are examples of service functions. Different services may have different SFC requirements. However, a service function can be common to different services, e.g., the contents of web services and video services may all be cached at the same node. A network node where a service function is instantiated at is called a service function node (SFN).

The most common approach to solve the problems with traversal constraints is to parse flows into flow segments according to their traversal constraints and apply the traditional multi-commodity flow (MCF) problem formulation [1] with flow segments as commodities. Such an approach does not scale with increasing number of flows/flow segments both in terms of algorithm complexity and the memory requirements.

On the other hand, in a wireless cellular network with ideal backhaul connections, the lack of capacity is mainly due to the access connection limitation, that is, the wireless access problem and low channel quality between UEs and BSs. However, in the envisioned 5G networks with small cells deployed in residential and office buildings, the assumption of ideal backhaul is more optimistic than realistic, especially considering the fact that a main portion of the small cells (i.e., femtocells) will be deployed by customers rather than service providers. Recently, there has been increasing interest in the literature in the study of limited backhaul capacities in future 5G HetNets. Figure 1 illustrates the admission control process and its main input parameters.

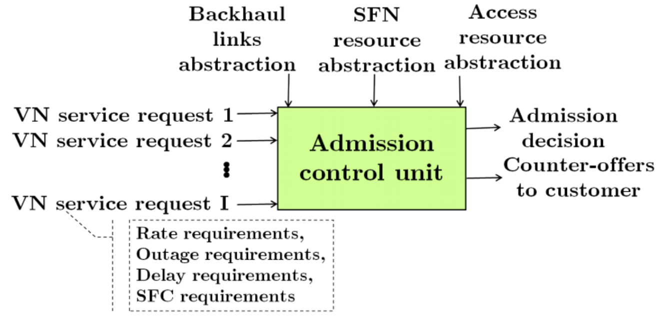

Fig. 1. Admission control process: admission decision is based on VN service requests and available resources.

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A. Contributions of This Paper

In this paper, we propose an efficient, tractable, and scalable service-segment-based admission control method for wireless VNs which incorporates network access limitations, network backhaul limitations, and service function limitations. The main contributions of this paper are summarized as follows:

- A novel wireless VN admission control method is proposed which incorporates both the access limitations and backhaul limitations for general network topologies.
- This problem is formulated as a joint convex optimization problem which is computationally tractable.
- Service segments and SFC constraints are included in the problem both in the sense of service function orders and the resource requirements at each service function node.
- A service-segment-based approach is put forth which is more scalable than its flow-segment-based counterpart.

B. Related Works

Admission control is a well investigated concept and there is a great volume of literature focusing on various aspects of it [2]. However, since the idea of network virtualization is relatively new [3], [4], admission control for VN requests, also referred to as VN embedding, has been a hot research topic recently [5]–[7]. Indeed, admission control for single users and single sessions is studied very well but the admission control of groups of users and VN requests is relatively recent.

Although there is a rich literature on VN admission control in wired networks, there are few works on wireless VN admission control [8]–[13]. In [8] and [9], fixed snapshots of UEs are considered, but the statistical specifications of traffic demand are not incorporated. In [10], [11], [13], the authors consider statistical arrival of VN requests with statistical demands. However, the demand and the resources are not specified in rate or delay, but rather in number of channels. To be more accurate, these papers assume that customers require a specific number of wireless channels, and the network allocates a number of channels to each UE upon availability, i.e., the spectral efficiency is not included in the allocation scheme and rate satisfaction is not the ultimate objective in these works.

C. Organization of the Paper

The remaining of this paper is organized as follows. In Section II, the system model considered in this paper is described and the VN admission control problem with joint optimization is introduced. Sections III and IV, present the proposed flow-based and segment-based methods for VN admission control, respectively. The simulation results are presented in Section V. The paper is concluded in Section VI.

II. System Model and Problem Definition

We consider the downlink of a cellular wireless network with general access and backhaul topologies. A geographical region $Q \subset \mathbb{R}^2$ (i.e., two dimensional plane) is assumed as the network layout that is served by a set of BSs B. Each BS is characterized by its available bandwidth W_k, $k \in K = \{1, ..., |B|\}$. Let $q \in Q$ denote a location on the network layout. We assume that demand arrivals for the *i*th VN service, $v_i \in V$, at location q, d_{iq}, are random variables that follow arbitrary distributions with mean λ_{iq}, where V is the set of VN services. This provides flexibility for inhomogeneous, i.e., heterogeneous, traffic characterization [14]–[19]. To avoid unnecessary technical difficulties, in this paper, we assume that the region Q is divided into a set of small (e.g., 5m $\times$ 5m) bins (areas) $a \in A$, and the demand for each bin is defined as

$$d_{ij} = \int_{q \in a_j} d_{iq}, j \in J, i \in I, \tag{1}$$

where $J = \{1, ..., |A|\}$ and $I = \{1, ..., |V|\}$.

In this paper, we assume that the BSs transmit with constant (not necessarily equal) power; we leave power control extension as a future work. The signal-to-interference-plus-noise ratio (SINR) for the center of bin $a_j \in A$ if it is connected to BS $b_k \in B$ is defined as

$$\gamma_{jk} = \frac{P_{jk}}{P_N + \sum_{j' \in J \setminus j} P_{j'k}}, \tag{2}$$

where P_{jk} is the effective received power from BS $b_k, k \in K$ at bin a_j, and P_N is the noise power.

The received power can be calculated based on path-loss exponent and channel models as well as the other environmental parameters. In practical cases, it also can be collected from field measurements. Spectral efficiency (SE) can be any arbitrary function of SINR in this method. In our simulations, we assume that it follows the Shannon's formula as follows:

$$\eta_{jk} = \log_2(1 + \gamma_{jk}). \tag{3}$$

While the simplest (and mostly used) UE-BS association is based on maximum received power and maximum SINR (max-SINR), technology advances in future wireless networks allow for general multiple association in which a UE (or a bin in this paper) can be associated to multiple BSs, or to a BS with low SINR but high available bandwidth. It is shown in the recent literature [20] that general association can result in significant improvement in network key parameter indicators, specially in future HetNets where small cells have limited coverage areas hence lower load compared to macro cells. Therefore, the VN admission control method proposed in this paper allows for general UE-BS association.

We assume that QoS expectations are specified by each VN customer by two main parameters: the required traffic demand which is specified by the traffic demand matrix d, or more specifically, by the distribution parameters of traffic in each bin, and the required maximum outage O_i. In addition, the SFC order and service function resource requirements are specified by each VN service customer. Figure 2 shows a sample network with access, backhaul, and SFNs.

The backhaul network is modeled as a directional graph $G(N, L)$, where N and L are the sets of nodes and links in the network, respectively. Let $L_{in}(n)$ and $L_{out}(n)$ be the sets of links terminating and originating at node $n \in N$, respectively. There are multiple services present in the network. There is a service function chain associated with each service. A service function chain is an ordered sequence of (service function) nodes in G. We consider a service as a set of flows

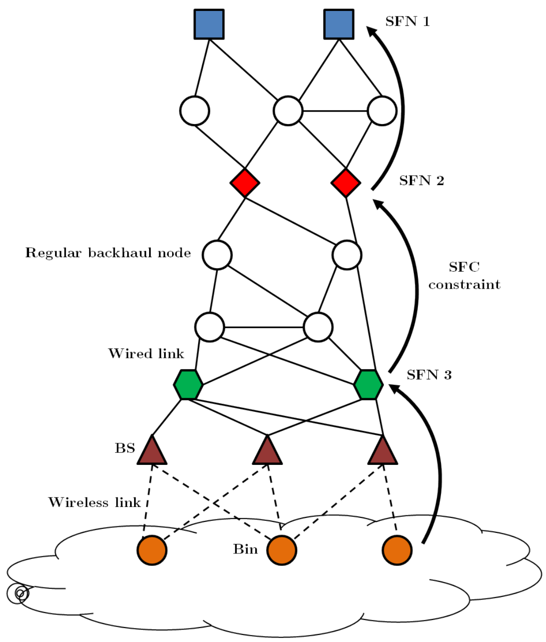

Fig. 2. We consider a wireless cellular network with general access and backhaul topologies and SFC constraints.

with some common SFC requirements, i.e., all flows within a service have some common traversal constraints. In the case of multiple deployment/instantiation of the same service function at network nodes, we assume fixed association of service flows to one of the multiple service function nodes.

We denote a flow with traversal constraint with $S \rightarrow N_1 \rightarrow \quad \rightarrow N_m \rightarrow D$ where S and D are the flow source and destination, respectively, and $N_1, ..., N_m$ are intermediate nodes that the flow must visit in the specified order before reaching its destination. Accordingly, a service is denoted by $S \rightarrow N_1 \rightarrow \quad \rightarrow N_m \rightarrow D$, where S is the set of source nodes of flows within the service and D is the set of destinations of flows within the service.

III. Flow-Based VN Admission Control

In this section, a flow-segment-based admission control approach is described.

A. Flow Segmentation

In order to address the flow traversal constraints, each flow is parsed according to its traversal constraints into multiple flow segments. This approach is illustrated in Figure 3 for a flow which must go through service function nodes A and B before reaching its destination.

B. Flow-Segment-Based VN Admission Control

A flow-based admission control method for VN requests can be applied to address the flow traversal constraints by using flow-segments as commodities. In other words, in the flow-segment-based admission control, the flow commodities are flow segments rather than the original flows. The demand of each flow segment is equal to the corresponding original flow.

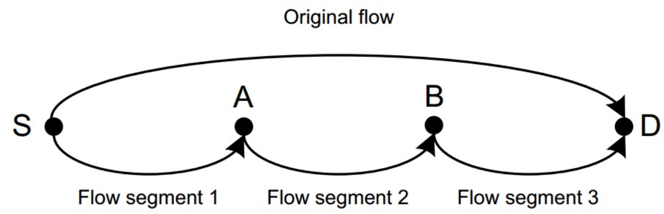

Fig. 3. Flow segmentation: the original flow is parsed into smaller flow segments based on the SFC constraints.

For a flow-segment-based admission control method, first, note that VN service admission control problem is a feasibility problem in which we are not trying to improve an objective function as the ultimate goal is to determine if some admission control constraints can be satisfied or not. Let x_{ijk} denote the amount of rate received from BS $b_k \in B$ at bin $a_j \in A$ by VN $v_i \in V$. We call this flow f_ij. Also let z_ijl denote the amount of traffic for flow f_ij passing through link $l \in L$. The objective of the optimization is to find a solution x^* and z^* that satisfies optimization constraints.

The optimization constraints can be divided into two categories: the access constraints, and the backhaul constraints. First, the access constraints are presented and then the backhaul constraints follow.

The first constraint is that the sum of the resources allocated by each BS to all bins cannot exceed its available bandwidth:

$$\text{C1:} \quad M_k = \sum_{i \in I} \sum_{j \in J} \frac{x_{ijk}}{\eta_{jk}} \leq W_k, \forall k \in K, \tag{4}$$

where M_k is the amount of used resources from BS k, W_k is the available resources, and η_{jk} is the effective spectral efficiency experienced from BS k at bin j.

Secondly, the outage constraint is that outage probability for all customers must be less than the maximum allowed outage. The outage probability for a VN $v_i \in V$ for one bin $a_j \in A$ is equal to the probability that the demand for this VN in this bin is higher than its received rate from all BSs:

$$\xi_{ij} = \mathbb{P}\mathrm{r}\{d_{ij} \geq r_{ij}\}, \tag{5}$$

where r_{ij} is the total rate received by VN $v_i \in V$ at bin $a_j \in A$:

$$\text{C2:} \quad r_{ij} = \sum_{k \in K} x_{ijk}, \forall i \in I, \forall j \in J. \tag{6}$$

To calculate the outage probability for a VN over the entire network, one way is to take the normal average. However, since different bins have different traffic demands, a better measure is a weighted average:

$$\xi_i = \frac{1}{\lambda_i} (\sum_{j \in J} \lambda_{ij} . \xi_{ij}), \tag{7}$$

where $\lambda_i = \sum_{j \in J} \lambda_{ij}$ is the total demand for VN $v_i \in V$. Therefore, the outage constraint is

$$\text{C3:} \quad \xi_i \leq O_i, \forall i \in I. \tag{8}$$

The service function requirements at each SFN can also be considered in the optimization problem. Let H be the set of all SFNs in the network. Also assume that each VN service v_i requires u_ih resource usage at SFN $h \in H$. The SFN resource constraint can be stated as

$$\text{C4:} \quad \sum_{i \in I} u_{ih} \leq U_h, \forall h \in H, \tag{9}$$

where U_h is the total resources available at SFN $h \in H$.

The first backhaul constraint is that the flow conservation law must hold at all network nodes:

$$\text{C5:} \quad \sum_{l \in L_{in}(n)} z_{ijl} = \sum_{l \in L_{out}(n)} z_{ijl}, \forall n \in N, \forall i \in I, \forall j \in J. \tag{10}$$

For the BSs, since they are at the border between backhaul and access, the flow conservation law is stated as follows:

$$\text{C6:} \quad \sum_{l \in L_{in}(k)} z_{ijl} = x_{ijk}, \forall k \in K, \forall i \in I, \forall j \in J. \tag{11}$$

Finally, the last backhaul constraint is that the total traffic passing through each link cannot exceed the link capacity:

$$\text{C7:} \quad \sum_{i \in I} \sum_{j \in J} z_{ijl} \leq C_l, \forall l \in L, \tag{12}$$

where C is the link capacity matrix.

In summary, the flow-segment-based VN admission control problem can be cast as the following problem:

$$\text{find}_{x,z} \quad x, z \quad \text{subject to: C1-C7.} \tag{13}$$

IV. Service-Based VN Admission Control

The flow-based approach described in Section III does not scale with increasing number of flows/flow segments in terms of admission control algorithm complexity. In this section, we propose a low-complexity admission control approach by taking advantage of the property that all flows within a service have the same traversal constraints.

A. Service Segmentation

We parse flows according to their SFC requirements into flow segments as described in Section III-A. A service segment is defined as a set of flow segments with common source node. Alternative definitions of service segment may be considered in different problems. For instance, in a network problem which is involved in uplink streams, a service segment can be defined to be a set of flows with common destination. However, in our problem, since we are considering downlink, defining service segments based on common source nodes helps to reduce the problem size. Figure 4 illustrates the bundling of flow segments with common source nodes under service segments.

In this paper, we assume that all of the traffic for each VN service goes through the same service chain. In other words, each VN service has a specific SFC. Therefore, the set of service segments matches the VN service segments.

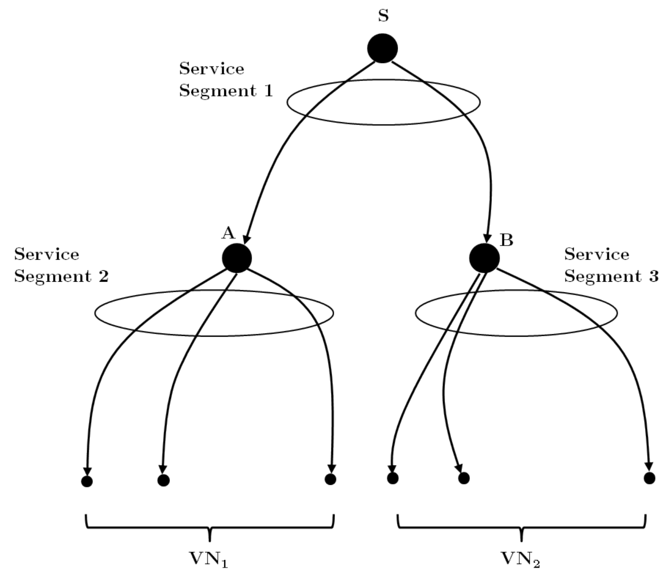

Fig. 4. Flow segments with common source node are categorized under a service segment.

B. Service-Segment-Based VN Admission Control

The service-segment-based admission control optimization formulation is presented in this section. The access constraints remain the same as flow-segment-based method due to the fact that the QoS requirements of VN services must be satisfied in a per bin basis. However, the backhaul constraints for the service-segment-based admission control are highly efficient and more scalable than the flow-segment-based method. Indeed, the number of variables is decreased considerably.

First, note that the constraint (10) will change as follows:

$$\text{C8:} \quad \sum_{l \in L_{in}(n)} y_l = \sum_{l \in L_{out}(n)} y_l, \forall n \in N, \tag{14}$$

where y_l represents the total amount of traffic passing through link l. It is clear that the size of variable y, compared to z in flow-based method, is reduced by a $|I| \times |J|$ magnitude.

Secondly, (11) can be stated in a summarized form as follows:

$$\text{C9:} \quad \sum_{l \in L_{in}(k)} y_l = x_{ijk}, \forall k \in K, \forall i \in I, \forall j \in J. \tag{15}$$

Finally, (12) is transformed in a very simple inequality as

$$\text{C10:} \quad y_l \leq C_l, \forall l \in L. \tag{16}$$

Therefore, the service-segment-based admission control optimization problem can be stated as follows:

$$\text{find}_{x,y} \quad x, y \quad \text{subject to: C1-C4, C8-C10.} \tag{17}$$

It can be shown easily that (17) has a significantly lower number of optimization variables and optimization constraints compared to (13). Table I shows the number of optimization variables and constraints in both flow-based and service-based admission control methods. In Table I, α is defined as

$$\alpha = |K| + |I| \times |J| + |I| + |I| \times |J| \times |K| + |L|. \tag{18}$$

TABLE I

COMPARISON BETWEEN FLOW-BASED AND SERVICE-BASED METHODS IN TERMS OF NUMBER OF VARIABLES AND CONSTRAINTS.

Criteria	Flow-based	Service-based
Var.	$\lvert I\rvert \times \lvert J\rvert \times (\lvert K\rvert + \lvert L\rvert)$	$\lvert I\rvert \times \lvert J\rvert \times \lvert K\rvert + \lvert L\rvert$
Cons.	$\alpha + \lvert I\rvert \times \lvert J\rvert \times \lvert N\rvert$	$\alpha + \lvert N\rvert$

C. Convexity of the Problem

A mathematical optimization problem can be solved efficiently and methodically if it can be cast as a convex problem [21]. A problem is convex if the objective function and the constraints are convex functions. In this section, we show that the problem (13) is convex and can be solved efficiently with well-known convex optimization tools and optimization methods such as interior point methods. Interior point methods are a certain class of algorithms that solve linear and nonlinear convex optimization problems [21].

In (13) and (17), all of the constraints except C3 (8) are linear functions of the optimization variables. So, the problem is convex if the constraint C3 is a convex function of optimization variables. First, note that C3 is convex if ξ_i is a convex function of r. Also note that ξ_i is a weighted sum of ξ_{ij}. So, it is convex if ξ_{ij} are convex functions w.r.t. r. From (5) it is clear that ξ_{ij} is the complementary CDF of d_{ij} at r_{ij}. Therefore, (13) and (17) are convex optimization problems if d has a concave CDF. It can be easily shown that the exponential distribution and uniform distribution have concave CDFs. The CDF of the exponential distribution is defined as

$$F_X(x) = 1 - \exp(-\Lambda x), \tag{19}$$

for parameter Λ, which has the following second derivative:

$$\frac{d^2F}{dx^2} = -\Lambda^2 \exp(-\Lambda x), \tag{20}$$

which is always negative.

The CDF of uniform distribution is defined as

$$F_X(x) = \frac{x-a}{b-a}, \tag{21}$$

for parameters a and b, which is a linear function.

It can also be shown that Gaussian distribution has a concave CDF for $r \geq \lambda$. The CDF of the Gaussian distribution is defined as

$$F_X(x) = \frac{1}{2}\left[1 + \text{erf}(\frac{x-\mu}{\sqrt{2}\sigma})\right], \tag{22}$$

for μ and σ, where erf(.), error function, is defined as

$$\text{erf}(x) = E(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt, \tag{23}$$

which has the following second derivative:

$$\frac{d^2E}{dx^2} = \frac{-4x}{\sqrt{\pi}} \exp(-x^2), \tag{24}$$

which is negative for positive x. Poisson distribution can be approximated by Gaussian distribution with very high accuracy for $\lambda > 1000$ [22].

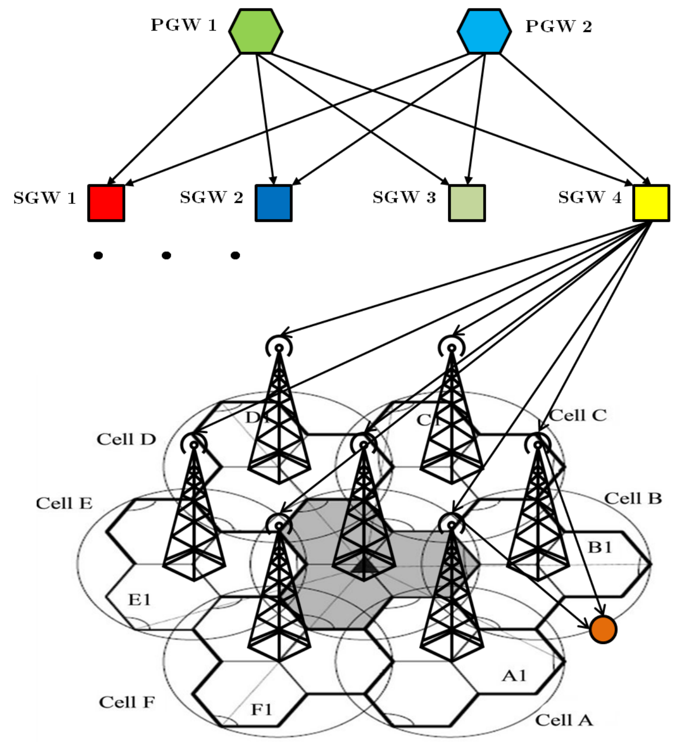

Fig. 5. We consider a network with 2 PGWs, 4 SGWs, 7 BSs, and 21 sectors.

V. Simulation Results

In this section, we compare admission control algorithm computation times as a measure of complexity for service-based and flow-based formulations. We also verify that service-based approach incurs no loss in optimality.

A. Simulation Setup

The proposed methods in this paper are general and can be applied to general access network and backhaul network topologies. In our simulation we consider the network topology illustrated in Figure 5. We assume that there are 4 VN service requests each to be served by a service gateway (SGW) and a packet gateway (PGW). The SGWs perform a first level of aggregation on the VN data and the PGWs perform a second level of aggregation on top of that.

There are 4 SGWs and 2 PGWs in the network backhaul. Each VN has a dedicated SGW. VNs v_1 and v_2 are served by PGW1 and VNs v_3 and v_4 are served by PGW2. The access network comprises 7 BSs each having 3 sectors, 21 sectors in total. All BSs have full connection with all SGWs and all SGWs have two connections to the PGWs. All BSs have 10 MHz bandwidth available for serving VN requests. We assume that the outage requirement for all VNs is 2 percent. Noise power is assumed to be -174 dB.

B. Complexity Comparison

We use the admission control algorithm computation times as a measure of computational complexity. Figures 6 and

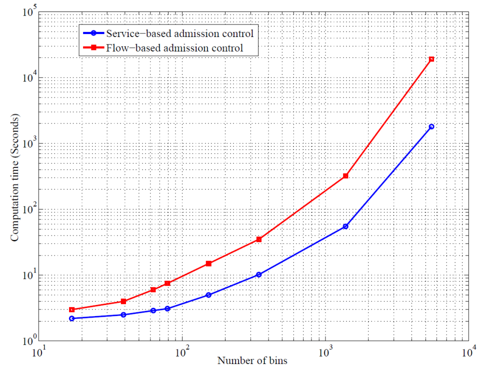

Fig. 6. Computation time: the vertical axis shows the computation times for flow-based and service-based admission control algorithms. The horizontal axis shows the number of flows.

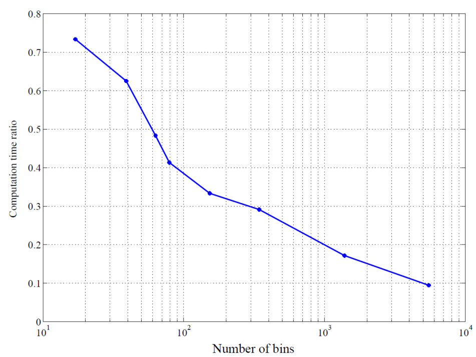

Fig. 7. Computation time ratio: the ratio of computation time of service-based admission control algorithm to the computation time of flow-based algorithm is illustrated versus the number of flows.

7 compare the running times of service-based and flow-based algorithms as a function of the number of flows in the network. Figure 6 shows the actual running times, in log scale, for both schemes as a function of the number of flows. Figure 7 shows the ratio of service-based running time to flow-based running time as a function of the number of flows. According to Figures 6 and 7, with more than 5000 flows in the network, the service-based algorithm is about 11 times faster than the flow-based algorithm. The problem complexity increases polynomially with the number of flows. However, the complexity of service-based does not increase significantly as the number of flows increases. This is due to the fact that the number of service segments is fixed in the network backhaul and does not increase as the number of flows increases. Furthermore, the number of backhaul constraints in the service-based admission control problem does not change as the number of flows increases.

VI. Concluding Remarks

We considered the problem of admission control of VN service requests in a multi-service network. We formulated the problem as a joint optimization of access and backhaul which is convex and tractable. We showed that service-based formulation reduces the problem size and computation time.

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