Non-coherent Multi-Layer Constellaions for Unequal Error Protection

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in collaboration with Kareem M. Attiah[2], Ramy H. Gohary[3], and Halim Yanikomergolu[3]

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  Define the sets $\mathcal{S}^1 = \{s_1^1, \ldots, s_{\mathcal{N}_1}^1\}$ and $\mathcal{S}^2 = \{s_1^2, \ldots, s_{\mathcal{N}_2}^2\}$, where the set $\mathcal{S}^1$ ($\mathcal{S}^2$) contains the more (less) protected subsymbols.

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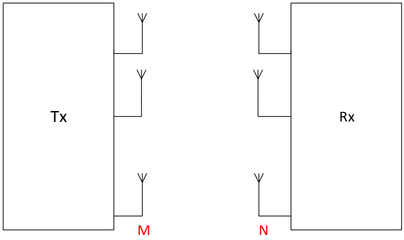
- The chosen supersymbol s_i is mapped onto the pair $\left(s_i^1, s_i^2\right)$ by some bijective function $f : \mathcal{S} \to \mathcal{S}^1 \times \mathcal{S}^2$.

(2) Non-Coherent Signaling



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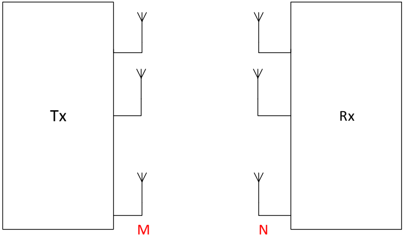

- Channel state information (CSI) is unknown at both transmitter and receiver.

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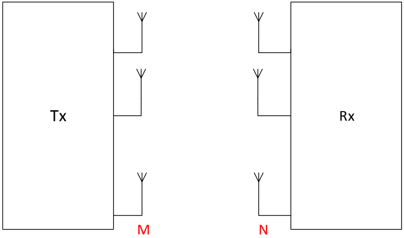

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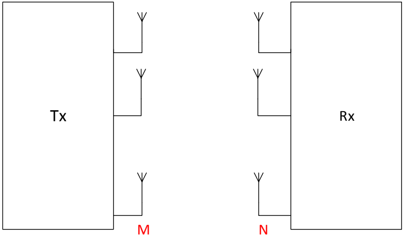
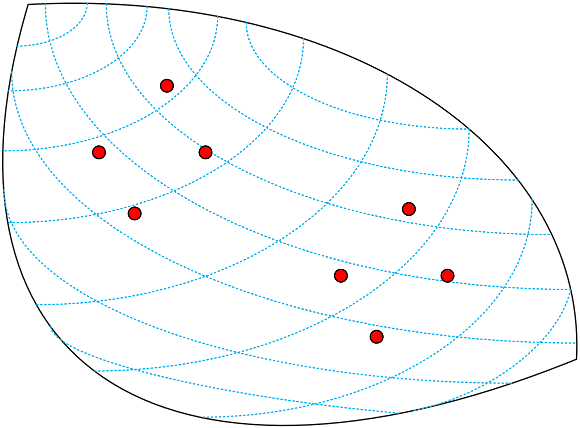
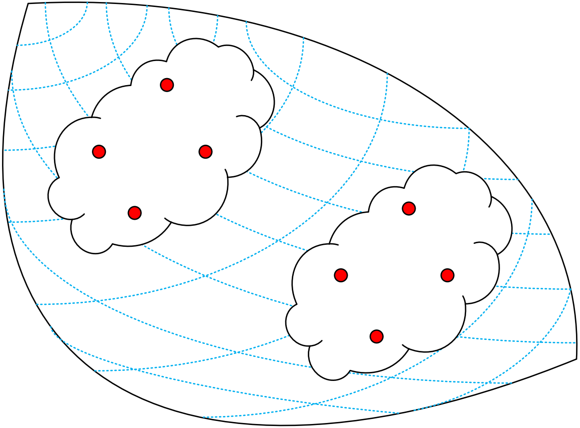


- Channel state information (CSI) is unknown at both transmitter and receiver.
- Transmission takes place over T symbol durations. Channel coefficients are assumed constant during that interval. We only consider $T > M$.
- Transmitted symbols are represented by points on the Grassmannian manifold $\mathbb{G}_{T,M}(\mathbb{C})$.

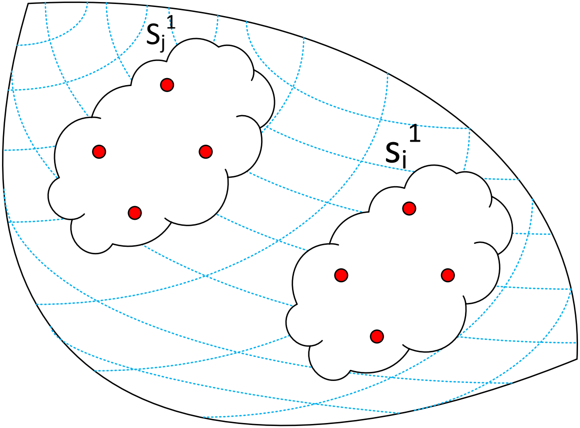
Set Partitioning





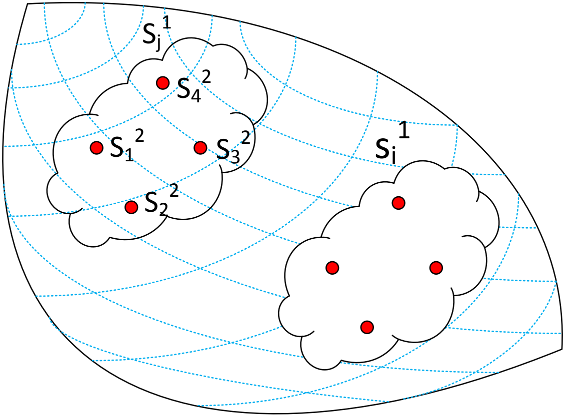
The constellation is decomposed into disjoint subsets.

Set Partitioning



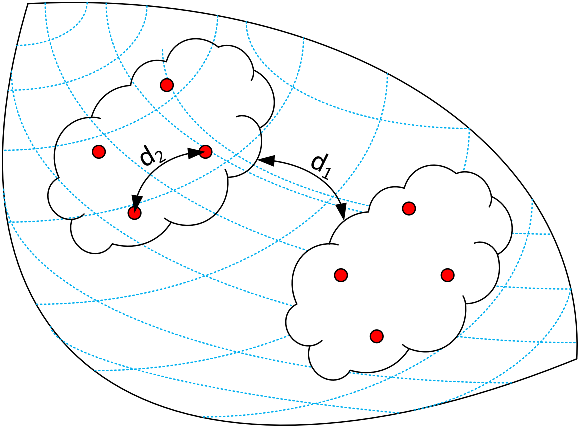
More important symbols encode subsets.

Set Partitioning



Less important symbols encode points within a chosen subset.

Set Partitioning



d_1 (d_2) controls the error probability of the more (less) protected symbols.

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How to Measure the Distance between Constellation Points?

- For unitary signaling, the asymptotic error probability of mistaking $\mathbf{X}_i$ for $\mathbf{X}_j$ under GLRT detection is given by[1]

$$P_{ij} = (d_{cp}(\mathbf{X}_i, \mathbf{X}_j)\gamma T)^{-MN} M^{MN} \binom{2MN-1}{MN},$$

$$d_{cp}(\mathbf{X}_i, \mathbf{X}_j) = \left(\prod_{k=1}^{M} \sin^2\theta_k\right)^{\frac{1}{M}}.$$

$\{\theta_i\}_{i=1}^{M}$ are the principle angles between the subspaces associated with $\mathbf{X}_i$ and $\mathbf{X}_j$.

[1] M. Brehler and M. K. Varanasi, Asymptotic error probability analysis of quadratic receivers in rayleigh-fading channels with applications to a unified analysis of coherent and noncoherent space-time receivers, IEEE Transactions on Information Theory.

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- The Chordal Product Distance[2] (not a true distance!) can be employed to define the coding gain of the more (less) protected symbols.

$$d_1 = \min_{i \neq j} d_{cp}(\Omega_i, \Omega_j),$$

$$d_2 = \min_{\mathbf{X},\mathbf{Y} \in \mathcal{C}} d_{cp}(\mathbf{X}, \mathbf{Y}).$$

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- Our design approach is entirely based on the more intuitive chordal distance. However, the connection between both metrics is essential since the coding gains are expressed in terms of the product chordal distance.

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Design Approach (1)[3]

- For an initial layer ξ_1, we define a sequence of $L-1$ children layers $\xi_2, \xi_3, \ldots, \xi_L$.

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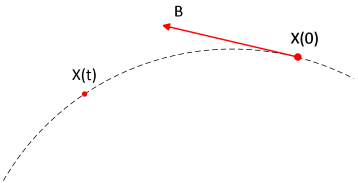
- Starting at $\mathbf{X} = \mathbf{X}(0)$, the motion of a point traveling along some direction $\mathbf{B}$ is given by

$$\mathbf{X}(\mathbf{t}) = [\mathbf{X} \quad \mathbf{X}_\perp] \begin{bmatrix} \mathbf{V} \cos \mathbf{\Sigma} t \\ \mathbf{U} \sin \mathbf{\Sigma} t \end{bmatrix}.$$

Where $\mathbf{B} = \mathbf{U\Sigma V}^\dagger$ is the SVD of $\mathbf{B}$, $\mathbf{X}_\perp$ is the orthogonal complement of $\mathbf{X}$ and t is a step parameter.



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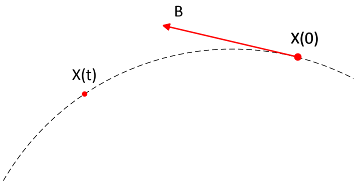

- Elements of ξ_i are found by transitioning along K geodesics emanating from each member in the i^{th} parent layer.

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$$\xi_i = \left\{ \begin{bmatrix} \mathbf{X}_p & \mathbf{X}_{p\perp} \end{bmatrix} \begin{bmatrix} \mathbf{V}_k \cos \mathbf{\Sigma}_k t_i \\ \mathbf{U}_k \sin \mathbf{\Sigma}_k t_i \end{bmatrix} \middle| \forall \mathbf{X}_p \in \mathcal{C}_i, \forall k = 1, \ldots, K \right\}.$$

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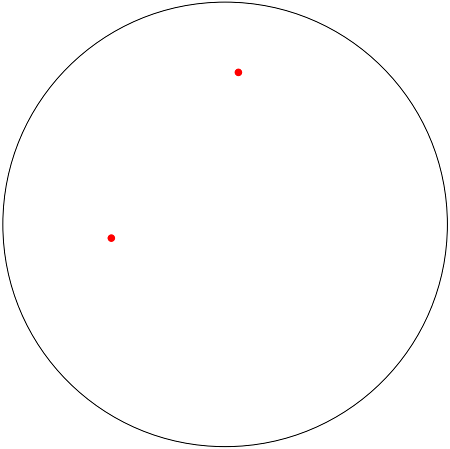
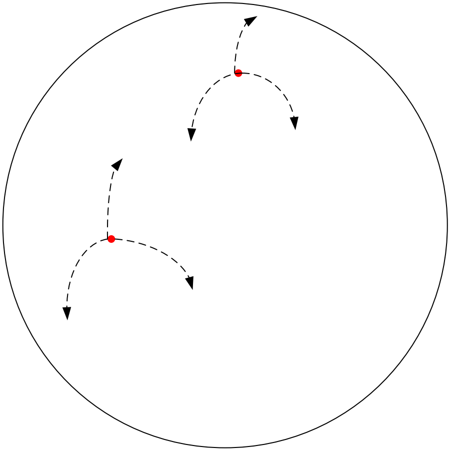
- The union set of all ξ_i forms the constellation.
- The size of the constellation is given by $|\mathcal{C}| = N'(K+1)^{L-1}$, where N' is the initial constellation size and K is the number of geodesic directions.



Initial constellation.

Design Approach (3)



Geodesic directions ensure maximal spacing.

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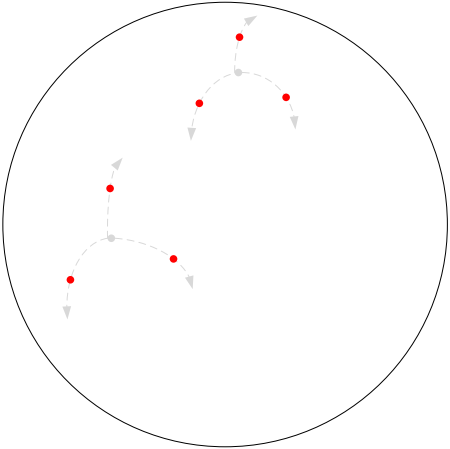

Children points are found along the geodesic directions.

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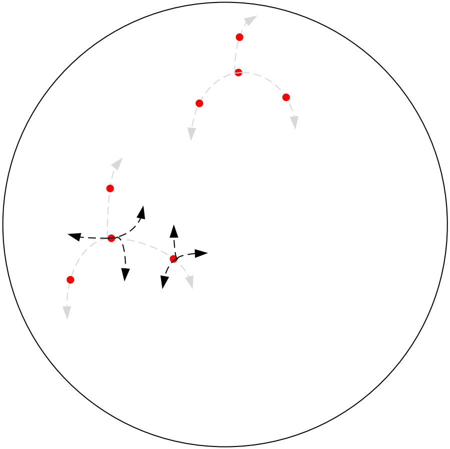

The process can be further repeated to add more points.

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- With careful selection of this parameter, a specific coding gain can be ensured for the more protected layer over the Equal Error Protection scheme.
- We propose two ways of doing this:
  1. Geometric Mean Approximation.
  2. Polynomial Regression.

Geometric Mean Approximation

- A lower bound on minimum subset distance d_1 is established in terms of $\beta = \sin t$.

$$d_1 \gtrapprox \frac{1}{M}(d_{pack} - 2\sqrt{M}\beta)^2 - \frac{M-1}{2M^2}(d_{pack} - 2\sqrt{M}\beta)^4,$$

where $d_{pack} = \frac{M(T-M)}{T}\sqrt{\frac{N'}{N'-1}}$.

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where $d_{pack} = \frac{M(T-M)}{T}\sqrt{\frac{N'}{N'-1}}$.
- Setting the desired coding gain to be equal to the RHS guarantees that the actual coding gain is at least as large as the desired value.
- This bound is rather crude when the constellation size is large.

Polynomial Regression

- Assume

$$d_1 = \sum_{i=1}^{n} a_i \beta^i.$$

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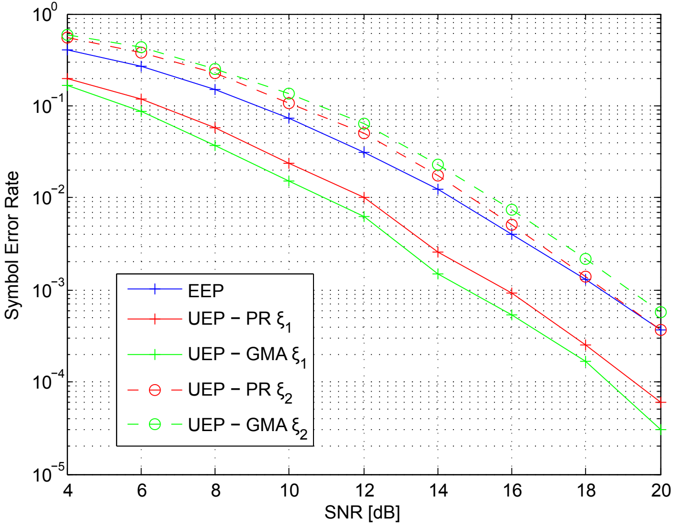
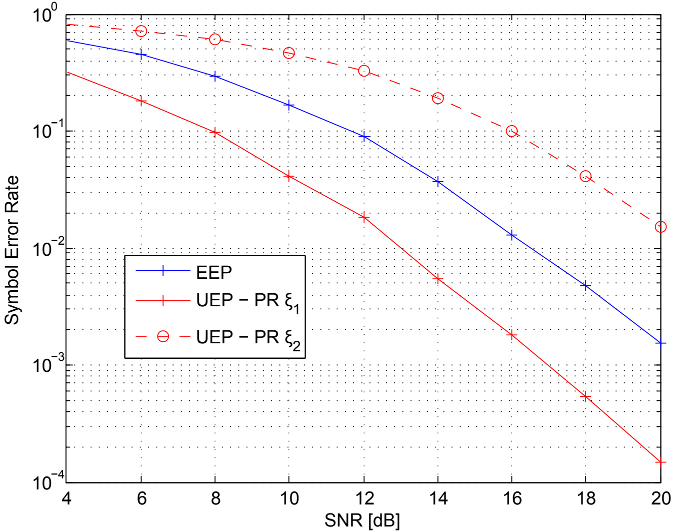
- With enough observations ($l \geq n$), we can estimate the unknown coefficients a_i.
- Performance analysis suggests that this is the more reliable approach.

- R = 1.5 bpcu, 33% of data is important



Performance Analysis

- R = 2 bpcu, 50% of data is important



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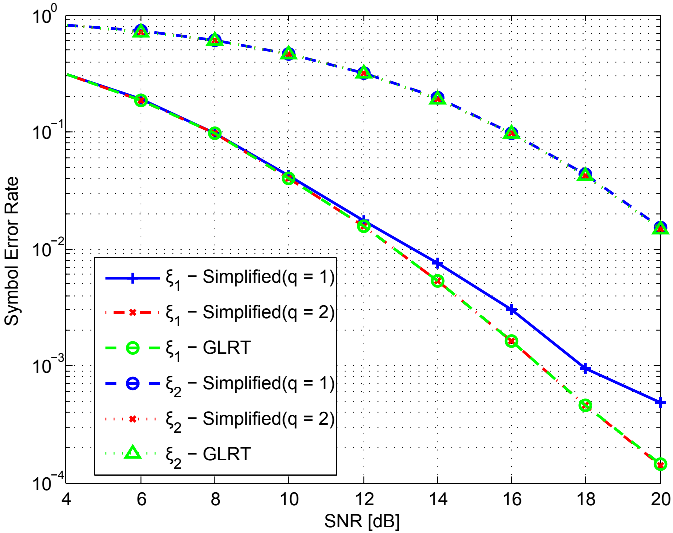
- The decoder then investigates the children of the q largest ML metric. The process is repeated for $L > 2$.
- A decision is made in favor of some $\mathbf{X}$, if $\mathbf{X}$ maximizes the ML metric over all examined points.
- A reduction factor of $(\frac{q}{K})^{L-1}$ is realized in the number of operations needed.

- $K = 15$.


Symbol Error Rate
SNR [dB]
10^0
10^-1
10^-2
10^-3
10^-4
4
6
8
10
12
14
16
18
20
ξ1 − Simplified(q = 1)
ξ1 − Simplified(q = 2)
ξ1 − GLRT
ξ2 − Simplified(q = 1)
ξ2 − Simplified(q = 2)
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