Reduced Complexity Optimal Detection of Binary Faster-than-Nyquist Signaling

Ebrahim Bedeer*, **Halim Yanikomeroglu***, and Mohamed Hossam Ahmed**

*Carleton University, Ottawa, ON, Canada
**Memorial Univeristy, St. John's, NL, Canada

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Agenda

- Introduction
- System Model
- Novel FTN Detection Schemes
  - Quasi-Optimal Detection (High SE)
  - Symbol-by-Symbol Detection (Low SE)
- Conclusions

Introduction

Introduction

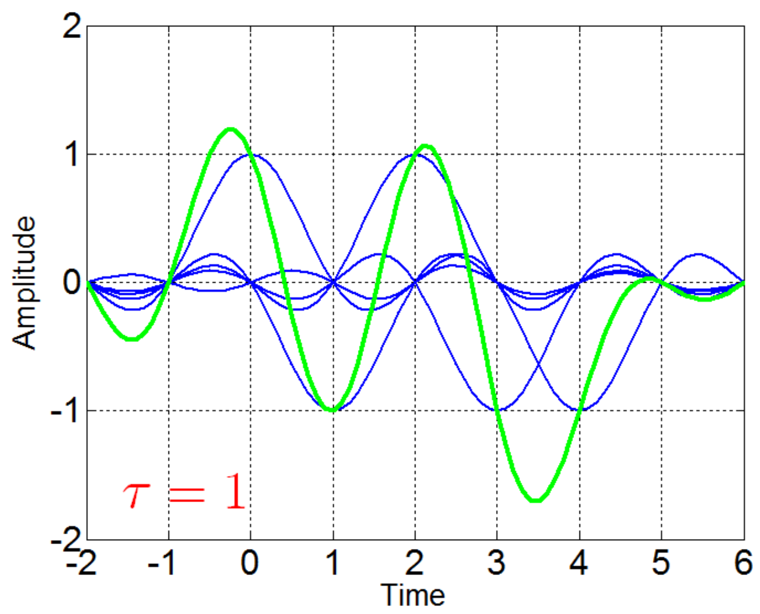
- Nyquist limit is more of a guideline than a rule.
- Nyquist limit simplifies receive design by avoiding ISI.
- Faster-than-Nyquist (FTN signaling) intentionally introduce ISI to improve SE.
- FTN signaling *concept* exists since 1968 [Saltzberg-68].
- FTN signaling *term* coined by Mazo in 1975 [Mazo-75].
- Mazo limit: FTN does not affect minimum distance of uncoded sinc binary transmission up to a certain range.

Saltzberg B. Intersymbol interference error bounds with application to ideal bandlimited signaling. IEEE Transactions on Information Theory. July 1968; 14(4):563-8.

Mazo JE. Faster-than-Nyquist signaling. The Bell System Technical Journal. 1975 Oct;54(8):1451-62.

FTN Signaling Basic Idea

$$s(t) = \sqrt{E_s} \sum_n a_n \, g(t - n\,T)$$

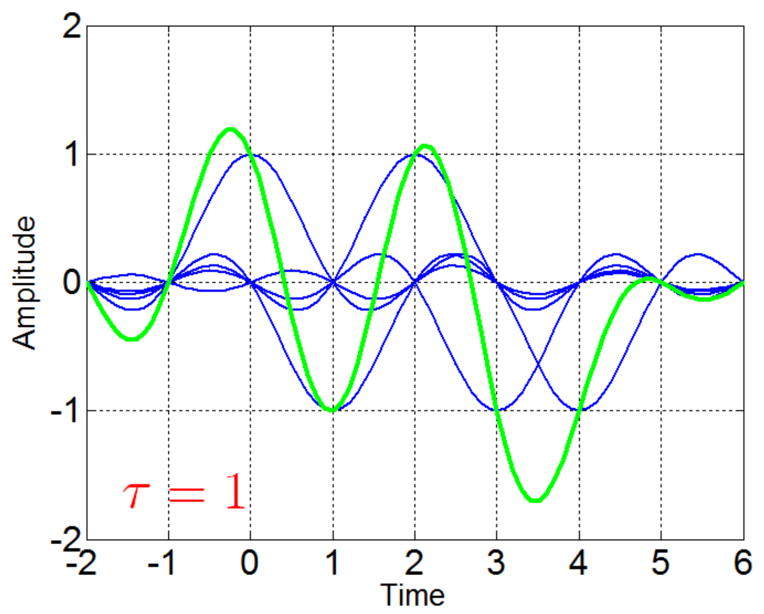


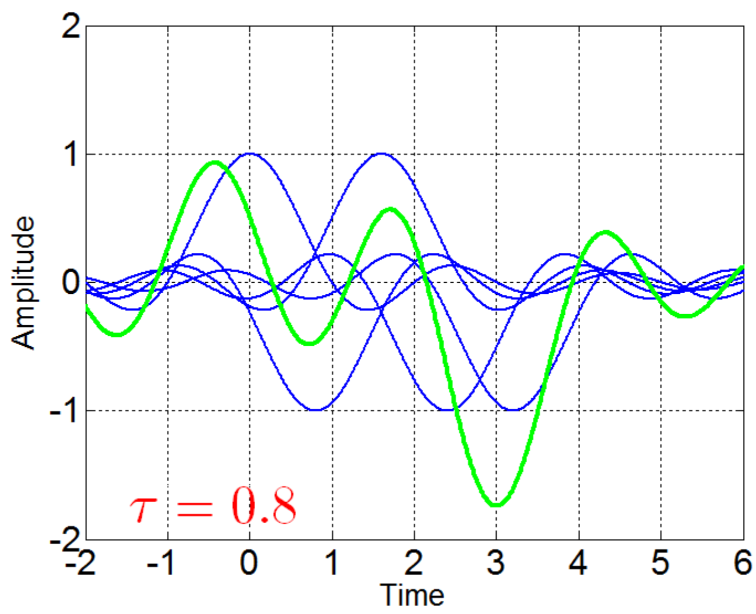
$2 \quad \text{symbols/s/Hz}$

FTN Signaling Basic Idea

$$s(t) = \sqrt{E_s} \sum_n a_n \, g(t - n\,T)$$

$$s(t) = \sqrt{E_s\,\tau} \sum_n a_n \, g(t - n\,\tau\,T)$$





2 symbols/s/Hz

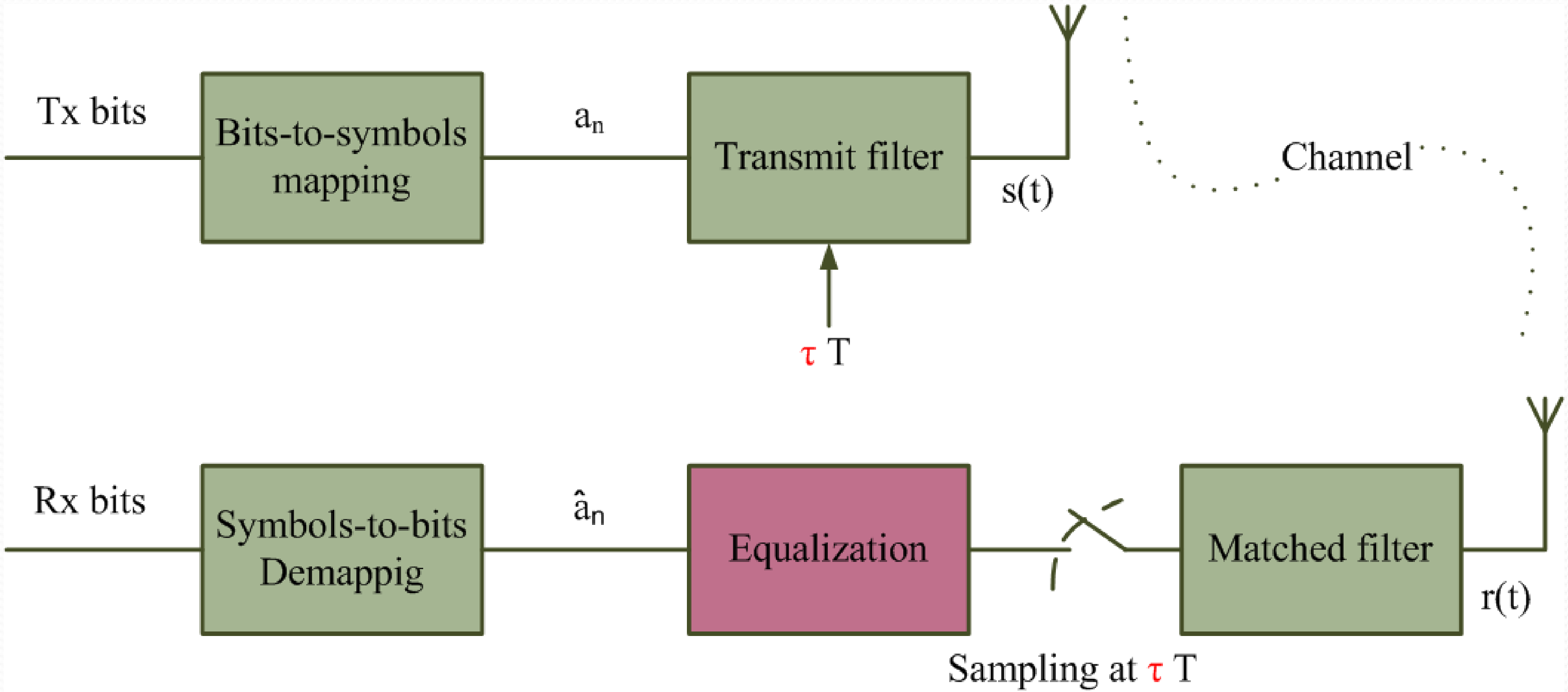
$\frac{2}{\tau}$ symbols/s/Hz

Extension of Mazo Limit

- Other pulse shapes (root-raised cosine, Gaussian, …)
- Non-binary transmission
- Frequency domain

System Model

FTN Block Diagram



FTN Detection Problem

- $\boldsymbol{y} = \boldsymbol{G}\,\boldsymbol{a} + \boldsymbol{w}$
  - $\boldsymbol{y}$**:** received sample vector
  - $\boldsymbol{G}$**:** ISI matrix
  - $\boldsymbol{a}$**:** transmit data symbols
  - $\boldsymbol{w}$**:** received noise samples $(0, \sigma^2\boldsymbol{G})$

- Detection Problem:
  - $\hat{a} = \arg\min_{\boldsymbol{a}\in\boldsymbol{D}} \quad (\boldsymbol{G}^{-1}\boldsymbol{y} - \boldsymbol{a})^{\mathrm{T}}\boldsymbol{G}(\boldsymbol{G}^{-1}\boldsymbol{y} - \boldsymbol{a})$

Modified Sphere Decoding (MSD)

- Noise covariance matrix can be exploited to develop MSD.
- Estimated data symbols can be found using MSD as

$$\left\lceil z_N - \frac{d}{R_{N,N}} \right\rceil \le a_N \le \left\lfloor z_N + \frac{d}{R_{N,N}} \right\rfloor .$$

$$a_{N-1} \ge \left\lceil z_{N-1} - \frac{\hat{d} - R_{N-1,N}(z_N - a_N)}{R_{N-1,N-1}} \right\rceil ,$$

$$a_{N-1} \le \left\lfloor z_{N-1} + \frac{\hat{d} + R_{N-1,N}(z_N - a_N)}{R_{N-1,N-1}} \right\rfloor ,$$

Simulation Results

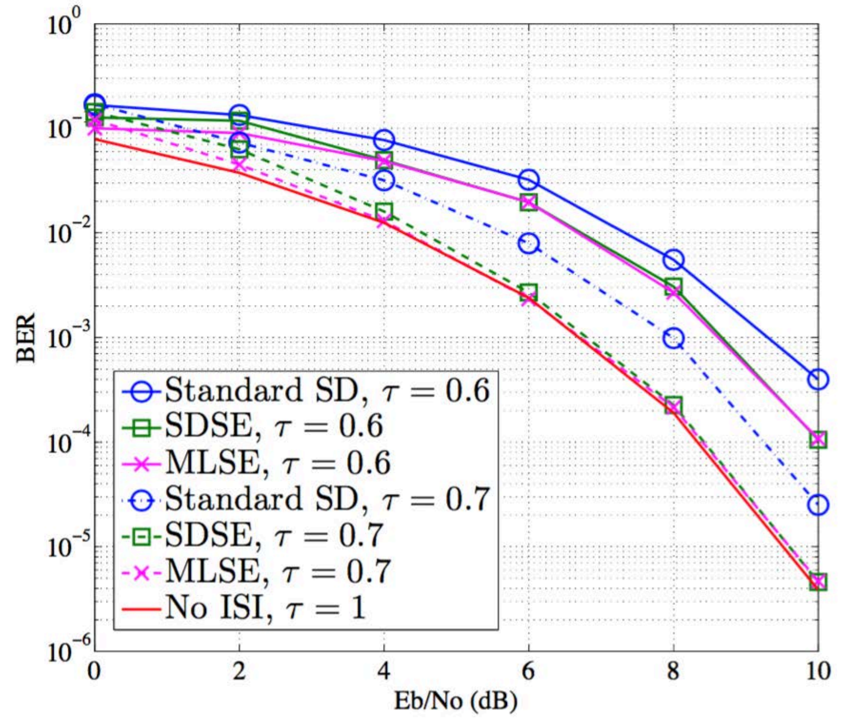


Fig. 2: BER performance of binary FTN detection versus $\frac{E_b}{N_o}$ using the standard SD-based and proposed SDSEs at $\beta = 0.3$ and $\tau = 0.6$ and 0.7.

Simulation Results

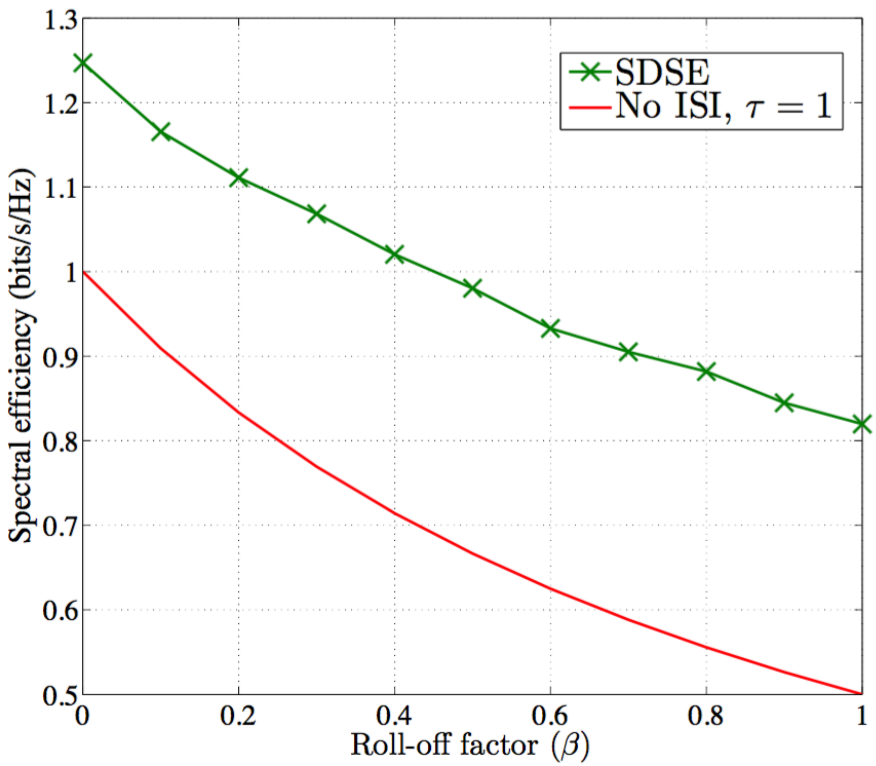
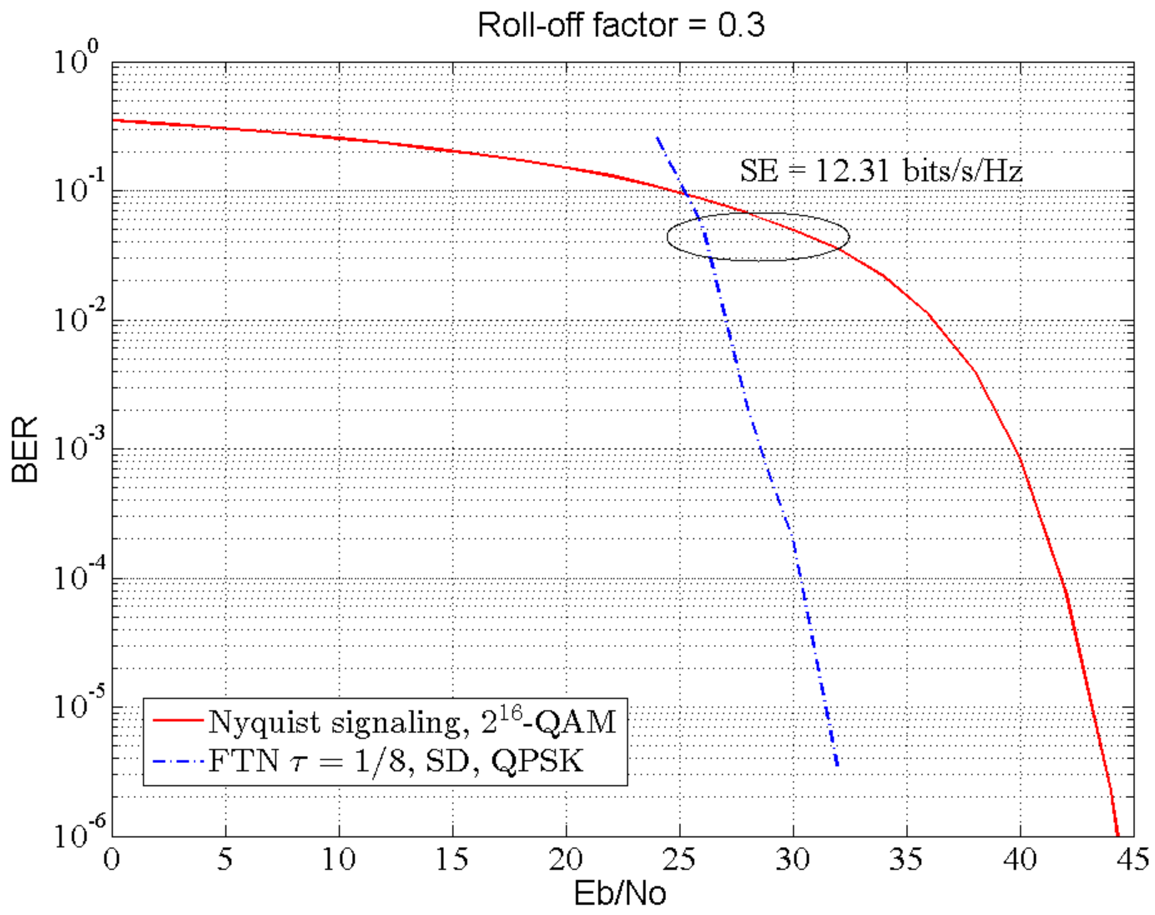


Fig. 4: Spectral efficiency comparison of binary FTN signaling versus β using the proposed SDSE and Nyquist signaling at BER = 10^{-4}.

Simulation Results



Symbol-by-Symbol Detection of FTN Signaling

Ebrahim Bedeer, Mohamed Ahmed, and Halim Yanikomeroglu, "A very low complexity successive symbol-by-symbol sequence estimator for binary faster-than-Nyquist signaling", accepted in *IEEE Access,* DOI: 10.1109/ACCESS.2017.2663762

Successive Symbol-by-Symbol Sequence Estimation (SSSSE)

- Received sample

$$y_k = \underbrace{G_{1,L}\, a_{k-L+1} + \ldots + G_{1,2}\, a_{k-1}}_{\text{ISI from previous } L-1 \text{ symbols}} + \underbrace{G_{1,1}\, a_k}_{\text{Current symbol to be estimated}} + \underbrace{G_{1,2}\, a_{k+1} + \ldots + G_{1,L}\, a_{k+L-1}}_{\text{ISI from upcoming } L-1 \text{ symbols}} .$$

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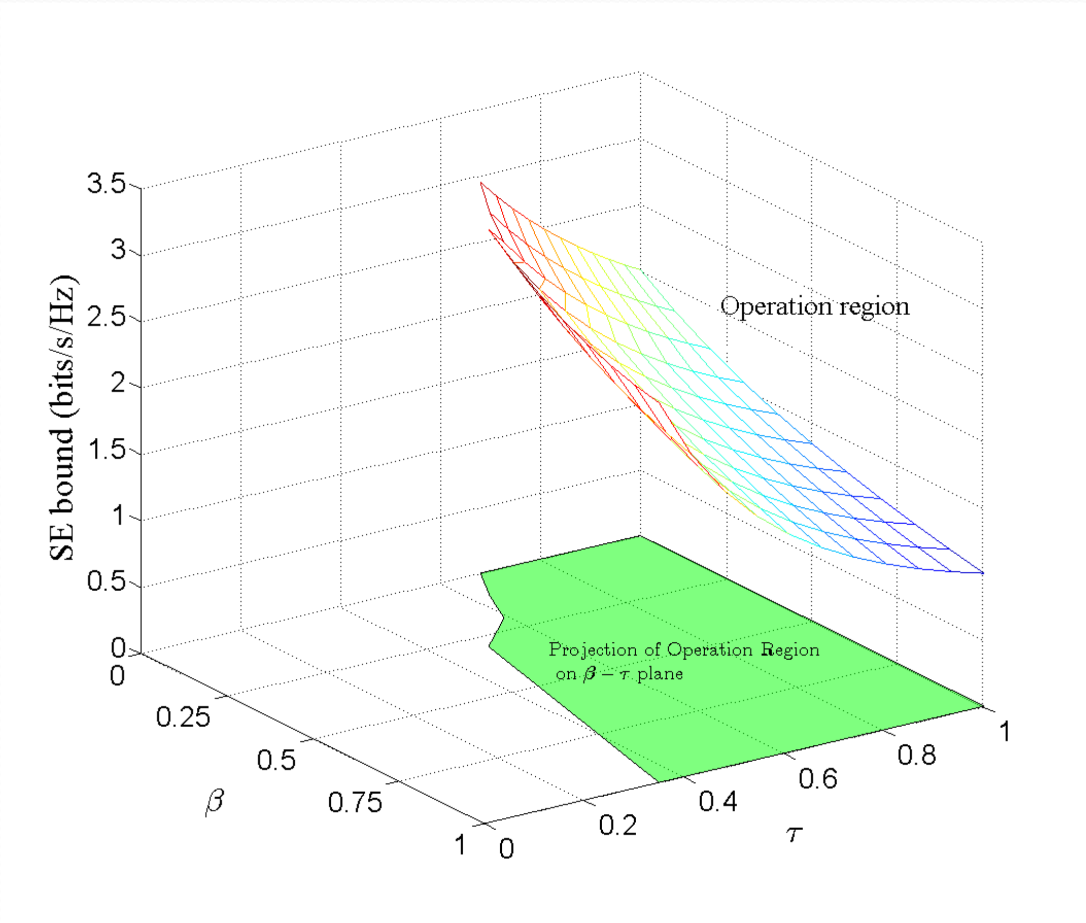
- Perfect estimation condition for QPSK FTN signaling

$$|G_{1,1}\, \Re\{a_k\}| > |G_{1,2}\, \Re\{a_{k+1}\} + \ldots + G_{1,L}\, \Re\{a_{k+L-1}\}|,$$

$$|G_{1,1}\, \Im\{a_k\}| > |G_{1,2}\, \Im\{a_{k+1}\} + \ldots + G_{1,L}\, \Im\{a_{k+L-1}\}|,$$

Operating region of SSSSE



Successive Symbol-by-Symbol Sequence Estimation (SSSSE)

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$$|G_{1,1}\, \Im\{a_k\}| > |G_{1,2}\, \Im\{a_{k+1}\} + \ldots + G_{1,L}\, \Im\{a_{k+L-1}\}|,$$

- Estimated symbol

$$\hat{a}_k = \text{quantize}\left\{y_k - (G_{1,L}\, \hat{a}_{k-L+1} + \ldots + G_{1,2}\, \hat{a}_{k-1})\right\},$$

Successive Symbol-by-Symbol with go-back-*K* Sequence Estimation (SSSgb*K*SE)

- Received sample

$$y_k = \underbrace{G_{1,L}\, a_{k-L+1} + \ldots + \underbrace{G_{1,K+1}\, a_{k-K} + \ldots + G_{1,2}\, a_{k-1}}_{\text{Previous } K \text{ symbols to be re-estimated}}}_{\text{ISI from previous } L-1 \text{ symbols}} + \underbrace{G_{1,1}\, a_k}_{\text{Current symbol to be re-estimated}}$$

$$+ \underbrace{G_{1,2}\, a_{k+1} + \ldots + G_{1,L}\, a_{k+L-1}}_{\text{ISI from upcoming } L-1 \text{ symbols}}.$$

- Estimated symbol

$$\hat{a}_k = \text{quantize}\Big\{ y_k - \big(\underbrace{G_{1,L}\, \hat{a}_{k-L+1} + \ldots + \underbrace{G_{1,K+1}\, \hat{\hat{a}}_{k-K} + \ldots + G_{1,2}\, \hat{\hat{a}}_{k-1}}_{\text{ISI from previous } K \text{ data symbols with improved estimation accuracy}}}_{\text{ISI from the previous } L-1 \text{ data symbols}} \big) \Big\}.$$

Simulation Results

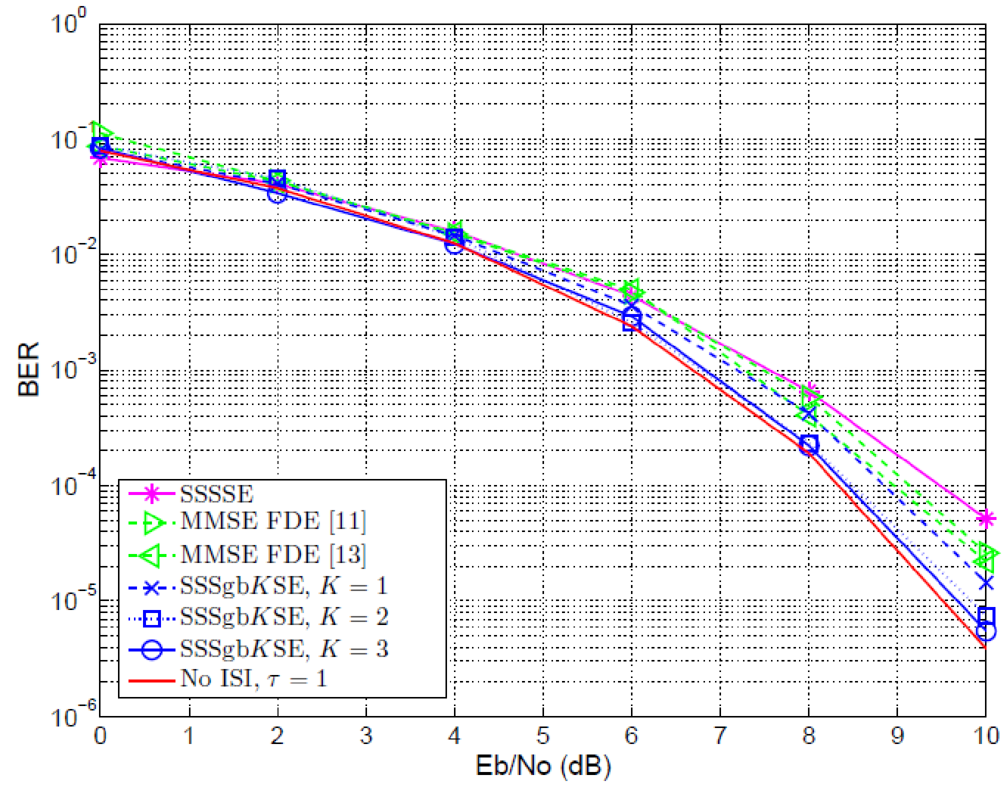

Fig. 4: BER performance of QPSK FTN sequence estimation as a function of $\frac{E_b}{N_o}$ using the proposed SSSSE, proposed SSSgb$K$SE, and FDEs in [11], [13] at $\beta = 0.3$ and SE of 1.71 bits/sec/Hz.

[11] S. Sugiura, "Frequency-domain equalization of faster-than-Nyquist signaling," *IEEE Wireless Commun. Lett.*, vol. 2, no. 5, pp. 555–558, Oct. 2013.

[13] T. Ishihara and S. Sugiura, "Frequency-domain equalization aided iterative detection of faster-than-Nyquist signaling with noise whitening," in *Proc. IEEE International Conference on Communications (ICC)*, May 2016, pp. 1–6.

Simulation Results

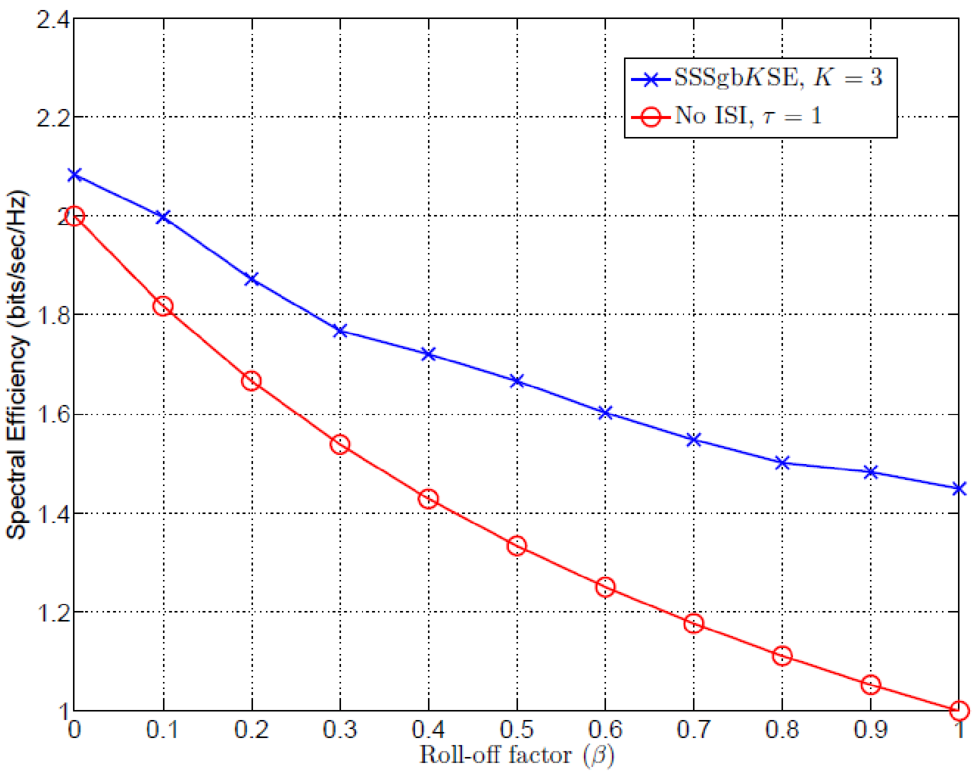

Fig. 6: Spectral efficiency of QPSK Nyquist and FTN signaling as a function of β using the proposed SSSgb$K$SE at BER = 10^{-4}.

Conclusions

Conclusions

- FTN signaling is promising to increase the SE
- Tradeoff between performance and complexity
- Gain of FTN increases at higher values of SE