Reduced Complexity Optimal Detection of Binary Faster-than-Nyquist Signaling

Ebrahim Bedeer*, Halim Yanikomeroglu*, and Mohamed Hossam Ahmed**

*Carleton University, Ottawa, ON, Canada
**Memorial University, St. John’s, NL, Canada

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Agenda

- Introduction
- System Model
- Novel FTN Detection Schemes
  - Quasi-Optimal Detection *(High SE)*
  - Symbol-by-Symbol Detection *(Low SE)*
- Conclusions
Introduction
Introduction

- Nyquist limit is more of a guideline than a rule.
- Nyquist limit simplifies receive design by avoiding ISI.
- Faster-than-Nyquist (FTN signaling) intentionally introduce ISI to improve SE.
- FTN signaling concept exists since 1968 [Saltzberg-68].
- FTN signaling term coined by Mazo in 1975 [Mazo-75].
- Mazo limit: FTN does not affect minimum distance of uncoded sinc binary transmission up to a certain range.


FTN Signaling Basic Idea

\[ s(t) = \sqrt{E_s} \sum_n a_n g(t - n T) \]

\[ \tau = 1 \]

2 symbols/s/Hz
FTN Signaling Basic Idea

\[ s(t) = \sqrt{E_s} \sum_{n} a_n g(t - n T) \]

\[ s(t) = \sqrt{E_s \tau} \sum_{n} a_n g(t - n \tau T) \]

\[ \tau = 1 \]

2 symbols/s/Hz

\[ \frac{2}{\tau} \] symbols/s/Hz

\[ \tau = 0.8 \]
Extension of Mazo Limit

- Other pulse shapes (root-raised cosine, Gaussian, ...)
- Non-binary transmission
- Frequency domain
System Model
FTN Block Diagram

 Tx bits → Bits-to-symbols mapping → $a_n$ → Transmit filter → $s(t)$ → Channel → Matched filter → $r(t)$

 Rx bits → Symbols-to-bits Demapping → $\hat{a}_n$ → Equalization → Sampling at $\tau T$
FTN Detection Problem

- \( y = G \alpha + w \)
  - \( y \): received sample vector
  - \( G \): ISI matrix
  - \( \alpha \): transmit data symbols
  - \( w \): received noise samples \((0, \sigma^2 G)\)

- Detection Problem:
  - \( \hat{\alpha} = \arg \min_{\alpha \in D} (G^{-1}y - \alpha)^T G (G^{-1}y - \alpha) \)
Modified Sphere Decoding (MSD)

- Noise covariance matrix can be exploited to develop MSD.
- Estimated data symbols can be found using MSD as

\[
\begin{bmatrix}
z_N - \frac{d}{R_{N,N}} \\
\end{bmatrix} \leq a_N \leq \begin{bmatrix}
z_N + \frac{d}{R_{N,N}} \\
\end{bmatrix}.
\]

\[
a_{N-1} \geq \begin{bmatrix}
z_{N-1} - \frac{\hat{d} - R_{N-1,N}(z_N - a_N)}{R_{N-1,N-1}} \\
\end{bmatrix},
\]

\[
a_{N-1} \leq \begin{bmatrix}
z_{N-1} + \frac{\hat{d} + R_{N-1,N}(z_N - a_N)}{R_{N-1,N-1}} \\
\end{bmatrix},
\]
Simulation Results

Fig. 2: BER performance of binary FTN detection versus $\frac{E_b}{N_0}$ using the standard SD-based and proposed SDSEs at $\beta = 0.3$ and $\tau = 0.6$ and 0.7.
Simulation Results

Fig. 4: Spectral efficiency comparison of binary FTN signaling versus $\beta$ using the proposed SDSE and Nyquist signaling at BER = $10^{-4}$. 
Simulation Results

Roll-off factor = 0.3

BER vs Eb/No for different modulation schemes:
- Nyquist signaling, $2^{16}$-QAM
- FTN $\tau = 1/8$, SD, QPSK

SE = 12.31 bits/s/Hz
Symbol-by-Symbol Detection of FTN Signaling

Ebrahim Bedeer, Mohamed Ahmed, and Halim Yanikomeroglu, “A very low complexity successive symbol-by-symbol sequence estimator for binary faster-than-Nyquist signaling”, accepted in *IEEE Access*, DOI: 10.1109/ACCESS.2017.2663762
Successive Symbol-by-Symbol Sequence Estimation (SSSSE)

- Received sample

\[ y_k = G_{1,L} a_{k-L+1} + \ldots + G_{1,2} a_{k-1} + G_{1,1} a_k + G_{1,2} a_{k+1} + \ldots + G_{1,L} a_{k+L-1} \]

- ISI from previous \( L - 1 \) symbols
- Current symbol to be estimated
- ISI from upcoming \( L - 1 \) symbols
Successive Symbol-by-Symbol Sequence Estimation (SSSSE)

- Received sample

\[ y_k = G_{1,L} a_{k-L+1} + \ldots + G_{1,2} a_{k-1} + G_{1,1} a_k + G_{1,2} a_{k+1} + \ldots + G_{1,L} a_{k+L-1}. \]

- Perfect estimation condition for QPSK FTN signaling

\[ |G_{1,1} \Re\{a_k\}| > |G_{1,2} \Re\{a_{k+1}\} + \ldots + G_{1,L} \Re\{a_{k+L-1}\}|, \]
\[ |G_{1,1} \Im\{a_k\}| > |G_{1,2} \Im\{a_{k+1}\} + \ldots + G_{1,L} \Im\{a_{k+L-1}\}|, \]
Operating region of SSSSE
Successive Symbol-by-Symbol Sequence Estimation (SSSSE)

- Received sample

\[ y_k = G_{1,L} a_{k-L+1} + \cdots + G_{1,2} a_{k-1} + G_{1,1} a_k + G_{1,2} a_{k+1} + \cdots + G_{1,L} a_{k+L-1}. \]

- Perfect estimation condition for QPSK FTN signaling

\[
|G_{1,1} \Re\{a_k\}| > |G_{1,2} \Re\{a_{k+1}\} + \cdots + G_{1,L} \Re\{a_{k+L-1}\}|, \\
|G_{1,1} \Im\{a_k\}| > |G_{1,2} \Im\{a_{k+1}\} + \cdots + G_{1,L} \Im\{a_{k+L-1}\}|, \\
\]

- Estimated symbol

\[ \hat{a}_k = \text{quantize}\{y_k - (G_{1,L} \hat{a}_{k-L+1} + \cdots + G_{1,2} \hat{a}_{k-1})\}, \]
Successive Symbol-by-Symbol with go-back-$K$ Sequence Estimation (SSSgb$K$SE)

- Received sample

\[ y_k = G_{1,L} a_{k-L+1} + \ldots + G_{1,K+1} a_{k-K} + \ldots + G_{1,2} a_{k-1} + G_{1,1} a_k \]

- Estimated symbol

\[ \hat{a}_k = \text{quantize}\left\{ y_k - (G_{1,L} \hat{a}_{k-L+1} + \ldots + G_{1,K+1} \hat{a}_{k-K} + \ldots + G_{1,2} \hat{a}_{k-1}) \right\} \]
Fig. 4: BER performance of QPSK FTN sequence estimation as a function of $\frac{E_b}{N_0}$ using the proposed SSSSE, proposed SSSgbKSE, and FDEs in [11], [13] at $\beta = 0.3$ and SE of 1.71 bits/sec/Hz.

Fig. 6: Spectral efficiency of QPSK Nyquist and FTN signaling as a function of $\beta$ using the proposed SSSgbKSE at $BER = 10^{-4}$. 
Conclusions
Conclusions

- FTN signaling is promising to increase the SE
- Tradeoff between performance and complexity
- Gain of FTN increases at higher values of SE