



# Unified and Non-parameterized Statistical Modeling of Temporal and Spatial Traffic Heterogeneity in Wireless Cellular Networks

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## Motivation:

- Problem definition, contributed solution and novelty

## Contributions and novelties:

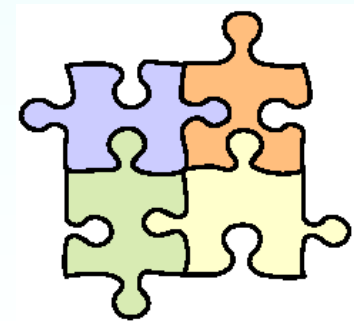
- Modeling and fitting procedure; for generator and measurement aspect
- Traffic is adjustable by just one first, second order, and correlation parameter

## Traffic generation process:

- Unified “Traffic Generator Input Parameters” (TGIPs)
- Umbrella for diverse models of point processes (PP)

## Results and Conclusions:

- Experimental results of traffic generation
- Performance results in cellular networks
- Performance improvement by clever placement of small cells
- Future work



## Problem definition:

- In wireless cellular networks path loss & SINR depend on the spatial distribution of users
- An adjustable and systematic model for a heterogeneous traffic (user) distribution is not available

## Relevant literature:

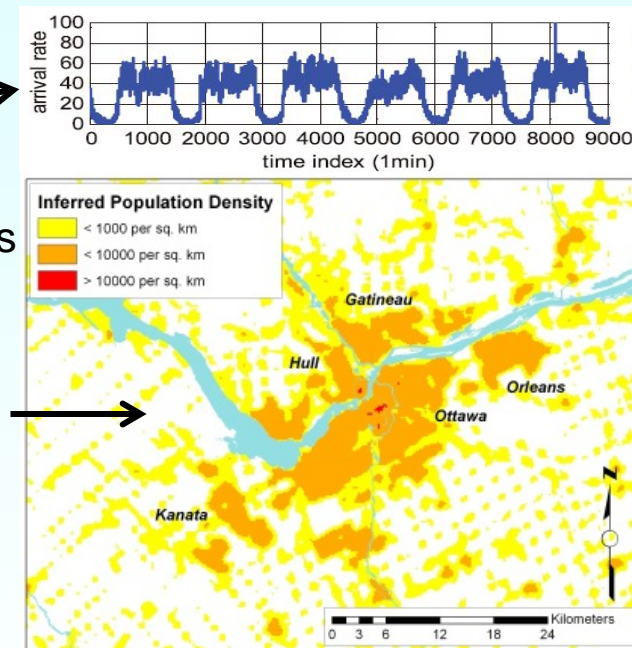
- In time domain traffic modeling has been investigated well
- Stochastic geometry is used for the location of BSs in HetNets

## Solution:

- Use stochastic geometry to model **UT** traffic in space domain
- Include point processes and random tessellations

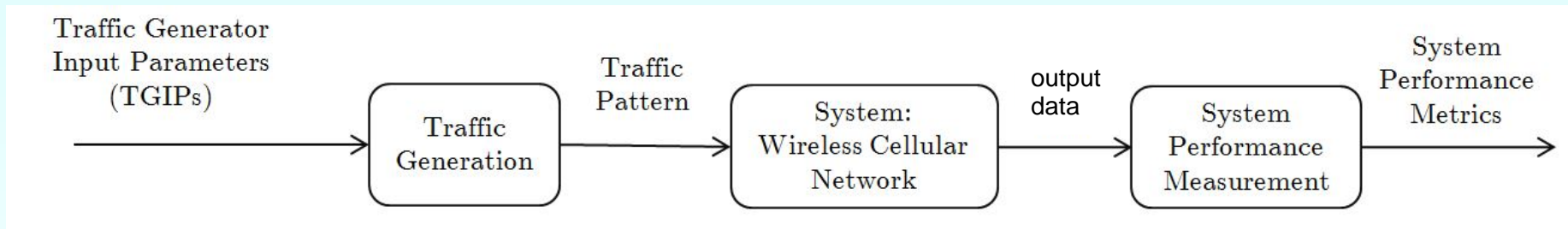
## Novelty:

- Comparison and analogy of traffic modeling in the time domain and space domain
- Introduction of unified and accurate metrics for modeling traffic in both domains
- Simply, **CoV** for adjustable heterogeneity and  $\rho$  for cross-correlation of UTs to BSs



This is the high-level view on what we want to achieve:

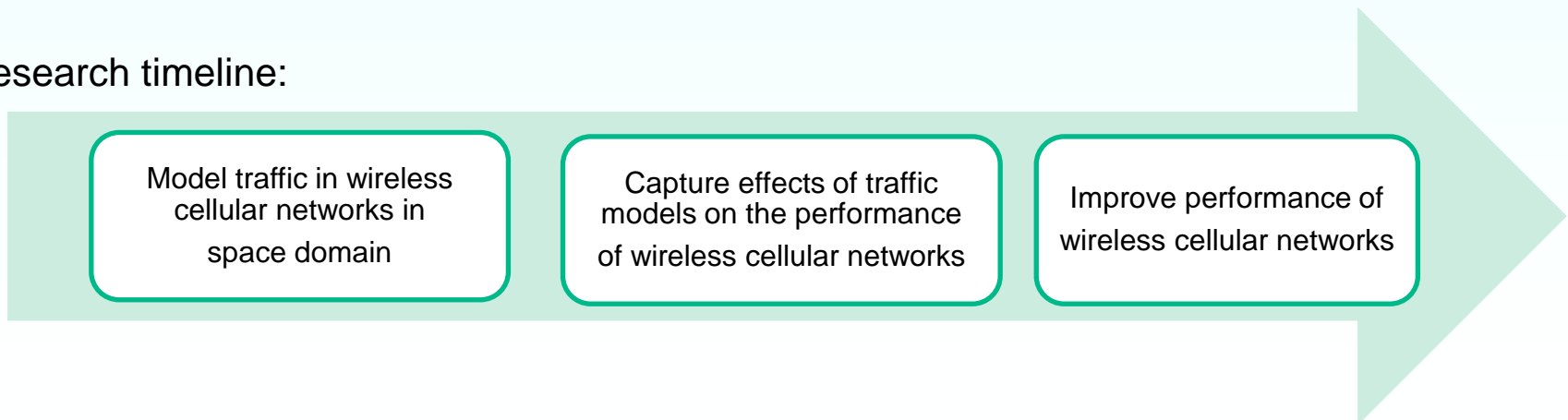
- Traffic in
- Performance out



Goal:

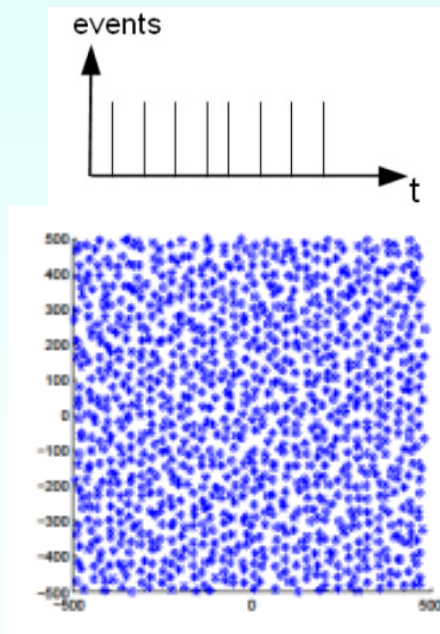
Only one simple yet versatile input parameter for heterogeneity

Research timeline:

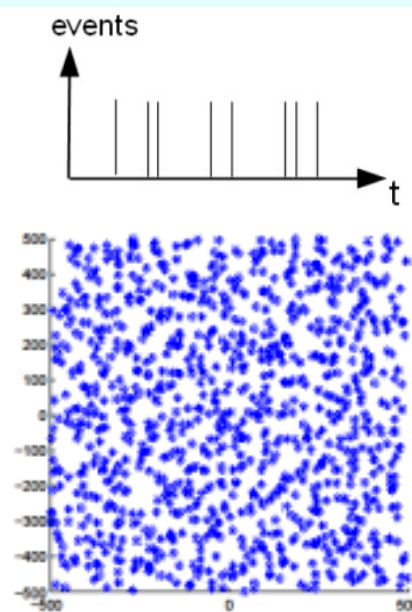




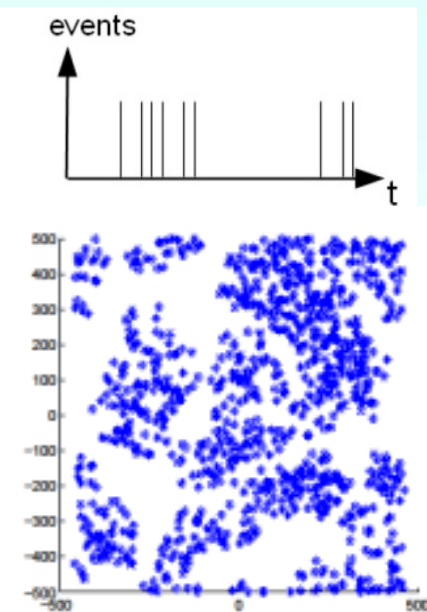
Sub-Poisson



Poisson

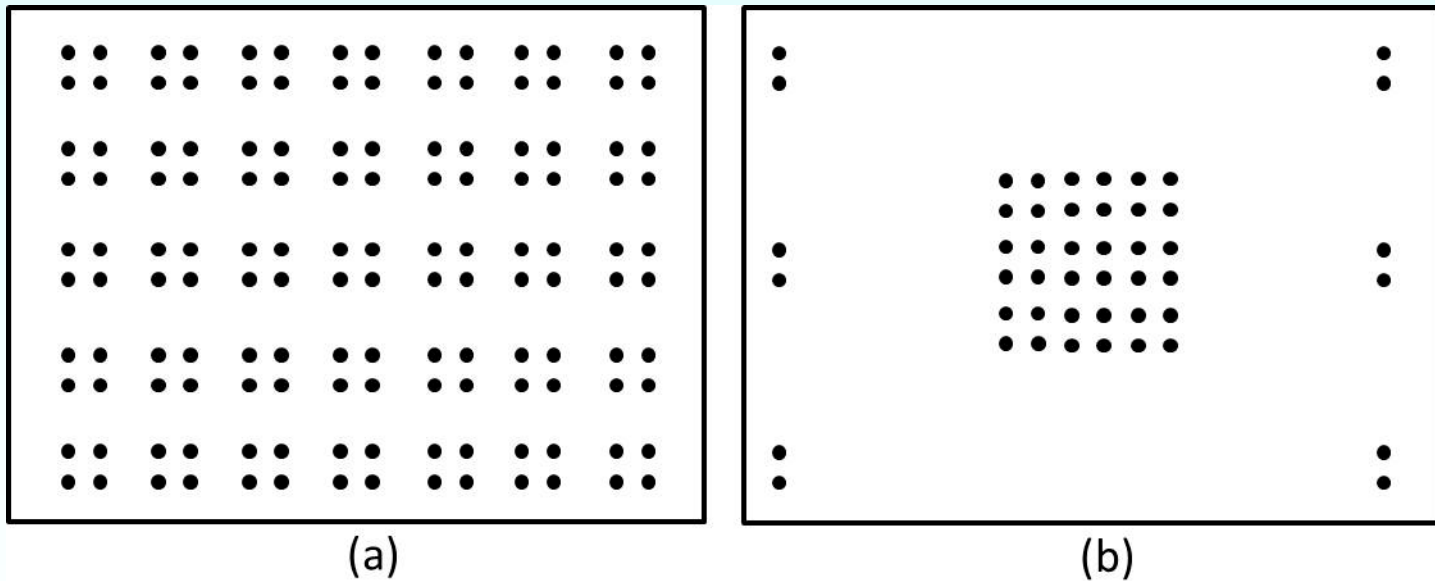


Super-Poisson



# Why some metrics are **unsuitable**

Nearest neighbor distance measure can not capture the heterogeneity of point process







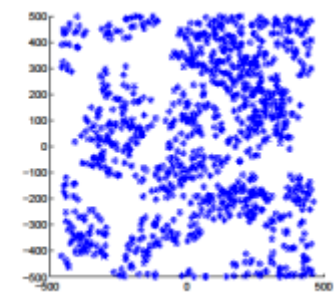
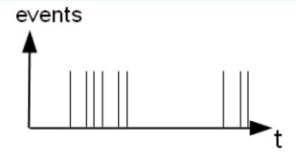
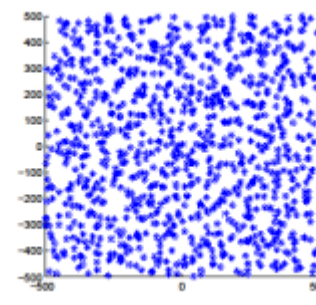
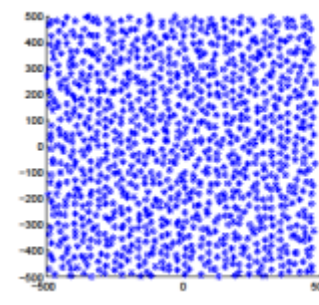
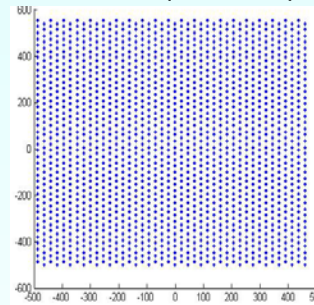
Complete randomness (CoV=1): Poisson →

Sub-Poisson (CoV<1): perturbation → Sub-Poisson (CoV<1)      Poisson (CoV=1)      Super-Poisson (CoV>1)

Maximum homogeneity (CoV=0): Lattice

$$\text{CoV} := \text{std}/\text{mean} = \sigma/\mu$$

Lattice (CoV=0)



Super-Poisson (CoV>1):

- Time domain: Markov-based **hierarchical** processes, e.g. MMPP (top level hierarchy) (number of replicas) (shift in space)

- Space domain: Hierarchical, too

- Clustering perturbation
- Physics inspired: Gravity (Astronomy: Galaxies)

Point Process	Parent Process	Replication Kernel	Displacement Kernel
Binomial Point Process	One-Point	Deterministic	Uniform
Voronoi-Perturbed Lattices	Lattice	General	General in Voronoi Cell
Simple Perturbed Lattice	Lattice	No Replication	Uniform in Voronoi Cell
Gaussian Perturbed Lattice	Lattice	No Replication	Gaussian
Generalized Shot-noise Cox	General	Poisson	General
Poisson-Poisson Cluster PP	Poisson	Poisson	General
Matern Point Process	Poisson	Poisson	Uniform in Ball
Thomas Point Process	Poisson	Poisson	Gaussian in Ball
Neyman-Scott Point Process	Poisson	General	General
Cox	General Levy	Poisson	Uniform
Log-Gaussian Cox	Log-Gaussian Levy	Poisson	Uniform

## Metric:

### Time domain:

- Density based: Interval counts (rates)
- Distance based: Inter-arrival time

### Space Domain:

- Density based: Ripley-k, pair correlation, moments, void prob.
- Distance based: nearest-neighbor

**New Approach:** Use properties of **Voronoi** & **Delaunay**

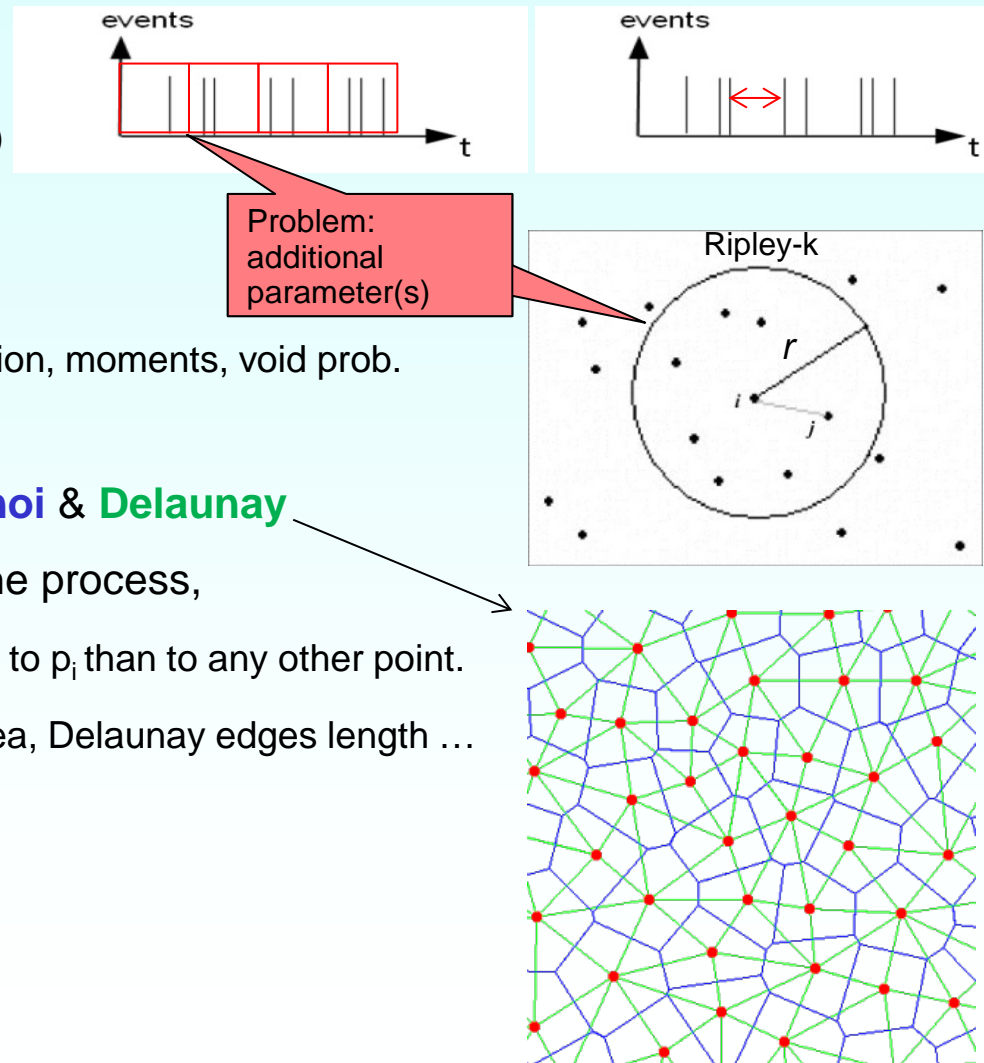
Voronoi cell := for each point  $p_i$  of the process,

the region consisting of all area closer to  $p_i$  than to any other point.

Unified metric can be: Voronoi cell area, Delaunay edges length ...

## Statistical property:

CoV := std/mean ( $C := \sigma/\mu$ )

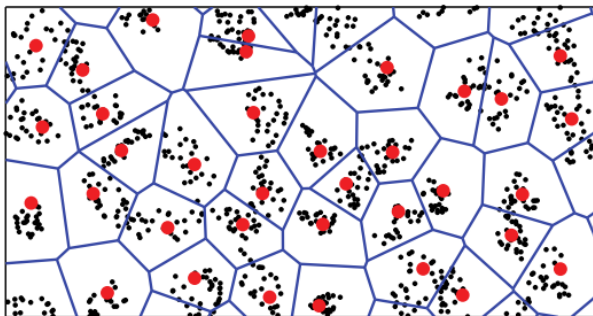
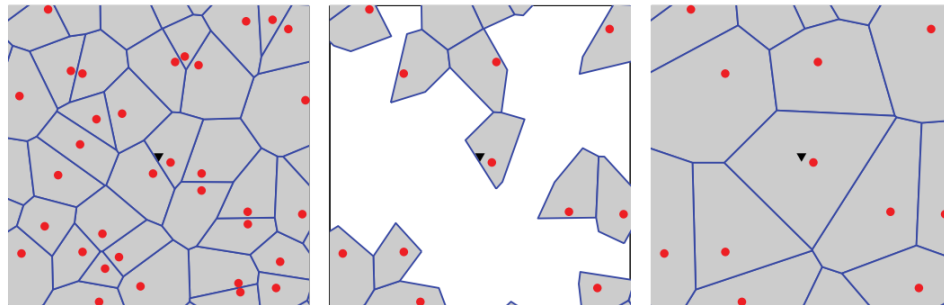
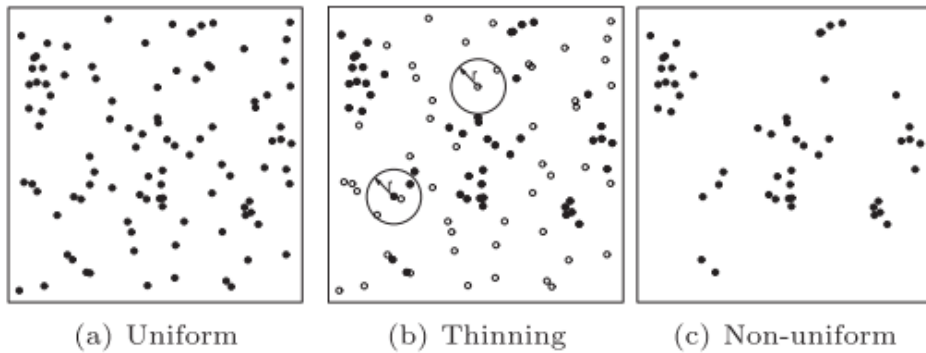




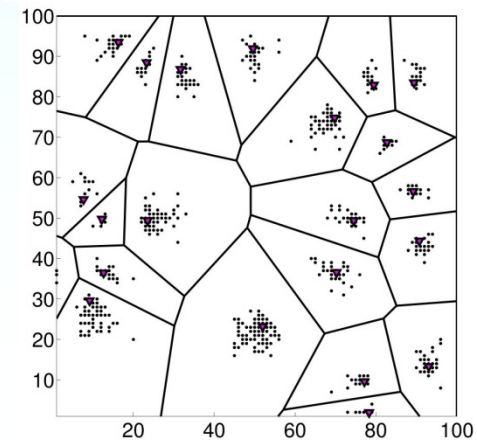
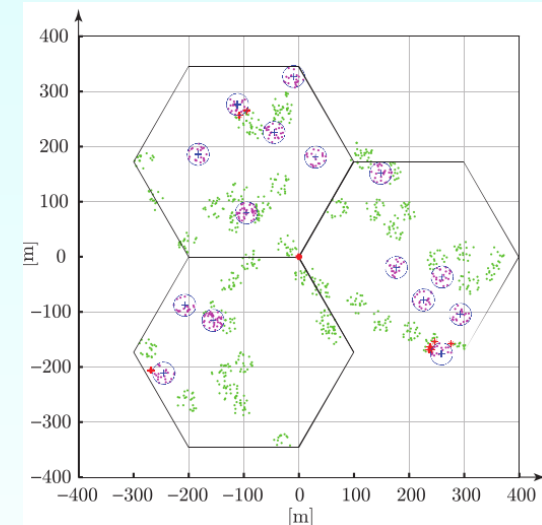
❑ Mostly homogeneously distributed (PPP) or even fixed number and location in a cell

❑ Heterogeneous user distribution examples:

- Thinning on PPP

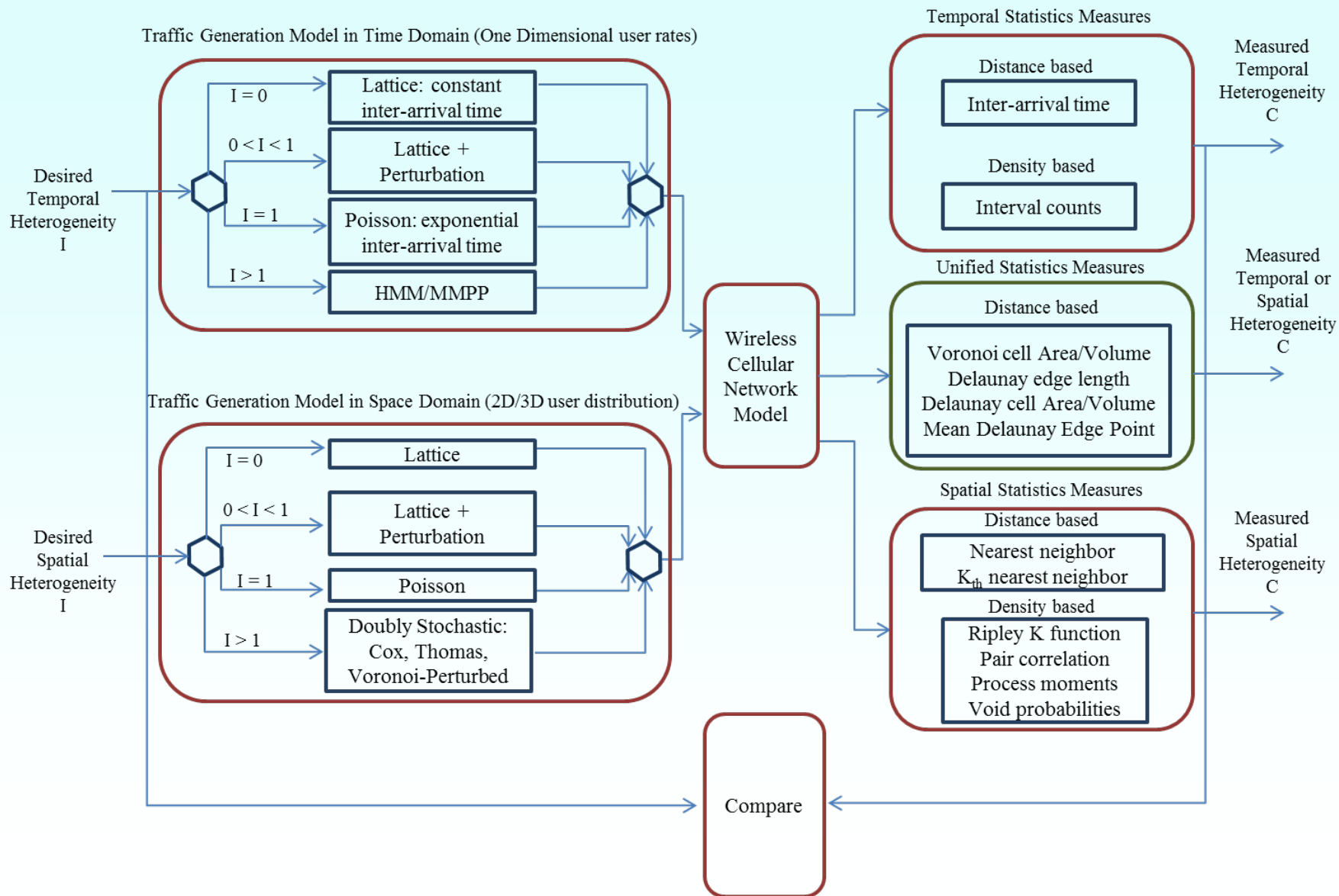


- Poisson Cluster Process





## Detailed Model





## Different PPP Metrics and their properties, 2D versus 3D

Distance based metrics	Analogue in the time domain	Statistics	1D	2D	3D
Nearest-neighbor distance (G)	$\min\{I_i, I_{i+1}\}$	Mean ( $\mu$ )	$0.5 \lambda^{-1}$	$0.5\Lambda^{-0.5}$	$0.5539\Lambda^{-0.33}$
		Variance ( $\sigma^2$ )	$0.25 \lambda^{-2}$	$0.0683\Lambda^{-1}$	$0.04049\Lambda^{-0.66}$
		CoV (C)	1	0.6535	0.364
Voronoi cell area/volume (V)	$\frac{I_i + I_{i+1}}{2}$	Mean ( $\mu$ )	$\lambda^{-1}$	$\Lambda^{-1}$	$\Lambda^{-1}$
		Variance ( $\sigma^2$ )	$\lambda^{-2}$	$0.28\Lambda^{-2}$	$0.18\Lambda^{-2}$
		CoV (C)	1	0.529	0.424
Delaunay cell area/volume (T)	$I_i$	Mean ( $\mu$ )	$\lambda^{-1}$	$0.5\Lambda^{-1}$	$0.147\Lambda^{-0.5}$
		Variance ( $\sigma^2$ )	$\lambda^{-2}$	$0.443\Lambda^{-2}$	$0.015\Lambda^{-1}$
		CoV (C)	1	0.879	0.833
Delaunay cell edge length (E)	$I_i$	Mean ( $\mu$ )	$\lambda^{-1}$	$1.131\Lambda^{-0.5}$	$1.237\Lambda^{-0.33}$
		Variance ( $\sigma^2$ )	$\lambda^{-2}$	$0.31\Lambda^{-1}$	$0.185\Lambda^{-0.66}$
		CoV (C)	1	0.492	0.347

$\Lambda$ : is the mean density of the point process [e.g., points/m<sup>2</sup>, points/m<sup>3</sup>]

$C_x$ : is the CoV of that random process assuming a volume filled with PPP of density  $\Lambda$ .



Measured  
Output Metrics:

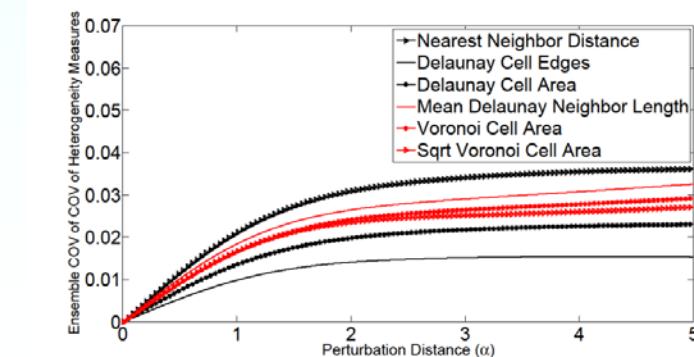
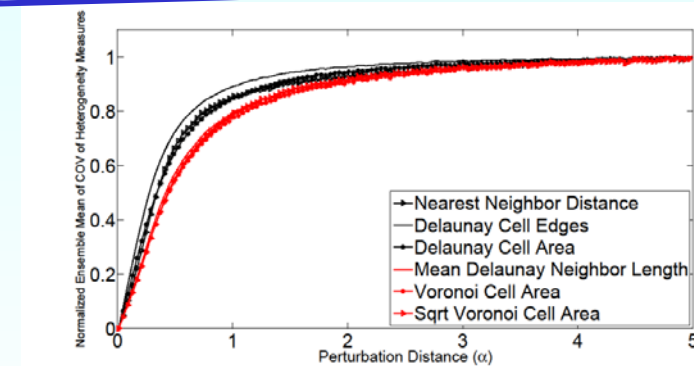
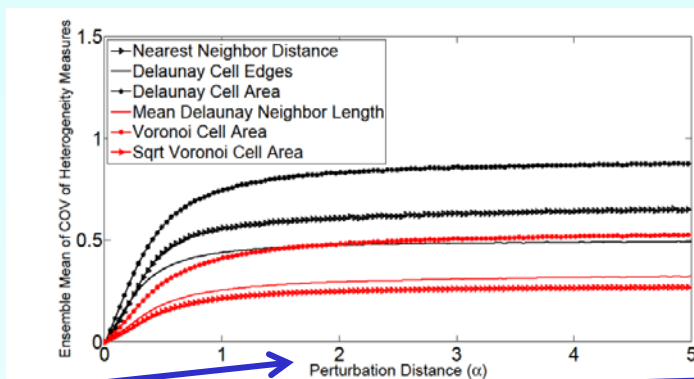
Ensemble Mean  
of CoV

Internal parameters:  
(TGIPs)

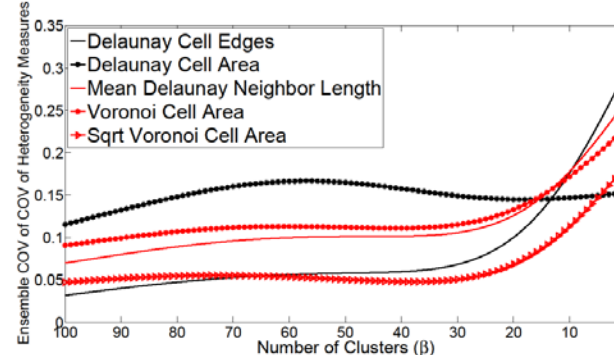
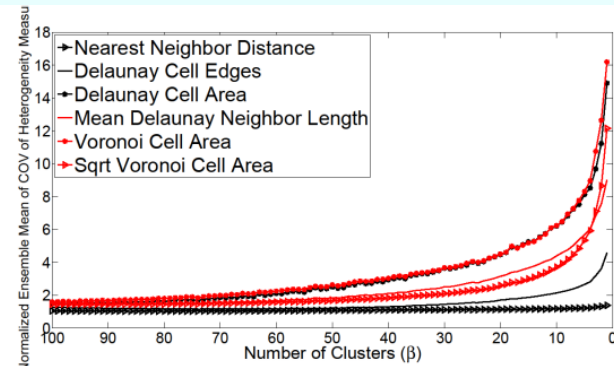
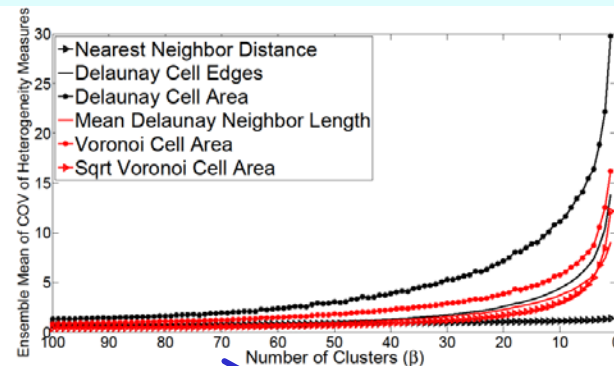
Normalized Mean  
of CoV  
(interval [0..1])

Ensemble CoV  
of CoV  
(means:  
quality, accuracy)

Sub-Poisson (CoV<1)



Super-Poisson (CoV>1)

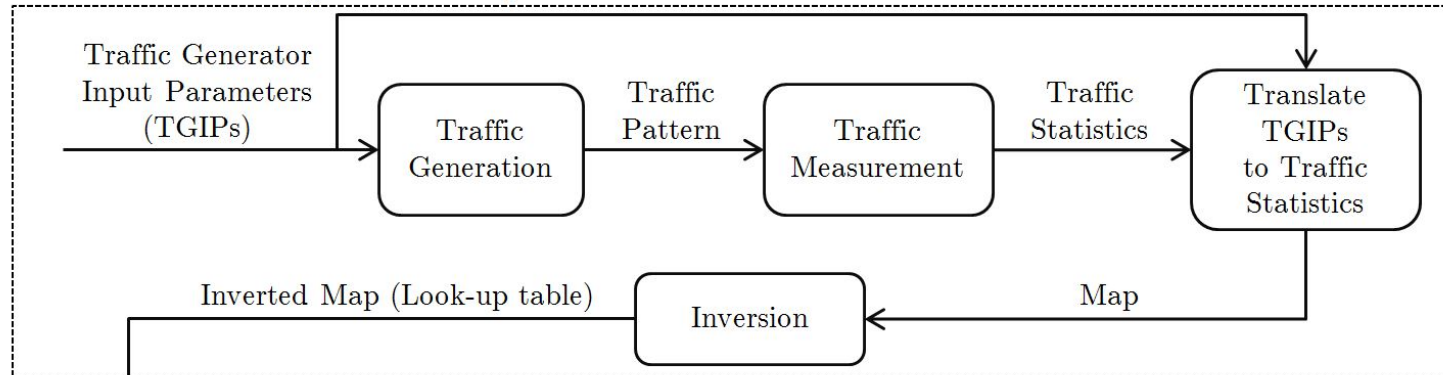


Perturbation distance

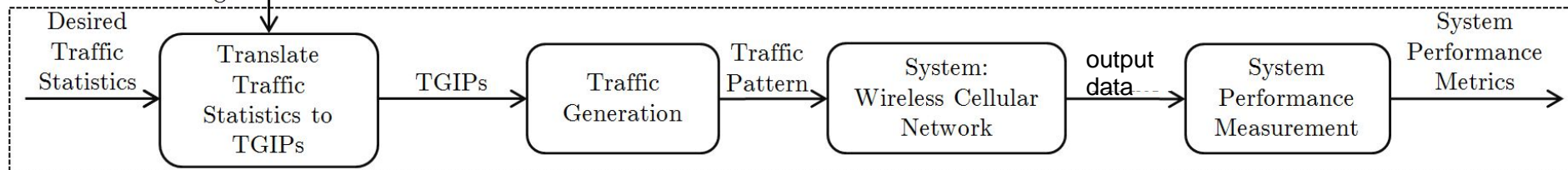
Number of clusters



Off-line calculation to generate look-up table:



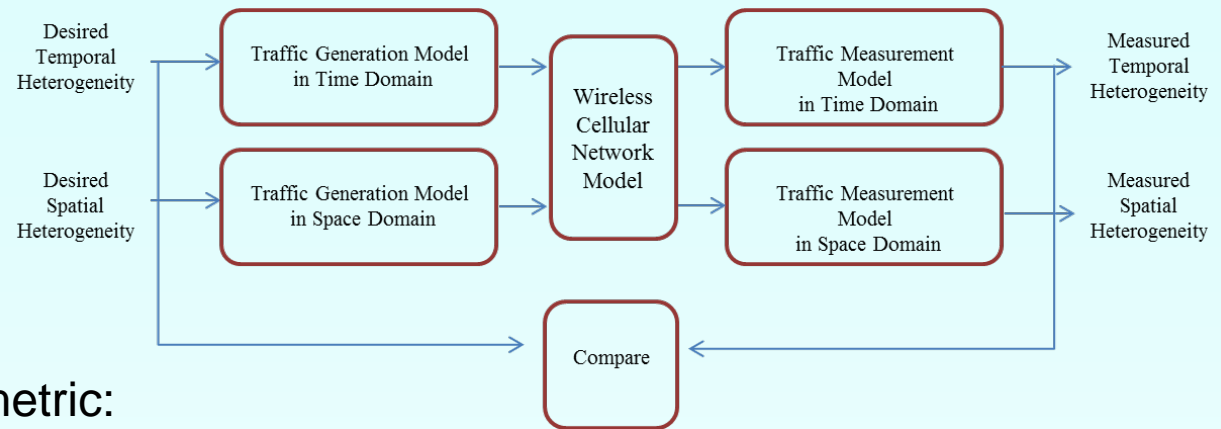
Statistical Modeling:



This is the generic procedure (for time and space domain)

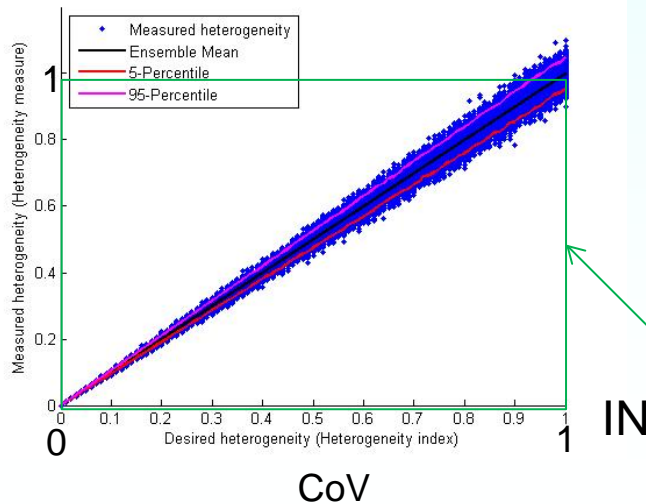
# Simulation Results

(in space domain)

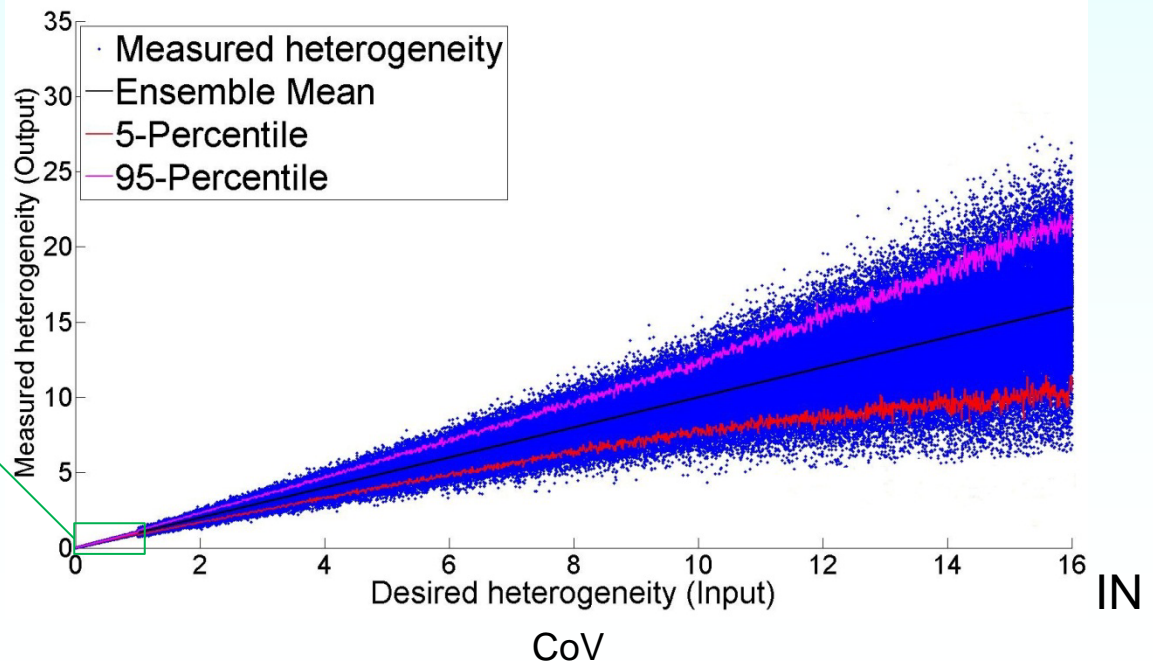


Using Voronoi cell area as metric:

OUT (CoV for sub-Poissonian case)



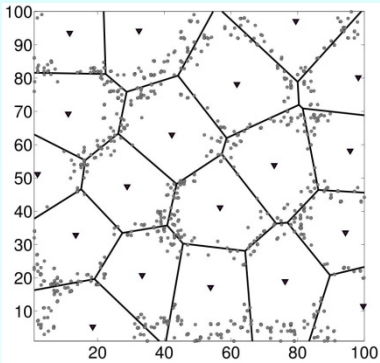
OUT (CoV for super-Poissonian case)



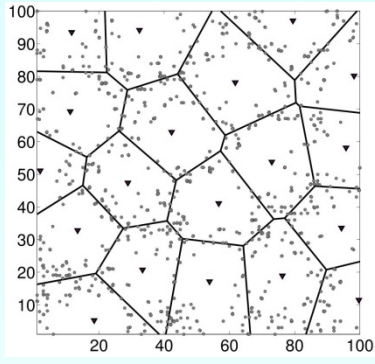




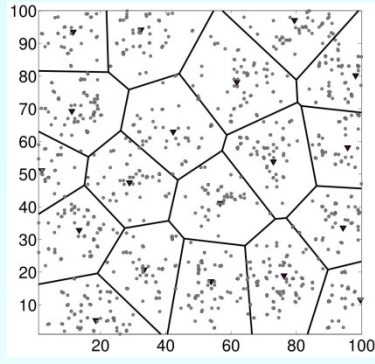
$K=1, b=-0.9$



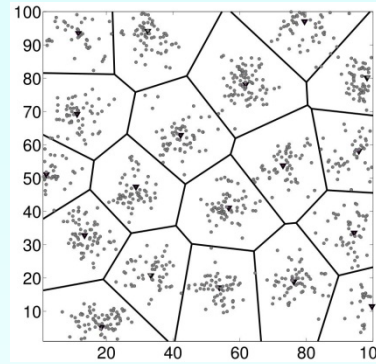
$K=1, b=-0.5$



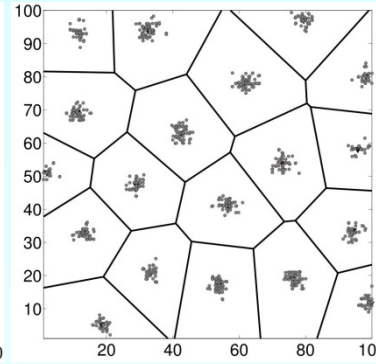
$K=1, b=0$



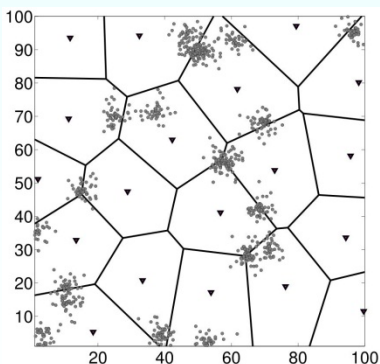
$K=1, b=0.5$



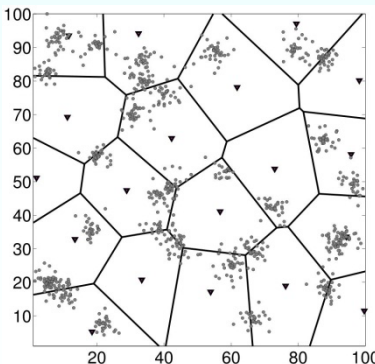
$K=1, b=0.9$



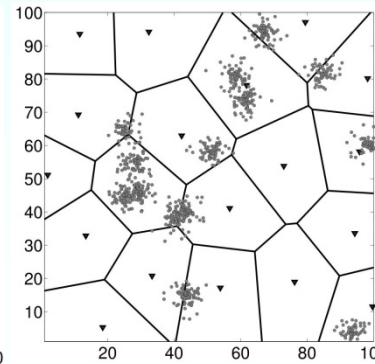
$K=50, b=-0.9$



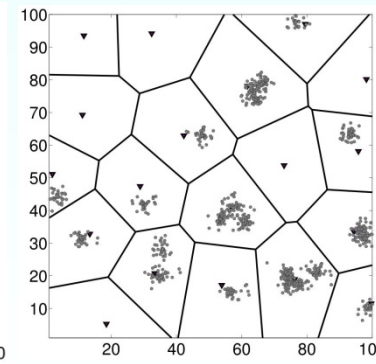
$K=50, b=-0.5$



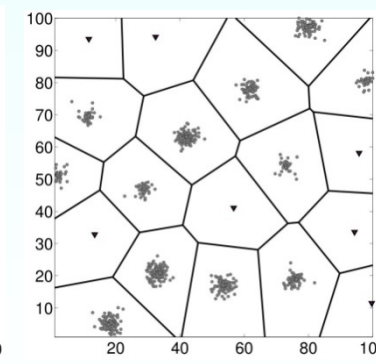
$K=50, b=0$



$K=50, b=0.5$

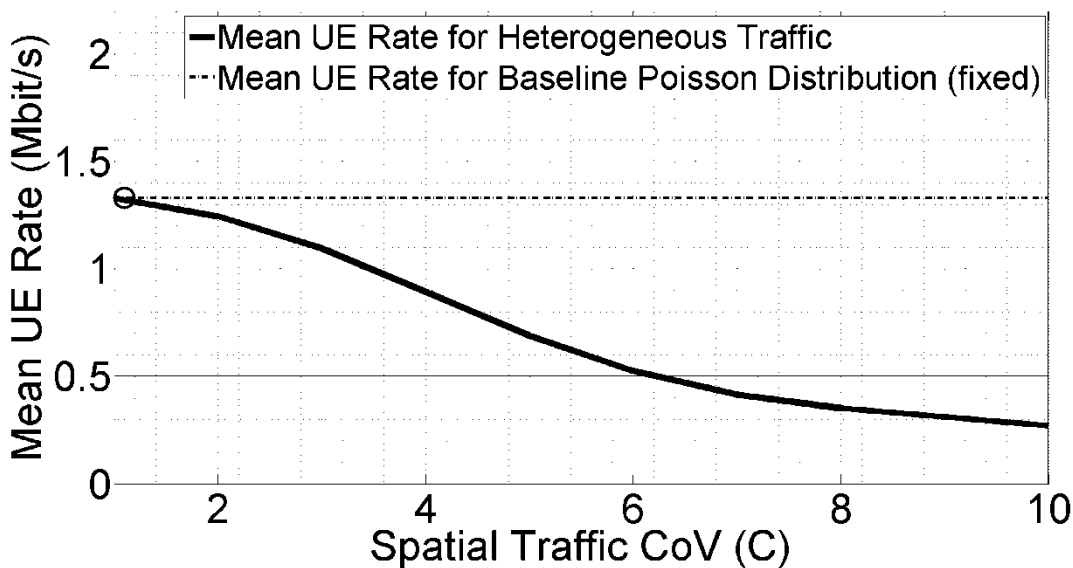


$K=50, b=0.9$



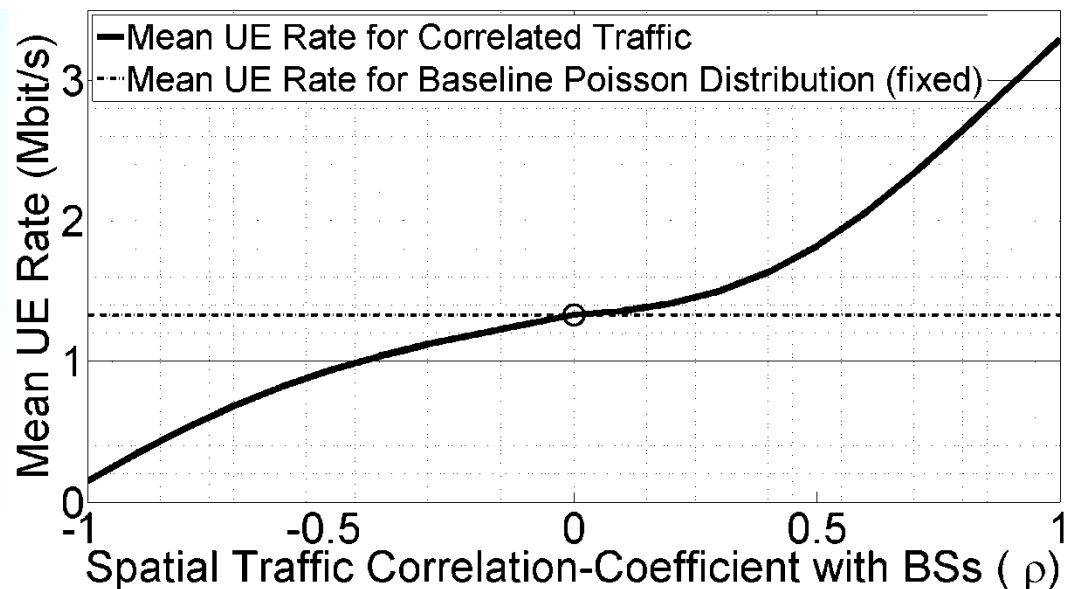
$$K = N_u / N_c$$

Globecom 2014



Result:  
Quantitative performance results on  
how heterogeneity affects  
spectral efficiency

Result:  
Quantitative performance results on  
how UT-BS correlation (affinity)  
affects spectral efficiency



Globecom 2014

We propose:

- accurate and unified traffic measures (instead of 10 methods)
- adjustable continuously from  $\text{CoV}=0, \dots, \infty$  (only one parameter)
- first-order parameter  $\Lambda$  (mean user density) is unchanged
- in space domain and time domain
- simplify traffic measurements (one metric only!)
- enable modeling traffic in combined domain

Study cellular network performance in HetNets

- Traffic generation models:
  - Voronoi-Thomas
  - Weighted Voronoi
  - Correlation between BSs and users
- Combined traffic model in time and space (future work)
- **HetHetNets**
- Intercell Load Coordination (**ICLC**)
- User-In-The-Loop (**UIL**)

IEEE Communications Magazine, Feb 2014  
<http://en.wikipedia.org/wiki/User-in-the-loop>