Selective DF Relaying in Multi-Relay Networks With Different Modulation Levels

Hamza Umit Sokun
Akram Bin Sediq
Halim Yanikomeroglu

Carleton University
Canada
{husokun, akram,halim}@sce.carleton.ca

Salama Ikki

Lakehead University
Canada
sikki@lakeheadu.ca

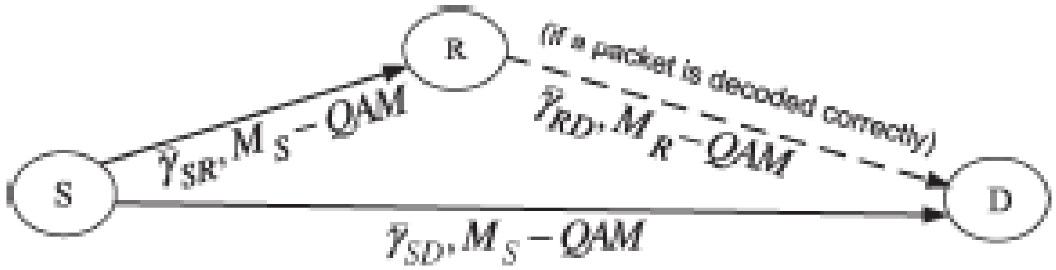
Outline

- Motivation, Background, and Context
- Contributions
- Error Rate Performance Analysis
- Asymptotic Performance Analysis
- Simulation Results
- Summary and Future Work

Motivation

- Common assumption in cooperative relaying literature:
  Same modulation levels by the source and relays
  - Poor spectrally efficiency
- Allow different modulation levels at the relays opportunistically
  - Better spectrally efficiency
- Performance analysis → Protocol design
- Interest in terminal relaying in 3GPP

Background



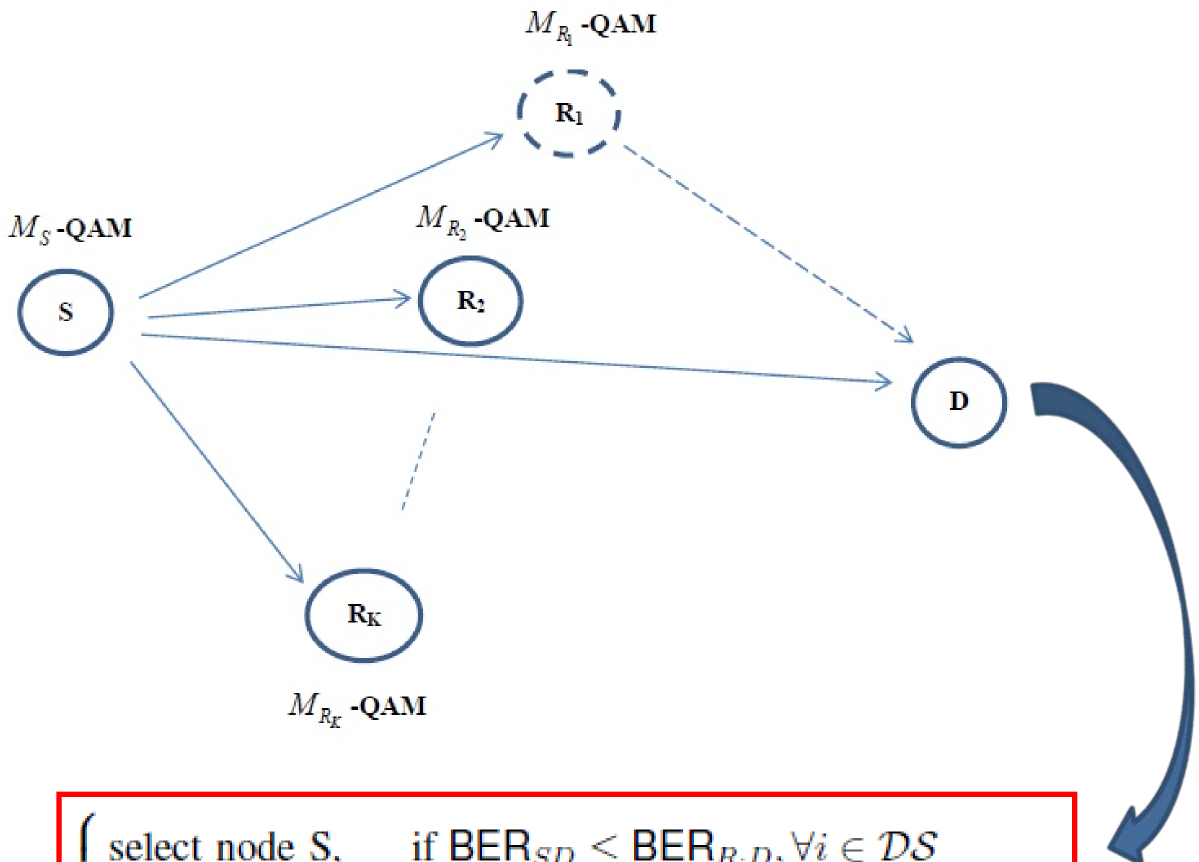
1 relay

BER-based selection is better than SNR-based selection, when the signals at branches have different modulation levels.

A. Bin Sediq and H. Yanikomeroglu, "Performance analysis of selection combining of signals with different modulation levels in cooperative communications," *IEEE Trans. Veh. Technol.*, vol. 60, no. 4, pp. 1880–1887, May 2011.

A. Bin Sediq and H. Yanikomeroglu, "Selection combining of signals with different modulation levels in Nakagami-m fading", *IEEE Commun. Letters*, vol. 16, no. 5, pp. 752-755, May 2012.

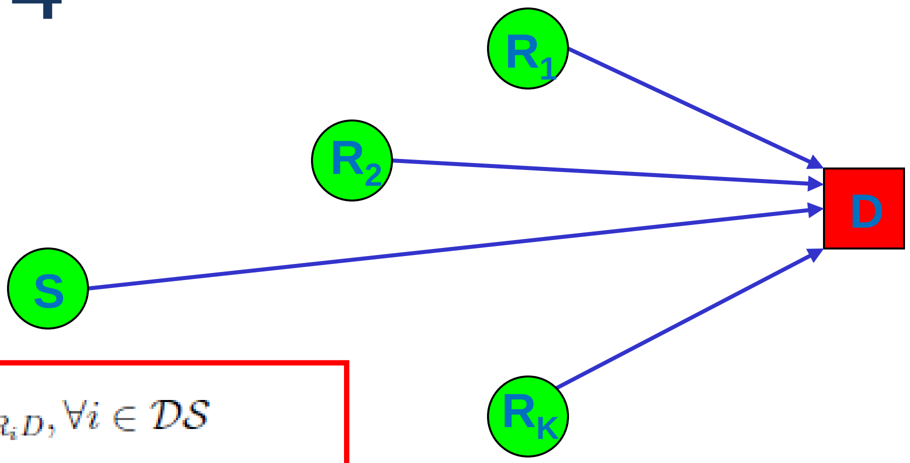
Context



Multiple relays

$$\begin{cases} \text{select node } S, & \text{if } \mathrm{BER}_{SD} \leq \mathrm{BER}_{R_iD}, \forall i \in \mathcal{DS} \\ \text{select node } \mathrm{R}_i, & \begin{array}{l} \text{if } \mathrm{BER}_{SD} > \mathrm{BER}_{R_iD} \text{ and} \\ \mathrm{BER}_{R_jD} > \mathrm{BER}_{R_iD},\ j \neq i, \forall i, j \in \mathcal{DS}. \end{array} \end{cases}$$

Contribution 1/4



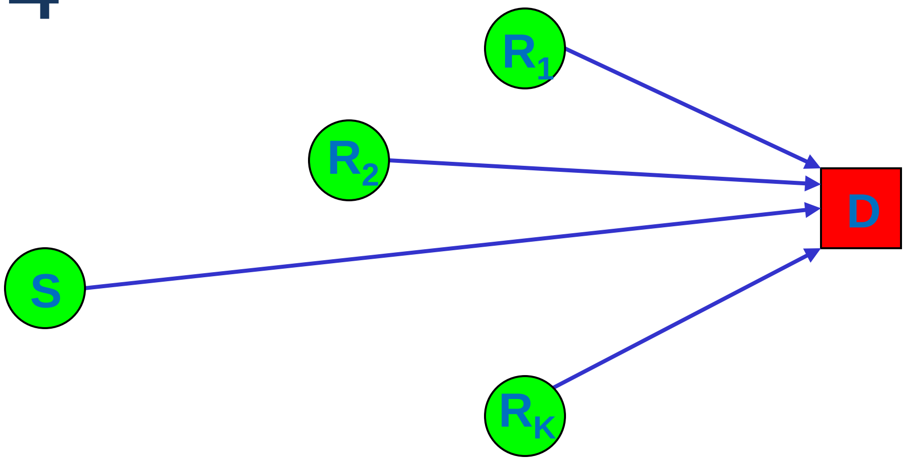
- Find biased SNRs

$$\begin{cases} \text{select node S}, & \text{if } \mathsf{BER}_{SD} \le \mathsf{BER}_{R_iD}, \forall i \in \mathcal{DS} \\ \text{select node } \mathrm{R}_i, & \begin{array}{l} \text{if } \mathsf{BER}_{SD} > \mathsf{BER}_{R_iD} \text{ and} \\ \mathsf{BER}_{R_jD} > \mathsf{BER}_{R_iD},\ j \neq i, \forall i, j \in \mathcal{DS}. \end{array} \end{cases}$$

$$\text{Select node S}, \quad \text{if } \gamma_{SD} = \max\left(\gamma_{SD}, \rho_1\gamma_{R_1D}, \ldots, \rho_K\gamma_{R_KD}\right), \text{ where } \rho_i = \frac{d^2_{M_{R_i}}}{d^2_{M_S}}$$

$$d_{M_{R_i}} = \sqrt{\frac{3}{2(M_{R_i}-1)}}, \quad M_{R_i} \ge 4$$

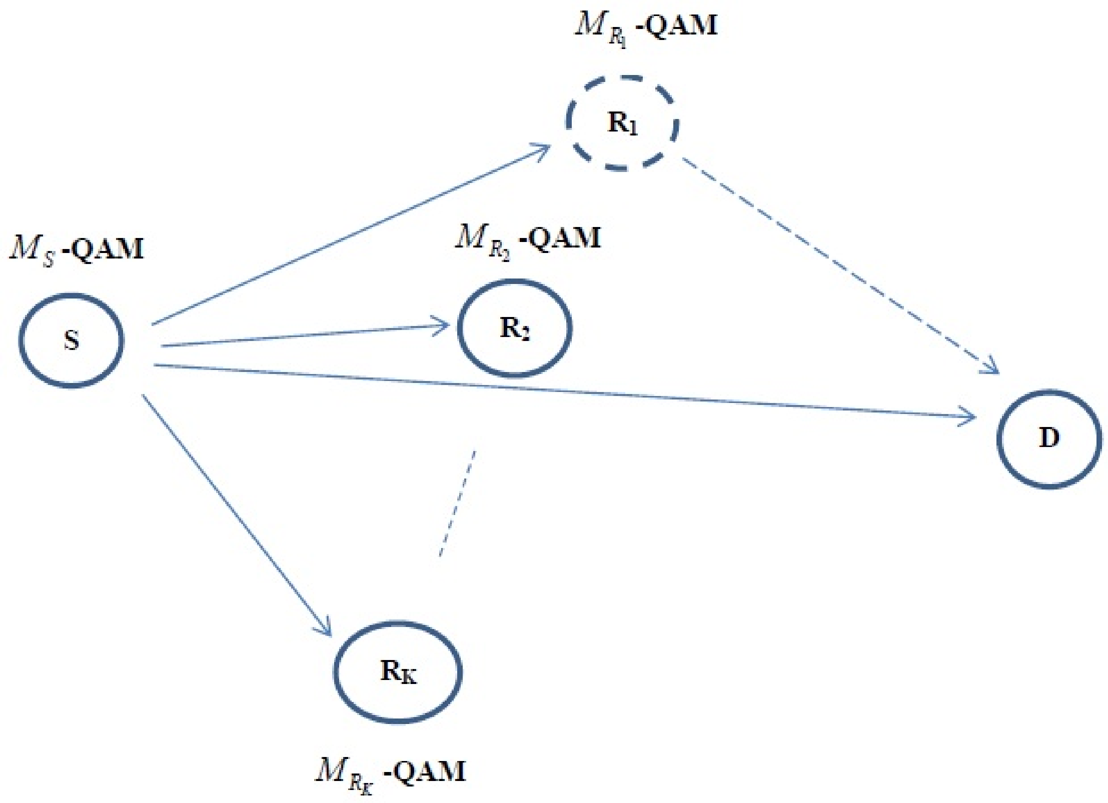
Contribution 2/4



- Find BER for selection combining
- Not straightforward
- Relevance to
  - CoMP
  - HARQ
  - Relay

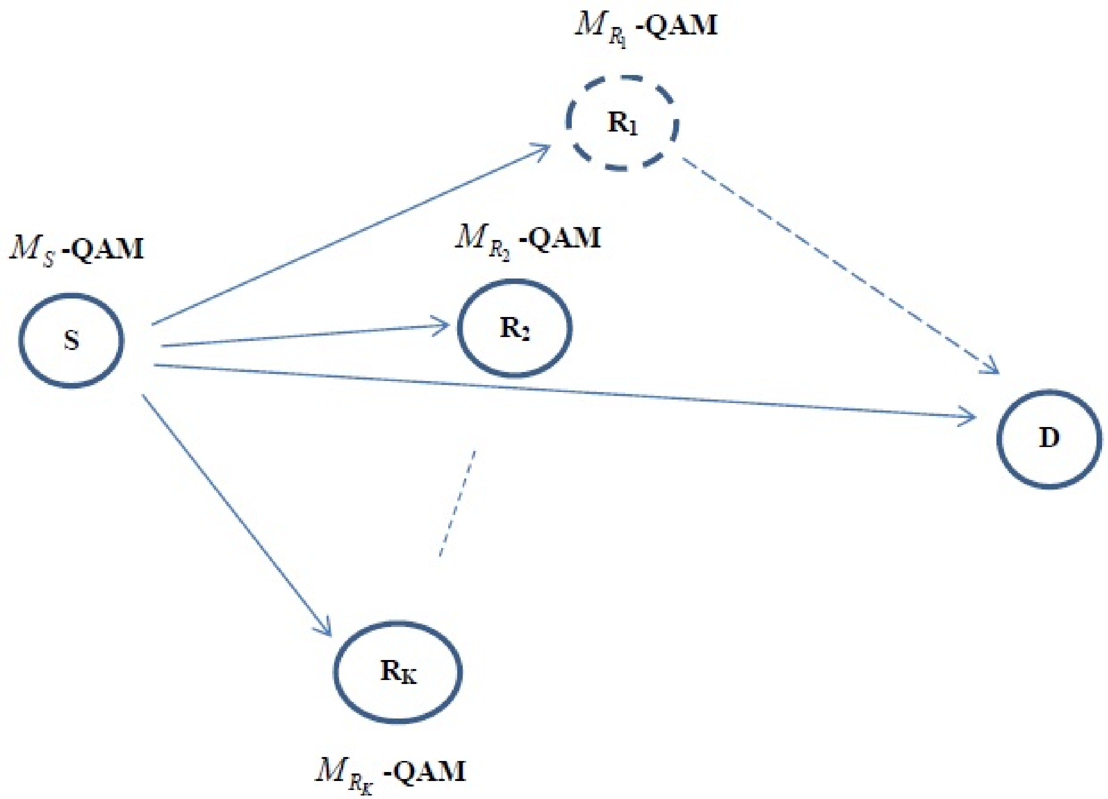
Contribution 3/4

- Find E2E BER in a network with selection combining



Contribution 4/4

- Find asymptotic E2E BER in a network with selection combining



Preliminaries

Point-to-Point AWGN BER

$$BER_{M_i}\left(\gamma_{ij}\right) \approx c_{M_i} Q\left(\sqrt{2d_{M_i}^2 \gamma_{ij}}\right),$$

$$\text{where } \left(c_{M_i}, d_{M_i}\right) = \begin{cases} (1,1), & M_i = 2, \\ \left(\dfrac{2-2/\sqrt{M_i}}{\log_2 \sqrt{M_i}}, \sqrt{\dfrac{3}{2\left(M_i - 1\right)}}\right), & M_i \geq 4, \end{cases}$$

Point-to-Point Rayleigh BER

$$BER_{ij} \approx \frac{1}{2} c_{M_i} \left(1 - \sqrt{\frac{d_{M_i}^2 \overline{\gamma}_{ij}}{1 + d_{M_i}^2 \overline{\gamma}_{ij}}}\right)$$

Average Packet Error Rate

$$PER_{SR_i} = 1 - (1 - SER_{SR_i})^{\frac{N}{\log_2 M_s}}$$

$$\approx 1 - \left(1 - \frac{1}{2} c_{M_s} \log_2\left(M_s\right) \left(1 - \sqrt{\frac{d_{M_s}^2 \overline{\gamma}_{SR_i}}{1 + d_{M_s}^2 \overline{\gamma}_{SR_i}}}\right)\right)^{\frac{N}{\log_2 M_s}}$$

where $SER \approx BER \log_2 M_s$ for Gray-coded constellations

Error Rate Performance (1/3)

End-to-End Average BER

$$BER = \left(\prod_{k=1}^{K} PER_{SR_k} \right) BER_{SD} + \sum_{r=1}^{K} \sum_{m=1}^{|P_r(S_{all})|} \left(\prod_{e_i \in P_{r,m}(S_{all})} \left(1 - PER_{SR_{e_i}}\right) \right) \left(\prod_{e_o \notin P_{r,m}(S_{all})} PER_{SR_{e_o}} \right) BER_{comp_{P_{r,m}(S_{all})}}$$

For example, for a two-relay scenario, it is given as

$$BER = PER_{SR_1} PER_{SR_2} BER_{SD} + (1 - PER_{SR_1}) PER_{SR_2} BER_{comp_{\{1\}}} + (1 - PER_{SR_2}) PER_{SR_1} BER_{comp_{\{2\}}} + (1 - PER_{SR_2})(1 - PER_{SR_1}) BER_{comp_{\{1,2\}}}$$

- $|P_r(S_{all})|$ represents the cardinality of S_{all} ,
- $P_r(S_{all})$ is r-th element power set of , i.e.,S_{all} ,
- $P_{r,m}(S_{all})$ is m-th element of $P_r(S_{all})$,
- S_{all} is the set of all relays' indexes, i.e., $S_{all} = \{1, \ldots, K\}$,
- PER_{SR_i} is the average packet error ratio in link $S - R_i$,
- BER_{SD} is average BER in link $S - D$,
- $BER_{comp_{DS}}$ is average BER conditioned on the decoding set at destination terminal after selection combining.

Error Rate Performance (2/3)

End-to-End Average BER Conditioned on the Decoding Set

$$BER_{comp,inst} \approx \begin{cases} c_{M_S} Q(\sqrt{2d_{M_S}^2 \gamma_{SD}}) & \text{if } \gamma_{SD} \geq \rho_i \gamma_{R_iD}, \quad \mathrm{i}=1,2,\ldots,K \\ c_{M_{R_i}} Q(\sqrt{2d_{M_{R_i}}^2 \gamma_{RD}}) & \text{if } \gamma_{SD} < \rho_i \gamma_{R_iD} \quad \text{and } \gamma_{R_jD} < \beta_{ij}\gamma_{R_iD} \quad j \neq i, \quad j=1,2,\ldots,K, \quad \text{for } \mathrm{i}=1,2,\ldots,K \end{cases}$$

where $\rho_i = \frac{d_{M_{R_i}}^2}{d_{M_S}^2}$ and $\beta_{ij} = \frac{d_{M_{R_i}}^2}{d_{M_{R_j}}^2}$ are biasing factors.

An approximate and simpler implementation of the instantaneous BER.

$$BER_{comp_{\{\text{decoding set}\}}} = BER_{\gamma_{SD} \geq \rho_1\gamma_{R_1D},\ldots,\rho_K\gamma_{R_KD}} + \sum_{i=1}^{K} BER_{\gamma_{SD}<\rho_i\gamma_{R_iD} \text{ and } \gamma_{R_jD}<\beta_{ij}\gamma_{R_iD} \;, j\neq i,\, j=1,2,\ldots,K}$$

1 $$BER_{\gamma_{SD}\geq\rho_i\gamma_{R_iD}} = \int_{\gamma_{SD}=0}^{\infty}\int_{\gamma_{R_1D}=0}^{\rho_1^{-1}\gamma_{SD}} \cdots \int_{\gamma_{R_KD}=0}^{\rho_K^{-1}\gamma_{SD}} c_{M_s} Q\left(\sqrt{2d_{M_s}^2\gamma_{SD}}\right)\left[\frac{1}{\overline{\gamma}_{SD}} e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}}\right]\left[\prod_{i=1}^{K}\frac{1}{\overline{\gamma}_{R_iD}} e^{-\frac{\gamma_{R_iD}}{\overline{\gamma}_{R_iD}}}\right] d_{\gamma_{SD}}\, d_{\gamma_{R_1D}} \ldots d_{\gamma_{R_KD}}$$

2 $$BER_{\gamma_{SD}<\rho_i\gamma_{R_iD} \text{ and } \gamma_{R_jD}<\beta_{ij}\gamma_{R_iD} \;, j\neq i,\, j=1,2,\ldots,K}$$

$$= \int_{\gamma_{R_iD}=0}^{\infty}\int_{\gamma_{SD}=0}^{\rho_i\gamma_{R_iD}}\int_{\gamma_{R_1D}=0}^{\beta_{i1}\gamma_{R_iD}} \cdots \int_{\gamma_{R_KD}=0}^{\beta_{iK}\gamma_{R_iD}} c_{M_{R_i}} Q\left(\sqrt{2d_{M_{R_i}}^2\gamma_{R_iD}}\right)\left[\frac{1}{\overline{\gamma}_{R_iD}} e^{-\frac{\gamma_{R_iD}}{\overline{\gamma}_{R_iD}}}\right]\left[\frac{1}{\overline{\gamma}_{SD}} e^{-\frac{\gamma_{SD}}{\overline{\gamma}_{SD}}}\right]\left[\prod_{j=1}^{K-1}\frac{1}{\overline{\gamma}_{R_jD}} e^{-\frac{\gamma_{RjD}}{\overline{\gamma}_{R_jD}}}\right] d_{\gamma_{R_iD}}\, d_{\gamma_{SD}}\, d_{\gamma_{R_1D}} \ldots d_{\gamma_{R_KD}}$$

Error Rate Performance (3/3)

End-to-End Average BER

$$
\begin{aligned}
BER = {} & \left(\prod_{k=1}^{K} \left[1 - \left(1 - \frac{1}{2} c_{M_s} \log_2\left(M_s\right) \left(1 - \sqrt{\frac{d_{M_s}^2 \bar{\gamma}_{SR_k}}{1 + d_{M_s}^2 \bar{\gamma}_{SR_k}}} \right) \right)^{\frac{N}{\log_2 M_s}} \right] \right) \left(\frac{1}{2} c_{M_S} \left(1 - \sqrt{\frac{d_{M_S}^2 \bar{\gamma}_{SD}}{1 + d_{M_S}^2 \bar{\gamma}_{SD}}} \right) \right) \\
& + \sum_{r=1}^{K} \sum_{m=1}^{\left|P_r\left(S_{all}\right)\right|} \left[\prod_{e_i \in P_{r,m}\left(S_{all}\right)} \left(\left(1 - \frac{1}{2} c_{M_s} \log_2\left(M_s\right) \left(1 - \sqrt{\frac{d_{M_s}^2 \bar{\gamma}_{SR_{e_i}}}{1 + d_{M_s}^2 \bar{\gamma}_{SR_{e_i}}}} \right) \right) \right)^{\frac{N}{\log_2 M_s}} \times \prod_{e_o \notin P_{r,m}\left(S_{all}\right)} \left(1 - \left(1 - \frac{1}{2} c_{M_s} \log_2\left(M_s\right) \left(1 - \sqrt{\frac{d_{M_s}^2 \bar{\gamma}_{SR_{e_o}}}{1 + d_{M_s}^2 \bar{\gamma}_{SR_{e_o}}}} \right) \right) \right)^{\frac{N}{\log_2 M_s}} \right. \\
& \times \left[I\left(\infty, c_{M_s}, d_{M_s}^2, \bar{\gamma}_{SD}\right) + \sum_{k=1}^{\left|P_{r,m}\left(S_{all}\right)\right|} \sum_{y=1}^{\binom{\left|P_{r,m}\left(S_{all}\right)\right|}{k}} (-1)^k I\left(\infty, \frac{c_{M_s}}{\bar{\gamma}_{SD}}, d_{M_s}^2, \frac{k+1}{HM\left\{\bar{\gamma}_{SD}, P_{k,y}\left\{S\right\}\right\}} \right) \left(\frac{k+1}{HM\left\{\bar{\gamma}_{SD}, P_{k,y}\left\{S\right\}\right\}} \right) \right. \\
& \left. \left. + \sum_{i=1}^{\left|P_{r,m}\left(S_{all}\right)\right|} \left[\mathrm{I}\left(\infty, c_{M_{R_i}}, d_{M_{R_i}}^2, \bar{\gamma}_{R_iD}\right) + \sum_{k=1}^{\left|P_{r,m}\left(S_{all}\right)\right|} \sum_{y=1}^{\binom{\left|P_{r,m}\left(S_{all}\right)\right|}{k}} (-1)^k I\left(\infty, \frac{c_{M_{R_i}}}{\bar{\gamma}_{R_iD}}, d_{M_{R_i}}^2, \frac{k+1}{HM\left\{\bar{\gamma}_{R_iD}, P_{k,y}\left\{S_x\right\}\right\}} \right) \left(\frac{k+1}{HM\left\{\bar{\gamma}_{R_iD}, P_{k,y}\left\{S_x\right\}\right\}} \right) \right] \right] \right]
\end{aligned}
$$

Asymptotic Performance (1/2)

$$BER = \left(\prod_{k=1}^{K} PER_{SR_k}\right) BER_{SD} + \sum_{r=1}^{K}\sum_{m=1}^{|P_r(S_{all})|}\left(\prod_{e_i \in P_{r,m}(S_{all})}\left(1-PER_{SR_{e_i}}\right)\right)\left(\prod_{e_o \notin P_{r,m}(S_{all})} PER_{SR_{e_o}}\right) BER_{comp_{P_{r,m}(S_{all})}}$$

1 $$PER_{SR_i} \overset{SNR\to\infty}{\approx} \frac{Nc_{M_S}}{4d_{M_S}^2 \sigma_{SR_i}^2 SNR},$$

2 $$1-PER_{SR_i} \overset{SNR\to\infty}{\approx} 1-\frac{Nc_{M_S}}{4d_{M_S}^2 \sigma_{SR_i}^2 SNR} \overset{SNR\to\infty}{\approx} 1,$$

3 $$BER_{SD} \overset{SNR\to\infty}{\approx} \frac{c_{M_S}}{4d_{M_S}^2 \sigma_{SD}^2 SNR}$$

4 $$BER_{comp_{\{\text{decoding set}\}}} = BER_{\gamma_{SD}\ge \rho_1\gamma_{R_1D},\ldots,\rho_K\gamma_{R_KD}} + \sum_{i=1}^{K} BER_{\gamma_{SD}<\rho_i\gamma_{R_iD} \text{ and } \gamma_{R_jD}<\beta_{ij}\gamma_{R_iD}\ ,\ j\ne i,\ j=1,2,\ldots,K}$$

$$BER_{\gamma_{SD}\ge\rho_i\gamma_{R_iD},\ i=1,2,\ldots,K} \overset{SNR\to\infty}{=} \int_{\gamma_{SD}=0}^{\infty}\int_{\gamma_{R_1D}=0}^{\rho_1^{-1}\gamma_{SD}}\cdots\int_{\gamma_{R_2D}=0}^{\rho_K^{-1}\gamma_{SD}} c_{M_s} Q\left(\sqrt{2d_{M_s}^2\gamma_{SD}}\right)\frac{1}{\overline{\gamma}_{SD}}\left[\prod_{i=1}^{K}\frac{1}{\overline{\gamma}_{R_iD}}\right] d_{\gamma_{SD}}\, d_{\gamma_{R_1D}}\ldots d_{\gamma_{R_KD}}$$

$$= \left[\prod_{i=1}^{K}\frac{\rho_i^{-1}}{\overline{\gamma}_{R_iD}}\right]\frac{c_{M_s}\Gamma(K+1.5)}{2\sqrt{\pi}\,\overline{\gamma}_{SD}(1+K)\left(d_{M_s}^2\right)^{K+1}}$$

$$BER_{\gamma_{SD}<\rho_i\gamma_{R_iD} \text{ and } \gamma_{R_jD}<\beta_{ij}\gamma_{R_iD}\ ,\ j\ne i,\ j=1,2,\ldots,K} \overset{SNR\to\infty}{=} \sum_{i=1}^{K}\left[\prod_{\substack{j=1\\ j\ne i}}^{K}\frac{\beta_{ij}}{\overline{\gamma}_{R_jD}}\right]\frac{\rho_i c_{M_i}\Gamma(K+1.5)}{2\sqrt{\pi}\,\overline{\gamma}_{SD}\overline{\gamma}_{R_iD}(1+K)\left(d_{M_i}^2\right)^{K+1}}$$

Asymptotic Performance (2/2)

Asymptotic BER

$$BER \overset{SNR\to\infty}{=} \left(\prod_{k=1}^{K} \frac{Nc_{M_S}}{4d_{M_S}^2 \sigma_{SR_k}^2}\right) \frac{c_{M_S}}{4d_{M_S}^2 \sigma_{SD}^2} \frac{1}{SNR^{K+1}}$$

$$+\sum_{r=1}^{K} \sum_{m=1}^{|P_r(S_{all})|} \left(\prod_{e_o \notin P_{r,m}(S_{all})} \frac{Nc_{M_S}}{4d_{M_S}^2 \sigma_{SR_{e_o}}^2 SNR}\right)$$

$$\times\left[\left[\prod_{i=1}^{|P_{r,m}(S_{all})|} \frac{\rho_i^{-1}}{\sigma_{R_iD}^2}\right] \frac{c_{M_s}\Gamma(|P_{r,m}(S_{all})|+1.5)}{2\sqrt{\pi}\sigma_{SD}^2(1+|P_{r,m}(S_{all})|)\left(d_{M_s}^2\right)^{|P_{r,m}(S_{all})|+1}} \frac{1}{SNR^{|P_{r,m}(S_{all})|+1}}\right.$$

$$\left.+\sum_{i=1}^{|P_{r,m}(S_{all})|}\left[\prod_{\substack{j=1\\ j\neq i}}^{|P_{r,m}(S_{all})|} \frac{\beta_{ij}}{\sigma_{R_jD}^2}\right] \frac{\rho_i c_{M_i}\Gamma(|P_{r,m}(S_{all})|+1.5)}{2\sqrt{\pi}\sigma_{SD}^2\sigma_{R_iD}^2(1+|P_{r,m}(S_{all})|)\left(d_{M_i}^2\right)^{|P_{r,m}(S_{all})|+1}} \frac{1}{SNR^{|P_{r,m}(S_{all})|+1}}\right]$$

Simulation Results

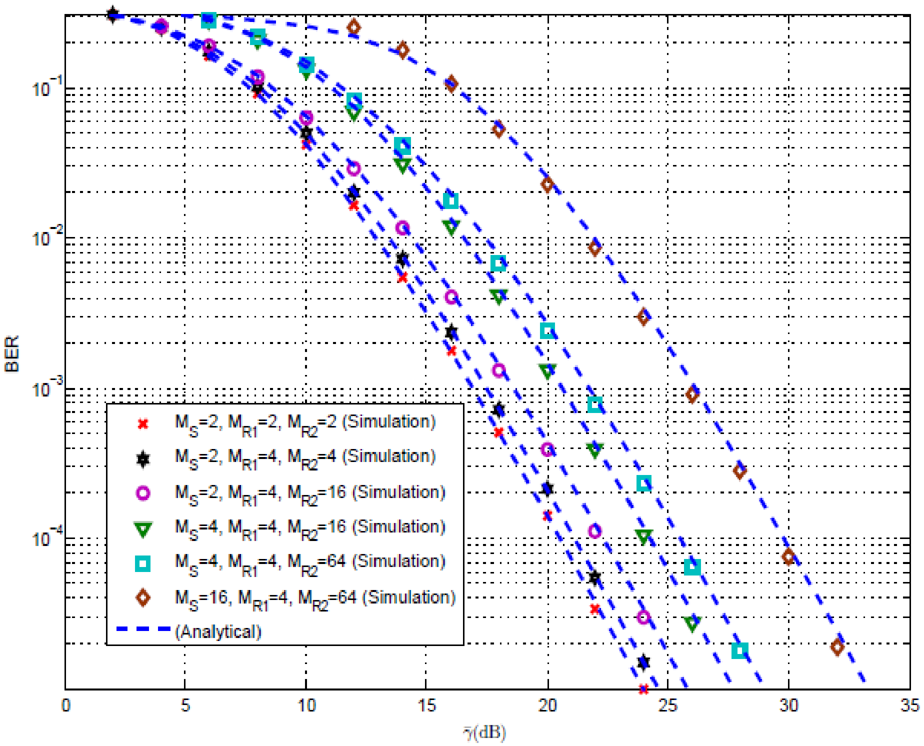
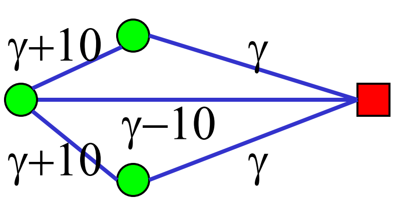


Fig. 1. BER performance of BER-based selection scheme for two-relay scenario, $\gamma_{SR_1} = \gamma + 10, \gamma_{SR_2} = \gamma + 10, \gamma_{SD} = \gamma - 10, \gamma_{R_1D} = \gamma, \gamma_{R_2D} = \gamma,$ assuming $N = 264$ bits. It is clear from the figures that the derived BER expressions and the simulation results are in excellent agreement.



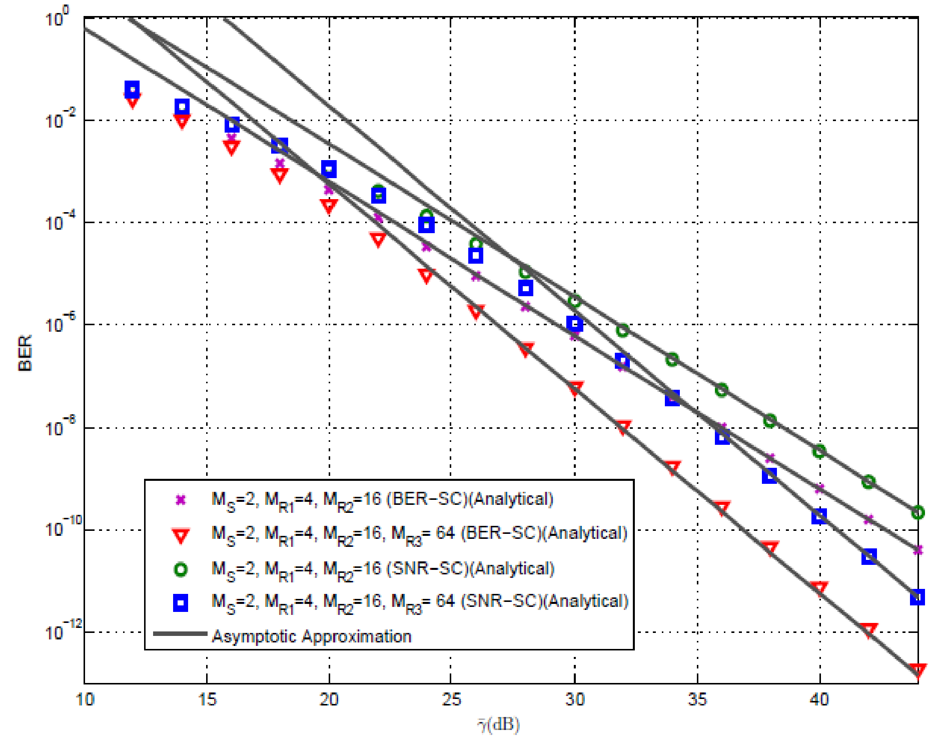


Fig. 2. Asymptotic BER performance of BER-based selection scheme for two-relay and three-relay scenarios, $\gamma_{SR_1} = \gamma + 10, \gamma_{SR_2} = \gamma + 10, \gamma_{SR_3} = \gamma + 10, \gamma_{SD} = \gamma - 10, \gamma_{R_1D} = \gamma,$ $\gamma_{R_2D} = \gamma, \gamma_{R_3D} = \gamma,$ assuming $N = 264$ bits. Although both BER-based selection scheme and SNR-based selection scheme achieve the same diversity order, BER-based selection scheme achieves higher SNR gain for all cases.

Summary

Selection combining of signals with different modulation levels in a relay network

- Biased SNRs for selection decision
- BER for selection combining
- E2E BER in a network
- Asymptotic E2E BER in a network

Future Work 1

- ICC 2014 + channel estimation errors + power control.

- H. U. Sokun, A. Bin Sediq, H. Yanikomeroglu, and S. Ikki, "Impact of Channel Estimation Errors in Selective Decode-and-Forward Relaying with Different Modulation Levels in a Multi-Relay Network", under review in *IEEE Trans. Commun.*

Future Work 2

- Modulation level and transmission mode joint selection.

- H. U. Sokun, H. Yanikomeroglu, and A. Bin Sediq, “Modulation level and transmission mode joint selection in two-hop decode-and-forward cooperative relaying”, under preparation.

- H. U. Sokun, H. Yanikomeroglu, and A. Bin Sediq, “Spectrally efficient selective decode-and-forward relaying in multi-relay adaptive cooperative systems”, under preparation.

S. Hares, H. Yanikomeroglu, and B. Hashem, “Diversity and AMC (adaptive modulation and coding)-aware routing in TDMA multihop networks”, *IEEE Globecom 2003*.

Future Work 3

- Modulation level and transmission mode (route) joint selection with maximum-likelihood receiver.

A. Bin Sediq and H. Yanikomeroglu, “Performance analysis of soft-bit maximal ratio combining in cooperative relay networks”, *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 4934-4939, Oct. 2009.

Future Work 4

- Joint space-time coding and routing decisions

Thank you!

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