Selective DF Relaying in Multi-Relay Networks With Different Modulation Levels

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Outline

• Motivation, Background, and Context
• Contributions
• Error Rate Performance Analysis
• Asymptotic Performance Analysis
• Simulation Results
• Summary and Future Work
Motivation

• Common assumption in cooperative relaying literature:
  Same modulation levels by the source and relays
  – Poor spectrally efficiency

• Allow different modulation levels at the relays opportunistically
  – Better spectrally efficiency

• Performance analysis → Protocol design

• Interest in terminal relaying in 3GPP
BER-based selection is better than SNR-based selection, when the signals at branches have different modulation levels.


Context

Multiple relays

- Select node $S$, if $\text{BER}_{SD} \leq \text{BER}_{R_iD}, \forall i \in DS$
- Select node $R_i$, if $\text{BER}_{SD} > \text{BER}_{R_iD}$ and $\text{BER}_{R_iD} > \text{BER}_{R_jD}, j \neq i, \forall i, j \in DS$
Contribution 1/4

- Find biased SNRs

\[
\begin{align*}
\text{Select node } S, & \quad \text{if } \text{BER}_{SD} \leq \text{BER}_{R_i,D}, \forall i \in D_S \\
\text{select node } R_i, & \quad \text{if } \text{BER}_{SD} > \text{BER}_{R_i,D} \text{ and } \\
& \quad \text{BER}_{R_j,D} > \text{BER}_{R_i,D}, \quad j \neq i, \forall i, j \in D_S.
\end{align*}
\]

\[
\text{Select node } S, \quad \text{if } \gamma_{SD} = \max \left( \gamma_{SD}, \rho_1 \gamma_{R_1,D}, \ldots, \rho_K \gamma_{R_K,D} \right), \quad \text{where } \rho_i = \frac{d_{MR_i}^2}{d_{MS}^2}.
\]

\[
d_{MR_i} = \sqrt{\frac{3}{2(M_{R_i} - 1)}}, \quad M_{R_i} \geq 4
\]

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Contribution 2/4

- Find BER for selection combining
- Not straightforward
- Relevance to CoMP, HARQ, Relay
Contribution 3/4

• Find E2E BER in a network with selection combining
Contribution 4/4

- Find asymptotic E2E BER in a network with selection combining
**Preliminaries**

**Point-to-Point AWGN BER**

\[
BER_{M_i}(\gamma_{ij}) \approx c_{M_i}Q\left(\sqrt{2d^2_{M_i}\gamma_{ij}}\right),
\]

where \(c_{M_i}, d_{M_i}\) = \[
\begin{cases} 
(1,1), & M_i = 2, \\
\left(\frac{2 - 2/\sqrt{M_i}}{\log_2 \sqrt{M_i}}, \sqrt{\frac{3}{2(M_i-1)}}\right), & M_i \geq 4,
\end{cases}
\]

**Point-to-Point Rayleigh BER**

\[
BER_{ij} \approx \frac{1}{2} c_{M_i} \left(1 - \sqrt{\frac{d^2_{M_i}\gamma_{ij}}{1 + d^2_{M_i}\gamma_{ij}}}\right)
\]

**Average Packet Error Rate**

\[
PER_{SR_i} = 1 - (1 - SER_{SR_i})^{\frac{N}{\log_2 M_s}}
\]

\[
\approx 1 - \left(1 - \frac{1}{2} c_{M_s} \log_2 (M_s) \left(1 - \sqrt{\frac{d^2_{M_s}\gamma_{SR_i}}{1 + d^2_{M_s}\gamma_{SR_i}}}\right)\right)^{\frac{N}{\log_2 M_s}}
\]

where \(SER \approx BER \log_2 M_s\) for Gray-coded constellations
Error Rate Performance (1/3)

End-to-End Average BER

\[
BER = \left( \prod_{k=1}^{K} \text{PER}_{SR_k} \right) \text{BER}_{SD} + \sum_{r=1}^{K} \sum_{m=1}^{|P_r(S_{all})|} \left( \prod_{e_l \in P_{r,m}(S_{all})} \left( 1 - \text{PER}_{SR_{e_l}} \right) \right) \left( \prod_{e_o \notin P_{r,m}(S_{all})} \text{PER}_{SR_{e_o}} \right) \text{BER}_{\text{comp}_{P_{r,m}(S_{all})}}
\]

For example, for a two-relay scenario, it is given as

\[
BER = \text{PER}_{SR_1} \text{PER}_{SR_2} \text{BER}_{SD} + (1 - \text{PER}_{SR_1}) \text{PER}_{SR_2} \text{BER}_{\text{comp}_{\{1\}}} + (1 - \text{PER}_{SR_2}) \text{PER}_{SR_1} \text{BER}_{\text{comp}_{\{2\}}} \\
+ (1 - \text{PER}_{SR_2})(1 - \text{PER}_{SR_1}) \text{BER}_{\text{comp}_{\{1,2\}}}
\]

- \(|P_r(S_{all})|\) represents the cardinality of \(S_{all}\),
- \(P_r(S_{all})\) is \(r\)-th element power set of \(S_{all}\), i.e., \(S_{all}\),
- \(P_{r,m}(S_{all})\) is \(m\)-th element of \(P_r(S_{all})\),
- \(S_{all}\) is the set of all relays’ indexes, i.e., \(S_{all} = \{1, \ldots, K\}\),
- \(\text{PER}_{SR_i}\) is the average packet error ratio in link \(S - R_i\),
- \(\text{BER}_{SD}\) is average BER in link \(S - D\),
- \(\text{BER}_{\text{comp}_{DS}}\) is average BER conditioned on the decoding set at destination terminal after selection combining.
Error Rate Performance (2/3)

End-to-End Average BER Conditioned on the Decoding Set

\[
BER_{\text{comp,inst}} \approx \begin{cases} 
    c_M Q(\sqrt{2d_{M_i}^2 \gamma_{SD}}) & \text{if } \gamma_{SD} \geq \rho_i \gamma_{R,D}, \quad i = 1, 2, \ldots, K \\
    c_M Q(\sqrt{2d_{M_i}^2 \gamma_{RD}}) & \text{if } \gamma_{SD} < \rho_i \gamma_{R,D} \quad \text{and } \gamma_{R,D} < \beta_j \gamma_{R,D} \quad j \neq i, \quad j = 1, 2, \ldots, K, \quad \text{for } i = 1, 2, \ldots, K 
\end{cases}
\]

where \( \rho_i = \frac{d_{M_i}^2}{d_{M}^2} \) and \( \beta_j = \frac{d_{M_i}^2}{d_{M_j}^2} \) are biasing factors.

An approximate and simpler implementation of the instantaneous BER.

\[
BER_{\text{comp,decoding set}} = BER_{\gamma_{SD} \geq \rho_i \gamma_{R,D}, \ldots, \rho_K \gamma_{R,K,D}} + \sum_{i=1}^{K} BER_{\gamma_{SD} < \rho_i \gamma_{R,D} \quad \text{and } \gamma_{R,D} < \beta_j \gamma_{R,D} \quad j \neq i, \quad j = 1, 2, \ldots, K}
\]

1. \[
BER_{\gamma_{SD} \geq \rho_i \gamma_{R,D}} = \int \int \cdots \int c_M Q(\sqrt{2d_{M_i}^2 \gamma_{SD}}) \left[ \frac{1}{\gamma_{SD}} \frac{\gamma_{SD}}{\bar{\gamma}_{SD}} \prod_{i=1}^{K} \frac{1}{\gamma_{R,D}} \frac{\gamma_{R,D}}{\bar{\gamma}_{R,D}} \right] d_{\gamma_{SD}} d_{\gamma_{R,D}} \cdots d_{\gamma_{R,K,D}}
\]

2. \[
BER_{\gamma_{SD} < \rho_i \gamma_{R,D} \quad \text{and } \gamma_{R,D} < \beta_j \gamma_{R,D} \quad j \neq i, \quad j = 1, 2, \ldots, K}
\]

\[
= \int \int \cdots \int c_M Q(\sqrt{2d_{M_i}^2 \gamma_{R,D}}) \left[ \frac{1}{\gamma_{R,D}} \frac{\gamma_{R,D}}{\bar{\gamma}_{R,D}} \prod_{j=1}^{K-1} \frac{1}{\gamma_{R,D}} \frac{\gamma_{R,D}}{\bar{\gamma}_{R,D}} \right] d_{\gamma_{R,D}} d_{\gamma_{SD}} d_{\gamma_{R,D}} \cdots d_{\gamma_{R,K,D}}
\]
Error Rate Performance (3/3)

End-to-End Average BER

\[
BER = \prod_{k=1}^{K} \left[ 1 - \left( 1 - \frac{1}{2} c_{M_s} \log_2 (M_s) \right) \left( 1 - \frac{d_{M_s}^2 \rho_{SR_k}}{\sqrt{1 + d_{M_s}^2 \rho_{SR_k}^2}} \right)^{-\frac{N}{\log_2 M_s}} \right] \left( 1 - \frac{d_{M_s}^2 \rho_{SD}}{\sqrt{1 + d_{M_s}^2 \rho_{SD}^2}} \right)
\]

\[
+ \sum_{r=1}^{K} \sum_{m=1}^{N \log_2 M_s} \prod_{e_i \in P_{r,m}(S_{all})} \left[ 1 - \frac{1}{2} c_{M_s} \log_2 (M_s) \left( 1 - \frac{d_{M_s}^2 \rho_{SR_i}}{\sqrt{1 + d_{M_s}^2 \rho_{SR_i}^2}} \right)^{-\frac{N}{\log_2 M_s}} \right] \times \prod_{e_o \neq P_{r,m}(S_{all})} \left[ 1 - \frac{1}{2} c_{M_s} \log_2 (M_s) \left( 1 - \frac{d_{M_s}^2 \rho_{SR_o}}{\sqrt{1 + d_{M_s}^2 \rho_{SR_o}^2}} \right)^{-\frac{N}{\log_2 M_s}} \right]
\]

\[
\times I \left( \infty, c_{M_s}, d_{M_s}^2, \rho_{SD}^2 \right) + \sum_{k=1}^{k} \sum_{y=1}^{k} (-1)^k I \left( \infty, \frac{c_{M_s}}{\rho_{SD}}, d_{M_s}^2, \frac{k+1}{HM \{ \rho_{SD}, P_{k,y} \{ S \} \}} \right) \left( \frac{k+1}{HM \{ \rho_{SD}, P_{k,y} \{ S \} \}} \right)
\]

\[
+ \sum_{i=1}^{k} \left[ I \left( \infty, c_{M_{R_i}}, d_{M_{R_i}}^2, \rho_{RD}^2 \right) + \sum_{k=1}^{k} \sum_{y=1}^{k} (-1)^k I \left( \infty, \frac{c_{M_{R_i}}}{\rho_{RD}}, d_{M_{R_i}}^2, \frac{k+1}{HM \{ \rho_{RD}, P_{k,y} \{ S \} \}} \right) \left( \frac{k+1}{HM \{ \rho_{RD}, P_{k,y} \{ S \} \}} \right) \right]
\]
Asymptotic Performance (1/2)

\[
BER = \left( \prod_{k=1}^{K} \text{PER}_{SR_k} \right) \text{BER}_{SD} + \sum_{r=1}^{K} \sum_{m=1}^{\left| P_r(S_{all}) \right|} \left( \prod_{e \in P_r(m) \cap (S_{all})} (1 - \text{PER}_{SR_{e_i}}) \right) \prod_{e_o \notin P_r(m) \cap (S_{all})} \text{PER}_{SR_{e_o}} \right) \text{BER}_{comp_{P,m}(S_{all})}
\]

1. \( \text{PER}_{SR_i} \xrightarrow{\text{SNR} \to \infty} \frac{N c_{M_S}}{4d^2 \sigma^2_{SR_i} \text{SNR}} \)
2. \( 1 - \text{PER}_{SR_i} \xrightarrow{\text{SNR} \to \infty} 1 - \frac{N c_{M_S}}{4d^2 \sigma^2_{SR_i} \text{SNR}} \approx 1 \)
3. \( \text{BER}_{SD} \xrightarrow{\text{SNR} \to \infty} \frac{c_{M_S}}{4d^2 \sigma^2_{SD} \text{SNR}} \)

4. \( \text{BER}_{SD}^{\text{comp}_{\text{decoding set}}} = \text{BER}_{SD}^{\gamma_{SD} \geq \rho_i \gamma_{R_D}, \ldots \rho_K \gamma_{R_D}} + \sum_{i=1}^{K} \text{BER}_{SD}^{\gamma_{SD} < \rho_i \gamma_{R_D} \text{ and } \gamma_{R_D} < \beta_{ij} \gamma_{R_D}}, j \neq i, j=1,2,\ldots,K} \)

\[
\text{BER}_{SD}^{\gamma_{SD} \geq \rho_i \gamma_{R_D}, i=1,2,\ldots,K} = \int_{\gamma_{SD}=0}^{\infty} \int_{\gamma_{R_D}=0}^{\rho_i^{-1} \gamma_{SD}} \ldots \int_{\gamma_{R_D}=0}^{\rho_K^{-1} \gamma_{SD}} c_{M_S} Q\left(\sqrt{2d^2_{M_S} \gamma_{SD}}\right) \frac{1}{\gamma_{SD}} \left[ \prod_{i=1}^{K} \frac{1}{\gamma_{R_D}} \right] d_{\gamma_{SD}} d_{\gamma_{R_D}} \ldots d_{\gamma_{R_D}}
\]

\[
= \left[ \prod_{i=1}^{K} \frac{\rho_i^{-1}}{\gamma_{R_D}} \right] \frac{c_{M_S} \Gamma(K+1.5)}{2\sqrt{\pi} \gamma_{SD} (1+K) \left(d^2_{M_S}\right)^{K+1}}
\]

\[
\text{BER}_{SD}^{\gamma_{SD} < \rho_i \gamma_{R_D} \text{ and } \gamma_{R_D} < \beta_{ij} \gamma_{R_D}}, j \neq i, j=1,2,\ldots,K} = \sum_{i=1}^{K} \left[ \prod_{j=1, j \neq i}^{K} \frac{\beta_{ij}}{\gamma_{R_D}} \right] \frac{\rho_i c_{M_S} \Gamma(K+1.5)}{2\sqrt{\pi} \gamma_{SD} \gamma_{R_D} (1+K) \left(d^2_{M_S}\right)^{K+1}}
\]
Asymptotic BER

\[
BER^{\text{SNR} \to \infty} = \left( \prod_{k=1}^{K} \frac{Nc_{M_S}}{4d_{M_S}^2 \sigma_{SR_k}^2} \right) \frac{c_{M_S}}{4d_{M_S}^2 \sigma_{SD}^2} \frac{1}{SNR^{K+1}} + \sum_{r=1}^{K} \sum_{m=1}^{\infty} \left( \prod_{e_o \not\in \mathcal{P}_{r,m}(S_{all})} \frac{Nc_{M_S}}{4d_{M_S}^2 \sigma_{SR_{e_o}}^2 SNR} \right) + \sum_{i=1}^{\infty} \left[ \prod_{j=1}^{\infty} \frac{|P_{r,m}(S_{all})|}{\sigma_{R_{i,j}}^2} \right] \frac{c_{M_i}}{2\sqrt{\pi} \sigma_{SD}^2 (1 + |P_{r,m}(S_{all})|)} \left( d_{M_i}^2 \right) + 1 \right] \frac{1}{SNR^{\mathcal{P}_{r,m}(S_{all})+1}}
\]
Simulation Results

Fig. 1. BER performance of BER-based selection scheme for two-relay scenario,
\[ \gamma_{SR_1} = \gamma + 10, \gamma_{SR_2} = \gamma + 10, \gamma_{SD} = \gamma - 10, \gamma_{R,D} = \gamma, \gamma_{R2,D} = \gamma, \]
assuming \( N = 264 \) bits. It is clear from the figures that the derived BER expressions and the simulation results are in excellent agreement.

Fig. 2. Asymptotic BER performance of BER-based selection scheme for two-relay and three-relay scenarios,
\[ \gamma_{SR_1} = \gamma + 10, \gamma_{SR_2} = \gamma + 10, \gamma_{SR_3} = \gamma + 10, \gamma_{SD} = \gamma - 10, \gamma_{R,D} = \gamma, \gamma_{R2,D} = \gamma, \gamma_{R3,D} = \gamma, \]
assuming \( N = 264 \) bits. Although both BER-based selection scheme and SNR-based selection scheme achieve the same diversity order, BER-based selection scheme achieves higher SNR gain for all cases.
Summary

Selection combining of signals with different modulation levels in a relay network

- Biased SNRs for selection decision
- BER for selection combining
- E2E BER in a network
- Asymptotic E2E BER in a network
Future Work 1

• ICC 2014 + channel estimation errors + power control.

Future Work 2

- Modulation level and transmission mode joint selection.


Future Work 3

- Modulation level and transmission mode (route) joint selection with maximum-likelihood receiver.

Future Work 4

• Joint space-time coding and routing decisions
Thank you!

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