# Optimal Design and Power Allocation for Multicarrier Decode and Forward Relays

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## Introduction

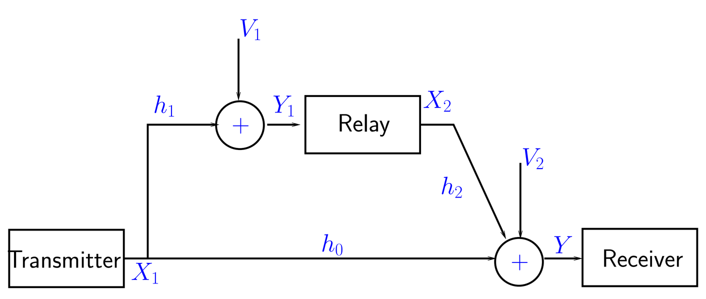



Figure: Gaussian relay channel.

- Multicarrier system assisted by multiple relays, one for each subcarrier.
- Decode-and-forward (DF) full-duplex.
- DF relaying achieves capacity of degraded channels.
- DF generally outperforms AF and CF when source-relay link stronger than source-destination link.
- In DF (Cover and Gamal '79):
  - relay decodes codeword transmitted by source;
  - uses Wyner-Ziv binning to determine bin index of source codeword;
  - relay transmits codeword corresponding to bin index in next block;
  - destination combines information in consecutive block to recover message.
- For scalar Gaussian relay channel, DF optimal source and relay codebooks Gaussian with particular correlation.

## Considered Scenario

- Multicarrier system assisted by multiple relays, one for each subcarrier.
- Total source power and total relays power less than budgets.
- How to determine optimal power allocation across subcarriers at source and relays together with optimal correlation coefficients between source and relay codebooks?

## Approach and Results

- Draw insight from single carrier DF.
- Formulate multicarrier design as an optimization problem.
- Problem non-convex
- Approach: Analyze KKT system.
- KKT system necessary but not sufficient.
- Capture non-convexity in one polynomial in the KKT system.
- Develop algorithm to find optimal solution.

## DF Relaying in Single-Carrier Communication Systems

- Relay received signal: $Y_1 = X_1h_1 + V_1$.
- Destination received signal: $Y = X_1h_0 + X_2h_2 + V_2$.
- Signal $X_2$ depends on $Y_1$ received in previous block but not current one.

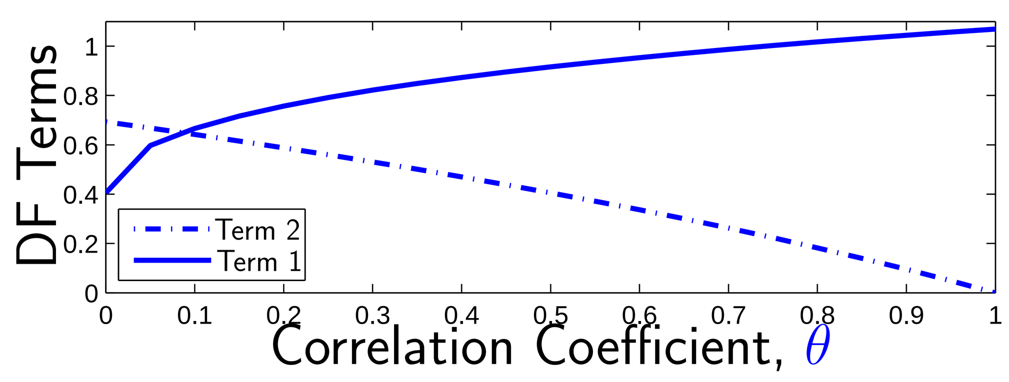
## DF Relaying in Single-Carrier Communication Systems (cont'd)

- Maximum data rate of DF achieved when $X_1$ and $X_2$ Gaussian with correlation [Madsen '05, Wang '05]

$$\theta = \frac{\mathrm{E}^2\{X_1X_2\}}{\mathrm{E}\{X_1^2\}\,\mathrm{E}\{X_2^2\}}.$$

- $X_1 = \sqrt{\frac{\theta P_1}{P_2}}X_2 + X_{10}$; $X_{10}$ Gaussian and independent.
- Using $g_i \triangleq |h_i|^2$, $i = 0, 1, 2$, maximum DF rate (single carrier)

$$\max_{0\leq\theta\leq 1} \min\{\frac{1}{2}\log(1 + g_0P_1 + g_2P_2 + 2\sqrt{\theta g_0g_2P_1P_2}), \frac{1}{2}\log(1 + (1-\theta)g_1P_1)\}.$$




## DF relaying in multicarrier communication systems

- System operating over $N$ orthogonal carriers.
- Let $P_T$ and $P_R$ source and relay power budgets.
- Let $\alpha_i = 1 - \theta_i$, $a_i = \frac{g_1}{g_0} - 1$, $i = 1, \ldots, N$. Maximum DF rate:

$$\begin{aligned}\max_{\{P_{1i}\}_{i=1}^N,\{P_{2i}\}_{i=1}^N,\{\alpha_i\}_{i=1}^N} \ & \frac{1}{2}\sum_{i=1}^{N}\log(1 + \alpha_i g_{1i}P_{1i}),\\ \text{subject to } & \sum_{i=1}^{N} P_{1i} \leq P_T,\\ & \sum_{i=1}^{N} P_{2i} \leq P_R,\\ & \alpha_i \leq 1, \qquad i = 1, \ldots, N,\\ & \sqrt{P_{2i}} \geq \sqrt{\frac{g_{0i}P_{1i}}{g_{2i}}}(\sqrt{a_i\alpha_i} - \sqrt{1-\alpha_i}), \qquad i = 1, \ldots, N.\end{aligned}$$

- Objective and last constraint non-convex.
- Last constraint captures the carriers with $\alpha_i = 1$ and $\alpha_i < 1$.
- Analyze using KKT conditions.
- KKT conditions provide necessary but not sufficient conditions.
- Optimal solution can be found by examining all solutions of KKT system.

## Subcarriers with $g_{0i} > g_{1i}$

- SNR observed by relay is less than SNR observed by destination.
- DF strategy cannot assist communication.
- Switch relay off better than DF.
- Source performs water-filling on those subcarriers.

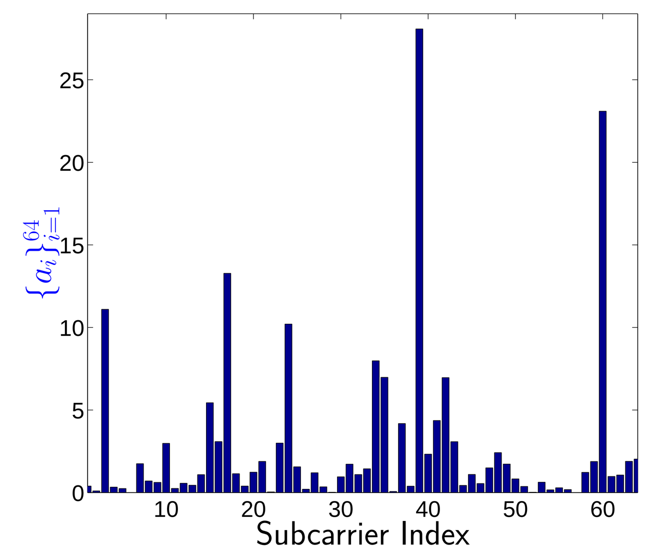
## Subcarriers with $g_{0i} < g_{1i}$

- KKT analysis used to obtain optimal power allocation and correlation coefficients
- Algorithm:
  - Suppose $\mu$ ratio of Lagrange multipliers corresponding to source and relay power constraints given.
  - Show that correlation coefficients $\{\alpha_i\}$ readily expressible in term of $\mu$.
  - Show that subcarrier powers readily expressible in term of $\mu$ and $\{\alpha_i\}$.
  - How to find $\mu$?
    - Iteratively—Works well but no guarantees
    - Analytically: exhaustive search over the roots of

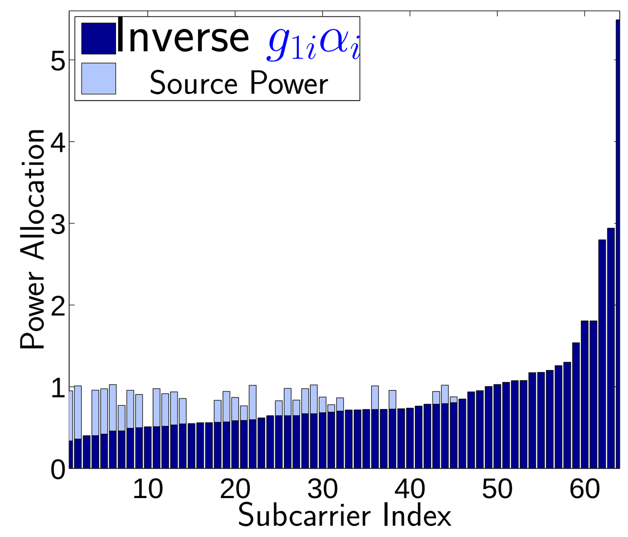
$$P_R = \sum_{i\in\mathcal{I}_+} \frac{\frac{a_ig_{2i}}{g_{0i}}\mu^2}{a_i + (1 + \frac{g_{2i}}{g_{0i}}\mu)^2}\left(\frac{P_T + \sum_{j\in\mathcal{I}_+}\frac{a_j + (1+\frac{g_{2j}}{g_{0j}}\mu)^2}{g_{1j}(1+\frac{g_{2j}}{g_{0j}}\mu)^2}}{1 + \sum_{\substack{j\in\mathcal{I}_+\\ j\neq i}}\frac{a_j + (1+\frac{g_{2j}}{g_{0j}}\mu)^2}{(1+\frac{g_{2j}}{g_{0j}}\mu)^2 + a_j(1+\frac{g_{2j}}{g_{0j}}\mu)}} - \frac{a_i + (1 + \frac{g_{2i}}{g_{0i}}\mu)^2}{g_{1i}(1 + \frac{g_{2i}}{g_{0i}}\mu)^2}\right).$$

## Numerical Example

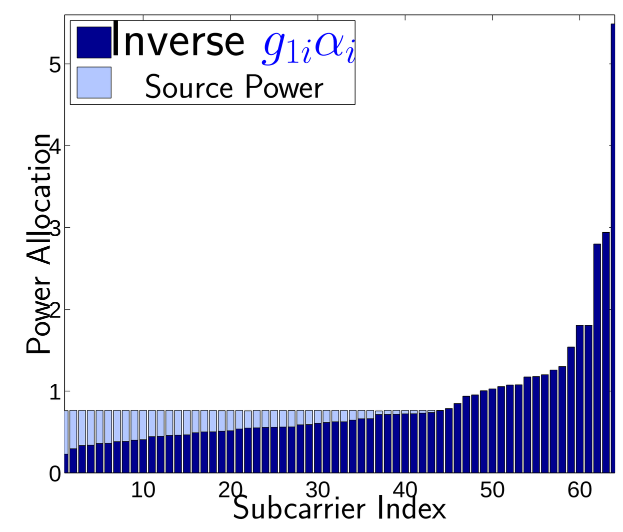
- Case of $N = 64$ subcarriers with $g_{0i} < g_{1i}$.
- Objective: Find optimal $\{\alpha_i\}_{i=1}^{64}$, $\{P_{1i}\}_{i=1}^{64}$ and $\{P_{2i}\}_{i=1}^{64}$.



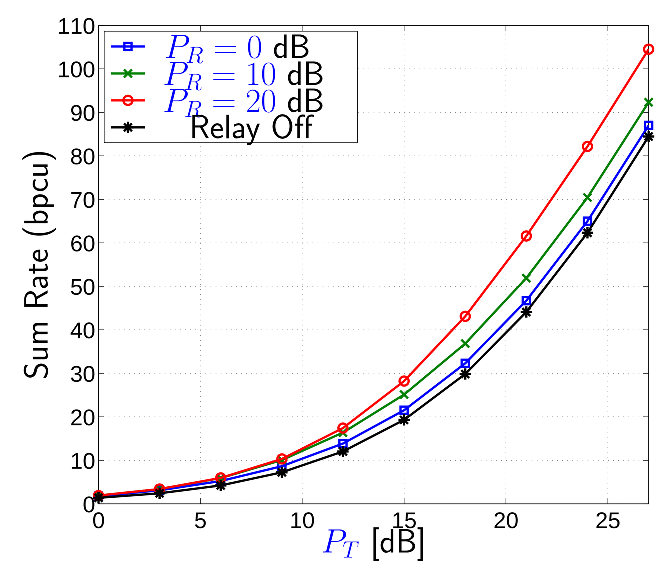

(a) $a_i$, $i = 1, \ldots, 64$.




(b) Power allocation ($P_R = 10$)




(c) Power allocation ($P_R = 20$)




(d) Sum rate

Figure: Optimal rates, correlation, and power allocation

- In agreement with analysis, as $P_R$ increases, optimal source power allocation approaches water-filling.

## Conclusions

- We analyzed the problem of joint optimization of the codebook design and power allocation for full-duplex multicarrier DF relaying.
- Problem nonconvex but can be solved efficiently using analysis of KKT system.