Space-Time Block Codes over the Stiefel Manifold

Mohammad T. Hussien[§] Karim G. Seddik[†] Ramy H. Gohary[‡] Mohammad Shaqfeh[*] Hussein Alnuweiri[*] and Halim Yanikomeroglu[‡]

[§]Department of Electrical Engineering, Alexandria University, Egypt
[†]Electronics Engineering Department, American University in Cairo, AUC Avenue, New Cairo 11835, Egypt
[‡]Department of Systems and Computer Engineering, Carleton University, Ottawa, Canada
[*]Department of Electrical Engineering, Texas A&M University in Qatar, Doha, Qatar

Motivation

- In multi-resolution broadcast communication systems two classes of information can be identified:
  - **The LR information**: the basic information that must be reliably communicated to both classes of receivers.
  - **The HR information**: the incremental information that only the HR receivers have access to.
- In this multi-resolution broadcast communication systems two classes of receivers can be identified:
  - **LR receivers** which do not have access to channel state information (CSI) and can only perform non-coherent detection.
  - **HR receivers** which have access to reliable CSI and can perform coherent detection.
- For MIMO systems operating at high SNRs, the LR information can be reliably communicated using constellations designed on the Grassmann manifold which are able to achieve the capacity of the non-coherent MIMO channel of the LR receiver.
- Since these Grassmannian constellations are unitarily-invariant, the incremental HR information must be encoded using square unitary matrices. Such matrices represent elements of the unitary group.
- Various techniques are available designing space-time Grassmannian constellations. However, techniques for designing codes on the unitary group are generally scarce.
- In this paper we focus on developing two techniques for designing unitarily-constrained square space-time block codes directly on the group of square unitary matrices.
  - **The greedy design**: the points of the constellation are designed sequentially.
  - **The direct design**: the points are designed jointly which yields constellations with more favourable properties but with higher design complexity.
- In comparison with conventional "unitarily-constrained" space-time block codes, the ones generated by the proposed approaches possess more favourable distance spectra and subsequently better performance.

Preliminaries and System Model

- For $T \geq M$, the Stiefel manifold $\mathbb{S}_{T,M}(\mathbb{C})$ is defined as the set of all unitary $T \times M$ matrices, that is,
$$\mathbb{S}_{T,M}(\mathbb{C}) = \{\mathbf{Q} \in \mathbb{C}^{T\times M} : \mathbf{Q}^{\mathcal{H}}\mathbf{Q} = \mathbf{I}_M\}. \quad (1)$$
The Stiefel manifold $\mathbb{S}_{T,M}(\mathbb{C})$ is submanifold of $\mathbb{C}^{T\times M}$ of $TM - M^2/2$ complex dimensions.
- The Grassmann manifold $\mathbb{G}_{T,M}(\mathbb{C})$ is defined as the quotient space of $\mathbb{S}_{T,M}(\mathbb{C})$ with respect to the equivalence relation that renders two elements $\mathbf{P}, \mathbf{Q} \in \mathbb{S}_{T,M}(\mathbb{C})$ equivalent if their T-dimensional column vectors span the same subspace. The Grassmann manifold has $M(T-M)$ complex dimension.
- We consider a broadcast MIMO communication system with M transmit antennas with two classes of receivers operating over the block Rayleigh flat-fading channel.
- The communication system can be modeled as
$$\begin{aligned}\mathbf{Y}_i &= \mathbf{X}\mathbf{H}_i + \mathbf{W}_i \\ &= \mathbf{U}\mathbf{A}\mathbf{H}_i + \mathbf{W}_i,\ i = 1, 2, \cdots,\end{aligned} \quad (2)$$
where $\mathbf{Y}_i$ is the $T \times N_i$ received matrix of the i-th receiver, and the channel is assumed to be constant over a coherence interval of T time slots.
$\mathbf{X} = \mathbf{U}\mathbf{A}$ is the $T \times M$ transmitted matrix which contains both the LR and the HR information.
- the LR information is encoded in the subspace spanned by the matrix $\mathbf{U}$ and the HR information is encoded in the $M \times M$ unitary matrix $\mathbf{A} \in \mathbb{U}_M$.
- The matrices $\mathbf{H}_i$ and $\mathbf{W}_i$ represent the channel and noise observed by receiver i, and the elements of these matrices are i.i.d circularly-symmetric zero mean complex Gaussian random variables. The entries of $\mathbf{H}_i$ have unit variance and the entries of $\mathbf{W}_i$ have variance N_0.

HR Layer Code Construction

A. The Greedy Design

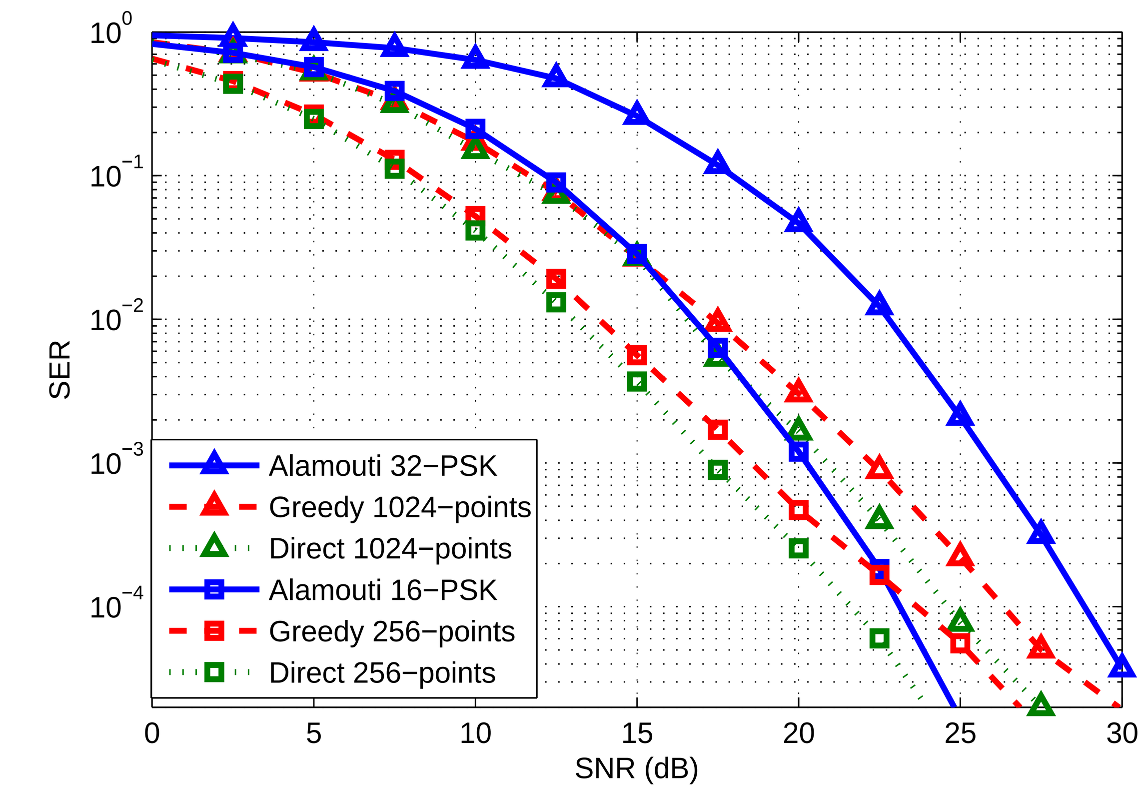
- The constellation of $|\mathcal{C}|$ points on $\mathbb{U}_M$ is designed in a point-by-point manner.
- The optimization problem for designing the i-th constellation point can be formulated as follows.
$$\begin{aligned}\mathbf{A}_i &= \arg \max_{\mathbf{A}\mathbf{A}^{\mathcal{H}}=\mathbf{I}} \min_{1\leq j\leq i-1} \|\mathbf{A} - \mathbf{A_j}\|_F \\ &= \arg \min_{\mathbf{A}\mathbf{A}^{\mathcal{H}}=\mathbf{I}} \left(\log\left(\sum_{j=1}^{i-1} \mathrm{e}^{\mathrm{Tr}^n(\Re(\mathbf{A}_j^{\mathcal{H}}\mathbf{A}))}\right)\right)^{\frac{1}{n}}. \end{aligned} \quad (3)$$
- To solve this problem, we use this algorithm:

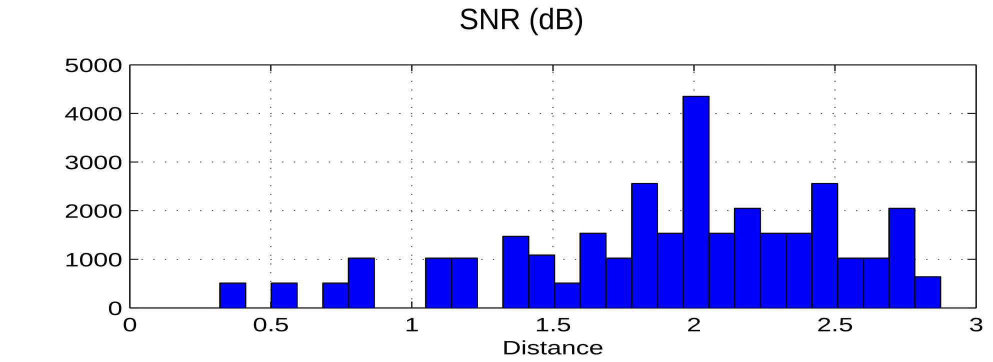
Algorithm 1 Gradient Descent Algorithm with curvi-linear search

Given an initial point $\mathbf{A}_o \in \mathbb{U}_M$
Initialization: Set $k \leftarrow 0$, $\epsilon \geq 0$ and $0 \leq \rho_1 \leq \rho_2 \leq 1$
while true **do**
  Prepare: Generate $\mathbf{S} = \mathbf{G_A}\mathbf{A}^{\mathcal{H}} - \mathbf{A}\mathbf{G}_{\mathbf{A}}^{\mathcal{H}}$ where $\mathbf{G_A}$ is the derivative of the cost function f w.r.t $\mathbf{A}$.
  Compute the step size τ_k: Call line search along the path $\mathbf{Y}(\tau) = \left(\mathbf{I} + \frac{\tau}{2}\mathbf{S}\right)^{-1}\left(\mathbf{I} - \frac{\tau}{2}\mathbf{S}\right)\mathbf{A}$ to obtain a step size τ_k that satisfies the Armijo-Wolfe conditions $f(\mathbf{Y}(\tau)) \leq f(\mathbf{Y}(0)) + \rho_1\tau f_\tau'(\mathbf{Y}(0))$ and $f_\tau'(\mathbf{Y}(\tau)) \geq \rho_2 f_\tau'(\mathbf{Y}(0))$.
  Update: $\mathbf{A}_{k+1} \leftarrow \mathbf{Y}(\tau_k)$
  Stopping Check:
  if $\|\nabla f_{k+1}\| \leq \epsilon$ **then** stop
  else $k \leftarrow k+1$ and **continue**

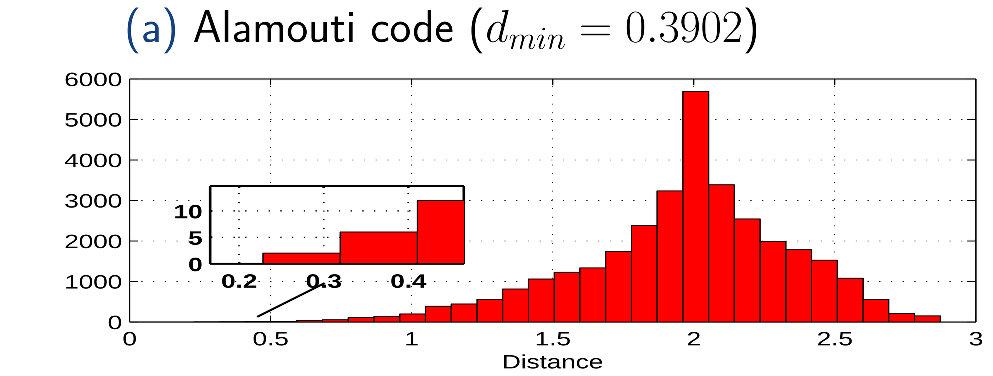
B. The Direct Design

- Here we jointly design all the $|\mathcal{C}|$ constellation points on the unitary group $\mathbb{U}_M$.
- The problem can be formulated as follows.
$$\{\mathbf{A}_r\}_{r=1}^{|\mathcal{C}|} = \arg \min_{\mathbf{A}_r\mathbf{A}_r^{\mathcal{H}}=\mathbf{I}} \left(\log\left(\sum_{i=1}^{|\mathcal{C}|-1}\sum_{j=i+1}^{|\mathcal{C}|} \mathrm{e}^{\mathrm{Tr}^n(\Re(\mathbf{A}_j^{\mathcal{H}}\mathbf{A}_i))}\right)\right)^{\frac{1}{n}}. \quad (4)$$
- To solve this optimization problem, we again use the gradient descent algorithm.

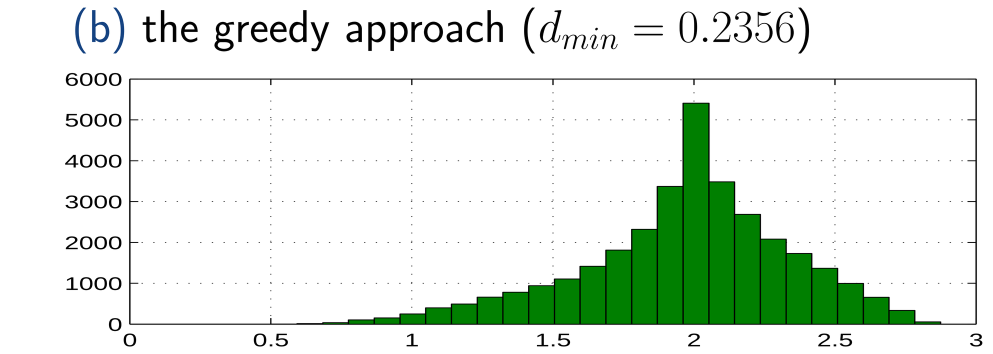




(a) Alamouti code ($d_{min} = 0.3902$)



(b) the greedy approach ($d_{min} = 0.2356$)



(c) the direct approach ($d_{min} = 0.5641$)

Conclusions

- We propose new unitary space-time block codes designed using the gradient descent algorithm over Stiefel manifold with two different approaches.
- The designed codes are shown to outperform the unitarily-constrained conventional space-time block codes, and some non-unitarily constrained space-time block codes at low and medium SNRs.
- These designed unitary codes can be used as the high resolution codes for any arbitrary number of transmit antennas M in a layered coding settings.