A Signal Space Diversity-Based Time Division Broadcast Protocol in Two-Way Relay Systems

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***Abstract*—This paper considers a two-way relay system with time division broadcast (TDBC) protocol. In such protocol two end-sources exchange information with each other through help of a half-duplex decode-and-forward relay. To enhance the spectral efficiency in this system, we propose a novel scheme which incorporates signal space diversity with TDBC protocol. In the proposed scheme, original data symbols are rotated by a certain angle before being transmitted, and then the end-sources and the relay cooperate for transmitting in-phase and quadrature components of two consecutive rotated symbols. Thereby, the number of transmitted symbols are doubled over three time slots, which results in an increase in spectral efficiency. In particular, for this system we first derive a closed-form expression for the end-to-end (E2E) error probability with an arbitrary constellation to acquire all the resulting non-uniform constellation cases. Then, using the derived expression, we optimize the rotation angle at the different values of the signal-to-noise ratio to further improve the E2E error probability performance. Numerical results corroborate the theoretical analysis and show that the scheme proposed herein provides not only higher spectral efficiency, but also higher reliability transmission.**

***Index Terms*—Cooperative communication, two-way relaying, time division broadcast (TDBC), signal space diversity, decode-and-forward, error probability.**

I. INTRODUCTION

Relay-based cooperative communication has been recognized as an important enabling technology for future wireless networks. As a matter of fact, simple cooperative communication techniques (mainly through operator-deployed fixed relays) have already been incorporated into several recent standards, such as 3GPP LTE-Advanced, IEEE 802.16j and IEEE 802.16m [1], [2]. It is expected that more sophisticated cooperative communication techniques with user-terminal relays will be incorporated in 5G standards [3].

In a classical one-way relay system, a source transmits information to a destination through help of half-duplex relay(s), which may use either a decode-and-forward (DF) relaying or an amplify-and-forward (AF) relaying. The use of relay-based cooperative transmission offers significant performance benefits, including being able to achieve spatial diversity through node cooperation and extending coverage without requiring large transmitter powers. Despite these benefits, one-way relay techniques suffer from the loss in spectral efficiency due to the half-duplex relay constraint. To enhance the spectral efficiency, two-way relay systems have been proposed and analysed [4], [5], in which two end-sources exchange information with each other through help of half-duplex relay(s).

In a two-way relay system, the information exchange between two end-sources can take place over four time slots, where source A transmits information to source B via relay R in the first two time slots with source B remaining idle, and then source B communicates to source A via relay R in the last two time slots with source A remaining idle. Such transmission scheme obviously achieves low spectral efficiency. In a more efficient two-way relay system, two end-sources can exchange the information over three time slots, where source A and source B transmit to relay R over the first and second time slots, respectively, and in the third time slot, relay R transmits a function of the received signals to source A and source B. This three-time slot protocol is known as time-division broadcast (TDBC) [5]. TDBC protocol can achieves the full diversity order exploiting the direct link between source A and source B; it provides a reliable transmission. On the other hand, the information exchange between two end-sources can take place over two time slots, where in the first time slot, source A and source B simultaneously transmit to relay R, and in the second time slot, relay R forwards the processed received signals to both source A and source B. This two-time slot protocol is known as physical network coding (PNC) [4]. Note that PNC protocol is more spectral efficient in comparison to TDBC protocol, since it requires less time slots. However, PNC protocol has a less reliable transmission in comparison to TDBC protocol, since it does not exploit the direct link.

To alleviate the loss in spectral efficiency and promote the attainment of diversity in three-time slot two-way relay system, we utilize signal space diversity (SSD) [6], which is also known as modulation diversity. In the SSD technique, to achieve signal space diversity, original data symbols are first rotated by a certain angle prior to transmission and then in-phase and quadrature phase (I/Q) interleaving is applied to the rotated symbols to guarantee that in-phase and quadrature phase components are transmitted through an independent realization of the channel. In [7], [8], the SSD technique has been applied to an one-way relay system and the error

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probability performance of the SSD technique in a single DF relaying cooperative system has been investigated. Then, in [9], the error performance of SSD technique has been studied in multi-relay scenarios. However, the aforementioned works have used loose bounds to determine the performance of E2E error probability with arbitrary constellation. Furthermore, as far as we know that there have been no analytical results on the error probability of TDBC protocol with SSD in the literature.

In this paper, we propose a novel scheme that combines SSD technique with TDBC protocol without incurring additional complexity. We first obtain a close-form expression for the E2E error probability with non-uniform constellation. Then, in a way to minimizing the E2E error probability, the optimal rotation angles are found at different values of signal-to-noise ratio (SNR). Finally, we verify the theoretical analysis through Monte Carlo simulations.

II. System Description

In this section, we first present the system model, and then describe the TDBC protocol with SSD technique.

A. System Model

We consider a two-way relay network scheme with two end-sources and one relay. In this scheme, two end-sources accomplish exchange of information through a decode-and-forward relay over three time slots. We denote by A, B and R, the first source, the second source and the relay, respectively. Each terminal has a single antenna for both transmission and reception and operates in a half-duplex mode. Assuming that the channels are reciprocal and undergoes independent not identical distributed Rayleigh fading, the complex Gaussian channel gains of the links $A \leftrightarrow R$, $B \leftrightarrow R$ and $A \leftrightarrow B$ are denoted by $h_{A,R} \sim \mathcal{N}(0, \Omega_{A,R})$, $h_{B,R} \sim \mathcal{N}(0, \Omega_{B,R})$ and $h_{A,B} \sim \mathcal{N}(0, \Omega_{A,B})$. In addition, the additive white Gaussian noise (AWGN) terms of all links are assumed to have zero-mean and equal variance (N_0).

B. TDBC Protocol with Signal Space Diversity

In the previously discussed two-way relay scheme, we consider TDBC protocol and SSD technique jointly. In the considered SSD two-way relay scheme, original symbols are rotated by a certain angle before being transmitted from both source A and source B, and then source A and source B cooperate with relay R to send the real and imaginary parts of the rotated symbols.

In the two-dimensional signal space, there exist rotations in which the in-phase component and the quadrature component of the transmitted signal carry enough information to uniquely represent the original signal [7]. Let χ be a constellation generated by applying a transformation Θ to an ordinary constellation shown in Fig. 1. Hence, the transformation Θ is given as

$$\Theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \tag{1}$$

where θ is the rotation angle in two-dimensional signal space.

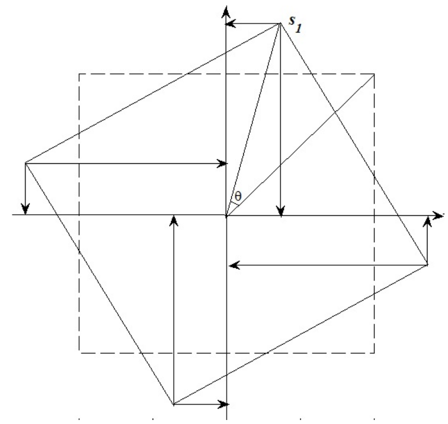

Fig. 1. A constellation generated by applying a transformation Θ to QPSK constellation.

1) First time slot operation: Let $\boldsymbol{s}^A = (s_1^A; s_2^A)$ be a pair of signal points from the original constellation (i.e., $s_1^A; s_2^A \in \chi$), that corresponds to the source A message. Hence, $s_1^A = \Re\{s_1^A\} + j\Im\{s_1^A\}$ and $s_2^A = \Re\{s_2^A\} + j\Im\{s_2^A\}$, where $\Re\{.\}$ and $\Im\{.\}$ are the in-phase and quadrature components of the corresponding signal points, respectively. The new constellation point that will be sent from source A is formed by interleaving the components of s_1^A and s_2^A as follows:

$$\lambda_A = \Re\{s_1^A\} + j\Im\{s_2^A\}. \tag{2}$$

Clearly, λ_A belongs to the expanded constellation Λ defined as follows:

$$\Lambda = \Re\{\chi\} \times \Im\{\chi\}, \tag{3}$$

where $\times$ denotes the Cartesian product of two sets. It is noted that each member of the expanded constellation consists of two components each of which uniquely identifies a particular member of χ. Hence, decoding a member of the expanded constellation results in decoding two different members of the original constellation (e.g., s_1^A and s_2^A).

Thus, the received signals in the first time slot at source B and the relay R can be written as

$$y_{A\to B} = h_{A,B}\sqrt{E_A}\lambda_A + n_B, \tag{4}$$

$$y_{A\to R} = h_{A,R}\sqrt{E_A}\lambda_A + n_R, \tag{5}$$

where E_A denotes the transmit source power.

2) Second time slot operation: Let $\boldsymbol{s}^B = (s_1^B; s_2^B)$ be a pair of signal points from the original constellation (i.e., $s_1^B; s_2^B \in \chi$), that corresponds to the source B message. Similarly, the new constellation point that will be sent from source B in this time slot is formed by interleaving the components of s_1^B and s_2^B as follows:

$$\lambda_B = \Re\{s_1^B\} + j\Im\{s_2^B\}. \tag{6}$$

Thus, the received signals in the second time slot at source A and the relay R can be written as

$$y_{B\to A} = h_{A,B}\sqrt{E_B}\lambda_B + n_A, \tag{7}$$

$$y_{B\to R} = h_{B,R}\sqrt{E_B}\lambda_B + \tilde{n}_R, \tag{8}$$

where E_B denotes the transmit source power.

3) Third time slot operation: If relay R detects both signals correctly, it forwards the processed signals received in the previous time slots to both sources A and B. Otherwise, relay R remains silent. It is presumed that error detection coding is utilized to detect errors at relay R. Assuming correct detection, the new constellation point is formed by interleaving the components of s_1^A and s_2^A and s_1^B and s_2^B as follows:

$$\lambda_R = \Re\{s_2^A\} + j\Im\{s_1^A\} + \Re\{s_2^B\} + j\Im\{s_1^B\}. \tag{9}$$

Therefore, the received signals at both sources A and B can be written as

$$\hat{y}_{R\to A} = \sqrt{E_R} h_{A,R} \lambda_R + \tilde{n}_A, \tag{10}$$

$$\hat{y}_{R\to B} = \sqrt{E_R} h_{B,R} \lambda_R + \tilde{n}_B, \tag{11}$$

where E_R denotes the transmit relay power.

Since sources A and B know their data, it can be cancelled out, and thus the modified received signals at sources A and B can be expressed as

$$y_{R\to A} = \sqrt{E_R} h_{A,R} \left[\Re\{s_2^B\} + j\Im\{s_1^B\}\right] + \tilde{n}_A, \tag{12}$$

$$y_{R\to B} = \sqrt{E_R} h_{B,R} \left[\Re\{s_2^A\} + j\Im\{s_1^A\}\right] + \tilde{n}_B. \tag{13}$$

Considering the direct and the cooperative links, the received signals at source B from the first and third time slots can be written as

$$y_{A\to B} = \sqrt{E_A} h_{A,B} \left[\Re\{s_1^A\} + j\Im\{s_2^A\}\right] + n_B, \tag{14}$$

$$y_{R\to B} = \sqrt{E_R} h_{B,R} \left[\Re\{s_2^A\} + j\Im\{s_1^A\}\right] + \tilde{n}_B. \tag{15}$$

By comparing the received signals at source B, it is observed that the received signal from relay R contains components of the original signal that are not included in the received signal from source A. Thus, from source B point of view, different components of each member of the original signal (i.e. s_1^A and s_2^A), are affected by independent channel fading.

To detect the original message, source B reorders the received components so that the corresponding components of each signal point in $\boldsymbol{s}^A$ join together. Let $\boldsymbol{r}^{A\to B} = (r_1^{A\to B}; r_2^{A\to B}; r_3^{A\to B}; r_4^{A\to B})$ be source B's signal after reordering the received components [9]. Hence,

$$r_1^{A\to B} = \Re\{h_{A,B}^* y_{A\to B}\} = \sqrt{E_A}|h_{A,B}|^2 \Re\{s_1^A\} + \hat{n}_{B_1}, \tag{16a}$$
$$r_2^{A\to B} = \Im\{h_{A,B}^* y_{A\to B}\} = \sqrt{E_A}|h_{A,B}|^2 \Im\{s_2^A\} + \hat{n}_{B_2}, \tag{16b}$$
$$r_3^{A\to B} = \Re\{h_{B,R}^* y_{R\to B}\} = \sqrt{E_R}|h_{B,R}|^2 \Re\{s_2^A\} + \hat{n}_{B_3}, \tag{16c}$$
$$r_4^{A\to B} = \Im\{h_{B,R}^* y_{R\to B}\} = \sqrt{E_R}|h_{B,R}|^2 \Im\{s_1^A\} + \hat{n}_{B_4}, \tag{16d}$$

where $\hat{n}_{B_1}$, $\hat{n}_{B_2}$, $\hat{n}_{B_3}$, and $\hat{n}_{B_4}$ are additive noise components.

Lastly, source B applies a maximum likelihood (ML) detector on the reordered signal to detect the source message. Hence, the source B's maximum likelihood decision rule can be given as

$$\hat{s}_1^A = \arg\min_{s^A \in \chi} \left[\left|r_1^{A\to B} - \sqrt{E_A}|h_{A,B}|^2 \Re\{s^A\}\right|^2 + \left|r_4^{A\to B} - \sqrt{E_R}|h_{B,R}|^2 \Im\{s^A\}\right|^2\right], \tag{17}$$

$$\hat{s}_2^A = \arg\min_{s^A \in \chi} \left[\left|r_3^{A\to B} - \sqrt{E_R}|h_{B,R}|^2 \Re\{s^A\}\right|^2 + \left|r_2^{A\to B} - \sqrt{E_A}|h_{A,B}|^2 \Im\{s^A\}\right|^2\right]. \tag{18}$$

Following the same procedure, the detection at source A can be obtained exactly in the same way.

For the detection at relay R, it is noted that the signal received from source A in the first time slot and from source B in the second time slot consists of signal points from the expanded signal constellation that uniquely represent the source A and B messages. Hence, relay R can detect both received signals and generate the signal that should be sent in the third time slot based on (9).

Similar to the detection at the sources, relay R applies ML detector in order to detect s_1^A and s_2^A from λ_A as follows:

$$\hat{\lambda}_A = \arg\min_{\lambda_A \in \Lambda} \left[y_{A\to R} - \sqrt{E_A} h_{A,R} \lambda_A\right]. \tag{19}$$

Knowing λ_A leads to know s_1^A and s_2^A. In addition, relay R uses ML detector in order to detect s_1^B and s_2^B from λ_B as follows:

$$\hat{\lambda}_B = \arg\min_{\lambda_B \in \Lambda} \left[y_{B\to R} - \sqrt{E_B} h_{B,R} \lambda_B\right]. \tag{20}$$

Knowing λ_B leads to know s_1^B and s_2^B.

Finally, we should mention that there is a scenario in which the relay cannot detect the source A and B messages, correctly. In this situation, both sources rely on the direct link only to detect both signals using the expanding constellation. So, the ML detection rule at source B can be written as

$$\hat{\lambda}_A = \arg\min_{\lambda_A \in \Lambda} \left[y_{A\to B} - \sqrt{E_A} h_{A,B} \lambda_A\right]. \tag{21}$$

Knowing λ_A leads to know s_1^A and s_2^A. The same rule can be applied at source A. Note that the proposed protocol doubles the spectral efficiency in the system by sending four symbols in three time slots rather than two symbols in three time slots.

III. Error Performance Analysis

Here we aim to derive the E2E error probability for the system model described in the preceding section. Specifically, we obtain symbol error rate (SER) expression at i-th source, $i \in \{A,\ B\}$. The average SER at i-th source can be expressed as

$$\overline{P}_i(e) = P_{off} P_{direct}^{i\to j} + (1 - P_{off}) \overline{P}_i(e|R),\ i \neq j,\ j \in \{A,\ B\}, \tag{22}$$

where P_{off} denotes the probability of the scenario when relay R does not decode both sources' symbols, i.e., from i-th and j-th sources, correctly, $P_{direct}^{i\to j}$ gives the average SER for transmission between i-th and j-th sources, and $\overline{P}_i(e|R)$ corresponds to average SER at i-th source when the relay detect both the sources' symbols correctly. Hence, TDBC protocol with SSD technique can be seen as a combination of two possible cases depending on the role of relay R; it is either active or silent.

A. Computation of $P_{direct}^{i\to j}$

In a direct link scenario, e.g., $A \to B$, source A transmits the symbols selected from expanded M^2-ary constellation Λ to source B with the transmit power E_A. Thus, the received instantaneous SNR at source B can be given by $\gamma_B = E_A|h_{A,B}|^2/N_0$. Then, the instantaneous SER expression at source B can be written in the form of a function of γ_B:

$$P_{direct}^{A\to B}(\gamma_B) = \sum_{k=0}^{M^2-1} \sum_{\substack{l=0 \\ l\neq k}}^{M^2-1} P_k \Pr\left\{ y_{A\to B} \in Z_{\Lambda(l)}(\gamma_B) \,|\, \lambda_A = \Lambda(k) \right\}, \quad (23)$$

where P_k is the probability of being transmitted k-th symbol, $\Lambda(k)$ is the k-th symbol in the expanded constellation and $Z_{\Lambda(k)}$ is the decision region of the symbol of $\Lambda(k)$. Note that $P_{direct}^{A\to B}$ consists of the sum all possibilities of that the transmitted $\Lambda(l)$ symbol drops into $Z_{\Lambda(k)}$.

For equiprobable signalling case, $P_k = 1/M^2$, $Z_{\Lambda(k)}$ can be expressed as [10]

$$Z_{\Lambda(k)}(\gamma_B) = \left\{ \begin{array}{l} y_{A\to B} : L_{k,l}\left(\frac{y_{A\to B}}{h_{A,B}}\right) < 0, \\ l\neq k, l = 0,\ldots, M^2-1 \end{array} \right\}, \quad (24)$$

where

$$L_{k,l}(y_{A\to B}) = \Re\left[y_{A\to B} c_{k,l}^*\right] + d_{k,l}, \quad (25a)$$
$$c_{k,l} = \Lambda(l) - \Lambda(k), \quad (25b)$$
$$d_{k,l} = \frac{1}{2}\left[|\Lambda(k)|^2 - |\Lambda(l)|^2\right]. \quad (25c)$$

In the upcoming steps, $L_{k,l}(y_{A\to B})$ is replaced with $\bar{L}_{k,l}(y_{A\to B})$ which is a normalized version with respect to $|c_{k,l}|$. By utilizing geometric trajectory on 2-D space shown in [11], $\Pr\left\{ y_{A\to B} \in Z_{\Lambda(l)}(\gamma_B) \,|\, \lambda_A = \Lambda(k) \right\}$ can be formulated as

$$\begin{aligned} &\Pr\left\{ y_{A\to B} \in Z_{\Lambda(l)}(\gamma_B) \,|\, \lambda_A = \Lambda(k) \right\} \\ &= \sum_{t=1}^{T_l} \pm \mathrm{Q}\left(\pm L_{l,p_t}(\Lambda(k))\sqrt{2\gamma_B}, \pm L_{l,p_{t+1}}(\Lambda(k))\sqrt{2\gamma_B}; \right. \\ &\quad \left. \pm\Re\left[c_{l,p_t}, c_{l,p_t}^*\right]\right), \end{aligned} \quad (26)$$

where T_l denotes the lines bounding the decision region $Z_{\Lambda(l)}$, the neighbour decision regions of the symbol $\Lambda(l)$ are expressed by $\Lambda(p_t)$ and $\Lambda(p_{t+1})$ and $\mathrm{Q}(\cdot,\cdot;\cdot)$ is complementary CDF of a bivariate Gaussian (BVG) variable [12]. The detailed information about the sign $\pm$ and summation terms can be found in [13].

Note that all steps during analysis can be also applied for BER expression by just adding $\frac{1}{m} d_H(\Lambda(l), \Lambda(k))$ in (23), where $d_H(.)$ is the Hamming distance.

The average $\overline{P}_{direct}^{A\to B}$ can be found by taking average of (23) over γ_B as

$$\begin{aligned} \overline{P}_{direct}^{A\to B} &= \mathbb{E}_{\gamma_B}\left[P_{direct}^{A\to B}(\gamma_B)\right] \\ &= \int_0^\infty P_{direct}^{A\to B}(\gamma_B) f_{\gamma_B}(\gamma_B)\, d\gamma_B \\ &= \sum_{k=0}^{M^2-1} \sum_{\substack{l=0 \\ l\neq k}}^{M^2-1} \sum_{t=1}^{T_l} \pm \int_0^\infty \mathrm{Q}(a,b;\rho) f_{\gamma_B}(\gamma_B)\, d\gamma_B, \end{aligned} \quad (27)$$

where $a = \pm\bar{L}_{l,p_t}(\Lambda(k))\sqrt{2\gamma_B}$, $b = \pm\bar{L}_{l,p_{t+1}}(\Lambda(k))\sqrt{2\gamma_B}$, $\rho = \pm\Re\left[c_{l,p_t}, c_{l,p_t}^*\right]$, $\mathbb{E}$ denotes the expectation operator, and also the probability density function (PDF) of γ_B is denoted by $f_{\gamma_B}(\gamma_B)$. Since the channels are assumed to experience Rayleigh fading, $f_{\gamma_B}(\gamma_B)$ can be given as $f_{\gamma_B}(\gamma_B) = \frac{1}{\bar{\gamma}_B}\exp\left(-\frac{\gamma_B}{\bar{\gamma}_B}\right)$, where the average SNR at source B is $\bar{\gamma}_B = E_A\Omega_{AB}/N_0$.

The resulting average $\overline{P}_{direct}^{A\to B}$ can be expressed as

$$\begin{aligned} \overline{P}_{direct}^{A\to B} &= \sum_{k=0}^{M^2-1} \sum_{\substack{l=0 \\ l\neq k}}^{M^2-1} \sum_{t=1}^{T_l} \frac{1}{2\pi} \int_0^{\upsilon(a,b,\rho)} d\theta \left\{ \int_0^\infty d\gamma_B \right. \\ &\quad \left. \exp\left(-\frac{a^2}{2\sin^2\theta}\right) \frac{1}{\bar{\gamma}_B} \exp\left(-\frac{\gamma_B}{\bar{\gamma}_B}\right) \right\} \\ &\quad + \frac{1}{2\pi} \int_0^{\upsilon(b,a,\rho)} d\theta \left\{ \int_0^\infty d\gamma_B \exp\left(-\frac{b^2}{2\sin^2\theta}\right) \frac{1}{\bar{\gamma}_B} \exp\left(-\frac{\gamma_B}{\bar{\gamma}_B}\right) \right\}, \\ &= \sum_{k=0}^{M^2-1} \sum_{\substack{l=0 \\ l\neq k}}^{M^2-1} \sum_{t=1}^{T_l} \upsilon(\alpha_1,\alpha_2,\rho) + \upsilon(\alpha_2,\alpha_1,\rho) \\ &\quad - \frac{\alpha_1 \tan^{-1}\left(\sqrt{\frac{\bar{\gamma}_B\alpha_1^2+2}{\bar{\gamma}_B}}\tan(\upsilon(\alpha_1,\alpha_2,\rho))\right)}{\sqrt{\frac{\bar{\gamma}_B\alpha_1^2+2}{\bar{\gamma}_B}}} \\ &\quad - \frac{\alpha_2 \tan^{-1}\left(\sqrt{\frac{\bar{\gamma}_B\alpha_2^2+2}{\bar{\gamma}_B}}\tan(\upsilon(\alpha_2,\alpha_1,\rho))\right)}{\sqrt{\frac{\bar{\gamma}_B\alpha_2^2+2}{\bar{\gamma}_B}}}, \end{aligned} \quad (28)$$

where $\tan^{-1}(\cdot)$ represents the inverse of the tangent function, $\alpha_1 = \sqrt{2}L_{l,p_t}(\Lambda(k))$, $\alpha_2 = \sqrt{2}L_{l,p_t+1}(\Lambda(k))$ and $\upsilon(\alpha_1,\alpha_2,\rho)$ is defined by

$$\upsilon(\alpha_1,\alpha_2,\rho) = \begin{cases} \tan^{-1}\left(\frac{\alpha_1\sqrt{1-\rho^2}}{\alpha_2-\rho\alpha_1}\right), & \rho\alpha_1 \leq \alpha_2, \\ \tan^{-1}\left(\frac{\alpha_1\sqrt{1-\rho^2}}{\alpha_2-\rho\alpha_1}\right) + \pi, & \alpha_2 < \rho\alpha_1, \\ \tan^{-1}\left(\frac{1+\rho}{1-\rho}\right), & \alpha_1 = 0,\ \alpha_2 = 0. \end{cases} \quad (29)$$

B. Computation of P_{off}

The probability that the relay does not decode both sources A and B correctly and remains silent in third time slot is expressed as

$$P_{off} = 1 - \left(1 - \overline{P}^{A\to R}_{direct}\right)\left(1 - \overline{P}^{B\to R}_{direct}\right), \tag{30}$$

where $\overline{P}^{A\to R}_{direct}$ and $\overline{P}^{B\to R}_{direct}$ can be obtained using (28).

C. Computation of $\overline{P}_i\left(e\,|R\right)$

In a cooperative scenario, the average SER of the cooperative link, e.g., $A \to R \to B$, can be given as

$$\begin{aligned}\overline{P}_B\left(e\,|R\right) &= \mathbb{E}_{\gamma_{coop}}\left[P_B\left(e\,|R\right)\right] \\ &\approx \sum_{k=0}^{M-1}\sum_{\substack{l=0\\ l\neq k}}^{M-1}\sum_{t=1}^{T_l} \pm \int_0^\infty \mathrm{Q}\left(a,b;\rho\right) f_{\gamma_{coop}}\left(\gamma\right)d\gamma,\end{aligned} \tag{31}$$

where $a = \pm\bar{L}_{l,p_t}\left(\chi\left(k\right)\right)\sqrt{2\gamma_B}$, $b = \pm\bar{L}_{l,p_{t+1}}\left(\,\chi\left(k\right)\right)\sqrt{2\gamma_B}$, $\rho = \pm\Re\left[c_{l,p_t}, c^*_{l,p_t}\right]$, $\gamma_{coop} = \gamma_{R,B} + \gamma_{A,B}$ and also $f_{\gamma_{coop}}\left(\gamma\right)$ is the PDF of γ_{coop}, which can be expressed as [14]

$$\begin{aligned}f_{\gamma_{coop}}\left(\gamma\right) &= \frac{\bar{\gamma}/\bar{\gamma}_{A,R}}{\bar{\gamma}_{R,B}-\bar{\gamma}}\left(\frac{\bar{\gamma}_{R,B}}{\bar{\gamma}_{A,B}}\left[\exp\left(\frac{-\gamma}{\bar{\gamma}_{A,B}}\right)-\exp\left(\frac{-\gamma}{\bar{\gamma}_{R,B}}\right)\right]\right) \\ &- \frac{\bar{\gamma}}{\bar{\gamma}_{A,B}-\bar{\gamma}}\left(\frac{\bar{\gamma}_{R,B}}{\bar{\gamma}_{A,B}}\left[\exp\left(\frac{-\gamma}{\bar{\gamma}_{A,B}}\right)-\exp\left(\frac{-\gamma}{\bar{\gamma}_{R,B}}\right)\right]\right) \\ &+ \frac{\bar{\gamma}/\bar{\gamma}_{R,B}}{\bar{\gamma}_{A,B}-\bar{\gamma}}\left[\exp\left(\frac{-\gamma}{\bar{\gamma}_{A,B}}\right)-\exp\left(\frac{-\gamma}{\bar{\gamma}}\right)\right],\end{aligned} \tag{32}$$

where $\bar{\gamma} = \bar{\gamma}_{A,R}\bar{\gamma}_{R,B}/(\bar{\gamma}_{A,R}+\bar{\gamma}_{R,B})$.

The resulting average $\overline{P}_B\left(e\,|R\right)$ can be expressed as

$$\overline{P}_B\left(e\,|R\right) \approx \sum_{k=0}^{M-1}\sum_{\substack{l=0\\ l\neq k}}^{M-1}\sum_{t=1}^{T_l} \pm\left[A\left(\upsilon_1,\alpha_1\right)+A\left(\upsilon_2,\alpha_2\right)\right], \tag{33}$$

where $\alpha_1 = \sqrt{2}L_{l,p_t}\left(\chi(k)\right)$, $\alpha_2 = \sqrt{2}L_{l,p_t+1}\left(\chi(k)\right)$, $\upsilon_1 = \upsilon\left(\alpha_1,\alpha_2,\rho\right)$, $\upsilon_2 = \upsilon\left(\alpha_2,\alpha_1,\rho\right)$, and an auxiliary function $A\left(\upsilon_k,\alpha_k\right)$ is given by (34), $k\in\{1,\ 2\}$, on the top of the next page.

By substituting (28), (30) and (33) into (22), the final closed-form expression for the average SER, $\overline{P}_i(e)$, can be obtained.

D. Optimization of Rotation Angle

In the SSD technique, the optimum choice of the rotation angle is an important factor on the system performance, since it determines the distribution of the constellation symbols and the distance between constellation symbols in the expanded constellation Λ. Here we optimize the rotation angle, θ in order to minimize the E2E error probability. The optimum rotation angle can be found as

$$\theta_{opt} = \arg\min_{\theta\in(0^\circ,\ 45^\circ)} \overline{P}_i(e). \tag{35}$$

Since finding an analytical solution to the optimization problem is intractable, we resort to numerical optimization of the E2E error probability to obtain the optimal rotation angle. For this purpose, we have used Matlab optimization toolbox command "fmincon", which is designed to find the minimum of a given constrained nonlinear multivariable function.

IV. Simulation Results

This section presents some numerical results to demonstrate the performance of the TDBC protocol with SSD technique. Results are shown for source A of two-way relay network. However, all discussions provided hereafter apply for source B as well. We assume that the transmission power of all terminals is $E_A = E_B = E_R = E$.

In Fig. 2, the E2E error probability performance for different rotation angles is shown by considering the QPSK modulation, and the derived analytical expression are validated through Monte Carlo simulation results. As observed, our analytical results in excellent agreement with the simulation results and the performance of the system are significantly affected by the change of rotation angle. As benchmark, in Fig. 2, we provide a comparison against the conventional TDBC two-way relay system, called C-TDBC. For a fair comparison, 16-QAM modulation is considered for C-TDBC to achieve the same spectral efficiency over three time slots. From this figure, it can be seen that the proposed TDBC scheme (P-TDBC) with optimal rotation angles outperforms the C-TDBC over the entire range of SNRs due to achieving both signal space and spatial diversities. For instance, the achieved gain of P-TDBC in E/N_0 at SER of 10^{-3} is about 2 dB as compared to C-TDBC. In Table I, we presents the optimum values of θ_{opt} in degree for various values of E/N_0, which we use to generate the P-TDBC curve with θ_{opt}.

TABLE I
The optimum values of rotation angle

E/N_0(dB)	0	5	10	15	20	25	30
θ_{opt}(deg)	34.96	30.28	28.52	27.91	27.66	27.56	27.53

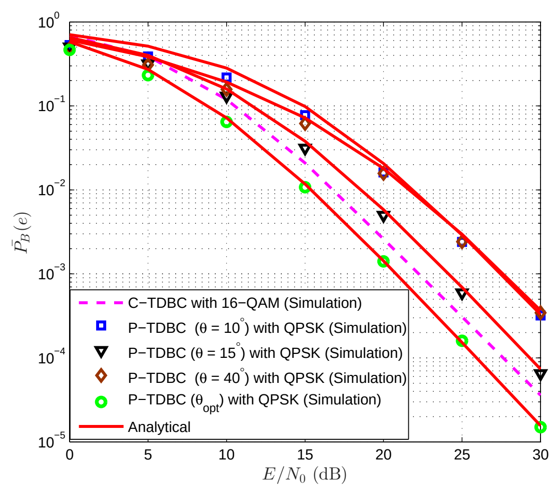

Fig. 2. SER performance of P-TDBC in compared to C-TDBC protocol.

In the following, we further discuss the effect of the rotation angle on the performance of the system. In Fig. 3, the E2E error probability is plotted versus the rotation angle at given different SNR values. It can be seen that the optimal rotation

$$
\begin{aligned}
A(\upsilon_k, \alpha_k) = {} & \bar{\gamma} \csc(\upsilon_k)^6 \Bigg(\alpha_k (1 - 2\cos\upsilon_k + \alpha_k a^2 \bar{\gamma}_{A,B}) \bar{\gamma}_{A,B} \bar{\gamma}_{R,B} (1 - 2\cos\upsilon_k + \alpha_k a^2 \bar{\gamma}_{R,B})(1 - 2\cos\upsilon_k + \alpha_k a^2 \bar{\gamma}) \\
& \times \Bigg\{ \upsilon_k {\alpha_k}^{-1} \sqrt{{\alpha_k}^2 + \frac{2}{\bar{\gamma}_{A,B}}} \bar{\gamma}_{A,B}^{\frac{3}{2}} \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{R,B}} \left(\bar{\gamma}_{R,B} - \bar{\gamma}\right) \sqrt{2 + {\alpha_k}^2 \bar{\gamma}} + \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{i,j}} \\
& \qquad \times \left[\upsilon_k {\alpha_k}^{-1} \left(\bar{\gamma}_{A,B} - \bar{\gamma}_{R,B}\right) \sqrt{{\alpha_k}^2 + \frac{2}{\bar{\gamma}}} \bar{\gamma}^{\frac{3}{2}} + \upsilon_k {\alpha_k}^{-1} \sqrt{{\alpha_k}^2 + \frac{2}{\bar{\gamma}_{R,B}}} \bar{\gamma}_{R,B}^{\frac{3}{2}} \left(\bar{\gamma} - \bar{\gamma}_{A,B}\right) \sqrt{2 + {\alpha_k}^2 \bar{\gamma}} \right] \Bigg\} \\
& + \left(-1 + 2\cos(\upsilon_k) + {\alpha_k}^2 \bar{\gamma}_{A,B}\right) \left(-1 + 2\cos(\upsilon_k) + {\alpha_k}^2 \bar{\gamma}_{R,B}\right) \left(\bar{\gamma}_{A,R} + \bar{\gamma}_{R,B}\right) \left(-1 + 2\cos(\upsilon_k) + {\alpha_k}^2 \bar{\gamma}\right) \\
& \times \Bigg\{ \upsilon_k \sqrt{{\alpha_k}^2 + \frac{2}{\bar{\gamma}_{A,B}}} \bar{\gamma}_{A,B}^{\frac{5}{2}} \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{R,B}} \left(\bar{\gamma}_{R,B} - \bar{\gamma}\right) \sqrt{2 + {\alpha_k}^2 \bar{\gamma}} + \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{A,B}} \Bigg[\upsilon_k \sqrt{{\alpha_k}^2 + \frac{2}{\bar{\gamma}_{R,B}}} \bar{\gamma}_{R,B}^{\frac{5}{2}} \left(-\bar{\gamma}_{R,B} + \bar{\gamma}\right) \\
& \times \sqrt{2 + {\alpha_k}^2 \bar{\gamma}} + \left(-\bar{\gamma}_{A,B} + \bar{\gamma}_{R,B}\right) \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{R,B}} - \upsilon_k \sqrt{{\alpha_k}^2 - \frac{2}{\bar{\gamma}}} \bar{\gamma}^{\frac{5}{2}} - \upsilon_k \left(-\bar{\gamma}_{A,B} + \bar{\gamma}\right) \left(-\bar{\gamma}_{R,B} + \bar{\gamma}\right) \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{R,B}} \Bigg] \Bigg\} \Bigg) \\
& \times \Big[2\pi \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{A,B}} \left(2 + {\alpha_k}^2 \csc^2(\upsilon_k) \bar{\gamma}_{A,B}\right) \bar{\gamma}_{A,R} \left(\bar{\gamma}_{A,B} - \bar{\gamma}_{R,B}\right) \bar{\gamma}_{R,B} \sqrt{2 + {\alpha_k}^2 \bar{\gamma}_{R,B}} \\
& \qquad \times \left(2 + {\alpha_k}^2 \csc^2(\upsilon_k) \bar{\gamma}_{R,B}\right) \left(\bar{\gamma}_{A,B} - \bar{\gamma}\right) \left(\bar{\gamma}_{R,B} - \bar{\gamma}\right) \sqrt{2 + {\alpha_k}^2 \bar{\gamma}} \left(2 + {\alpha_k}^2 \csc^2(\upsilon_k) \bar{\gamma}\right) \Big]^{-1}.
\end{aligned}
\tag{34}
$$

angle differs in terms of chosen SNR value and in high SNR values, the system performance is more sensitive to the change in the rotation angle.

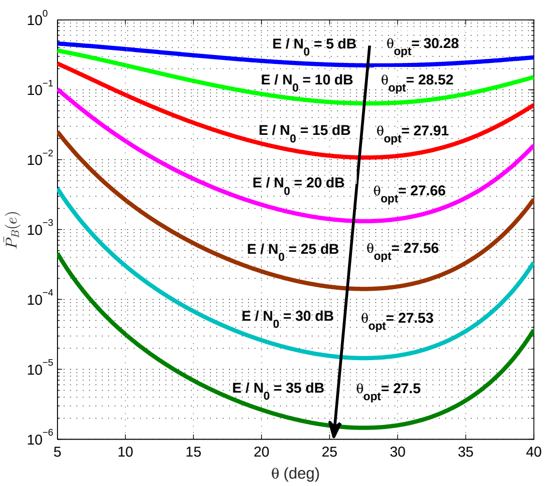

Fig. 3. The effect of choosing different rotation angles θ on the system performance at the different E/N_0 values.

V. Conclusions

This paper presented the study on the E2E error probability performance of three-time slot TDBC protocol with SSD technique in two-way relay systems. We derived the closed-form expression for the E2E error probability with arbitrary modulation. Based on the derived the E2E error probability expression, the optimal angle rotation was also discussed. We showed the sensitivity of the system to variations of the rotation angle; it is more noticeable, especially at high SNRs. Our analytical results implied that, with using SSD technique, the system can benefit from SSD in addition to spatial diversity in two-way cooperative systems. However, for full utilization of SSD, it is important to choose the optimal rotation angles.

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