The Impact of User Spatial Heterogeneity in Heterogeneous Cellular Networks

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***Abstract*—The spatial distribution of users in cellular networks plays an important role on network performance. It is becoming increasingly common in the recent literature to model the user locations according to a homogeneous Poisson point process, yet users are often spatially clustered in reality. In this paper, we investigate the impact of user spatial heterogeneity in downlink cellular networks, including macro-only networks and small-cell enhanced heterogeneous cellular networks (HCNs). In the generic point process introduced in this paper, the degree of user spatial heterogeneity is scalable, smoothly controlled, and is measured by a non-negative real number. Numerical results show that the network performance deteriorates significantly when user distribution becomes more heterogeneous while remaining uncorrelated with the base station locations. However, by deploying small-cells in the centers of the user hot-spots found by cluster analysis on non-uniform user points, we show that HCNs can benefit from a certain degree of user spatial heterogeneity.**

Index Terms*—Performance Evaluation, Stochastic Geometry, Heterogeneous Cellular Networks, small-cells, Cluster Analysis.

I. Introduction

The performance of wireless networks depends highly on their spatial configuration, not only because the signal-to-interference-plus-noise ratio (SINR) is related to transmitter-receiver distance, but also because the traffic load in spatial domain influences the overall resource utilization, and hence, network performance. In the context of heterogeneous cellular networks (HCNs), the traffic load plays a more significant role in user throughput compared to the commonly used SINR metric [1]. Recently, stochastic geometry has become increasingly popular in modeling the spatial distribution of the network entities. The locations of the network entities are abstracted to a point process (PP) based on their properties. The PPs that are commonly used in wireless networks are i) Poisson point process (PPP), ii) hard core point process (HCPP), and iii) Poisson cluster process (PCP). The PPP is the most popular PP due to its simplicity and tractability [2]. However, the research community mainly focuses on modeling the locations of base stations (BSs) rather than the users (mobile devices) [2], [3].

In the majority of the papers in wireless networks, the user spatial distribution is assumed to be random and uniform (homogeneous PPP) [3], and often with a fixed number of users, which becomes a conditional PPP, or equivalently, binomial PP (BPP) [4]. When the PPP model is used, the downlink analysis is performed at a typical user at origin according to Slivnyak's theorem [5], which states that the statistics seen from a PPP is independent from the test location. In recent years, some papers have introduced heterogeneity to user spatial distribution. For example, in the model used in [6], the user density decreases linearly with respect to the distance from the BS up to a certain distance beyond which users are uniformly distributed to the rest of the cell area. In [7], user locations are modeled by a PCP. A parent PP Φ uniformly spreads N_c cluster heads over the coverage region of a macrocell, and then users are dropped randomly and uniformly within a certain radius of each cluster head. Given a fixed total number of users, the degree of user spatial heterogeneity is controlled by changing the number of cluster heads. This method brings spatial heterogeneity to the user distribution, but due to the integer nature of the number of cluster heads, the spatial heterogeneity cannot be controlled in a continuous manner. In [8], both user and BS locations are modeled by homogeneous PPPs with different intensities in the first step, and heterogeneous user distribution is obtained by conditional thinning of BSs and the corresponding users in the Voronoi cells of BSs. This method facilitates the deduction of analytical expressions, yet the generated user distributions may not be entirely realistic because of the absence of users in the thinned areas and the identical intensity of users in the remaining areas.

The impact of non-uniform and BS-uncorrelated user distribution in a cellular CDMA network has been investigated in [9], indicating that uniform distribution can lead to an overestimation of the system capacity. In [6], the authors have shown that the performance enhances when users are concentrating around the BSs in WCDMA networks.

The distinguish feature of this paper is the introduction of a user spatial model that covers user distribution from homogeneous to highly heterogeneous. Moreover, based on the performance impact of user spatial heterogeneity on downlink network, a strategy for deploying small-cells in HCNs is developed. By using the statistical measure introduced in our previous work [10], the degree of user spatial heterogeneity is evaluated with a non-negative real number C (to be defined in Section III). The measure introduced in [10] makes our evaluation based on dynamic user spatial heterogeneity possible to compare with other scientific works. Our contributions are summarized as follows:

- A doubly stochastic Poisson process (DSPP), also known as the Cox process, is used to model the user locations. With a single parameter, the spatial

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heterogeneity is controlled smoothly in a broad range from uniform (PPP) to extremely heterogeneous.

- The effect of user spatial heterogeneity (captured by C) on the performance of downlink cellular networks is obtained. We find that the network performance metrics deteriorate significantly with increasing C if the user locations are uncorrelated with the locations of the macro and small-cell BSs.
- Cluster analysis on the non-uniform user points is utilized to find the cluster centroids as the potential locations for small-cells. Simulation results show that the network performance can improve substantially with increasing C if we take advantage of user spatial heterogeneity to deploy small-cells in the appropriate locations.

The rest of this paper is organized as follows: Section II introduces the network model and the generation of heterogeneous user distribution. A scalar measure of spatial heterogeneity is provided in Section III. Then, locations of small-cells with an application of clustering algorithm is discussed in Section IV. Numerical analysis is presented next in Section V, and this paper is concluded with the remarks in Section VI.

II. System Setup

A. Network Model

The locations of macrocell sites are fixed and form a hexagonal grid layout as shown in Fig. 1. 19 sites, each with 3 cells, with inter-site distance (ISD) of 500 meters, are configured in the system. The wrap-around technique is applied in the simulations to eliminate the boundary effect. The HCNs consist of two tiers with small-cells (not shown in Fig. 1) overlaid on the same area of macrocells. The macrocells adopt directional antennas while small-cells use omni antennas. The distribution of small-cells is either according to a BPP or user-distribution related, which will be discussed in Section IV.

B. User Distribution

To obtain the spatial heterogeneity of user distribution, this paper adopts the Cox process, a generalization of the PPP. Instead of being constant as in PPP, the intensity in Cox is itself a stochastic process [5]. For example, in a homogeneous PPP with intensity λ, the number of points in a bounded Borel set $B \subset \mathbb{R}^2$ is Poisson distributed with mean λA_B, where A_B is the area of B. While in the Cox process, the number of points in B is Poisson distributed with a mean intensity $\bar{\Lambda} = \int_B \Lambda(s)\,ds, s \in B$, where $\Lambda(\cdot)$ is an intensity function. From the definition, and also as the name DSPP implies, Cox process brings a second layer of randomness to the Poisson process by generalizing the constant intensity λ into a intensity function $\Lambda(\cdot)$. By varying $\Lambda(\cdot)$, we get different kinds of Cox processes. A Cox process is called a log Gaussian Cox process (LGCP) if $\Lambda(s) = \exp\{Y(s)\}$, where $Y = \{Y(s) : s \in \mathbb{R}^2\}$ is a real-valued Gaussian process (i.e., the joint distribution of any finite vector $(Y(s_1), ..., Y(s_n))$ is Gaussian) [11]. The distribution of Y is specified by the mean $\mu = E(Y(s))$, the variance $\sigma^2 = Var(Y(s))$, and the covariance $COV(Y(s_i), Y(s_j))^{\dagger}$.

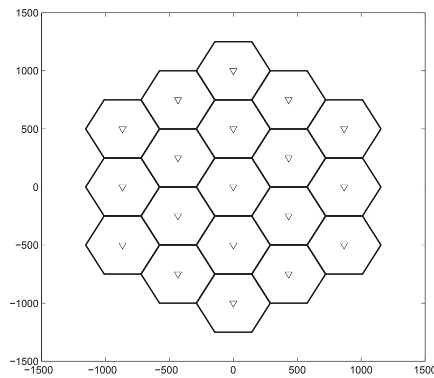

Fig. 1. Macrocell geometry with 19 sites and 57 cells. Macrocell sites are shown by triangles with each angle indicating the antenna boresight of the 3 cells in the same site.

In this paper, we assume $COV(Y(s_i), Y(s_j)) = 0$ for $i \neq j$, indicating no correlation within $\Lambda(\cdot)$. The distribution of $\Lambda(\cdot)$ is specified by the mean $m = \exp(\mu + \sigma^2/2)$ and the variance $v = \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$. Due to the exponential nature of the intensity function $\Lambda(\cdot)$, the LGCP provides a wide range of intensity values with a small variation in σ. When σ is equal to zero, Λ becomes constant, and the LGCP falls back to a homogeneous PPP. By increasing σ (and changing μ accordingly to keep the overall user intensity m constant), the intensity Λ becomes more fluctuating (higher v), resulting in higher spatial heterogeneity over the whole area. A realization of LGCP with different σ values is shown in Fig. 2. Note that σ can take any nonnegative real value continuously from 0 to infinity, which facilities the smooth control of the user spatial heterogeneity.

Implementation of LGCP involves two steps. First the Gaussian field is generated in the minimum square that contains all the 19 hexagons. We adopt the method in [11] by discretizing the square into tiles‡ and approximating the Gaussian process by the values of the corresponding Gaussian distribution on the tiles. Then the Gaussian field $\tilde{Y} = (\tilde{y}_{ij})_{(i,j \in I)}$ is obtained, where I represents all the tiles after discretization. In the second step, for the given Gaussian field $\tilde{Y}$, a homogeneous PPP with intensity $\tilde{\lambda} = \exp(\tilde{y}_{ij})$ is generated in each tile.

III. Degree of User Spatial Heterogeneity

Before we evaluate the network performance with respect to adjustable user spatial heterogeneity, we need a statistical measure to capture the degree of spatial heterogeneity.

This paper adopts the method introduced in our previous work [10], in which measures based on the Voronoi and Delaunay tessellations are proposed, and coefficient of

†The COV used here for covariance should not be confused with CoV used for coefficient of variation defined in Section III.

‡A coarse discretization results in a small sample size, and hence a decreased statistical variation, while a refined discretization may result in an unrealistic situation (e.g., many users squeezed in a small area) and higher computational complexity. For the numerical analysis in this paper, we found that it is sufficient to use 40×40 tiles (i.e., a total number of 1600 tiles for the square enclosing area).

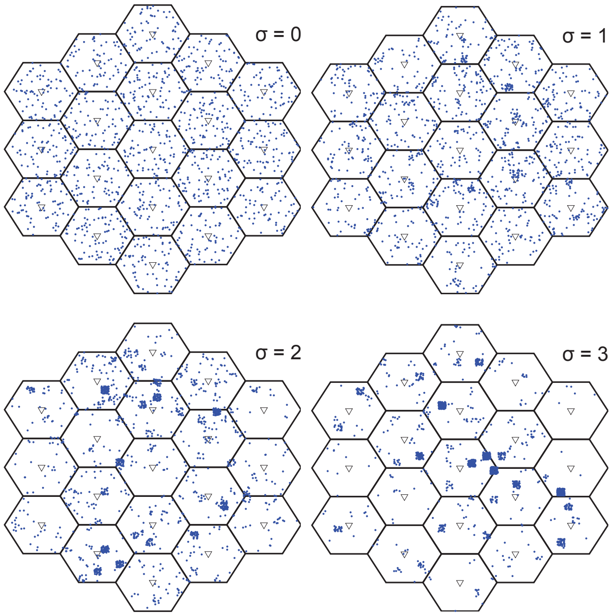

Fig. 2. An example of user distribution with different deviation in LGCP. The PPP is the special case of LGCP (when σ equals to zero). Each dot represents an active user in the system. With the measure discussed in Section III, the degree of spatial heterogeneity C equals to 1.00, 1.44, 2.70, 4.72, respectively.

variation (CoV), the normalized second-order statistic (the standard deviation divided by the mean), is suggested to be used to capture the main statistical properties of the measures. The statistics of PPP are used as the normalization factors to normalize those of the sub-Poisson processes and super-Poisson processes. Then, the user spatial heterogeneity can be represented by a non-negative real number C, the normalized CoV of different measures (e.g., the Voronoi cell area used in this paper). Based on the formulation, C is greater than 1 in super-Poisson processes, equal to 1 in PPP, and between 0 and 1 in sub-Poisson processes. The LGCP introduces more heterogeneity in PPP, so it is a super-Poisson point process in which PPP constitutes a special case (when $\sigma = 0$). For example, for the user distributions in Fig. 2, we first draw the Voronoi tessellations for the user points, and measure the area of each Voronoi cell, A_i. Then the CoV of A is calculated from the ratio of the standard deviation and the mean of A. Finally, the spatial heterogeneity level C is obtained from the normalized CoV [10].

IV. LOCATIONS OF SMALL-CELLS

With a spatially non-uniform user distribution, some areas of the network may have no user or only few users, and hence the resource of the BSs in those areas are either totally wasted or underused, while the so-called hot-spot areas may be congested with users inside suffering from low rates. One solution to this problem is deploying small-cells in the user hot-spots to offload traffic from macrocells. HCNs with small-cells overlaid on macrocells have intensely been researched in recent years, yet the distribution of small-cells is assumed to be a BPP in most of the papers. However, given an inhomogeneously distributed set of users as the case in this paper, how to find the hot-spots from the user distribution to deploy small-cells is a natural, yet non-trivial, question. This is especially true for the operator-planned picocells, which are deployed by the network operators based on the traffic distribution.

The cluster analysis technique groups data into clusters such that the objects in the same cluster are more similar to each other than to those in a different cluster. This is a main task in data mining and has played an important role in a wide variety of fields, including machine learning, image analysis, and information retrieval [12]. This paper uses the cluster analysis technique to find the user clusters as the potential locations for small-cells.

A. Basic k-means Algorithm

The k-means algorithm is one of the most popular clustering algorithms used in the cluster analysis. It is a prototype-based, partitional clustering technique that attempts to find a user-specified number of clusters (k) represented by their centroids. The centroids are the mean of the points that belong to the cluster. The basic algorithm [12] is described below.

Algorithm Basic k-means Algorithm

```
Select k points as initial centroids.
repeat
    Form k clusters by assigning each point to its closest centroid.
    Recompute the centroid of each cluster.
until Centroids do not change.
```

B. Preprocessing and Postprocessing

As we intend to use the centroids of the clusters to deploy small-cells, outlier users (isolated points) that are supposed to be served by macrocells are not taken into account. We apply preprocessing to eliminate the isolated points from affecting the locations of the centroids. A classification method of points in another density-based clustering algorithm, DBSCAN [12], is used. All points are defined as being a core point, a border point, or a noise point. Precisely, a point is a core point if the number of points within a certain radius of its neighborhood exceeds a threshold. A border point is a point that falls within the neighborhood of a core point. A noise point is any point that is neither a core point nor a border point. After the classification, the noise points (outlier users) are eliminated before applying the k-means algorithm.

The planned number of small-cells can be used as the value of k in the k-means algorithm. However, since the users may not naturally be clustered into k groups, the clusters that are obtained from the k-means algorithm may turn out to be too big for the coverage of a typical small-cell. In other words, a centroid may turn out to be in the middle of two or more natural user clusters when k is small. A simple yet effective way to avoid this situation is to enlarge k by splitting the clusters (by running clustering algorithm iteratively inside the cluster), a technique that is commonly used in the postprocessing for cluster analysis [12]. In our case, all clusters that have a larger radius than the typical coverage distance of the planned small-cells are split after the k-means clustering algorithm. After the postprocessing, k' (greater than k) clusters are obtained.

C. Selection Criteria

After postprocessing, the algorithm provides more than k clusters, potentially k' hot-spots. As only k small-cells are planned, a selection criterion is needed to choose k clusters from the k' clusters generated by the clustering algorithm.

A simple way is to determine the number of points n_i in each cluster i, and then to choose the top k out of k' clusters with respect to the number of points in them. However, when a cluster is close to a macrocell, a small-cell deployed in such a hot-spot will suffer substantial interference from the macrocell in a co-channel scenario. Even in a non-co-channel scenario, deploying small-cells close to the center of a macrocell is not as efficient as deploying them at the edge of a macrocell, as the latter improves the user spectral efficiency and provides more capacity at the same time.

In this paper, we use the ratio of the distance between a user and a macrocell, and that between a user and a potential small-cell, as a component in the objective function to select k hot-spots from k' clusters. We will also use the number of users criterion as the baseline method for comparison in Section V. Suppose that there are n_i users in a cluster i; for these n_i users, $d_j^{(m)}$ and $d_j^{(s)}$ represent the distances of user j to its closest macrocell, and to its cluster centroid (the location of the planned small-cell), respectively. The proposed selection criterion for cluster i is formulated as

$$U_i = \frac{1}{n_i} \sum_{j=1}^{n_i} \log \frac{d_j^{(m)}}{d_j^{(s)}}. \quad (1)$$

This objective function is basically derived from the Shannon formula and the path loss expression. The goal is to select k clusters that have maximum sum user rate, $\sum R_j$, which is proportional to the mean of the sum spectral efficiency $\frac{1}{n_i} \sum_j \log(1+SINR_j)$ when equal resource allocation is used. In our situation, it is reasonable to ignore the background noise and assume a high signal-to-interference-ratio (SIR) as users are close to the proposed small-cells. So the objective function becomes $\frac{1}{n_i} \sum_j \log(SIR_j)$. Let us assume that the dominant interferer, which is from the closest macrocell, is the only interferer, and that both signal power and interference power are calculated from the power-law based path-loss model with the same exponent. Then $\sum_j \log(SIR_j)$ becomes proportional to $\sum_j \log(d_j^{(m)}/d_j^{(s)})$, the sum-logarithm of the ratio of the user to closest-macro distance to the user to planned-small-cell distance. Since this objective function is formed under several assumptions, it is rather approximate. However, this is not a concern, because this objective function is not used for evaluation; it is rather used for ranking the candidates (clusters). If interference coordination between the macrocells and the small-cells within its coverage is used, the function can be adapted to have the second nearest neighbor macrocell as the main interference source.

V. Numerical Analysis

A. Simulation Setup

A static snapshot-based system-level simulator is used in this paper. The parameters used in the simulation are based on 3GPP LTE, but the conclusions reached in this paper are valid for any cellular network with a heterogeneous user spatial distribution and a small-cell enhanced network architecture. In this simulation, the case 6.2 in 3GPP release 9 [13] is the scenario used for HCNs, in which outdoor picocells are the small-cells that are layered on macrocells to cover hot-spots. Users associate with only one cell, macro or picocell, based on the strength of the received power. Biasing, also known as cell range expansion (CRE), is applied to picocells.

The channel follows the model 2 in [13] for both macrocells and outdoor picocells, in which a line-of-sight (LOS) and non-line-of-sight (NLOS) power-law path loss model is used. The downlink signal experiences log-normal shadowing, while the fast fading is assumed to be averaged out.

A user suffers interference from all the macrocells and picocells outside its own serving cell (which may be a macrocell or a picocell). As long as a cell (macro or pico) is serving one or more users, it is assumed that this cell is contributing to interference at the full transmit power level. However, if there are no users served by a particular cell, that cell is powered off to prevent unnecessary interference. Both macrocells and picocells share the same bandwidth, and no interference coordination or cancellation technique is used. Table I shows the key parameters used in the simulations.

TABLE I. Simulation Parameters

Parameter	Assumption
Macrocell layout	Hexagonal grid of 19 x 3 = 57 macrocells with wrap-around. ISD = 500 m
Picocell layout	1 or 2 picocells per macrocell, BPP or related to user distribution
Average user density	25 users / macrocell
System bandwidth	10 MHz (FDD) at 2 GHz
Shadowing	Log-normal, s.d. 4 for LOS, 6 for NLOS
Macrocell Tx power	46 dBm
Picocell Tx power	37 dBm
Antenna gain	Macrocell: 17 dBi, picocell: 5 dBi
CRE biasing value	2 dB
Traffic model	Full buffer
Resource allocation	Proportional fair

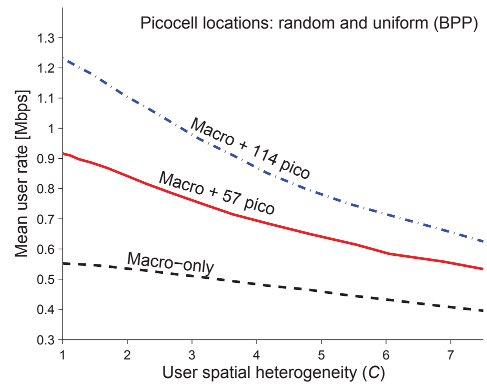

Fig. 3. Mean user rate versus user spatial heterogeneity under different network configurations. The user distribution is a PPP when C equals to 1.

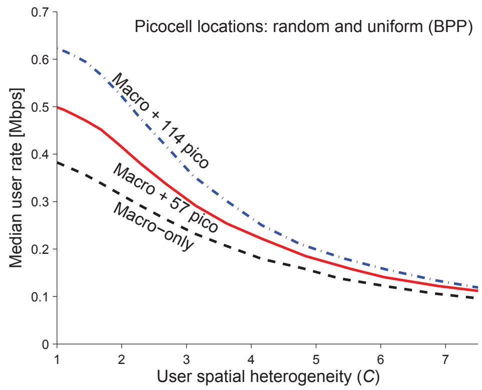

Fig. 4. Median user rate versus user spatial heterogeneity under different network configurations.

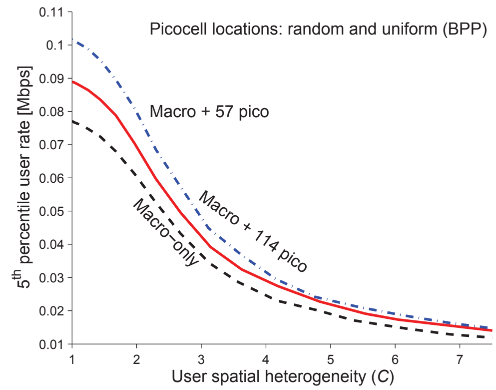

Fig. 5. $5^{\rm th}$ percentile of user rate versus user spatial heterogeneity under different network configurations.

B. Impact of User Spatial Heterogeneity

We change the value of σ in LGCP to get user distributions with different heterogeneities, which are measured by C, the normalized CoV of Voronoi cell area in the Voronoi tessellations of the user points. When C is equal to 1, the user distribution forms a PPP. Performance metrics are evaluated in three scenarios: macro-only network, pico-enhanced networks with the number of picocells equal to 57 and 114 (on average, 1 and 2 picocells per macrocell). Picocells are deployed according to a BPP in the macrocell covered area (as mentioned before, since the total number of picocells are fixed, the distribution of them is a conditional PPP, or equivalently, BPP [4]).

The metrics recommended in 3GPP [13] are used in the simulation, which are the mean rate, median rate, and the 5% worst user rate. Because the overall density of users in LGCP is kept constant, the mean user rate is proportional to the sum throughout of the network, while the median user rate separates users into two halves. The $5^{\rm th}$ percentile user rate is a metric commonly used to indicate the rates of low-SINR users, however, under non-uniform distribution of both traffic demand and traffic supply, the users that belong to this tail-rate user group may not necessarily be the low-SINR receivers, but the low share-of-resource receivers.

As we can see from Fig. 3, Fig. 4 and Fig. 5, the above mentioned three metrics all deteriorate significantly with the increase in user spatial heterogeneity. This is due to the fact that when users are more spatially heterogeneous, there is a high chance that parts of the network will be highly congested, resulting in very low user rates; while the other parts of the network will be underused or even totally empty. This is true for both macro-only networks and pico-enhanced HCNs where picocells are randomly deployed. In terms of sensitivity, the $5^{\rm th}$ percentile user rate is the most sensitive metric (curve goes down most rapidly), as the higher user spatial heterogeneity makes the share of resource for each user more divergent, resulting in a lower $5^{\rm th}$ percentile rate.

C. small-cell Deployment Strategy

This part evaluates the network performance with respect to user spatial heterogeneity under two different small-cell deployment strategies: 1) Random and uniform (BPP), and 2) centroids of clusters obtained by cluster analysis. The number of picocells is kept to 57 in this section. For cluster selection, two criteria are compared: the number of users in the cluster and the proposed objective function defined in (1).

As we can see from Fig. 6, in comparison to the BPP strategy shown in the bottom curve (identical to the middle curve in Fig. 3), the cluster center strategy with the proposed objective function discussed in Section IV improves the mean user rate by more than 50%. Besides, instead of decreasing monotonically with the increasing user spatial heterogeneity, the mean user rate increases first and then decreases when cluster analysis is applied for choosing picocell locations. The cluster center strategy performs better than the BPP strategy because of two reasons: 1) By bringing small-cells to the centers of the traffic demand, the load among cells becomes more balanced; 2) the spectral efficiency is improved with the distance between the transmitters and receivers shortened. However, a higher user spatial heterogeneity (more or larger user clusters) is beneficial to spectral efficiency improvement with appropriately deployed small-cells, but may also make the traffic load more imbalanced. When the traffic imbalance caused by user spatial heterogeneity outweighs the capacities of all the small-cells, the performance goes down. This observation gives us the insight that a certain degree of user spatial heterogeneity can be explored by correlating the locations of users and small-cells in HCNs.

D. Fairness Index

The widely used Jain's index is evaluated to quantify the rate fairness among all the users. Note that the fairness evaluated here is different from the commonly used measure that indicates whether users or applications are receiving a fair share of the system resources. In this paper, we use Jain's index to measure the fairness of all users in the system in terms of the rate. It is defined as

$$\mathcal{J}(x_1, x_2, ..., x_n) = \frac{(\sum_{i=1}^{n} x_i)^2}{n \cdot \sum_{i=1}^{n} x_i^2}, \qquad (2)$$

where n denotes the number of users and x_i denotes the user rate for user i. Figure 7 shows the fairness index versus user

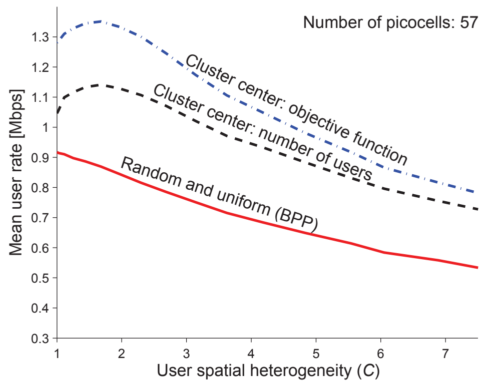

Fig. 6. Mean user rate versus user spatial heterogeneity with different small-cell deployment strategies and different cluster selection methods.

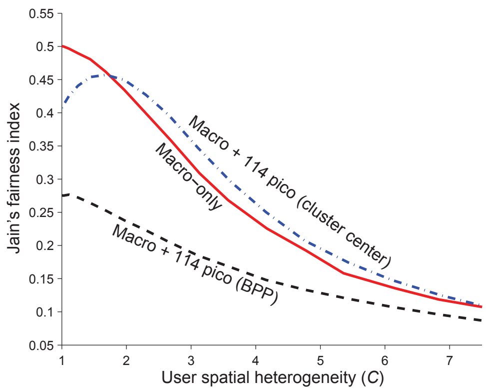

Fig. 7. Jain's fairness index of user rate versus user spatial heterogeneity under different network configurations.

spatial heterogeneity under different network configurations.

The fairness index goes down monotonically when users are more heterogeneously distributed in the marco-only networks, and in HCNs where small-cells are randomly and uniformly deployed (BPP); the Jain's index is even worse in the latter. The downwards trend is because of the fact that the increasing spatial heterogeneity causes more resource imbalance. The performance gap between the macro-only networks and HCNs is as a result of the randomly located picocells which serve only a small portion of the users, letting them reach high rates (which in turn worsens the fairness index). Similar to Fig. 6, when picocells are deployed based on the clustering algorithm introduced in Section IV, the fairness index rises first and then goes down with the increasing user spatial heterogeneity.

VI. Conclusion and Outlook

This paper investigates the impact of user spatial heterogeneity on network performance and develops a small-cell deployment strategy based on the observed impact. The spatial heterogeneity generated in this paper is scalable from homogeneous to extremely heterogeneous controlled by a single parameter, and the level of the heterogeneity is measured by a single non-negative real number. The numerical results show that if the locations of the users and small-cells are not correlated, network performance deteriorates significantly when the users are more heterogeneously distributed. Using the cluster analysis technique, the user cluster centers are obtained from the non-uniform user points as the potential small-cell locations. It is observed that if small-cells are deployed based on the user spatial distributions, the performance of HCNs can benefit from the increase in user spatial heterogeneity.

Due to the space limitations, results for finite user rate demands and those taking the limited back-haul capabilities of small-cells into account are not included in this paper. The correlation among users and the cross-correlation between the users and BSs can also be included in the user spatial model [14]. Alternatively, in order to pushing the traffic demand (user cluster) towards the traffic supply (small-cells), the user-in-the-loop technique [15] can be used as a solution to combat traffic imbalance problem in future wireless networks. This is achieved by encouraging users to move to spots with higher spectral efficiency or lower traffic load.

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