Trade-offs in Sum-Rate Maximization and Fairness in Relay-Enhanced OFDMA-based Cellular Networks

Davut Incebacak, Halim Yanikomeroglu, and Bulent Tavli

***Abstract*—Routing, subchannel scheduling, and power allocation are generally treated as separate problems in relay-enhanced OFDMA-based cellular networks. They are mostly modeled using non-linear constraints to maximize either sum rate or minimum rate. Although separation of problems simplifies modeling, it can lead to suboptimal solutions which can degrades network efficiency (i.e., low sum rate or low minimum rate). Also, models that include non-linear constraints generally belong to the NP-hard class. In this study, we jointly optimize routing, subchannel scheduling, and power allocation in relay-enhanced OFDMA-based cellular networks through a novel Linear Programming (LP) framework employing discrete power levels. Our framework is comprised of LP models for the following problems: Sum Rate Maximization (SRM), Max-Min Fairness (MMF), and Joint Sum Rate Maximization and Max-Min Fairness ($JSRM^3F$). We investigate the trade-offs in sum rate maximization and max-min fairness in terms of achievable maximum data rates and subchannel sharing by numerical evaluations of the LP models. We show that maximum data rates obtained with discrete power allocation are near-optimal even with a few discrete power levels. We provide upper bounds for joint maximization of sum rate and minimum rate. Furthermore, the results of this study reveal that fairness has a significant impact on subchannel sharing.**

***Index Terms*—cellular networks, linear programming, power level, fairness.**

I. Introduction

As the demand for high data rates increases in cellular networks, optimum resource allocation gains more importance. Current systems such as LTE [1] use orthogonal frequency division multiple access (OFDMA) as a multiple access scheme to provide fast and reliable data services. OFDMA enables efficient usage of resources by dividing the broadband channel into multiple narrowband subchannels. Multiple users are able to transmit simultaneously on different subcarriers that are orthogonal to each other.

Using relays is expected to enhance the performance of cellular networks [2]. However, the presence of relays complicates the resource allocation process. Relaying requires routing and scheduling on top of OFDMA which also needs optimum power allocation on each subcarrier to achieve the desired rate for each user. Channel gains of subcarriers for each user are, generally, non-identical and a subcarrier with a poor channel gain for one user may be in a good state for another user in the network. Hence, multiuser diversity can be exploited in conjunction with resource allocation. The sum rate can be maximized by scheduling each subcarrier to the user with the best channel gain and routing data accordingly. This solution is known to result in unfairness for users with prolonged bad channel conditions.

The literature on subcarrier allocation, transmission policy optimization, rate maximization, routing, and fairness for OFDMA systems is extensive and, thus, is beyond the scope of our work. However, we provide a brief literature overview by succinctly summarizing the papers most relevant to our study.

In [3], the optimum transmission policy is investigated to maximize throughput with a finite number of power levels and code rates for discrete adaptive transmission systems. Optimum transmission policy includes channel state space partition, power and rate allocation. In [4], maximization of uplink communication sum rate of a single cell is investigated under the assumption of binary power levels. It was shown that the optimum power allocation is either "on" or "off". In [5], OFDMA subcarrier allocation in chunks are studied over downlink channels. Binary integer optimization models are developed to investigate maximizing sum rate of downlink channels by allocating subcarriers in chunks. Discrete rates and discrete power levels are used in their binary optimization models. In [6], the problem of maximizing weighted sum rate is studied for downlink channel in a multi-cell data network. The base station adjusts the transmit power by considering mitigation of inter-cell interference and using coordinated scheduling and discrete power control. In [7] and [8], joint subchannel and power allocation are studied in relay-enhanced cellular networks. In [7], a heuristic algorithm is developed to maximize sum rate. The problem is solved in two steps: 1) Subchannel allocation and 2) Power allocation. In [8], a stochastic optimization problem is formulated to maximize average sum rate and provide minimum rates to mobile users. In [9], optimal design and efficiently computable bounds are investigated to maximize a weighted-sum rate of the data communicated over a generic OFDMA wireless network by determining optimal data routes, subchannel schedules, and power allocations. First, the problem is modeled as a mixed integer nonlinear program then relaxed to a convex optimization problem by allowing sharing the subchannels in time.

In [10], max-min fairness problem is studied to maximize the capacity of the worst node using flat transmit power. In [11], a two-level Lagrangian dual decomposition method is

D. Incebacak is with Kocaeli University, Kocaeli, Turkey (e-mail: davut.incebacak@kocaeli.edu.tr).

H. Yanikomeroglu is with Carleton University Ottawa, Canada (e-mail: halim@sce.carleton.ca).

B. Tavli is with TOBB University of Economics and Technology, Ankara, Turkey (e-mail: btavli@etu.edu.tr).

developed for efficient solution of max-min fairness problem.

Generally, routing, subchannel scheduling, and power allocation have been considered separately for the solution of sum rate maximization or max-min fairness. Better network performance can be obtained by jointly optimizing multiple functionalities [12]. In this paper, we study the trade-offs in sum rate maximization and max-min fairness by developing novel problem formulations using Linear Programming (LP). In this paper, our contributions are as follows:

- We jointly optimize routing, scheduling and power allocation in relay-enhanced OFDMA-based cellular networks using LP with discrete power levels.
- We develop LP models for the following problems: Sum Rate Maximization (SRM), Max-Min Fairness (MMF), and Joint Sum Rate Maximization and Max-Min Fairness ($JSRM^3F$).
- $JSRM^3F$ model gives the upper bound for joint maximization of sum rate and minimum rate in the network.
- We show that maximum data rates obtained with discrete power allocation are near-optimal even with a few number of discrete power levels especially when fairness is considered.
- We investigate subchannel sharing characteristics of SRM, MMF, and $JSRM^3F$. We show that subchannel sharing characteristics of relay-enhanced cellular networks are directly related to fairness.

II. Model

We consider an OFDMA-based cellular network with N nodes and a single base station. The network topology is represented by a directed graph, $G = (\mathrm{V}, \mathrm{A})$, where V is the set of all nodes and the base station which is defined as node-0. We also define set M which includes all nodes except node-0 (i.e., M = V$\backslash\{0\}$). $\mathrm{A} = \{(i,j) : i \in \mathrm{M}, j \in V - i, \}$ is the ordered set of arcs. Note that the definition of A implies that no node sends data to itself. Each node-i has a data rate of s_i to be routed through the base station using K subchannels ($k \in K \Rightarrow \{1, ..., K\}$). While transmitting, each node can use different power levels between 0 and P_i where P_i is the power budget of node-i ($0 < p^1 < p^2 < ... < p^t < ... < p^T = P_i$). The number of power levels is defined as T and the set of power levels is defined as P ($p^t \in$ P). Achievable data rate between node-i and node-j on subchannel-k with power level p^t is represented as $f_{i,j}^{k,p^t}$. The indicator variable $c_{i,j}^{k,p^t}$ shows whether node-i sends data to node-j using subchannel-k with power level p^t or not. $h_{i,j}^k$ denotes the channel gain between node-i and node-j over the subchannel-k. In the considered network, each node-i can be in the role of source, or relay (decode and forward). A zero-mean additive white gaussian noise with variance N_0 is added in each received signal. The total bandwidth of the network is W_0 and each subchannel has the same bandwidth of $W = W_0/K$.

First, we develop an LP model with the objective of maximizing the sum rate of the network ($R_T = \sum_{i \in \mathrm{M}} s_i$). The network flow is modeled in the form of a series of constraints presented in Fig. 1. All system variables with their acronyms and descriptions are presented in Table I.

Maximize $R_T = \sum_{i \in \mathrm{M}} s_i$
Subject to:

$$f_{i,j}^{k,p^t} = W log_2 \left(1 + \frac{p^t |h_{i,j}^k|^2}{W N_0}\right), \quad i \in \mathrm{M}, j \in \mathrm{V}, k \in \mathrm{K}, p^t \in \mathrm{P}, \tag{1}$$

$$\sum_{j \in V} \sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} c_{i,j}^{k,p^t} \leq 1, \qquad i \in \mathrm{M}, \tag{2}$$

$$\sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} \left(\sum_{j \in \mathrm{V}} c_{i,j}^{k,p^t} f_{i,j}^{k,p^t} - \sum_{j \in \mathrm{W}} c_{j,i}^{k,p^t} f_{j,i}^{k,p^t} \right) - s_i = 0, \quad i \in \mathrm{M}, \tag{3}$$

$$\sum_{j \in \mathrm{V}} \sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} c_{i,j}^{k,p^t} p^t - P_i \leq 0, \qquad i \in \mathrm{M}, \tag{4}$$

$$p^t \in \{0, p^1, p^2, ..., p^t, ..., p^T = P_i\}, \tag{5}$$

$$s_i \geq 0, \qquad i \in \mathrm{M}, \tag{6}$$

$$c_{i,j}^{k,p^t} \in \{0,1\}, \quad i \in \mathrm{M}, j \in \mathrm{V}, k \in \mathrm{K}, p^t \in \mathrm{P}. \tag{7}$$

Fig. 1. The MBIP model (SRM_b).

TABLE I
Terminology for MBIP and LP Models

Variable	Description
N	Number of nodes
$f_{i,j}^{k,p^t}$	Achievable data rate with power level p^t on subchannel-k between node-i and node-j
s_i	Data rate of node-i
P_i	Power budget of node-i
$c_{i,j}^{k,p^t}$	Indicator variable determines if subchannel-k on flow from node-i to node-j is used or not with power level p^t
$h_{i,j}^k$	Channel gain between node-i and node-j over subchannel-k
p^t	Power level between 0 and P_i
K	Number of subchannels
R_{min}	Minimum data rate in the network
$\max(R_{min})$	Achievable maximum value of R_{min}
R_T	Sum-rate of the network
$\max(R_T)$	Achievable maximum value of R_T
V	Set of nodes, including the base station as node-0
M	Set of nodes, except the base station (node-0)
A	Set of edges
T	Number of power levels
$p(\ell)$	Path loss component
S_ℓ	Shadowing component
W	Bandwidth of each subchannel
W_0	Total bandwidth of the network
P	Set of power levels

The data rate between node-i and node-j over subchannel-

k is determined by the allocated power to subchannel-k. According to the channel gain of node-i, different data rates can be achieved for each power level p^t. In constraint (1), each achievable data rate with power level p^t on subchannel-k between node-i and node-j is defined in the parameter $f_{i,j}^{k,p^t}$. Note that as the number of power levels goes to infinity, power allocation with discrete power levels converges to the continuous power allocation. Although continuous power allocation can be used to obtain the optimum data rates, it makes the problem non-linear and hence much more difficult to solve. Furthermore, adjusting the power level in a continuum is an unrealistic assumption (*i.e.*, modern cellular systems are equipped with transceivers that can operate at a finite number of discrete power levels). As we show later, we can get near-optimum data rates using few discrete power levels. In the OFDMA-based network scenario, interference is prevented by using each subchannel once in the network as enforced in constraint (2). Constraint (3) determines routing operations in the network and known as the flow conservation constraint which is satisfied for all nodes. If node-i is not a relay then the sum of outgoing flows is the total amount of data rate injected into the network by node-i. If node-i is a relay node, then the sum of outgoing flows from node-i equals to the sum of incoming flows to node-i plus the generated data at node-i. Constraint (4) limits the total transmit power used by each node by the total power budget P_i. Constraint (5) determines the set of power levels. Constraint (6) is the nonnegativity constraint for the variable s_i. $c_{i,j}^{k,p^t}$ determines the usage of subchannel-k with power level p^t between node-i and node-j. Hence, constraint (7) is a binary scheduling constraint and $c_{i,j}^{k,p^t}$ can be considered as a binary scheduling variable. Setting $c_{i,j}^{k,p^t}$ equal to 1 determines that subchannel-k is assigned to node-i to send data to node-j during the entire communication interval using power level p^t. Setting $c_{i,j}^{k,p^t}$ equal to zero indicates that node-i doesn't use subchannel-k to transmit data to node-j with power level p^t.

We called the model in Fig. 1 as Sum-Rate Maximization with binary scheduling variables (SRM_b). Since $c_{i,j}^{k,p^t}$ is binary, SRM_b is a Mixed Binary Integer Programming (MBIP) problem. In general, MBIP problems are in the NP-hard class due to their computational complexity [13]. If $c_{i,j}^{k,p^t}$ is used as a binary variable, each subchannel can only be used by one node. Allowing $c_{i,j}^{k,p^t}$ to lie in the interval $[0,1]$ and satisfying constraint (2), $c_{i,j}^{k,p^t}$ can be interpreted as a continuous scheduling variable. In this case, $c_{i,j}^{k,p^t}$ works as a time sharing parameter and enables node-i to send data to node-j using subchannel-k with power level p^t for a fraction $c_{i,j}^{k,p^t}$ of communication interval which can be expressed as

$$c_{i,j}^{k,p^t} \in [0,1], \qquad i \in \mathrm{M}, j \in \mathrm{V}, \mathrm{k} \in K, p^t \in \mathrm{P}. \tag{8}$$

By replacing constraint (7) with constraint (8) in SRM_b model, we developed Sum-Rate Maximization with continuous scheduling variables (SRM_c). Note that since $c_{i,j}^{k,p^t}$ is not binary, SRM_c is an LP model. LP models whose variables take continuous values are relatively easier to solve.

Since our objective is to maximize R_T in SRM_b and SRM_c models, nodes with low channel gains may not get as much resources as nodes with high channel gains. This unequal resource distribution results in unfairness among nodes. To distribute resources fairly among nodes, we, in constraint (9), introduce fairness parameter R_{min} which determines minimum required data rate for each node as

$$s_i \geq R_{min}, \qquad i \in \mathrm{M}. \tag{9}$$

Using constraints (1) - (6), (8), and (9), we develop MMF model with the objective of maximization of R_{min}.

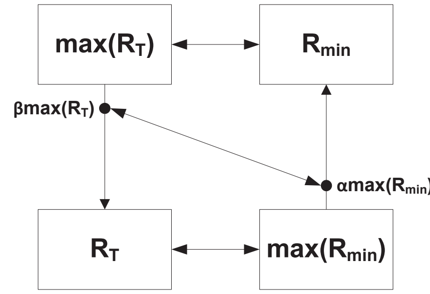

Fig. 2. Illustration of R_{min} and R_T values with conflicting objectives and expected values of α and β with joint optimization of R_{min} and R_T.

As depicted in Fig. 2, objectives of SRM_c and MMF are conflicting. If our objective is only maximizing R_T, some of the nodes in the network can get less data rate. In some cases they cannot get any data rate ($R_{min} = 0$). Hence, R_{min} value can be lower than the achievable maximum value of R_{min} (max(R_{min})). On the other hand, if our objective is only maximizing R_{min}, R_T can be lower than the achievable maximum value of R_T (max(R_T)). In order to investigate the trade-offs in maximizing R_T and R_{min}, two additional constraints are introduced as

$$s_i \geq \alpha \max(R_{min}), \qquad \forall i \in \mathrm{M}, \tag{10}$$

$$\sum_{i \in \mathrm{M}} s_i \geq \beta \max(R_T) \tag{11}$$

by getting the values of max(R_{min}) and max(R_T) from the SRM_c and MMF models. α and β in (10) and (11) are control variables for setting the minimum per node data rate and minimum aggregate data rate, respectively. Using constraints (1) - (6), (8), (10), and (11), we construct $JSRM^3F$ model that jointly maximizes R_T and R_{min}.

III. Analysis

We use GAMS (General Algebraic Modeling System) for the numerical analysis of the MBIP and LP models. GAMS consists of high-performance solvers for solving the MBIP and LP models efficiently. Hence, when we solve our MBIP and LP models using GAMS, one of these solvers is used to obtain the best solution.

We investigate scenarios in which N nodes are randomly deployed in a 100 m x 100 m square area. Power budget for all nodes is the same ($P_i = 10$ dBm or $P_i = 20$ dBm) for a given scenario. The total number of power levels is 32 and the number of subchannels is equal to 60. The parameters used in the analysis are presented in Table II.

TABLE II
PARAMETERS USED IN THE ANALYSIS

Parameter	Value
Network area	100 m X 100 m
N	20, 30
P_i	10 dBm, 20 dBm
K	60
W_0	20 MHz
f_c	3.4 GHz
T	1 to 32

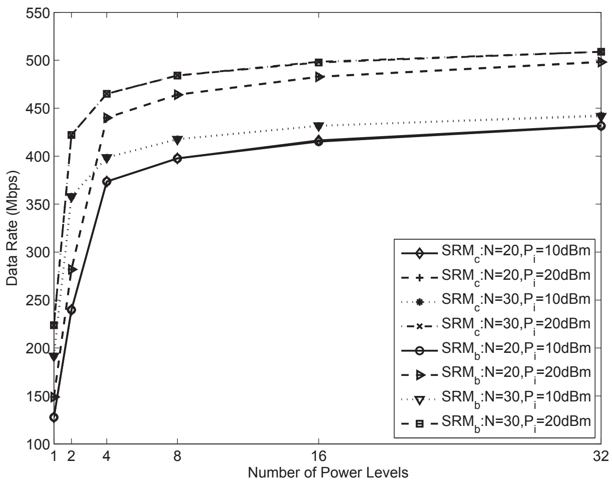

Fig. 3. Sum rates as a function of the number of power levels in the SRM_c and SRM_b models.

We consider an IMT-Advanced scenario [14] in which the total bandwidth and the thermal noise power density are set to 20 MHz and -174 dBm/Hz, respectively. The channel gains of subchannels are obtained by assuming that the subchannels experience the standard quasi-static frequency-flat Rayleigh fading with log-normal shadowing and pathloss components. As in [9], the subchannel gains are calculated by

$$|h_{i,j}^k|^2 = 10^{-0.1S_\ell - 0.1p(\ell)}|h_{i,j}^{\prime k}|^2 \quad i \in \mathrm{M}, j \in \mathrm{V}, \mathrm{k} \in K, \quad (12)$$

where S_ℓ is a Gaussian distributed random variable with 0 dB mean and a standard deviation of $\sigma_s = 4$ dB representing the shadowing component. $p(\ell)$ is the path loss component and modeled as $p(\ell) = 43.3\log_{10}(d_l) + 11.5 + 20\log_{10}(f_c)$ where d_l is the distance between two nodes in meters and f_c is the carrier frequency in GHz which is set to $f_c = 3.4$ GHz. $|h_{i,j}^{\prime k}|^2$ is a zero-mean unit-variance complex Gaussian-distributed random variable and corresponds to the Rayleigh fading component in the channel model. Each problem is solved for 50 deployments and the results are averaged. We analyze data rate and channel sharing characteristics for the SRM, MMF and $JSRM^3F$ models.

We first study the maximization of sum rate with the SRM_b and SRM_c models. In Fig. 3, the sum rates (Mbps) as a function of number of power levels are presented for $N = 20$ and $N = 30$ cases with $P_i = 10$ dBm and $P_i = 20$ dBm.

We observe that the sum rates obtained by employing the SRM_b and SRM_c models are almost the same and that the sum rate increases as the number of power levels, the number of nodes, and the power budget of nodes increase. However, once the utilized number of power levels exceeds 8, the increase in the sum rates becomes very low. In other words, we can approach the maximum sum rates achieved by continuous power allocation using discrete power levels which is a typical case of diminishing marginal gains. For example, when the number of power levels increases from 1 to 2, 2 to 4, 4 to 8, 8 to 16, and 16 to 32, the rates of increase in the sum rate are 88.67%, 10.18%, 4.12%, 2.93%, and 2.09%, respectively, for N=30, P_i=20 dBm.

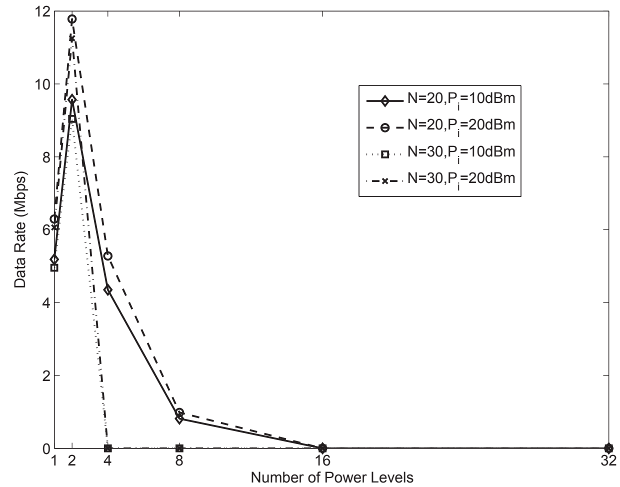

Fig. 4. Minimum rates as a function of the number of power levels in the SRM_c model.

In Fig. 4, the minimum rates in the network are presented as a function of the number of power levels for the SRM_c model. When N=20 and N=30, in all cases, there is at least one node that is not able to send data to other nodes or to the base station ($R_{min} = 0$) after the number of available power levels exceeds 8 and 4, respectively. Minimum date rate increases up to 11.78 Mbps for N=20, P_i=20 dBm. The reason for the lower minimum data rate for higher number power levels is that, with a larger degree of freedom for power level assignment, it is more likely to utilize the power budget of certain nodes with better channels to maximize the aggregate rate and to deny some nodes to inject data into the network. If the number of power levels are too few then the opportunity to fine tune the transmission power assignment is very limited.

In Fig. 5, the achievable maximum of the minimum data rates is illustrated by using the MMF model. As the number of power levels increases from one to four, minimum data rates increase, however, as the number of power levels exceeds four, the data rates stay constant. For example, when the number of power levels increases from 1 to 2, 2 to 4, 4 to 8, 8 to 16, and 16 to 32, the rates of increase in the minimum data rates are 89.31%, 9.49%, 0.16%, 0.03%, and 0.01%, respectively for N=30, P_i=20 dBm.

In Fig. 6, the sum rates obtained by using the MMF model are presented as functions of the number of power levels. As in Fig. 5, the sum rates increase as the number of power level increases, however, as the number of power levels exceeds four, the rate of increase in the minimum data rates goes to

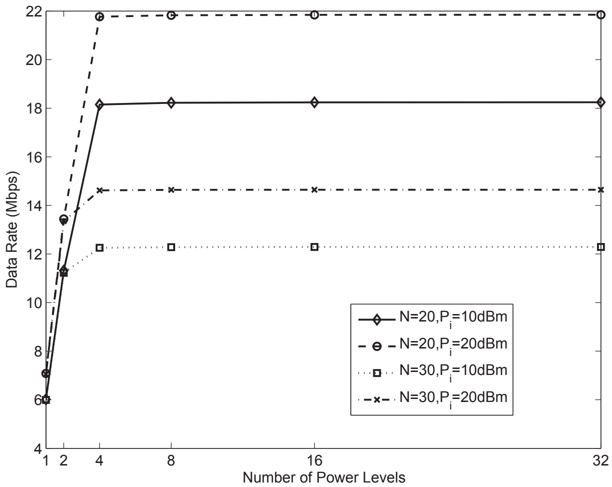

Fig. 5. Minimum rates as a function of the number of power levels in the MMF model.

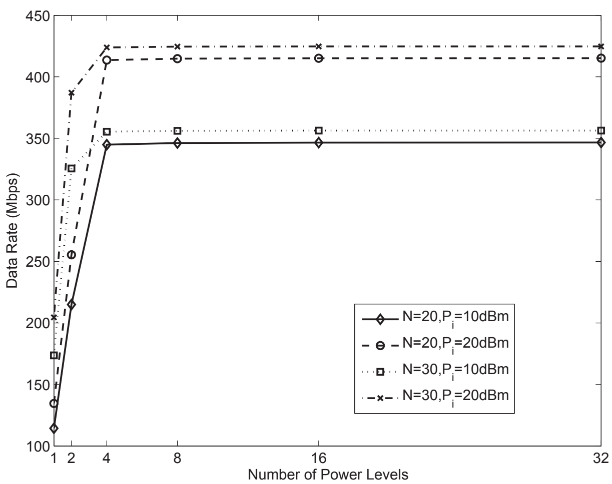

Fig. 6. Sum rates as a function of the number of power levels in the MMF model.

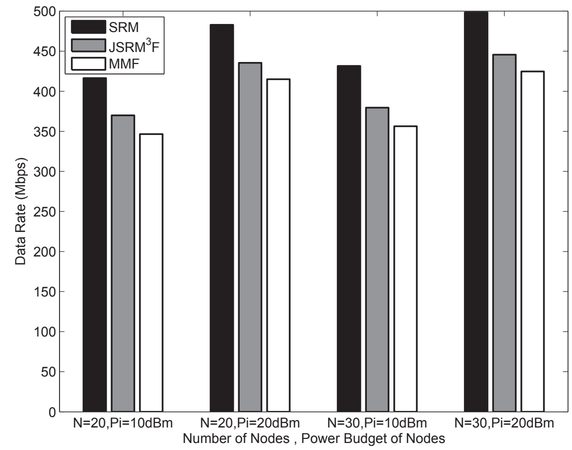

Fig. 7. Sum rates as a function of the number of nodes and the power budgets of nodes in the $JSRM^3F$ model with 16 power levels.

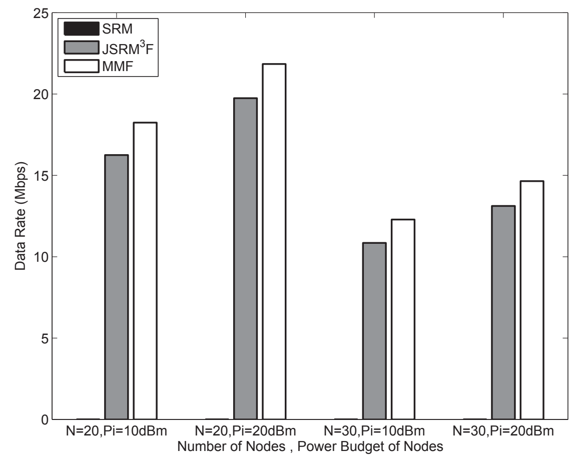

Fig. 8. Minimum rates as a function of the number of nodes and the power budgets of nodes in the $JSRM^3F$ model with 16 power levels.

zero.

We can conclude from Fig. 3 and Fig. 5 that the maximum data rates obtained with discrete power allocation are near-optimal even with a few discrete power levels. Indeed, for the MMF model more than four power levels are not necessary at all.

Sum rates obtained with SRM_c model are 24.47%, 19.99%, 23.93%, and 19.80% higher than the sum rates obtained with the MMF model using 16 power levels with parameter sets {N=20, P_i=10 dBm}, {N=20, P_i=20 dBm}, {N=30, P_i=10 dBm}, and {N=30, P_i=20 dBm{, respectively. On the other hand, the minimum data rates obtained with the SRM_c model using 8 power levels increase from 0.81 Mbps to 12.22 Mbps, from 0.98 Mbps to 21.83 Mbps, from 0 to 12.28 Mbps, and from 0 to 14.64 Mbps when the MMF model is employed for data sets {N=20, P_i=10 dBm}, {N=20, P_i=20 dBm}, {N=30, P_i=10 dBm}, and {N=30, P_i=20 dBm}, respectively. Hence, significant gains in the minimum data rate is achieved by relatively modest sacrifices from the aggregate rate by the MMF model.

In the SRM_c model, the minimum rate is sacrificed for the maximization of the aggregate data rate whereas in the MMF model the aggregate rate is sacrificed for the sake of providing a minimum level of data rate to all nodes in the network. In fact, both of these models lie at the opposite directions of the tradeoff curve. An interesting question at this point is: *Can we find a middle ground between these two extremes?* The answer for this question comes in the form a new model: $JSRM^3F$.

In the $JSRM^3F$ model, fairness can be provided both to the individual nodes and to the network as a whole. We use the $JSRM^3F$ model to maximize both the sum rate and the minimum rate, jointly. In Fig. 7 and Fig. 8, we present the minimum rate and the sum rate, respectively, for the SRM_c, MMF, and $JSRM^3F$ models. The $JSRM^3F$ model is solved by setting $\alpha = \beta$ ($\alpha = \beta \in [0, 1]$). The price paid for the high sum rate of the SRM_c model is the complete denial of access for certain nodes whereas in the MMF model, a modest decrease in the aggregate rate provides a significant data rate for the minimum rate assigned nodes. For example, with N=20, P_i=20 dBm, the SRM_c model gets 482.86 Mbps sum rate but zero minimum rate, yet, the MMF model gets 415.06 Mbps sum rate and 21.85 Mbps minimum rate. The

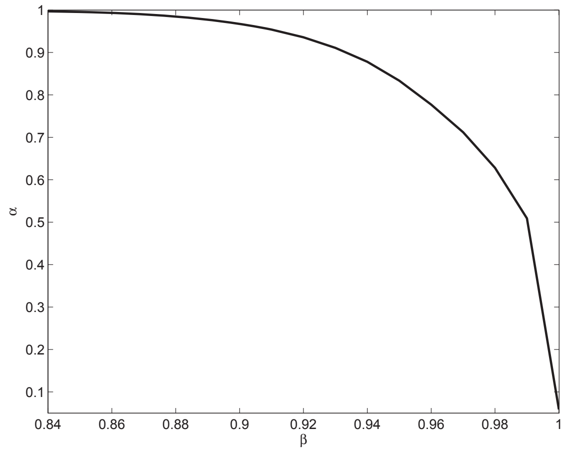

Fig. 9. Interrelation of α and β in the $JSRM^3F$ model with N=20, $P_i = 20$ dBm, 8 power levels.

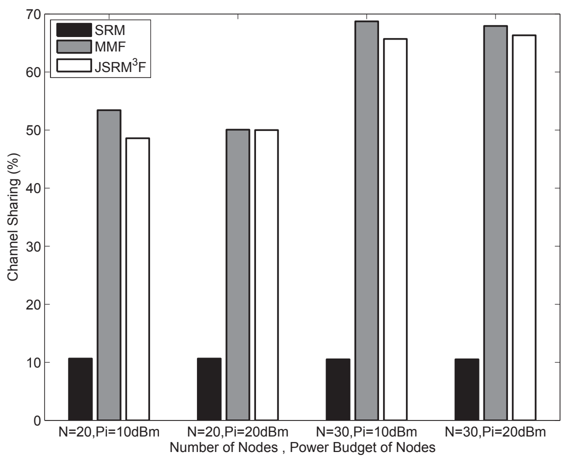

Fig. 10. Percentage of channel sharing in the SRM_c, MMF and $JSRM^3F$ models with 16 power levels.

$JSRM^3F$ model with $\alpha = \beta$ setting positions itself in between the SRM_c and MMF models in terms of both sum rate (435.44 Mbps) and minimum rate (19.74 Mbps). What the $JSRM^3F$ model achieves is the flexibility. In fact, the $JSRM^3F$ model brings one more degree of freedom which is not present neither in SRM_c nor in MMF. The operating curve of $JSRM^3F$ in α and β space with N=20, $P_i = 20$ dBm, and 8 power levels is shown in Fig. 9.

In Fig. 10, sharing of subchannels in time ($0 < c_{i,j}^{k,p^t} < 1$) is investigated using the SRM_c, MMF and $JSRM^3F$ models. When fairness is not considered, at most 10.63% of all the subchannels in the network are shared in time in the SRM_c model. However, when the MMF and $JSRM^3F$ models are used to provide max-min fairness, the percentage of sharing of all subchannels in time increases up to 67.93%.

IV. Conclusion

In this study, we present novel LP models utilizing discrete power levels, where routing, scheduling, and power allocation operations are jointly optimized, to investigate the trade-offs in sum rate maximization and max-min fairness for relay-enhanced cellular networks. We develop a model that we refer to as $JSRM^3F$ to explore the boundaries of the joint maximization of R_{min} and R_T. Our analysis reveals that the maximum data rates (both as $\max(R_{min})$ and $\max(R_T)$) obtained with discrete power allocation are near-optimal even with a few number of discrete power levels. Furthermore, when fairness is considered discrete power allocation is shown to provide the optimal performance. We show that subchannel sharing characteristics of relay-enhanced cellular networks are directly related to fairness. A few subchannels are shared in time, in the sum rate maximization models. However, the number of subchannels shared in time is high in the max-min fairness models.

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