Trade-offs in Sum-Rate Maximization and Fairness in Relay-Enhanced OFDMA-based Cellular Networks

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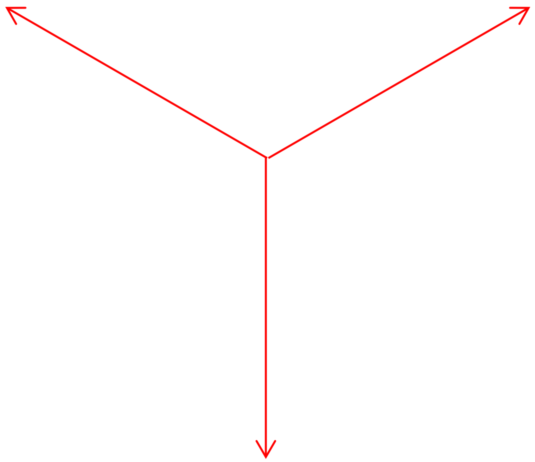
TUBITAK (The Scientific and Technological Research Council of Turkey)

Resource Allocation

Huge literature: **Perspective needed**

Resource Allocation

Variables
power
RB
link (routing)
…



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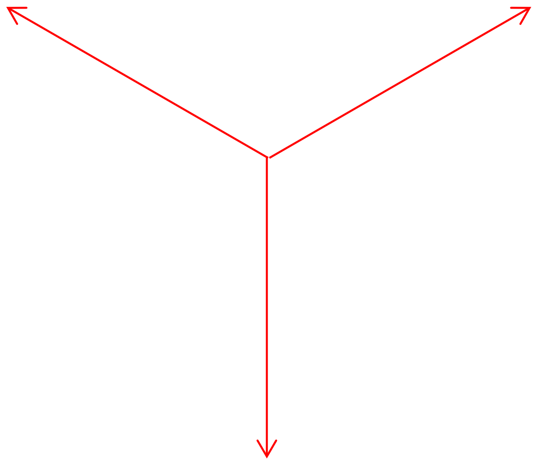

Objectives
max sum-rate
max min-rate
min sum-power
fairness
…

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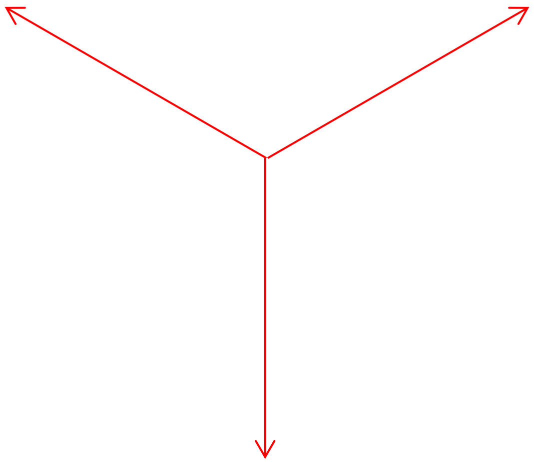

RAN architecture
one-cell
multi-cell
ICIC, CoMP, CRAN
relays, cooperation
…
ad hoc, reuse

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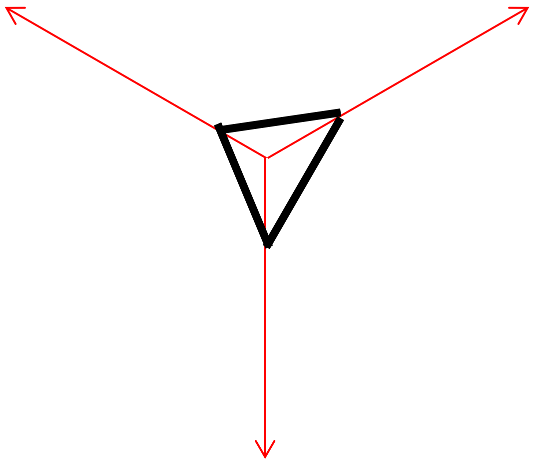

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Optimal solutions:
Only in simple settings

Advanced settings:
Not sufficiently explored

Problem Setting

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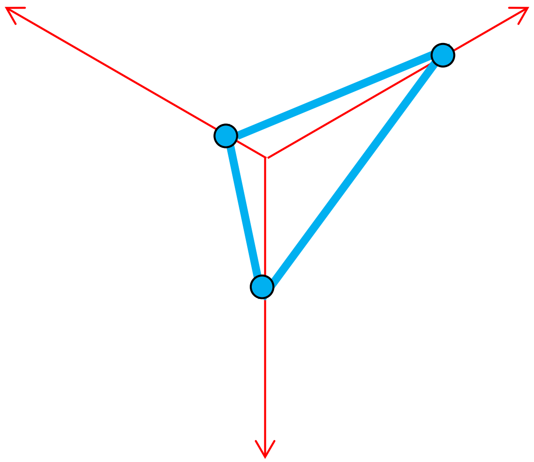

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A. Bin Sediq, R. Gohary, R. Schoenen, H. Yanikomeroglu, “Optimal tradeoff between sum-rate efficiency and Jain’s fairness index in resource allocation”, *IEEE Transactions on Wireless Communications*, July 2013.

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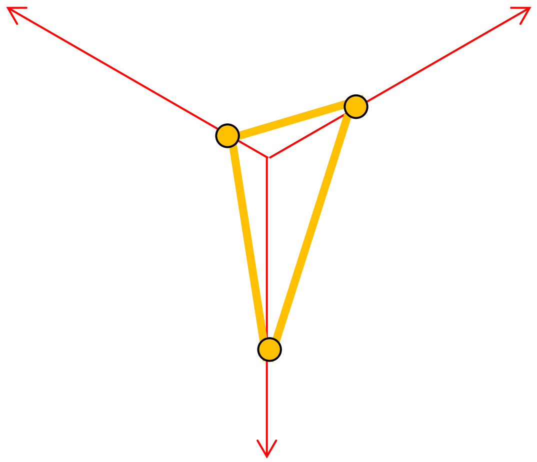

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A. Bin Sediq, R. Schoenen, H. Yanikomeroglu, G. Senarath, “Optimized distributed inter-cell interference coordination scheme using projected subgradient and network flow optimization”, to appear in *IEEE Transactions on Communications*, 2015.

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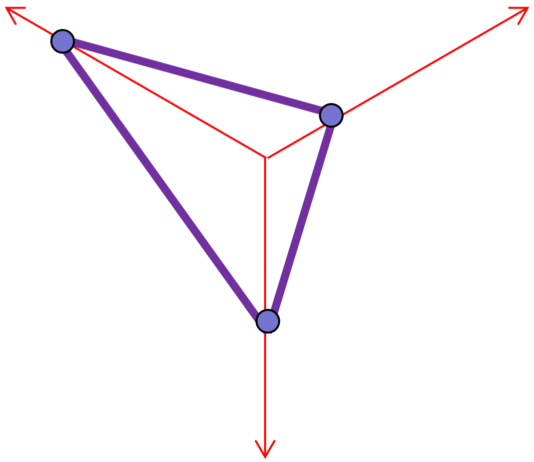

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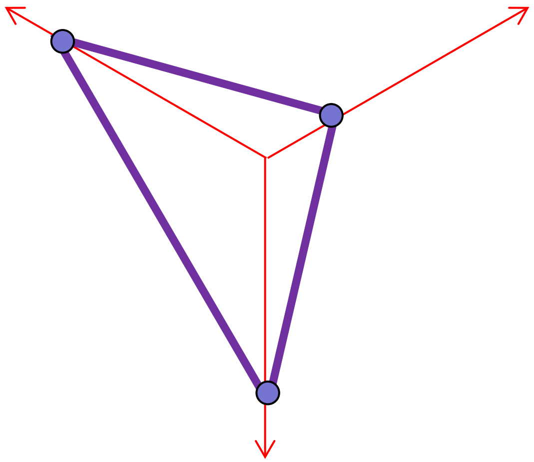

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R. Rashtchi, R. Gohary, H. Yanikomeroglu, "Generalized cross-layer designs for generic half-duplex multicarrier wireless networks with frequency reuse", under review in *IEEE Transactions on Wireless Communications* (submission: July 2014).

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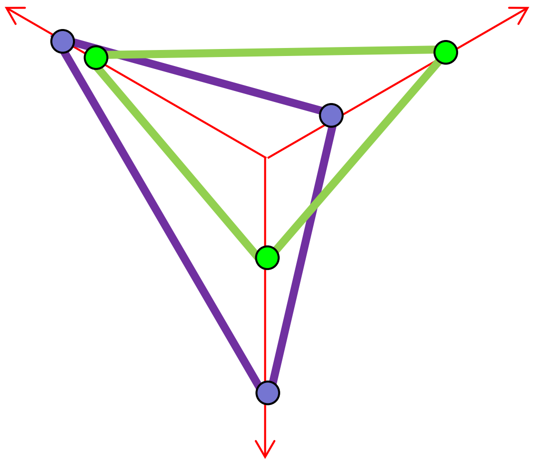

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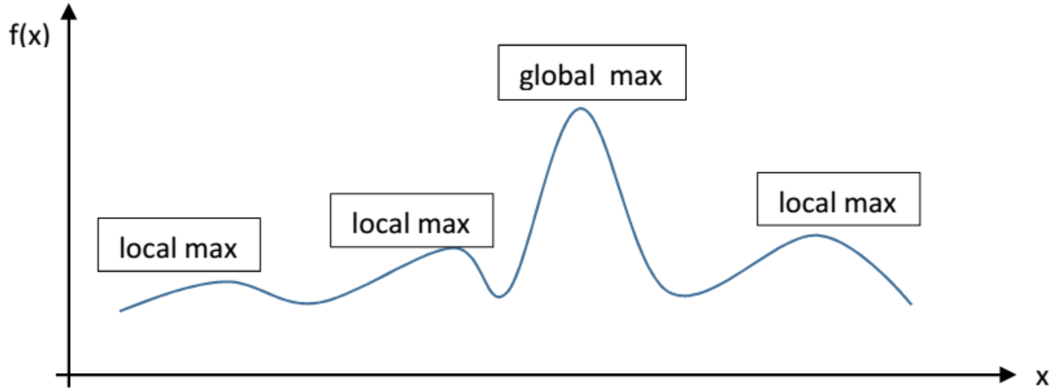
This paper

Background

- Allocation of resources is mostly modeled using NLP to maximize either sum-rate or minimum rate.
- Non-linearity is due to the capacity formula

$$Capacity = W\, log_2\left(1 + \frac{p_{ij}^k * \left(h_{ij}^k\right)^2}{W N_0}\right)$$

- NLP models generally belong to the class of NP-hard.
  - Works on very small settings



Problem Definition

- Resource allocation in cellular networks
  - ➢ Subchannel allocation
  - ➢ Power allocation
  - ➢ Routing

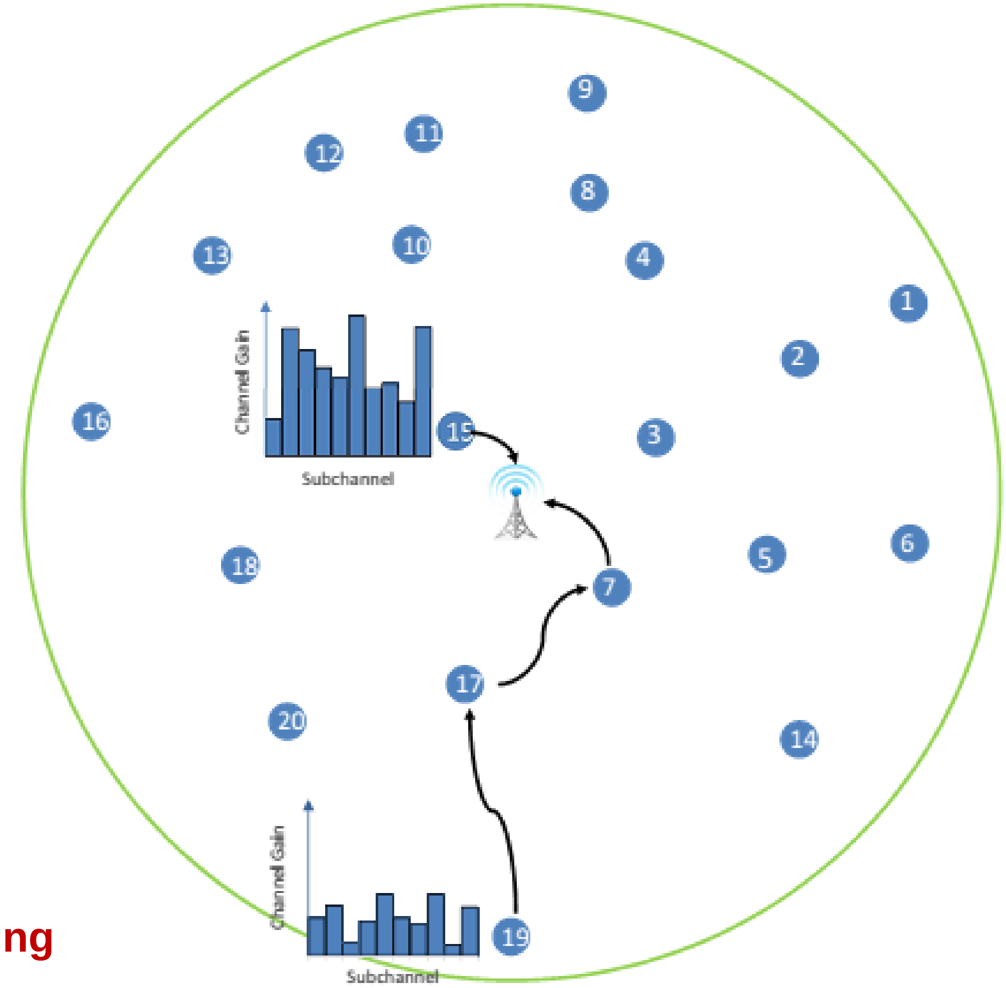
Joint design of power, subchannel allocation and routing to exploit the opportunities offered by network

- Objective
  - ➢ Sum Rate Maximization
  - ➢ Max-Min Fairness

Trade-off

- Joint Sum-Rate Maximization and Max-Min Fairness

- Computationaly complex NLP solutions for joint design
  - ➢ **Jointly optimize routing, scheduling and power allocation using <u>LP with discrete power levels</u>**

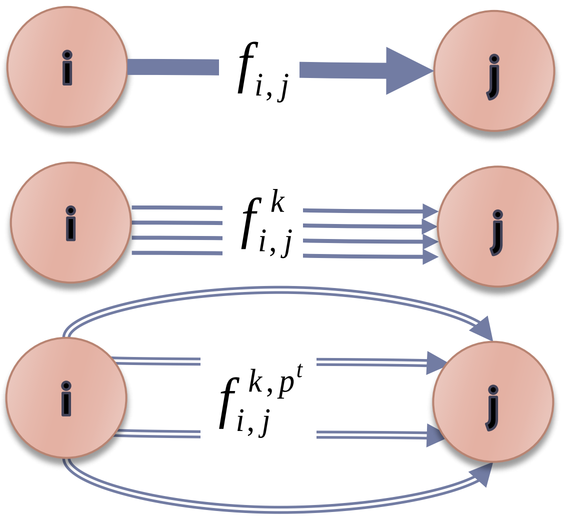


$$Capacity = W\,log_2\left(1 + \frac{p_{ij}^k * \left(h_{ij}^k\right)^2}{WN_0}\right)$$

Design Variables

- p^t power level between 0 and P_i (maximum power) $p^t \in \{0, p^1, p^2, ..., p^t, ..., p^T = P_i\}$.
- $h^k_{i,j}$ channel gain between node-*i* and node-*j* over subchannel-*k*.
- $c^{k,p^t}_{i,j}$ indicator variable determines if subchannel-*k* on flow from node-*i* to node-*j* is used or not with power level p^t
- $f^{k,p^t}_{i,j}$ achievable data rate with power level p^t on subchannel-*k* between node-*i* and node-*j*.

$$f^{k,p^t}_{i,j} = W log_2 \left(1 + \frac{p^t |h^k_{i,j}|^2}{W N_0}\right)$$



Sum-Rate Maximization with Binary Scheduling Variables (SRM$_b$)

Maximize $R_T = \sum_{i \in \mathrm{M}} s_i$
Subject to:

$$f_{i,j}^{k,p^t} = W log_2 \left(1 + \frac{p^t |h_{i,j}^k|^2}{W N_0}\right), \quad i \in \mathrm{M}, j \in \mathrm{V}, k \in \mathrm{K}, p^t \in \mathrm{P}, \qquad (1)$$

$$\sum_{j \in V} \sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} c_{i,j}^{k,p^t} \leq 1, \quad i \in \mathrm{M}, \qquad (2)$$

$$\sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} \left(\sum_{j \in \mathrm{V}} c_{i,j}^{k,p^t} f_{i,j}^{k,p^t} - \sum_{j \in \mathrm{W}} c_{j,i}^{k,p^t} f_{j,i}^{k,p^t} \right) - s_i = 0, \quad i \in \mathrm{M}, \qquad (3)$$

$$\sum_{j \in \mathrm{V}} \sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} c_{i,j}^{k,p^t} p^t - P_i \leq 0, \quad i \in \mathrm{M}, \qquad (4)$$

$$p^t \in \{0, p^1, p^2, ..., p^t, ..., p^T = P_i\}, \qquad (5)$$

$$s_i \geq 0, \quad i \in \mathrm{M}, \qquad (6)$$

$$c_{i,j}^{k,p^t} \in \{0,1\}, \quad i \in \mathrm{M}, j \in \mathrm{V}, k \in \mathrm{K}, p^t \in \mathrm{P}. \qquad (7)$$

Objective: Maximize Sum-Rate

(1) achievable data rates with power level pt on subchannel-k between node-i and node-j

(2) interference is prevented by using each subchannel once in the network

(3) flow conservation constraint which is satisfied for all nodes.

(4) limits the total transmit power used by each node

(5) determines the set of power levels

(6) nonnegativity constraint for data rates

(7) binary scheduling constraint

Sum-Rate Maximization with Continuous Scheduling Variables (SRM$_c$)

Maximize $R_T = \sum_{i \in \mathrm{M}} s_i$
Subject to:

$$f_{i,j}^{k,p^t} = W log_2 \left(1 + \frac{p^t |h_{i,j}^k|^2}{W N_0}\right), \quad i \in \mathrm{M}, j \in \mathrm{V}, k \in \mathrm{K}, p^t \in \mathrm{P}, \qquad (1)$$

$$\sum_{j \in V} \sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} c_{i,j}^{k,p^t} \leq 1, \qquad i \in \mathrm{M}, \qquad (2)$$

$$\sum_{k \in \mathrm{K}} \sum_{p^t \in \mathrm{P}} \left(\sum_{j \in \mathrm{V}} c_{i,j}^{k,p^t} f_{i,j}^{k,p^t} - \sum_{j \in \mathrm{W}} c_{j,i}^{k,p^t} f_{j,i}^{k,p^t} \right) - s_i = 0, \quad i \in \mathrm{M}, \qquad (3)$$

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$$p^t \in \{0, p^1, p^2, ..., p^t, ..., p^T = P_i\}, \qquad (5)$$

$$s_i \geq 0, \qquad i \in \mathrm{M}, \qquad (6)$$

$$c_{i,j}^{k,p^t} \in [0,1], \quad i \in \mathrm{M}, j \in \mathrm{V}, \mathrm{k} \in K, p^t \in \mathrm{P}. \qquad (8)$$

(1) achievable data rates with power level pt on subchannel-k between node-i and node-j

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(6) nonnegativity constraint for data rates

(8) continuous scheduling constraint

Sum-Rate Maximization with Continuous Scheduling Variables (SRM_c)

Maximize $R_T = \sum_{i \in M} s_i$
Subject to:

$$f_{i,j}^{k,p^t} = W log_2 \left(1 + \frac{p^t |h_{i,j}^k|^2}{W N_0} \right), \quad i \in M, j \in V, k \in K, p^t \in P, \qquad (1)$$

$$\sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p^t} \le 1, \quad i \in M, \qquad (2)$$

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Objective: Maximize Sum-Rate

(1) achievable data rates with power level pt on subchannel-k between node-i and node-j

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Sum-Rate Maximization with Continuous Scheduling Variables (SRM_c)

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Subject to:

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Note that since $c_{i,j}^{k,p^t}$ is not binary, SRMc is an LP model

Max-Min Fairness (MMF)

Maximize R_{min}

Subject to:

$$f_{i,j}^{k,p^t} = W log_2\left(1 + \frac{p^t |h_{i,j}^k|^2}{W N_0}\right), \quad i \in \mathrm{M}, j \in \mathrm{V}, k \in \mathrm{K}, p^t \in \mathrm{P}, \qquad (1)$$

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- In SRM_b and SRM_c models → fairness problem
- In constraint (9) fairness parameter R_{min} (minimum data rate generated by one node in the network) is introduced.

$$s_i \geq R_{min}, \qquad i \in \mathrm{M}. \qquad (9)$$

- Using (9), **Max-Min Fairness (MMF)** model is developed with the **objective of maximizing R_{min}**.

Joint Sum-Rate Maximization and Max-Min Fairness ($JSRM^3F$)

Maximize $(\alpha + \beta)$

Subject to:

$$f_{i,j}^{k,p^t} = W log_2 \left(1 + \frac{p^t |h_{i,j}^k|^2}{W N_0}\right), \quad i \in \mathrm{M}, j \in \mathrm{V}, k \in \mathrm{K}, p^t \in \mathrm{P}, \tag{1}$$

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- To investigate the trade-offs between maximizing R_T and R_{min}, two additional constraints are introduced as

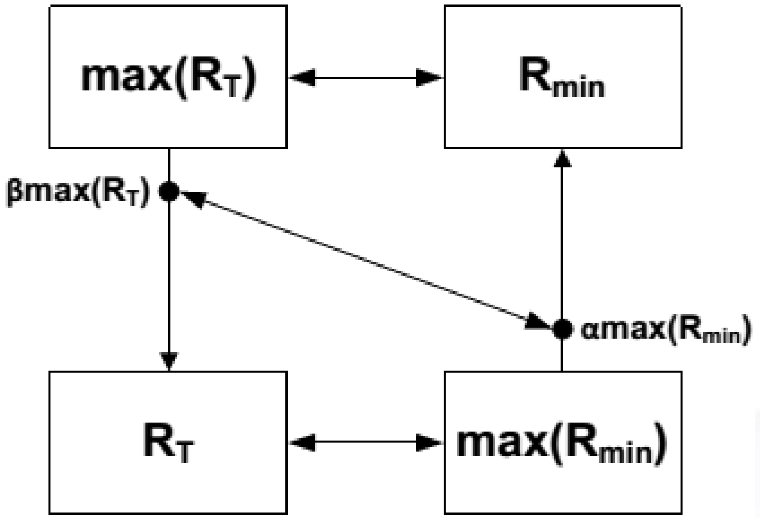
$$s_i \geq \alpha \max(R_{min}), \quad \forall i \in \mathrm{M}, \tag{10}$$

$$\sum_{i \in \mathrm{M}} s_i \geq \beta \max(R_T) \tag{11}$$

- Values of $\max(R_{min})$ and $\max(R_T)$ are from MMF and SRM_c models.
- α and β are controlling variables for the level of $\max(R_{min})$ and $\max(R_T)$.
- Using SRM_c and constraint (10) and (11)), $JSRM^3F$ model is developed that maximize $(\alpha + \beta)$.
  - $JSRM^3F$ model jointly maximizes R_T and R_{min}.

Joint Sum-Rate Maximization and Max-Min Fairness ($JSRM^3F$)

- Objectives of SRM_c and MMF are conflicting
  - Fairness is achieved at the cost of a decreased sum-rate



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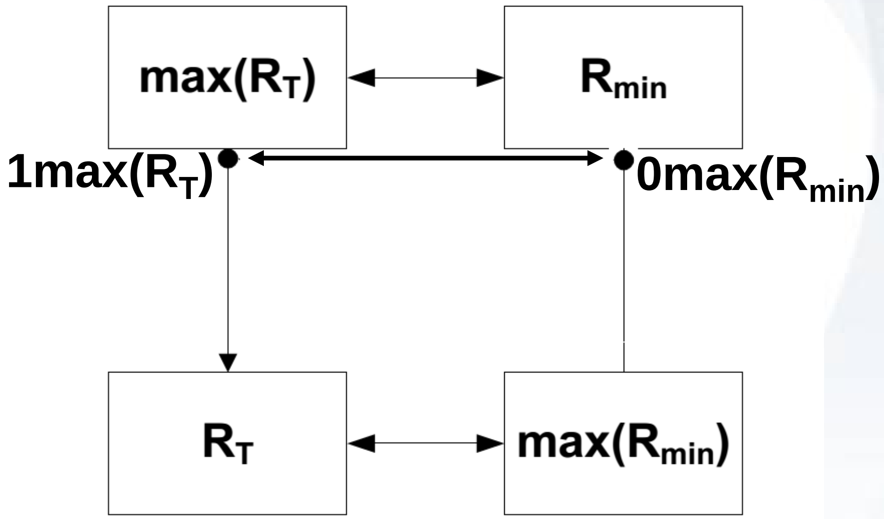

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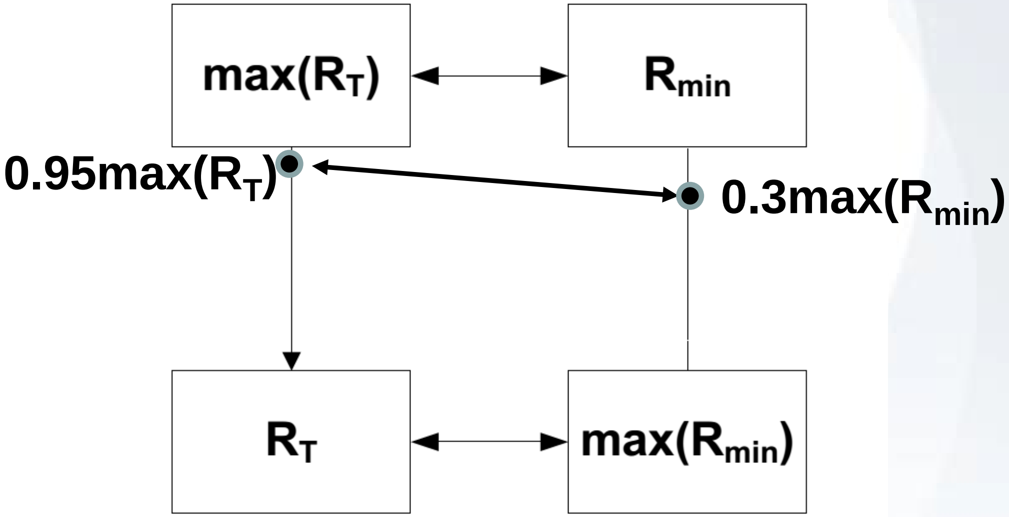

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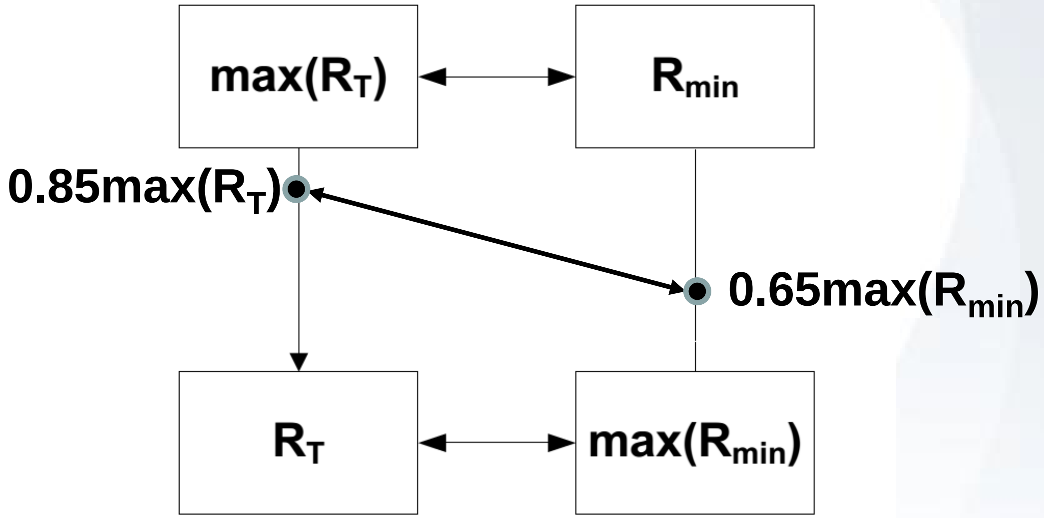

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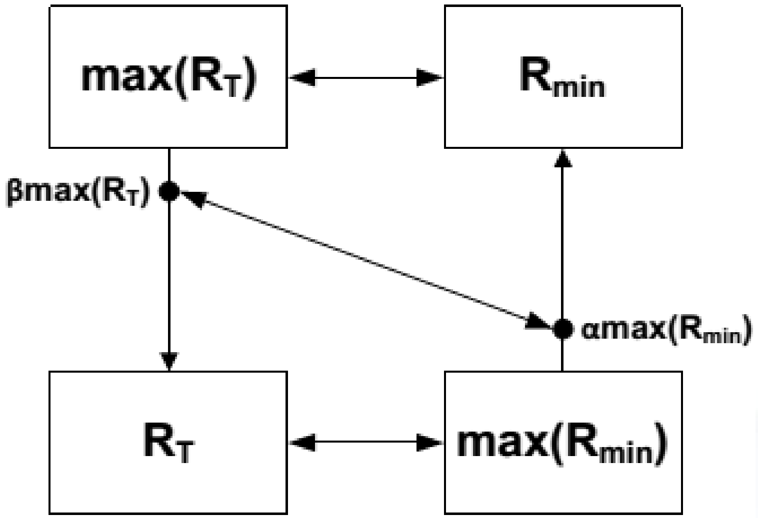
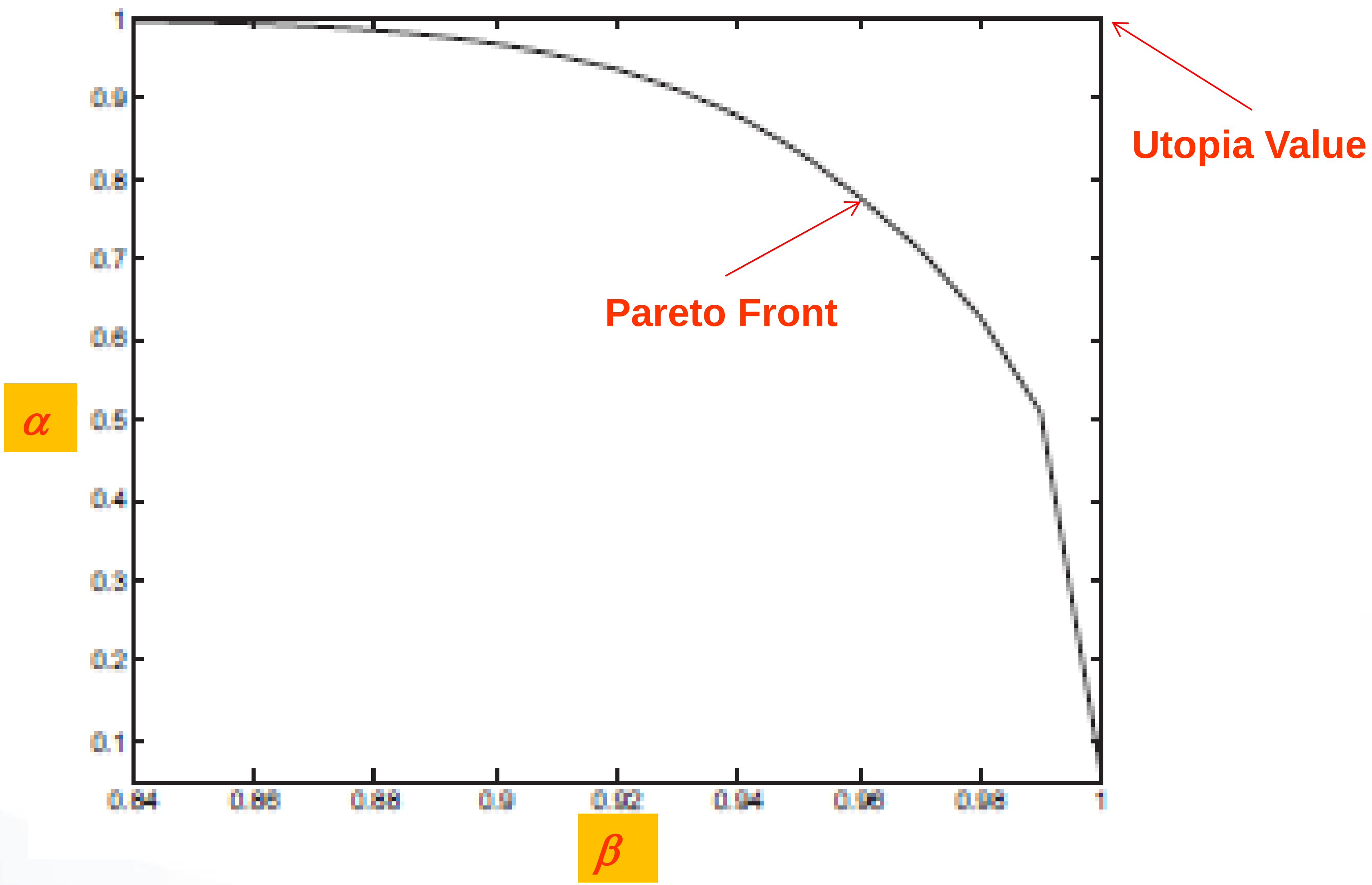


Pareto Front in Multiobjective Optimization



Simulations

- GAMS for the numerical analysis of the MBIP and LP models.

  General Algebraic Modeling System (GAMS) is a high-level modeling system for solving linear, nonlinear, and mixed-integer optimization problems.

- N nodes (20, 30) are randomly placed in a unit square area (100 m x 100 m).

- Power budget: Same for all nodes (P_i = 10 dBm, 20 dBm).

- No of power levels: 1 (on-off power control) to 32.

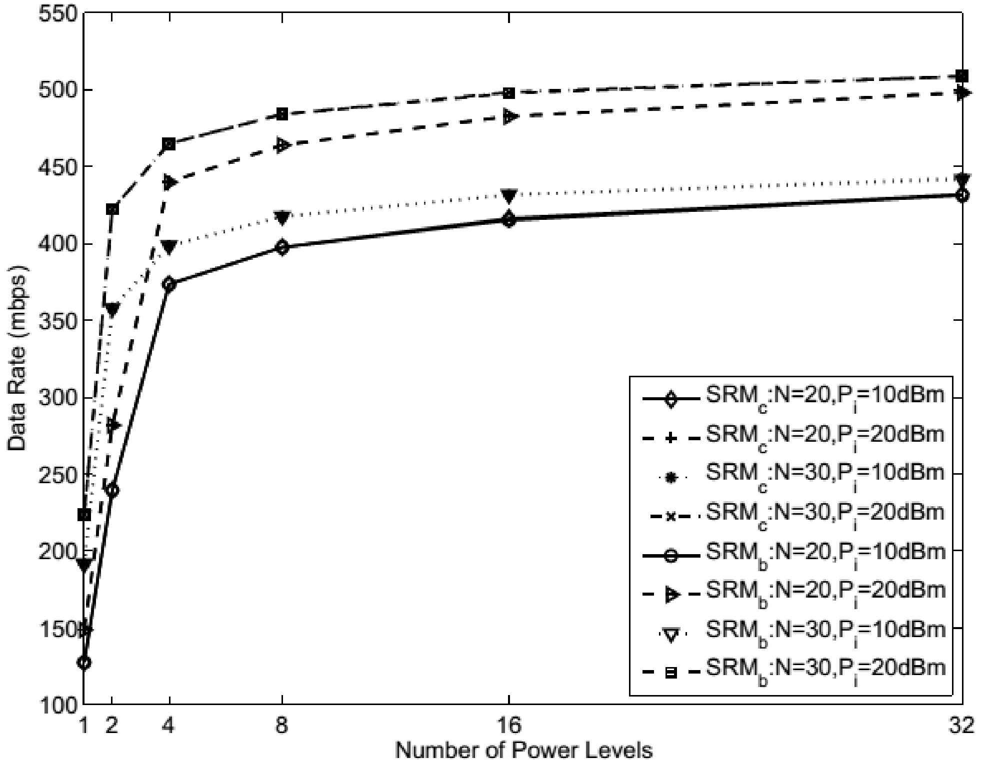
- AWGN (no interference), lognormal shadowing, Rayleigh fading.

- Total BW: W_0 (20 MHz) subchannel BW: $W = W_0/K$ $(K=60)$.

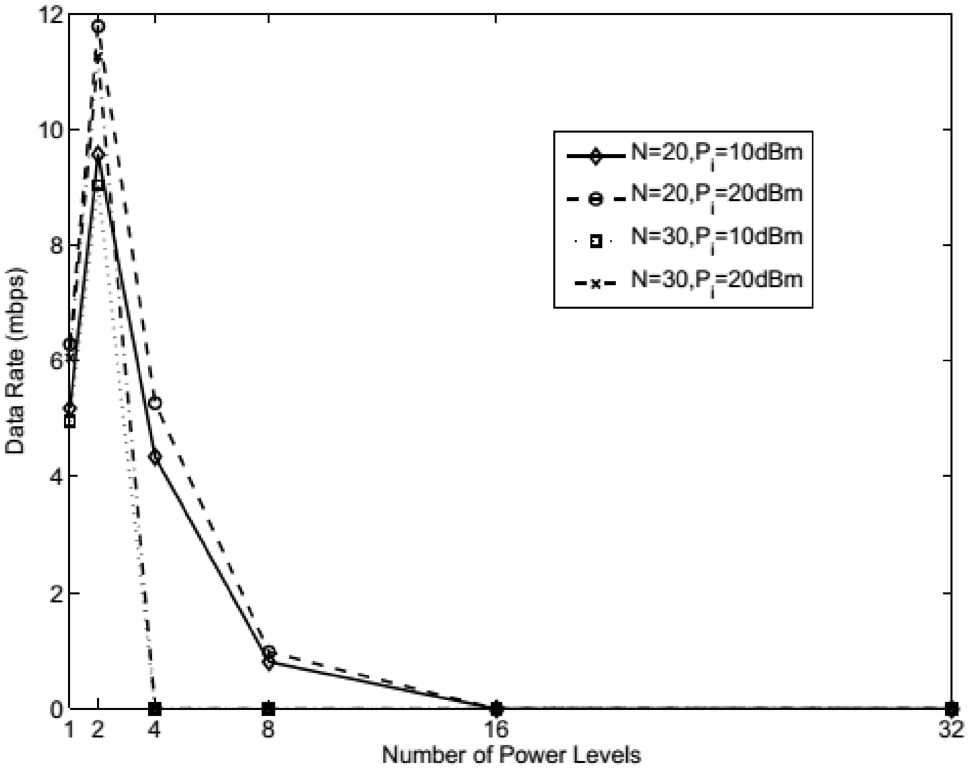
- Monte Carlo simulations with 50 drops.

Analysis – SRM Models

- Sum rates obtained by employing SRM_b and SRM_c models are almost the same
- Once the utilized no of power levels exceeds 8, increase in the sum rates becomes very low



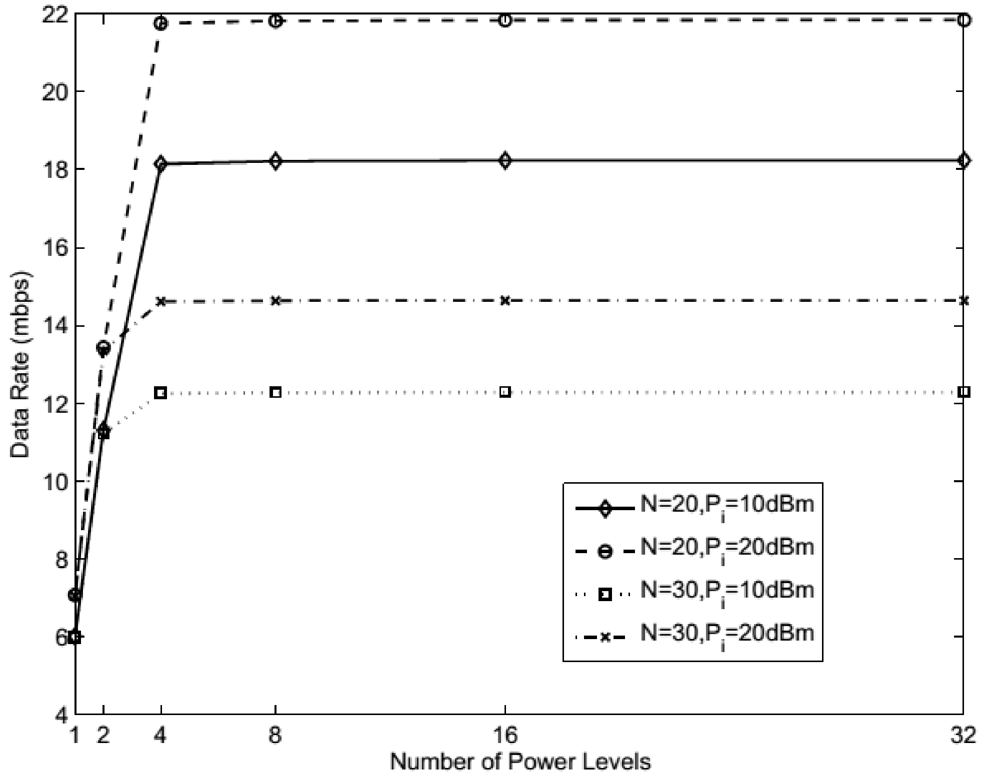
Sum-rates as a function of the number of power levels in the SRM_c and SRM_b models.



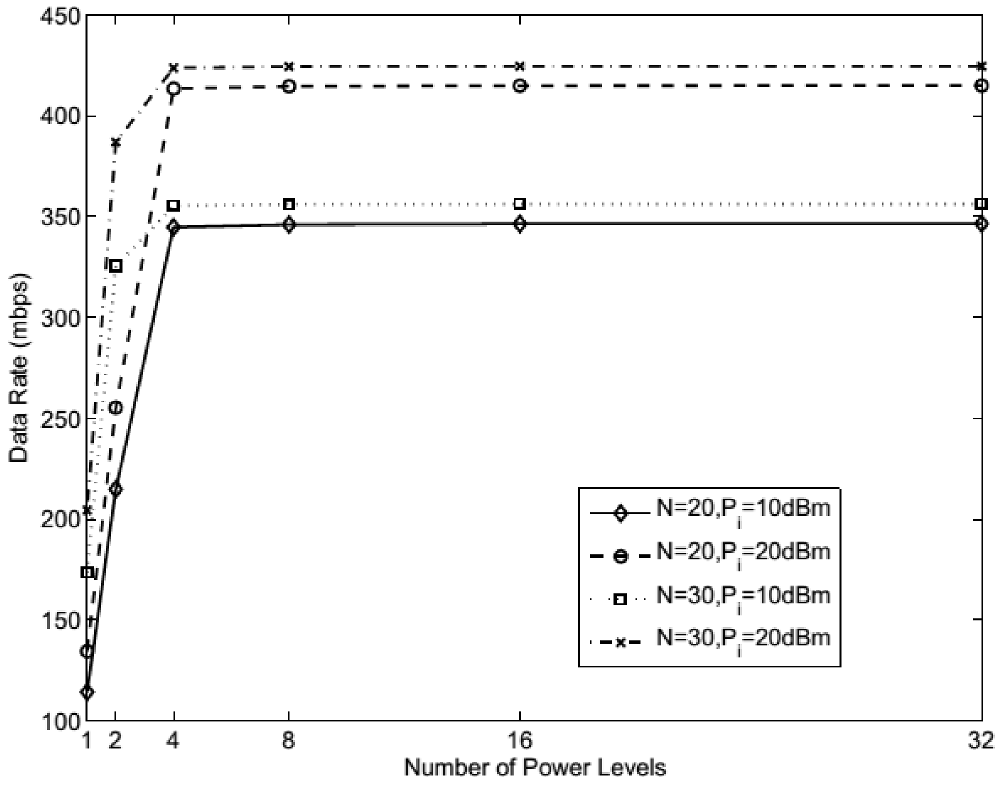
Minimum rates as a function of the number of power levels in the SRM_c model.

Analysis – MMF Model

- As the number of power levels exceeds four, the data rates stay constant in MMF model.



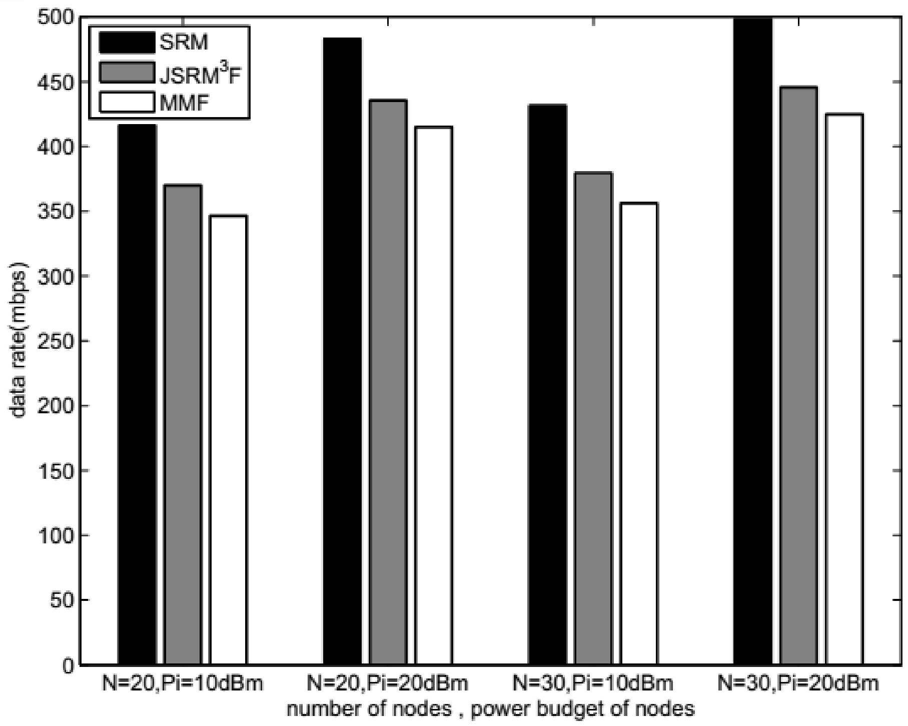
Minimum rates as a function of the number of power levels in the MMF model.



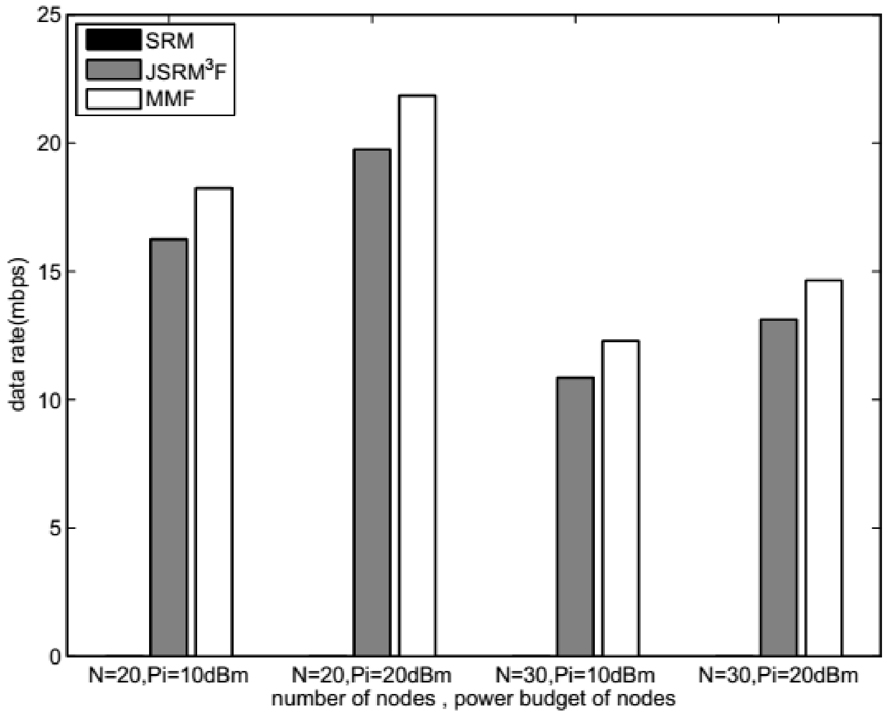
Sum-rates as a function of the number of power levels in the MMF model.

Analysis - JSRM^{3}F Model

- SRM$_c$ model: Minimum rate is sacrificed for maximization of the aggregate data rate
- MMF model: Aggregate rate is sacrificed for providing a minimum level of data rate to all nodes



Sum-rates as a function of the number of nodes and the power budgets of nodes in the JSRM^{3}F model with 16 power levels.

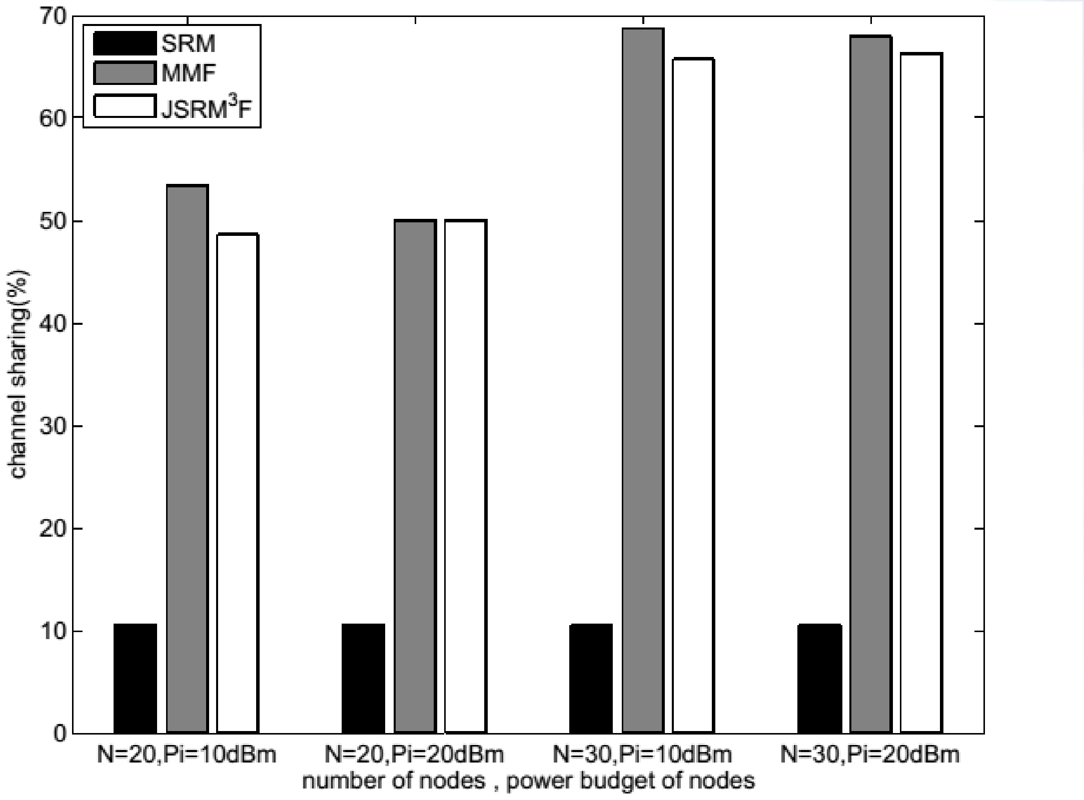


Minumum rates as a function of the number of nodes and the power budgets of nodes in the JSRM^{3}F model with 16 power levels.

Analysis – Channel Sharing

- Sharing of subchannels in time is investigated using SRM_c, MMF and $JSRM^3F$ models.

- When fairness is not considered, at most 10.63 % of all subchannels in the network are shared in time in SRM_c model.
- MMF and $JSRM^3F$ models are used to provide max-min fairness, percentage of sharing of all subchannels in time increases up to 67.93 %.



Percentage of channel sharing in the SRMc, MMF and $JSRM^3F$ models with 16 power levels

Concluding Remarks

- **Joint optimization**
  - Routing, subchannel scheduling and power allocation are jointly optimized.
- **Low complexity**
  - LP models are developed using discrete power levels.
  - Maximum data rates (both as max(R_{min}) and max(R_T)) obtained with discrete power allocation is near-optimal even with few number of discrete power levels.
- **Trade-off**
  - Trade-offs between sum-rate maximization and max-min fairness in relay-enhanced one-cell network is investigated.
- **Channel Sharing**
  - Subchannel sharing: Important when fairness is a concern.