Trade-offs in Sum-Rate Maximization and Fairness in Relay-Enhanced OFDMA-based Cellular Networks

Davut INCEBACAK  (Kocaeli U, Turkey)
Halim YANIKOMEROGLU  (Carleton U, Canada)
Bulent TAVLI  (TOBB U, Turkey)

TUBITAK (The Scientific and Technological Research Council of Turkey)
Huge literature: Perspective needed
Variables
power
RB
link (routing)
…
Resource Allocation

Variables
- power
- RB
- link (routing)
- ...

Objectives
- max sum-rate
- max min-rate
- min sum-power
- fairness
- ...

Resource Allocation

Variables
- power
- RB
- link (routing)
- ...

Objectives
- max sum-rate
- max min-rate
- min sum-power
- fairness
- ...

RAN architecture
- one-cell
- multi-cell
- ICIC, CoMP, CRAN
- relays, cooperation
- ...
- ad hoc, reuse
Resource Allocation

Variables
- power
- RB
- link (routing)
- ...

Objectives
- max sum-rate
- max min-rate
- min sum-power
- fairness
- ...

RAN architecture
- one-cell
- multi-cell
- ICIC, CoMP, CRAN
- relays, cooperation
- ...
- ad hoc, reuse

Optimal solutions:
- Only in simple settings

Advanced settings:
- Not sufficiently explored
**Problem Setting**

**Variables**
- power
- RB
- link (routing)
  ...  

**Objectives**
- max sum-rate
- max min-rate
- min sum-power
- fairness
  ...  

**RAN architecture**
- one-cell
- **multi-cell**
- ICIC, CoMP, CRAN
- relays, cooperation
  ...  
- ad hoc, reuse  

---

**Problem Setting**

### Variables
- power
- RB
- link (routing)

### RAN architecture
- one-cell
- multi-cell
- ICIC, CoMP, CRAN
- relays, cooperation
- ad hoc, reuse

### Objectives
- max weighted sum-rate
- max min-rate
- min sum-power
- fairness

---

Problem Setting

**Variables**
- power
- RB
- link (routing)

**Objectives**
- max weighted sum-rate
- max min-rate
- min sum-power
- fairness

**RAN architecture**
- one-cell
- multi-cell
- ICIC, CoMP, CRAN
- relays, cooperation
- ad hoc, reuse

---

Problem Setting

Objectives
max weighted sum-rate
max min-rate
min sum-power
fairness
...

Variables
power
RB
link (routing)
...

RAN architecture
one-cell
multi-cell
ICIC, CoMP, CRAN
relays, cooperation
...
ad hoc, reuse

R. Rashtchi, R. Gohary, H. Yanikomeroglu,

R. Rashtchi, R. Gohary, H. Yanikomeroglu,
“Generalized cross-layer designs for generic half-duplex multicarear wireless networks with frequency reuse”, under review in IEEE Transactions on Wireless Communications (submission: July 2014).
Problem Setting

Variables
power
RB
link (routing)
...

Objectives
max sum-rate
max min-rate
min sum-power
fairness
...

RAN architecture
one-cell
multi-cell
ICIC, CoMP, CRAN
relays, cooperation
...
ad hoc, reuse

This paper
• Allocation of resources is mostly modeled using NLP to maximize either sum-rate or minimum rate.

• Non-linearity is due to the capacity formula

\[
\text{Capacity} = W \log_2 \left( 1 + \frac{p_{ij}^k \times (h_{ij}^k)^2}{WN_0} \right)
\]

• NLP models generally belong to the class of NP-hard.
  ➢ Works on very small settings
Problem Definition

- Resource allocation in cellular networks
  - Subchannel allocation
  - Power allocation
  - Routing

- Objective
  - Sum Rate Maximization
  - Max-Min Fairness

- Computationally complex NLP solutions for joint design
  - Jointly optimize routing, scheduling and power allocation using LP with discrete power levels

Joint design of power, subchannel allocation and routing to exploit the opportunities offered by network

Trade-off
- Joint Sum-Rate Maximization and Max-Min Fairness
Design Variables

- $p^t_i$ power level between 0 and $P_i$ (maximum power) $p^t \in \{0, p^1, p^2, ..., p^t, ..., p^T = P_i\}$.
- $h_{i,j}^k$ channel gain between node-$i$ and node-$j$ over subchannel-$k$.
- $c_{i,j}^{k,p^t}$ indicator variable determines if subchannel-$k$ on flow from node-$i$ to node-$j$ is used or not with power level $p^t$.
- $f_{i,j}^{k,p^t}$ achievable data rate with power level $p^t$ on subchannel-$k$ between node-$i$ and node-$j$.

$$f_{i,j}^{k,p^t} = W \log_2 \left(1 + \frac{p^t |h_{i,j}^k|^2}{WN_0}\right)$$
Sum-Rate Maximization with Binary Scheduling Variables (SRM\textsubscript{b})

\textbf{Objective: Maximize Sum-Rate}

\begin{align*}
\text{Maximize } R_T &= \sum_{i \in M} s_i \\
\text{Subject to: }
\end{align*}

\( f_{i,j}^{k,p^t} = W \log_2 \left( 1 + \frac{p^t |h_{i,j}^k|^2}{WN_0} \right), \quad i \in M, j \in V, k \in K, p^t \in P, \quad (1) \)

\( \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p^t} \leq 1, \quad i \in M, \quad (2) \)

\( \sum_{k \in K} \sum_{p^t \in P} \left( \sum_{j \in V} c_{i,j}^{k,p^t} f_{i,j}^{k,p^t} - \sum_{j \in W} c_{j,i}^{k,p^t} f_{j,i}^{k,p^t} \right) - s_i = 0, \quad i \in M, \quad (3) \)

\( \sum_{j \in V} \sum_{k \in K} c_{i,j}^{k,p^t} p^t - P_i \leq 0, \quad i \in M, \quad (4) \)

\( p^t \in \{0, p^1, p^2, ..., p^T = P_i\}, \quad (5) \)

\( s_i \geq 0, \quad i \in M, \quad (6) \)

\( c_{i,j}^{k,p^t} \in \{0, 1\}, \ i \in M, j \in V, k \in K, p^t \in P. \quad (7) \)
Sum-Rate Maximization with Continuous Scheduling Variables (SRM\(_C\))

Maximize \( R_T = \sum_{i \in M} s_i \)

Subject to:

\[
\begin{align*}
    f_{i,j}^{k,p} &= W \log_2 \left( 1 + \frac{p_t |h_{i,j}^k|^2}{WN_0} \right), \\
    i \in M, j \in V, k \in K, p^t \in P, & \quad \text{(1)} \\
    \sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p} & \leq 1, \\
    i \in M, & \quad \text{(2)} \\
    \sum_{k \in K} \sum_{p^t \in P} \left( \sum_{j \in V} c_{i,j}^{k,p} f_{i,j}^{k,p} - \sum_{j \in W} c_{j,i}^{k,p} f_{j,i}^{k,p} \right) - s_i & = 0, \\
    i \in M, & \quad \text{(3)} \\
    \sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p} p^t - P_i & \leq 0, \\
    i \in M, & \quad \text{(4)} \\
    s_i & \geq 0, \\
    i \in M, & \quad \text{(6)} \\
    c_{i,j}^{k,p} & \in [0,1], \\
    i \in M, j \in V, k \in K, p^t \in P. & \quad \text{(8)}
\end{align*}
\]

(1) achievable data rates with power level \( p_t \) on subchannel-\( k \) between node-\( i \) and node-\( j \)

(2) interference is prevented by using each subchannel once in the network

(3) flow conservation constraint which is satisfied for all nodes.

(4) limits the total transmit power used by each node

(5) determines the set of power levels

(6) nonnegativity constraint for data rates

(8) continuous scheduling constraint
Maximize $R_T = \sum_{i \in M} s_i$

Subject to:

\[ f_{i,j}^{k,p^t} = \log_2 \left( 1 + \frac{p^t |h_{i,j}^k|^2}{WN_0} \right), \quad i \in M, j \in V, k \in K, p^t \in P, \quad (1) \]

\[ \sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p^t} \leq 1, \quad i \in M, \quad (2) \]

\[ \sum_{k \in K} \sum_{p^t \in P} \left( \sum_{j \in V} c_{i,j}^{k,p^t} f_{i,j}^{k,p^t} - \sum_{j \in W} c_{j,i}^{k,p^t} f_{j,i}^{k,p^t} \right) - s_i = 0, \quad i \in M, \quad (3) \]

\[ \sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p^t} p^t - P_i \leq 0, \quad i \in M, \quad (4) \]

\[ p^t \in \{0, p^1, p^2, ..., p^T = P_i\}, \quad (5) \]

\[ s_i \geq 0, \quad i \in M, \quad (6) \]

\[ c_{i,j}^{k,p^t} \in [0, 1], \quad i \in M, j \in V, k \in K, p^t \in P. \quad (8) \]

**Objective:** Maximize Sum-Rate

(1) achievable data rates with power level $p_t$ on subchannel-$k$ between node-$i$ and node-$j$

(2) interference is prevented by using each subchannel once in the network

(3) flow conservation constraint which is satisfied for all nodes.

(4) limits the total transmit power used by each node

(5) determines the set of power levels

(6) nonnegativity constraint for data rates

(8) continuous scheduling constraint
Sum-Rate Maximization with Continuous Scheduling Variables (SRM<sub>C</sub>)

Maximize \( R_T = \sum_{i \in M} s_i \)
Subject to:

\[
\sum_{i \in M} \sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p^t} \leq 1, \quad i \in M, \tag{2}
\]

\[
\sum_{k \in K} \sum_{p^t \in P} \left( \sum_{j \in V} c_{i,j}^{k,p^t} f_{i,j}^{k,p^t} - \sum_{j \in W} c_{j,i}^{k,p^t} f_{j,i}^{k,p^t} \right) - s_i = 0, \quad i \in M, \tag{3}
\]

\[
\sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p^t} p^t - P_i \leq 0, \quad i \in M, \tag{4}
\]

\[
p^t \in \{0, p^1, p^2, ..., p^t, ..., p^T = P_i\}, \tag{5}
\]

\[
s_i \geq 0, \quad i \in M, \tag{6}
\]

\[
c_{i,j}^{k,p^t} \in [0, 1], \quad i \in M, j \in V, k \in K, p^t \in P. \tag{8}
\]

Note that since \( c_{i,j}^{k,p^t} \) is not binary, SRM<sub>C</sub> is an LP model.
Max-Min Fairness (MMF)

\[ \text{Maximize } \quad R_{\text{min}} \]
\[ \text{Subject to: } \]
\[ f_{i,j}^{k,p_i} = W \log_2 \left( 1 + \frac{p_i^t |h_{i,j}^k|^2}{WN_0} \right), \quad i \in M, j \in V, k \in K, p_i \in P, \quad (1) \]
\[ \sum_{j \in V} \sum_{k \in K} \sum_{p_i \in P} c_{i,j}^{k,p_i} \leq 1, \quad i \in M, \quad (2) \]
\[ \sum_{k \in K} \sum_{p_i \in P} \left( \sum_{j \in V} c_{i,j}^{k,p_i} f_{i,j}^{k,p_i} - \sum_{j \in W} c_{i,j}^{k,p_i} f_{j,i}^{k,p_i} \right) - s_i = 0, \quad i \in M, \quad (3) \]
\[ \sum_{j \in V} \sum_{k \in K} \sum_{p_i \in P} c_{i,j}^{k,p_i} p_i - P_i \leq 0, \quad i \in M, \quad (4) \]
\[ p_i \in \{0, p_1^t, p_2^t, ..., p_i^t, ..., p_T^T = P_i \}, \quad (5) \]
\[ s_i \geq 0, \quad i \in M, \quad (6) \]
\[ c_{i,j}^{k,p_i} \in [0, 1], \quad i \in M, j \in V, k \in K, p_i \in P. \quad (8) \]

- In SRM\textsubscript{b} and SRM\textsubscript{c} models \( \rightarrow \) fairness problem

- In constraint (9) fairness parameter \( R_{\text{min}} \) (minimum data rate generated by one node in the network) is introduced.

\[ s_i \geq R_{\text{min}}, \quad i \in M. \quad (9) \]

- Using (9), Max-Min Fairness (MMF) model is developed with the **objective of maximizing** \( R_{\text{min}} \).
Joint Sum-Rate Maximization and Max-Min Fairness (JSRM$^3$F)

Maximize . \[(\alpha + \beta)\]
Subject to:

\[f_{i,j}^{k,p} = W \log_2 \left( 1 + \frac{p^t |h_{i,j}^k|^2}{WN_0} \right),\]
\[i \in M, j \in V, k \in K, p^t \in P,\] (1)

\[
\sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p} \leq 1, \quad i \in M,\]
(2)

\[
\sum_{k \in K} \sum_{p^t \in P} \left( \sum_{j \in V} c_{i,j}^{k,p} f_{i,j}^{k,p} - \sum_{j \in W} c_{i,j}^{k,p} f_{i,j}^{k,p} \right) - s_i = 0, \quad i \in M,\]
(3)

\[
\sum_{j \in V} \sum_{k \in K} \sum_{p^t \in P} c_{i,j}^{k,p} p^t - P_i \leq 0, \quad i \in M,\]
(4)

\[p^t \in \{0, p^1, p^2, ..., p^t, ..., p^T = P_i\},\]
(5)

\[s_i \geq 0, \quad i \in M,\]
(6)

\[c_{i,j}^{k,p} \in [0, 1], \quad i \in M, j \in V, k \in K, p^t \in P.\]
(8)

- To investigate the trade-offs between maximizing $R_T$ and $R_{min}$, two additional constraints are introduced as

\[s_i \geq \alpha \max(R_{min}), \quad \forall i \in M,\]
(10)

\[
\sum_{i \in M} s_i \geq \beta \max(R_T)\]
(11)

- Values of $\max(R_{min})$ and $\max(R_T)$ are from MMF and SRM$_c$ models.

- $\alpha$ and $\beta$ are controlling variables for the level of $\max(R_{min})$ and $\max(R_T)$.

- Using SRM$_c$ and constraint (10) and (11)), JSRM$^3$F model is developed that maximize $(\alpha + \beta)$.

- JSRM$^3$F model jointly maximizes $R_T$ and $R_{min}$. 
Objectives of SRM\textsubscript{c} and MMF are conflicting

- Fairness is achieved at the cost of a decreased sum-rate

\[
\begin{align*}
\max(R_T) & \quad \beta_{\max}(R_T) \\
R_T & \quad \alpha_{\max}(R_{\min}) \\
R_{\min} & \quad \max(R_{\min})
\end{align*}
\]
Joint Sum-Rate Maximization and Max-Min Fairness (JSRM$^3$F)

- Objectives of SRM$_c$ and MMF are conflicting
  - Fairness is achieved at the cost of a decreased sum-rate
Joint Sum-Rate Maximization and Max-Min Fairness (JSRM$^3$F)

- Objectives of SRM$_c$ and MMF are conflicting
  - Fairness is achieved at the cost of a decreased sum-rate
Joint Sum-Rate Maximization and Max-Min Fairness (JSRM$^3$F)

- Objectives of SRM$_c$ and MMF are conflicting
  - Fairness is achieved at the cost of a decreased sum-rate

\[
\begin{align*}
\text{max}(R_T) & \rightarrow R_{\text{min}} \\
0.85\text{max}(R_T) & \rightarrow R_T \\
0.65\text{max}(R_{\text{min}}) & \rightarrow \text{max}(R_{\text{min}})
\end{align*}
\]
Objectives of SRM$_c$ and MMF are conflicting

- Fairness is achieved at the cost of a decreased sum-rate
Pareto Front in Multiobjective Optimization

Pareto Front

Utopia Value

\( \alpha \)

\( \beta \)
GAMS for the numerical analysis of the MBIP and LP models.

General Algebraic Modeling System (GAMS) is a high-level modeling system for solving linear, nonlinear, and mixed-integer optimization problems.

- $N$ nodes (20, 30) are randomly placed in a unit square area (100 m x 100 m).
- Power budget: Same for all nodes ($P_i = 10$ dBm, 20 dBm).
- No of power levels: 1 (on-off power control) to 32.
- AWGN (no interference), lognormal shadowing, Rayleigh fading.
- Total BW: $W_0$ (20 MHz) subchannel BW: $W = W_0/K$ $(K=60)$.
- Monte Carlo simulations with 50 drops.
• Sum rates obtained by employing SRM\textsubscript{b} and SRM\textsubscript{c} models are almost the same
• Once the utilized no of power levels exceeds 8, increase in the sum rates becomes very low

**Analysis – SRM Models**

- **Sum-rates as a function of the number of power levels in the SRM\textsubscript{c} and SRM\textsubscript{b} models.**
- **Minimum rates as a function of the number of power levels in the SRM\textsubscript{c} model.**
As the number of power levels exceeds four, the data rates stay constant in MMF model.

Minimum rates as a function of the number of power levels in the MMF model.

Sum-rates as a function of the number of power levels in the MMF model.
Analysis - JSRM$^3$F Model

- SRM$_c$ model: Minimum rate is sacrificed for maximization of the aggregate data rate
- MMF model: Aggregate rate is sacrificed for providing a minimum level of data rate to all nodes

**Sum-rates** as a function of the number of nodes and the power budgets of nodes in the JSRM$^3$F model with 16 power levels.

**Minimum rates** as a function of the number of nodes and the power budgets of nodes in the JSRM$^3$F model with 16 power levels.
Analysis – Channel Sharing

• Sharing of subchannels in time is investigated using SRM<sub>c</sub>, MMF and JSRM<sup>3</sup>F models.

• When fairness is not considered, at most 10.63 % of all subchannels in the network are shared in time in SRM<sub>c</sub> model.

• MMF and JSRM<sup>3</sup>F models are used to provide max-min fairness, percentage of sharing of all subchannels in time increases up to 67.93 %.
Concluding Remarks

- **Joint optimization**
  - Routing, subchannel scheduling and power allocation are jointly optimized.

- **Low complexity**
  - LP models are developed using discrete power levels.
  - Maximum data rates (both as $\max(R_{\min})$ and $\max(R_T)$) obtained with discrete power allocation is near-optimal even with few number of discrete power levels.

- **Trade-off**
  - Trade-offs between sum-rate maximization and max-min fairness in relay-enhanced one-cell network is investigated.

- **Channel Sharing**
  - Subchannel sharing: Important when fairness is a concern.