Efficiently Computable Bounds on the Rates Achieved by A Cross Layer Design with Binary Scheduling in Generic OFDMA Wireless Networks

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***Abstract*—Future broadband communication networks are expected to be OFDMA-based with generic *ad hoc* topologies, wherein the wireless nodes play multiple roles, including transmission, reception and relaying. When the schedules by which the OFDMA subchannels are assigned to wireless links are binary, obtaining a characterization of the maximum rates that can be reliably communicated by these networks can be shown to be NP-hard. To circumvent this difficulty, we provide efficient means for computing two lower bounds on the achievable weighted sum rates. The first bound is obtained by using geometric programming approximation techniques, whereas the second bound is obtained by rounding the solutions of a relaxed version of the cross layer design problem. Finally, we consider an existing upper bound on the achievable weighted sum rates, and we use numerical simulations to show that the lower and upper bounds are relatively tight, especially at high signal-to-noise ratios.**

***Index Terms*—Cross layer design, convex optimization, geometric programming, ad hoc networks, scheduling.**

I. Introduction

To meet the prospective increase in demands on the high data rate services delivered by future broadband communication networks, it is expected that the wireless terminals in these networks will perform multiple tasks, which include transmitting, receiving and relaying data to other nodes [1]. As such, future communication networks are expected to be less structured and more responsive to instantaneous demands. A generic model that captures the dynamic topology of these networks is the *ad hoc* one [2]. This model provides a flexible representation of various communication network structures, including current cellular ones and sensor networks.

To facilitate the implementation of future wireless networks, these networks are likely to rely on orthogonal frequency division multiple access (OFDMA) for accessing the wireless medium. In addition to its practicality and resilience to multiuser interference and frequency-selective fading, OFDMA offers an effective means for sharing the frequency band between multiple terminals, depending on their channel conditions [3].

Another feature of future wireless networks is that the resources available for their operation are likely to be rather scarce. Such resources include the available frequency spectrum and the, typically low, power of the batteries to be used by the wireless terminals. Providing ubiquitous high data rate services under these stringent operation conditions leaves little room for wasting resources. Such wastage can be alleviated by judicious selection of the data routes, the subchannels schedules and the power allocations. Although making these selections separately facilitates the design of the network, they are interrelated and considering them conjointly results in more effective utilization of the available resources. Unfortunately, such joint considerations can result in prohibitive computational cost. This is especially true for networks with a large number of wireless terminals with various capabilities. Hence, it is desirable to develop joint designs that are, not only close to being optimal, but also efficiently computable.

Capitalizing on the advantages of considering multiple aspects jointly, several cross layer design techniques have been developed for wireless communication networks. For instance, jointly optimal subchannel schedules and power allocations have been obtained in [4] for an OFDMA-based system with time-shared subchannels. Optimization frameworks for obtaining jointly optimal routes and power allocations were developed in [5] when the subchannels assigned to the wireless terminals are orthogonal and in [6] when these subchannels are not necessarily orthogonal, but the rates are chosen from a discrete set. Using geometric programming (GP), an approach that generates locally optimal transmission schedules and power allocations was developed in [7] for code-division multiple access systems. Approaches that exploit the broadcast feature of the wireless medium to provide jointly optimized cross layer designs were developed in [8] and [9] using superposition coding and successive interference cancellation [9]. A heuristic for obtaining data routes, power allocations and binary schedules jointly when the terminals are restricted to being transmitters, receivers or relays was developed in [10]. Removing the latter restriction and allowing the subchannels to be time-shared, a jointly optimal cross layer design was provided in [11]. A tutorial that surveys some of the relatively recent cross layer designs is also available in [12].

In many practical scenarios, allowing the subchannels to be time-shared by multiple terminals increases the overhead required for establishing coherent communication between the wireless terminals. This is especially true in networks with time-varying channels. In these situations, it is more practical to restrict the subchannel schedules to be binary [13]. Such schedules result in assigning each subchannel to a particular wireless terminal for the entire signalling interval. One of the main drawbacks of this approach is that it results in an NP-hard mixed integer optimization problem [14] that is difficult to solve. To alleviate this problem, in this paper we develop bounds on the rates that can be achieved by binary scheduling. In particular, we consider the joint optimization of data routes, binary schedules and power allocations in an *ad hoc* OFDMA-based wireless network. The wireless terminals are assumed capable of performing multiple tasks simultaneously including sending, receiving and relaying data to other nodes using a half-duplex decode-and-forward relaying scheme. The design objective is to maximize a weighted sum of the rates injected and reliably communicated over the network.

Two approaches are developed for obtaining efficiently computable lower bounds on these rates. In the first approach, the restriction of the schedules to be binary is captured by imposing a set of constraints on the power allocations. Unfortunately, the resulting formulation is nonconvex. However, this formulation is amenable to an efficient GP-based approximation technique known as monomial approximation [15]. Using this technique locally optimal routes and power allocations are obtained, which can then be used to recover the binary schedules. To develop our second approach, we begin by considering the convex formulation developed in [11] for obtaining the optimal weighted sum rate when the subchannel schedules are not restricted to be binary. The formulation in [11] is a relaxation of the current one and hence can be used to bound the maximum weighted sum rate. Unlike the first approach, the formulation in [11] yields explicit continuous schedules. Hence, normalizing these schedules and rounding them yields potentially suboptimal binary schedules. With these schedules fixed, the cross layer design problem is shown to be convex and is used to provide the second lower bound. In the numerical investigations section we will compare the two lower bounds with the upper bound obtained in [11] and the true maximum obtained by exhaustive search over all possible binary schedules. Numerical results suggest that the gap between the obtained lower bounds and the upper bound is small and is reduced with the increase in power.

II. System Model and Preliminaries

In this section, we will describe the system model under consideration and will review the background material necessary for developing the formulations used to obtain the desired bounds. Most of the background material is available in [11], but is included herein for completeness.

We consider an *ad hoc* OFDMA-based wireless network with N nodes, each with one transmit and one receive antenna and a fixed power budget. In addition to transmitting and receiving data, the nodes can act as relays to assist other nodes using a half-duplex decode-and-forward relaying scheme. This assumption is generic since scenarios in which some of the nodes are barred from performing any of these tasks can be readily incorporated in the forthcoming formulations.

A. System Model

The considered network can be modelled by a weighted fully connected directed graph with N vertices and $L = N(N-1)$ inter-nodal links. Each link is composed of K subchannels, each with a non-negative weight representing the complex gain. Let the set of vertices in this graph be denoted by $\mathcal{N} \triangleq \{1, \ldots, N\}$ and the set of links be denoted by $\mathcal{L} \triangleq \{1, \ldots, L\}$. We will adopt the following convention for labelling the links [8]. For any $n, n' \in \mathcal{N}$, the link connecting node n to node n' is given by

$$\ell = \begin{cases} (N-1)(n-1) + n' - 1 & \text{if } n < n', \\ (N-1)(n-1) + n' & \text{otherwise.} \end{cases} \tag{1}$$

The sets of all incoming and outgoing links of any node $n \in \mathcal{N}$ are denoted by $\mathcal{I}(n)$ and $\mathcal{O}(n)$, respectively. The connectivity of the graph can be characterized by using the incidence matrix, $A = [a_{n\ell}]$, where $a_{n\ell} = 1$ if $\ell \in \mathcal{O}(n)$, $a_{n\ell} = -1$ if $\ell \in \mathcal{I}(n)$ and $a_{n\ell} = 0$ otherwise [5].

In the considered OFDMA-based network, the total bandwidth available for communication, W_0, is divided into K subchannels, each with a bandwidth of $W = \frac{W_0}{K}$. The set of all subchannels is denoted by $\mathcal{K} \triangleq \{1, \ldots, K\}$, and the complex gain of subchannel k of link ℓ connecting node n to node n' is denoted by $h_{nn'}^{(k)}$, which represents fading and transmit and receive antenna gains. These gains are assumed to remain essentially constant during each signalling interval. Since the graph is fully connected, noisy versions of each transmission are 'heard' by all the nodes in the network. The additive noise processes at the nodes are assumed to be independent with zero mean Gaussian distribution and variance N_0.

Using binary scheduling, each subchannel is used by at most one node, which implies that relaying is half-duplex.

B. GP-based Monomial Approximation

GP is a class of nonlinear optimization problems which, although nonconvex, can be readily transformed into efficiently solvable convex ones [15]. This class of problems will be instrumental in obtaining one of our lower bounds on the maximum weighted sum rate. Let $\boldsymbol{z} = (z_1, \ldots, z_n) \in \mathbb{R}_+^n$ be the vector of optimization variables in the considered GP. A function of the form $c_0 \prod_{i=1}^n z_i^{\alpha_i}$ is said to be a monomial in $\boldsymbol{z}$ if $c_0 > 0$ and $\{\alpha_i\}$ are arbitrary constants. Similarly, a function of the form $\sum_{j=1}^J c_j \prod_{i=1}^n z_i^{\alpha_{ij}}$ is said to be a posynomial in $\boldsymbol{z}$ if $\{c_j\}$ are positive and $\{\alpha_{ij}\}$ are arbitrary constants. Now, the standard form of a GP has the following form:

$$\begin{aligned} \min_{\mathbf{z}} \quad & f_0(\boldsymbol{z}), \\ \text{subject to} \quad & f_i(\boldsymbol{z}) \leq 1, \qquad i = 1, \ldots, m, \\ & g_i(\boldsymbol{z}) = 1, \qquad i = 1, \ldots, p, \end{aligned} \tag{2}$$

where $\{f_i\}$ are posynomials and $\{g_i\}$ are monomials.

A monomial approximation of a positive differentiable function $h(\boldsymbol{z}) > 0$ near $\boldsymbol{z}^{(0)}$ is the first order Taylor expansion in the logarithmic domain [15]. Defining $\gamma_i = \frac{z_i^{(0)}}{h(\boldsymbol{z}^{(0)})} \frac{\partial h}{\partial z_i}\big|_{\boldsymbol{z}=\boldsymbol{z}^{(0)}}$, the monomial approximation of $h(\boldsymbol{z})$ is given by

$$h(\boldsymbol{z}) \approx h(\boldsymbol{z}^{(0)}) \prod_{i=1}^{n} \big(z_i/z_i^{(0)}\big)^{\gamma_i}. \tag{3}$$

Having described the system model and the GP framework, in the next section we will use this model to derive constraints that must be satisfied by feasible data routes, binary schedules and power allocations. We will later show how these constraints can be cast in a GP compatible form.

III. Optimization Framework

In this section we will provide the mathematical characterization of the system constraints. We will use this characterization to cast the cross layer design as an optimization problem.

A. System Constraints

Let subchannel schedulings to links be determined by the binary matrix $B = [\beta_{\ell k}]$. In particular, setting the ℓk-th entry of this matrix, $\beta_{\ell k}$, to 1 or 0 determines whether subchannel k is assigned to link ℓ or not, respectively. Hence,

$$\beta_{\ell k} \in \{0, 1\}, \qquad \ell \in \mathcal{L},\ k \in \mathcal{K}. \tag{4}$$

Note that the restriction of the entries of B to be binary implies that no subchannel time-sharing is allowed; subchannel schedules are fixed for the entire signalling interval. As is typical in OFDMA-based networks, interference is avoided by restricting each subchannel to be used at most once across the entire network [14]. Hence, with (4) satisfied, this requirement can be expressed as

$$\sum_{\ell \in \mathcal{L}} \beta_{\ell k} \le 1, \qquad k \in \mathcal{K}. \tag{5}$$

We now consider the constraints that must be satisfied by the power allocated by the nodes to each subchannel. To do so, let $p_n^{(k)}$ denote the power allocated by node n to subchannel k. Then the elements of $\{p_n^{(k)}\}$ must satisfy

$$p_n^{(k)} \ge 0, \qquad n \in \mathcal{N},\ k \in \mathcal{K}. \tag{6}$$

Assuming that the nodes have limited power budgets, the elements of $\{p_n^{(k)}\}$ must also satisfy

$$\sum_{k \in \mathcal{K}} \sum_{\ell \in \mathcal{O}(n)} \beta_{\ell k} p_n^{(k)} \le P_n, \qquad n \in \mathcal{N}, \tag{7}$$

where P_n is the total power budget of node n. In (7), we have used the fact that only the subchannels scheduled to outgoing links contribute to the power consumed by every node $n \in \mathcal{N}$.

To determine the capacity, $C_{\ell k}$, of subchannel k on link ℓ, we assume that the nodes use Gaussian signalling. Invoking the fact that the restriction imposed on the subchannels to be scheduled to, at most, one link implies that this capacity is

$$C_{\ell k} = \beta_{\ell k} \log_2\big(1 + p_n^{(k)} |h_{nn'}^{(k)}|^2 / W N_0\big), \tag{8}$$

where ℓ is the label of the link connecting node n to node n'. For the capacity expression in (8) to be non-zero, $\beta_{\ell k}$ must be equal to 1, i.e., subchannel k must be scheduled to link ℓ.

To characterize the routing constraints, let $\mathcal{D} \triangleq \{1, \ldots, D\}$ be the set of all destination nodes; $\mathcal{D} \subseteq \mathcal{N}$. Let $s_n^{(d)}$ denote the rate of the data injected at node $n \in \mathcal{N}$ and intended for destination $d \in \mathcal{D}$. Finally, let $x_{\ell k}^{(d)}$ denote the flow intended for destination $d \in \mathcal{D}$ on subchannel $k \in \mathcal{K}$ of link $\ell \in \mathcal{L}$. Since, in our model, the graph representing the network is assumed to be directed, we must have

$$x_{\ell k}^{(d)} \ge 0, \qquad \ell \in \mathcal{L},\ k \in \mathcal{K},\ d \in \mathcal{D}, \tag{9}$$

$$s_n^{(d)} \ge 0, \qquad n \in \mathcal{N} \setminus \{d\},\ d \in \mathcal{D}, \tag{10}$$

where $\setminus$ denotes the set minus operation.

The injected rates, $\{s_n^{(d)}\}$, and the flows, $\{x_{\ell k}^{(d)}\}$ of every destination $d \in \mathcal{D}$ are related by the flow conservation law [5]. Using the incidence matrix, A, defined in Section II-A, this law results in the following constraints:

$$\sum_{\ell \in \mathcal{L}} \sum_{k \in \mathcal{K}} a_{n\ell} x_{\ell k}^{(d)} = s_n^{(d)}, \qquad n \in \mathcal{N} \setminus \{d\},\ d \in \mathcal{D}. \tag{11}$$

Successive application of the flow conservation law yields that the total data rate received by any destination node is given by $s_d^{(d)} = -\sum_{n \in \mathcal{N} \setminus \{d\}} s_n^{(d)}$, $d \in \mathcal{D}$.

For reliable communication, the aggregate rate on subchannel k of link ℓ cannot exceed $C_{\ell k}$ defined in (8), i.e.,

$$\sum_{d \in \mathcal{D}} x_{\ell k}^{(d)} \le W \beta_{\ell k} \log_2\Big(1 + \frac{p_n^{(k)} |h_{nn'}^{(k)}|^2}{W N_0}\Big),\ \ell \in \mathcal{L}, k \in \mathcal{K}. \tag{12}$$

Having characterized the constraints that must be satisfied by the network, we are now ready to write the cross layer optimization problem.

B. Problem Formulation

Let $w_n^{(d)}$ be the non-negative weight assigned to $s_n^{(d)}$. Our objective in the following formulation will be to maximize $\sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N} \setminus \{d\}} w_n^{(d)} s_n^{(d)}$, for some given $\{w_n^{(d)}\}$ satisfying $\frac{1}{D(N-1)} \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N} \setminus \{d\}} w_n^{(d)} = 1$. Maximizing the objective corresponding to all possible weights enables us to evaluate the region of injected rates that can be communicated over the network during a signalling interval. By varying $\{w_n^{(d)}\}$ over the (scaled) unit simplex, one can control the quality of the service assigned to each rate; a higher weight implies a higher priority to the corresponding rate. Using (4)–(7), (9)–(12), the cross layer design problem can be cast as

$$\max_{\{s_n^{(d)}\}, \{x_{\ell k}^{(d)}\}, \{\beta_{\ell k}\}, \{p_n^{(k)}\}} \sum_{d \in \mathcal{D}} \sum_{n \in \mathcal{N} \setminus \{d\}} w_n^{(d)} s_n^{(d)}, \tag{13a}$$

$$\text{subject to} \quad s_n^{(d)} \ge 0, \qquad n \in \mathcal{N} \setminus \{d\}, d \in \mathcal{D}, \tag{13b}$$

$$\sum_{\ell \in \mathcal{L}} \sum_{k \in \mathcal{K}} a_{n\ell} x_{\ell k}^{(d)} = s_n^{(d)}, \quad n \in \mathcal{N} \setminus \{d\}, d \in \mathcal{D}, \tag{13c}$$

$$x_{\ell k}^{(d)} \ge 0, \qquad \ell \in \mathcal{L},\ k \in \mathcal{K},\ d \in \mathcal{D}, \tag{13d}$$

$$\sum_{d \in \mathcal{D}} x_{\ell k}^{(d)} \le W \beta_{\ell k} \log_2\Big(1 + \frac{p_n^{(k)} |h_{nn'}^{(k)}|^2}{W N_0}\Big), \quad \ell \in \mathcal{L},\ k \in \mathcal{K}, \tag{13e}$$

$$\beta_{\ell k} \in \{0,1\}, \qquad \ell \in \mathcal{L},\, k \in \mathcal{K}, \tag{13f}$$
$$p_n^{(k)} \geq 0, \qquad n \in \mathcal{N}, k \in \mathcal{K}, \tag{13g}$$
$$\sum_{\ell \in \mathcal{L}} \beta_{\ell k} \leq 1, \qquad k \in \mathcal{K}, \tag{13h}$$
$$\sum_{k \in \mathcal{K}} \sum_{\ell \in \mathcal{O}(n)} \beta_{\ell k} p_n^{(k)} \leq P_n, \qquad n \in \mathcal{N}. \tag{13i}$$

This problem is in the form of a mixed integer program, which is known to be NP-hard [14] due to the binary constraints in (13f). Moreover the constraints in (13e) and (13i) are nonconvex; the right hand side (RHS) of (13e) is not jointly concave in $\{c_{\ell k}, p_n^{(k)}\}$ and the Hessian matrix of the left hand side (LHS) of (13i) is non-definite.

To avoid dealing directly with the NP-hardness of the formulation in (13), in the next section we will provide an analogous formulation in which the binary schedules are accounted for by equivalent constraints on the power allocations of the nodes. The resulting formulation shares common features with the GP standard form in (2). This will enable us to develop a GP-based approach to efficiently obtain a lower bound on the weighted sum rates. Another lower bound will be developed based on using a previously obtained upper bound, which relies on the relaxation of the binary constraints.

IV. Two Lower Bounds

In this section, we develop two efficiently computable lower bounds on the weighted sum rate generated by (13). In the first approach, the restriction of the schedules to be binary is captured by imposing a set of constraints on the power allocations. Although the new formulation is still NP-hard, it is amenable to the GP-based monomial approximation technique described in Section II-B. In the second approach, we consider the problem in (13) when the subchannel schedules are not restricted to be binary. Removing this restriction was used in [11] to develop a convex optimization framework for obtaining the optimal weighted sum rate when continuous, rather than binary, scheduling is used. Now, we note that, because of the relaxation, the weighted sum rate generated by the formulation in [11] is an upper bound on that generated by (13). A straightforward way to obtain a lower bound on the weighted sum rate when the schedules are binary is by normalizing and rounding the continuous schedules generated by the convex formulation in [11] and using the resulting schedules in solving the cross layer design problem. The details of these two approaches will be described next.

A. The GP-Based Approach

The binary scheduling constraints in (13f) and (13h) imply that at most one node can transmit on subchannel $k \in \mathcal{K}$. In other words, at most one node can allocate a strictly positive power to any subchannel $k \in \mathcal{K}$. Hence, the binary scheduling constraints can be equivalently expressed as

$$p_n^{(k)} p_{n'}^{(k)} = 0, \qquad k \in \mathcal{K},\ n, n' \in \mathcal{N},\ n \neq n'. \tag{14}$$

Enforcing this constraint yields to at most one 'active' node per subchannel, which makes the recovery of the binary schedules from the power allocations straightforward.

The replacement of the constraints in (13f) and (13h) with the ones in (14) does not affect the flow conservation and the non-negativity constraints in (13b)–(13d) and (13g). However, this replacement affects the capacity constraints in (13e) and the power budget constraints in (13i). Since by (14) only one element in $\{p_n^{(k)}\}_{n=1}^N$ is strictly positive for any subchannel $k \in \mathcal{K}$, the constraints in (13e) can be expressed as

$$\sum_{d \in \mathcal{D}} x_{\ell k}^{(d)} \leq W \log_2\Big(1 + \frac{p_n^{(k)} |h_{nn'}^{(k)}|^2}{W N_0}\Big), \quad \ell \in \mathcal{L},\, k \in \mathcal{K}, \tag{15}$$

and the constraint in (13i) can be expressed as

$$\sum_{k \in \mathcal{K}} p_n^{(k)} \leq P_n, \qquad n \in \mathcal{N}. \tag{16}$$

Replacing the constraints in (13e), (13f), (13h) and (13i) with those in (14)–(16) yields an optimization problem that, although nonconvex, is amenable to GP-based monomial approximation. To use this approximation, we use the exponential function to map $\{s_n^{(d)}\}$ and $\{x_{\ell k}^{(d)}\}$ to $\{t_n^{(d)}\}$ and $\{r_{\ell k}^{(d)}\}$, respectively. In particular, we have

$$s_n^{(d)} = \log_2 t_n^{(d)}, \qquad n \in \mathcal{N} \setminus \{d\},\, d \in \mathcal{D}, \tag{17}$$
$$x_{\ell k}^{(d)} = W \log_2 r_{\ell k}^{(d)}, \qquad \ell \in \mathcal{L},\, k \in \mathcal{K},\, d \in \mathcal{D}. \tag{18}$$

These are one-to-one mappings which enable the original variables to be readily recovered from the new ones. Using the new variables, the objective and the constraints in (13b)–(13d) can be readily expressed in GP compatible forms; cf. Section II-B. Now, using the new variables, the capacity constraints in (15) can be expressed as

$$W N_0 \prod_{d \in \mathcal{D}} r_{\ell k}^{(d)} \leq W N_0 + p_n^{(k)} |h_{nn'}^{(k)}|^2, \quad \ell \in \mathcal{L}, k \in \mathcal{K}. \tag{19}$$

Unfortunately, the RHS of each constraint in (19) is not a monomial and hence, does not conform to the GP framework[1]. To overcome this difficulty, we invoke the monomial approximation method in Section II-B, whereby (3) is used to approximate the RHSs of (19) with monomial functions near some initial power allocation, $\{p_{0,n}^{(k)}\}$.

We now consider the remaining constraints. The non-negativity constraints on the allocated powers in (13g) are inherently incorporated in the GP framework. For the constraints in (14), although the LHSs are in the form of monomials, they are not compatible with the GP framework because their RHSs are not equal to 1. This difficulty can be readily alleviated by replacing (14) with constraints of the form $p_n^{(k)} p_{n'}^{(k)} \leq \epsilon$, where ϵ is a small positive number. Finally, the constraints in (16) are compatible with the GP standard form.

Now, combining the above constraints and assuming that a feasible initial power allocation, $\{p_{0,n}^{(k)}\}$, is given, the maximum weighted sum rate around $\{p_{0,n}^{(k)}\}$ can be approximated

[1]This is expected because the original problem is NP-hard.

by solving the following GP.

$$\max_{\{t_n^{(d)}\},\{r_{\ell k}^{(d)}\},\{p_n^{(k)}\}} \quad \prod_{d\in\mathcal{D}}\prod_{n\in\mathcal{N}\setminus\{d\}} \big(t_n^{(d)}\big)^{w_n^{(d)}}, \tag{20a}$$

$$\text{subject to} \quad t_n^{(d)} \geq 1, \qquad n\in\mathcal{N}\setminus\{d\}, d\in\mathcal{D}, \tag{20b}$$

$$\prod_{\ell\in\mathcal{L}}\prod_{k\in\mathcal{K}} \big(r_{\ell k}^{(d)}\big)^{W a_{n\ell}} = t_n^{(d)}, \qquad n\in\mathcal{N}\setminus\{d\},\, d\in\mathcal{D}, \tag{20c}$$

$$r_{\ell k}^{(d)} \geq 1, \qquad \ell\in\mathcal{L},\, k\in\mathcal{K},\, d\in\mathcal{D}, \tag{20d}$$

$$WN_0 \prod_{d\in\mathcal{D}} r_{\ell k}^{(d)} \leq q_{\ell k}\big(p_n^{(k)}/p_{0,n}^{(k)}\big)^{\theta_{\ell k}}, \quad k\in\mathcal{K}, \ell\in\mathcal{L}, \tag{20e}$$

$$\sum_{k\in\mathcal{K}} p_n^{(k)} \leq P_n, \qquad n\in\mathcal{N}, \tag{20f}$$

$$p_n^{(k)} p_{n'}^{(k)} \leq \epsilon, \qquad n,n'\in\mathcal{N},\, k\in\mathcal{K},\, n\neq n', \tag{20g}$$

where $q_{\ell k} = WN_0 + p_{0,n}^{(k)}|h_{nn'}^{(k)}|^2$, and $\theta_{\ell k} = \frac{p_{0,n}^{(k)}|h_{nn'}^{(k)}|^2}{q_{\ell k}}$. Using a standard transformation, the problem in (20) can be readily converted to an efficiently solvable convex form [15].

The formulation in (20) can be used in an iterative fashion. Starting from a feasible $\{p_{0,n}^{(k)}\}$, the problem in (20) is transformed to a convex form and solved using an interior point solver. The output is then used in the following iteration as the initial power allocation. The convergence of this iterative procedure to a solution that satisfies the Karush-Kuhn-Tucker (KKT) conditions corresponding to the original cross layer design problem can be guaranteed under relatively mild conditions [16]. Since that problem is not convex, the KKT conditions are only necessary for optimality and the resulting solution is a lower bound on the achievable weighted sum rate.

B. The Rounding-Based Approach

To develop our second approach we consider the formulation in (13), but with the binary scheduling constraints in (13f) relaxed to the following constraints

$$\beta_{\ell k} \in [0,1], \qquad \ell\in\mathcal{L},\; k\in\mathcal{K}. \tag{21}$$

The weighted sum rate obtained by solving this relaxed version is an upper bound on the corresponding sum rate obtained by solving the original problem in (13). Using a change of variables, the relaxed problem was cast in [11] in a convex form, which using the technique in [11] yields the continuous schedules $\hat{\beta}_{\ell k} \in [0,1]$, $\ell\in\mathcal{L}$, $k\in\mathcal{K}$. To construct a set of, potentially suboptimal, binary schedules, $\{\tilde{\beta}_{\ell k}\}$ from continuous ones, $\{\hat{\beta}_{\ell k}\}$, for every $k\in\mathcal{K}$, we simply set the element of $\{\tilde{\beta}_{\ell k}\}_{\ell=1}^{L}$ corresponding to the largest element of $\{\hat{\beta}_{\ell k}\}_{\ell=1}^{L}$ to 1 and the other elements to 0. More specifically,

$$\tilde{\beta}_{\ell k} = \left\lfloor \frac{\hat{\beta}_{\ell k}}{\max_{\ell'\in\mathcal{L}} \hat{\beta}_{\ell' k}} \right\rfloor, \qquad \ell\in\mathcal{L},\, k\in\mathcal{K}. \tag{22}$$

We now use $\{\tilde{\beta}_{\ell k}\}$ as if they were the optimal subchannel schedules. With $\{\tilde{\beta}_{\ell k}\}$ fixed, the cross layer design problem in (13) can be cast in the following convex form:

$$\max_{\{s_n^{(d)}\},\{x_{\ell k}^{(d)}\},\{p_n^{(k)}\}} \quad \sum_{d\in\mathcal{D}}\sum_{n\in\mathcal{N}\setminus\{d\}} w_n^{(d)} s_n^{(d)}, \tag{23a}$$

$$\text{subject to} \quad s_n^{(d)} \geq 0, \quad n\in\mathcal{N}\setminus\{d\}, d\in\mathcal{D}, \tag{23b}$$

$$\sum_{\ell\in\mathcal{L}}\sum_{k\in\mathcal{K}} a_{n\ell} x_{\ell k}^{(d)} = s_n^{(d)}, \quad n\in\mathcal{N}\setminus\{d\}, d\in\mathcal{D}, \tag{23c}$$

$$x_{\ell k}^{(d)} \geq 0, \qquad \ell\in\mathcal{L},\, k\in\mathcal{K},\;\; d\in\mathcal{D}, \tag{23d}$$

$$\sum_{d\in\mathcal{D}} x_{\ell k}^{(d)} \leq W\tilde{\beta}_{\ell k}\log_2\Big(1+\frac{p_n^{(k)}|h_{nn'}^{(k)}|^2}{WN_0}\Big), \qquad \ell\in\mathcal{L},\, k\in\mathcal{K}, \tag{23e}$$

$$p_n^{(k)} \geq 0, \qquad n\in\mathcal{N}, k\in\mathcal{K}, \tag{23f}$$

$$\sum_{k\in\mathcal{K}}\sum_{\ell\in\mathcal{O}(n)} \tilde{\beta}_{\ell k} p_n^{(k)} \leq P_n, \qquad n\in\mathcal{N}. \tag{23g}$$

Notice that the only nonlinear constraint is the one in (23e), which, in spite of being nonlinear, is convex. Thus, we have shown that when $\{\tilde{\beta}_{\ell k}\}$ are fixed, the cross layer design problem is convex. Since this problem is formulated based on potentially suboptimal subchannel schedules, the weighted sum rate it generates is a lower bounds on the corresponding sum rate generated by the original design problem.

V. Numerical Investigations

We consider a numerical example of a network with $N = 4$ nodes and $L = 12$ links; cf. Figure 1. We assume that $\mathcal{D} = \{1,2\}$ and that two unit bandwidth subchannels are available for communication, i.e., $K = 2$ and $W = 1$. Our objective in this example is to maximize the sum rate, i.e., we set $w_n^{(d)} = 1$, for $n = 1,\ldots,4$, $d = 1,2$, $n\neq d$. The nodes are assumed to have identical power budgets, $P_n = P$, $n = 1,\ldots,N$. The subchannels are assumed to be quasi static and the noise variance is assumed to be $N_0 = 1$. The subchannel gains are drawn randomly from the standard circularly-symmetric Gaussian distribution. The squared absolute values of these gains are given in the following matrices:

$$\big[|h_{nn'}^{(1)}|^2\big] = \begin{bmatrix} 0.0000 & 0.5664 & 0.0332 & 0.5168 \\ 0.1395 & 0.0000 & 0.3282 & 0.3018 \\ 0.2503 & 0.2900 & 0.0000 & 0.2506 \\ 0.3032 & 0.7612 & 0.5595 & 0.0000 \end{bmatrix},$$

$$\big[|h_{nn'}^{(2)}|^2\big] = \begin{bmatrix} 0.0000 & 0.0667 & 0.1006 & 0.3451 \\ 0.2050 & 0.0000 & 0.5316 & 0.4219 \\ 0.2151 & 0.2295 & 0.0000 & 0.3659 \\ 0.4872 & 0.3195 & 0.2912 & 0.0000 \end{bmatrix}.$$

Setting $P/N_0 = 10$ dB, $\epsilon = 10^{-6}$ and $\{p_{0,n}^{(k)}\} = 10^{-4}$, the proposed GP-based approach converges to a sum rate of 4.93 bits per channel use (bpcu) within 5 iterations and the rounding-based approach yields a sum rate of 4.46 bpcu. To demonstrate the efficacy of the proposed bounds, we compare these results with the optimal solution of (13) and its upper bound obtained by the technique in [11]. To find the optimal solution of (13) we perform an exhaustive search over all possible 2^{LK} schedules. For each schedule a convex formulation similar to the one in (23) is solved; the schedule that yields the highest sum rate is the optimal one. The sum rate obtained by this approach is 5.29 bpcu. Using the convex

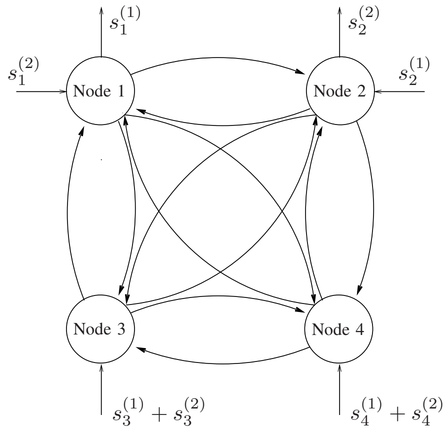

Fig. 1. Network schematic. Each link is composed of 2 subchannels.

TABLE I
A COMPARISON OF THE PROPOSED LOWER BOUNDS, THE OPTIMAL AND AN UPPER BOUND ON THE SUM RATE

	Round. Low. Bnd.	GP Low. Bnd.	Ex. Srch. Optimal	Time-Shar. Up. Bnd.
Sum Rate	4.46	4.93	5.29	6.07

formulation of the upper bound in [11] yielded a sum rate of 6.07 bpcu. This comparison is summarized in Table I.

To further investigate the tightness of the proposed lower bounds, in Figure 2 we compare the sum rate yielded by these bounds with the sum rate yielded by solving the original cross layer design problem using exhaustive search. We also show the sum rate yielded by the upper bound obtained in [11]. From this figure it can be seen that the gap between the lower and upper bounds decreases with the increase in P.

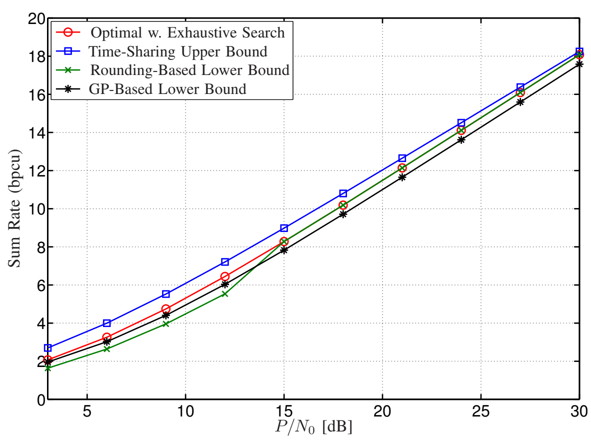

Fig. 2. Sum rate comparison: maximum, upper and proposed lower bounds.

VI. CONCLUSION

In this paper we considered the joint design of data routes, binary schedules and power allocations in a generic *ad hoc* OFDMA network. We developed two efficiently computable lower bounds on the maximum weighted sum of the rates that can be reliably communicated over this network. The first bound is based on a GP approximation of the original design problem, whereas the second bound is based on normalizing and rounding the solution of a relaxed version thereof. Numerical investigations suggest that the gap between these lower bounds and the upper bound is generally small and decreases with the increase in the power budgets.

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