Efficiently Computable Bounds on the Rates Achieved by A Cross Layer Design with Binary Scheduling in Generic OFDMA Wireless Networks

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Introduction

Future wireless communication systems

- Requirements:
  - Support large number of users
  - High data rates
  - Flexibility in quality of services (QoS)
- Challenges:
  - Availability of frequency spectrum
  - Total transmit power
  - Nature of the wireless channel
- Solution:
  - High-speed OFDM transmission
  - Efficient resource management
  - Cross layer optimization

Literature Review

- Xiao *et al*, 2004: Propose an optimization framework for routing and power allocation in networks with orthogonal channel assignments.
- Karakayali *et al*, 2007: Develop a heuristic algorithm for routing, scheduling and power allocation in OFDMA networks in which each node can be either a transmitter, a receiver or a relay.
- Huang *et al*, 2009: Find the optimal solution for time-shared scheduling and power allocation in an OFDMA structure.
- Rashtchi *et al*, 2012: Obtain the optimal design for joint routing, scheduling and power allocation in *ad hoc* OFDMA networks using time-sharing.

System Model

- An OFDMA ad hoc network with N nodes is considered.
- Nodes are able to send, receive and/or relay data.
- This network is represented by a complete weighted directed graph.
- The graph has $L = N(N-1)$ links.
- Each link consists of K subchannels.
- Each subchannel has $W = \frac{W_0}{k}$ bandwidth.
- There are D receivers across the network, $D \leq N$.

Assumptions

- Time-invariant channels during each signalling interval.
- Half duplex operation mode.
- Infinite backlog at source nodes.
- Interference avoidance by scheduling.
- Gaussian signalling.
- Decode-and-forward relaying scheme.

Scheduling and Power Allocation Constraints

$\beta_{\ell k}$ is the time schedule of subchannel k on link ℓ.

- Binary scheduling: $\beta_{\ell k} \in \{0, 1\}$: Integer constraint
- Interference avoidance: $\sum_{\ell} \beta_{\ell k} \leq 1$.

$p_n^{(k)}$ is the power allocated to subchannel k on node n.
$\mathrm{O}(n)$ is the set of links coming out from node n.

- Non-negative powers: $p_n^{(k)} \geq 0$.
- Limited power budget: $\sum_k \sum_{\ell \in \mathrm{O}(n)} \beta_{\ell k} p_n^{(k)} \leq P_n$.

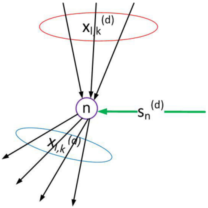
Routing Constraints

$s_n^{(d)}$ is the injected rate to node n for destination d.
$x_{\ell k}^{(d)}$ is the rate on link ℓk intended for destination d.

- Non-negative injected rate: $s_n^{(d)} \geq 0$.
- Non-negative internal rate: $x_{\ell k}^{(d)} \geq 0$.
- Flow-conservation law: $\sum_\ell \sum_k a_{n\ell} x_{\ell k}^{(d)} = s_n^{(d)}$.

$[a_{n\ell}]$ captures the connectivity of the graph:

$$a_{n\ell} = \begin{cases} 1, & \text{if } n \text{ is the start node of link } \ell \\ -1, & \text{if } n \text{ is the end node of link } \ell \\ 0, & \text{otherwise.} \end{cases}$$



Coupling Constraints

$h^{(k)}_{nn'}$ is the channel gain on subchannel k of link ℓ connecting nodes n to n'.

- Capacity constraint: the aggregate rate on ℓk-th subchannel must be less than its capacity,

$$\sum_{d} x^{(d)}_{\ell k} \leq W\beta_{\ell k} \log_2\left(1 + \frac{p^{(k)}_n |h^{(k)}_{nn'}|^2}{N_0 W}\right).$$

- Nonconvex constraint in $\{\beta_{\ell k}, p^{(k)}_n\}$.

Cross Layer Design with Binary Scheduling

$$\max_{\{s_n^{(d)}\},\{x_{\ell k}^{(d)}\},\{\beta_{\ell k}\},\{p_n^{(k)}\}} \quad \sum_{d} \sum_{n, n \neq d} w_n^{(d)} s_n^{(d)},$$

subject to: routing constraints,
scheduling constraints,
power allocation constraints,
capacity constraints.

$w_n^{(d)}$ is a non-negative weight associated to $s_n^{(d)}$,

- Spanning of the boundary of the achievable rate region.
- Controlling the quality of service: higher weight implies higher priority.

mixed integer nonlinear programming, known as NP-hard.

JRSPA: Binary Scheduling

$$\max_{\{s_n^{(d)}\},\{x_{\ell k}^{(d)}\},\{\beta_{\ell k}\},\{p_{\ell k}\}} \sum_d \sum_{n,n\neq d} w_n^{(d)} s_n^{(d)},$$

subject to:

$$x_{\ell k}^{(d)} \geq 0, \qquad \ell \in \mathcal{L},\ k \in \mathcal{K},\ d \in \mathcal{D},$$

$$\sum_\ell \sum_k a_{n\ell} x_{\ell k}^{(d)} = s_n^{(d)},\ \ s_n^{(d)} \geq 0, \qquad d \in \mathcal{D}, n \in \mathcal{N} \setminus \{d\},$$

$$\sum_d x_{\ell k}^{(d)} \leq W\beta_{\ell k} \log_2\left(1 + \frac{p_n^{(k)}|h_{nn'}^{(k)}|^2}{WN_0}\right), \qquad \ell \in \mathcal{L}, k \in \mathcal{K},$$

$$\beta_{\ell k} \in \{0,1\}, \qquad \ell \in \mathcal{L},\ k \in \mathcal{K},$$

$$p_n^{(k)} \geq 0, \qquad n \in \mathcal{N}, k \in \mathcal{K},$$

$$\sum_\ell \beta_{\ell k} \leq 1, \qquad k \in \mathcal{K},$$

$$\sum_k \sum_{\ell \in \mathrm{O}(n)} \beta_{\ell k} p_n^{(k)} \leq P_n, \qquad n \in \mathcal{N}.$$

Rounding-based Lower Bound

An Upper Bound

- Relax the binary schedules
  - $\beta_{\ell k} \in [0, 1]$: Continuous (or time-share) scheduling
- Cast the problem as convex[1]
  - Find optimal continuous schedules: $\hat{\beta}_{\ell k} \in [0, 1]$

Rounding-based Lower Bound

- Construct a set of binary schedules: $\tilde{\beta}_{\ell k} \in \{0, 1\}$
  - For each k, set the largest element equal to 1.
  - Set the rest equal to zero.
- Solve the problem with known binary schedules, $\{\tilde{\beta}_{\ell k}\}$.
  - The problem is convex.
  - It is efficiently solvable.

[1] Rashtchi, Gohary, Yanikomeroglu, ICC 2012

GP-Standard form

GP is a class of nonlinear optimization which, although nonconvex, can be readily transformed into convex one.

- $\boldsymbol{z} = (z_1, \ldots, z_n) \in \mathbb{R}_+^n$ are optimization variables.
- $c_0 \prod_{i=1}^{n} z_i^{\alpha_i}$ is a monomial function in $\boldsymbol{z}$ if:
  - $c_0 > 0$ and
  - $\{\alpha_i\}$ are arbitrary constants.
- $\sum_{j=1}^{J} c_j \prod_{i=1}^{n} z_i^{\alpha_{ij}}$ is a posynomial function in $\boldsymbol{z}$ if:
  - $\{c_j\}$ are positive and
  - $\{\alpha_{ij}\}$ are arbitrary constants.
- Standard GP has the following form:

$$\begin{aligned} \min_{\mathbf{z}} \quad & f_0(\boldsymbol{z}), \\ \text{subject to} \quad & f_i(\boldsymbol{z}) \leq 1, \qquad i = 1, \ldots, m, \\ & g_i(\boldsymbol{z}) = 1, \qquad i = 1, \ldots, p, \end{aligned}$$

where $\{f_i\}$ are posynomials and $\{g_i\}$ are monomials.

GP-based Lower Bound

- Binary scheduling can be captured as:
  - Replace the binary scheduling constraint with

$$p_n^{(k)} p_{n'}^{(k)} = 0, \quad \forall n, n', k, \ n \neq n'$$

  - Reformulate other constraints with $\beta_{\ell k}$

$$\sum_d x_{\ell k}^{(d)} \leq W \log_2 \left(1 + \frac{p_n^{(k)} |h_{nn'}^{(k)}|^2}{W N_0}\right),$$

$$\sum_k p_n^{(k)} \leq P_n.$$

- The result is amenable to GP-based monomial approximation.
  - Define new variables: $s_n^{(d)} = \log_2 t_n^{(d)}, \quad x_{\ell k}^{(d)} = W \log_2 r_{\ell k}^{(d)}$.
  - Bring the problem in to logarithmic domain.

GP-based Lower Bound (cont'd.)

$$\max_{\{t_n^{(d)}\},\{r_{\ell k}^{(d)}\},\{p_n^{(k)}\}} \prod_{d\in\mathcal{D}} \prod_{n\in\mathcal{N}\setminus\{d\}} \left(t_n^{(d)}\right)^{w_n^{(d)}},$$

$$\text{subject to} \quad r_{\ell k}^{(d)} \geq 1, \qquad \ell\in\mathcal{L},\ k\in\mathcal{K},\ d\in\mathcal{D},$$

$$\prod_{\ell\in\mathcal{L}} \prod_{k\in\mathcal{K}} \left(r_{\ell k}^{(d)}\right)^{Wa_{n\ell}} = t_n^{(d)}, \quad t_n^{(d)} \geq 1, \qquad n\in\mathcal{N}\setminus\{d\},\ d\in\mathcal{D},$$

$$WN_0 \prod_{d\in\mathcal{D}} r_{\ell k}^{(d)} \leq WN_0 + p_n^{(k)}|h_{nn'}^{(k)}|^2, \qquad \ell\in\mathcal{L}, k\in\mathcal{K},$$

$$\sum_{k\in\mathcal{K}} p_n^{(k)} \leq P_n, \qquad n\in\mathcal{N},$$

$$p_n^{(k)} p_{n'}^{(k)} = 0, \qquad n, n'\in\mathcal{N},\ k\in\mathcal{K},\ n\neq n',$$

GP-based Lower Bound (cont'd.)

In each iteration do the following until reach to a stable point:

- Choose a feasible starting point.
- Replace the non-GP constraints with their monomial approximations.
- Replace the scheduling constraints with $p_n^{(k)} p_{n'}^{(k)} \leq \epsilon$.
- Solve the resulting GP problem.
- Replace the initial point with the resulting point.

Pros:

- Converges to a locally optimal solution.
- The convergence is guaranteed.
- Each iteration is convex and hence efficiently solvable.

Cons:

- Highly depends on the starting point.

Numerical Example

$N = 4,\ L = 12$

$K = 2,\ W = 1,$

$D = \{1, 2\},$

$P_n = 10dB,\ \forall n$

$w_n^{(d)} = 1,\ \forall n, d$

$N_0 = 1,\ h_{nn'}^{(k)} \sim \mathcal{N}(0, 1),$

$\{P_{0,n}^{(k)}\} = 10^{-4},\ \epsilon = 10^{-6}.$

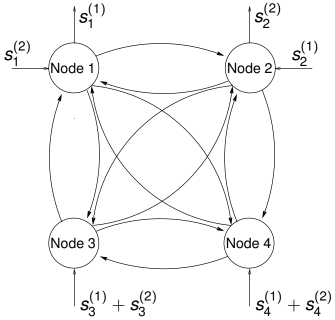


Figure: Each link is composed of 2 subchannels.

	Round. Low. Bnd.	GP Low. Bnd.	Ex. Srch. Optimal	Time-Shar. Up. Bnd.
Sum Rate	4.46	4.93	5.29	6.07

Numerical Results (cont'd.)

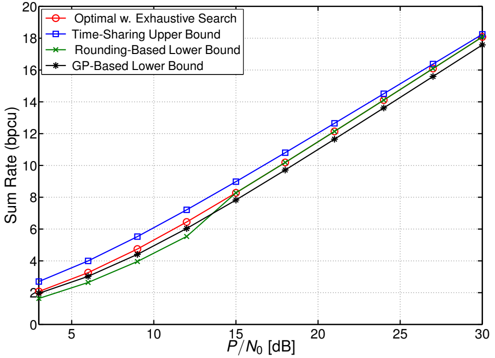


Figure: Sum rate comparison: maximum, upper and proposed lower bounds.

Conclusion

- We considered a cross layer design of routing, binary scheduling and power allocation in a generic OFDMA network.
  - Mixed integer nonlinear programming which is NP-hard.
- We proposed two efficiently computable lower bounds:
  - Rounding-based lower bound
    - Relax the binary scheduling constraint
    - Cast the problem as convex and solve it
    - Construct a set of binary schedules
    - solve the problem with these fixed schedules
  - Iterative GP-based lower bound
    - Reformulate the problem
    - Define new sets of variables in logarithmic domain
    - Approximate non-GP constraints with monomials
    - Solve the resulting GP problem