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On the Optimal Number of Hops in Infrastructure-based Fixed Relay Networks

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***Abstract*[*] - In this paper we analyze the optimal number and locations of fixed radio relay stations forming multi-hop links in infrastructure-based wireless networks. Under the assumptions of using orthogonal channels for the hops and all links having the same average path loss exponent, we show using the spectral efficiency as a metric that the optimal relay locations are at equal intervals along the straight line between source and destination whenever multihop is to be utilized. We show that a single-hop fixed radio link can be efficiently (from a spectral efficiency perspective) replaced with the multi-hop link only if the single-hop SNR has a relatively low value. We introduce a "multi-hop criterion" to quantify the single-hop SNR value below which a more efficient multi-hop replacement exists. We also determine the optimal number of hops for the multi-hop links having relays disposed in straight line at equal intervals.**

I. Introduction

It is generally accepted that the architecture of the present day cellular networks can not meet the stringent requirements envisioned for 4G cellular systems. Economically feasible solutions are likely to be based on some form of multi-hop relaying allowing uniform coverage at very high data rates and reducing the required number of expensive cell sites.

Multi-hop relaying with fixed relays is based on fixed relay stations deployed as part of the network infrastructure [1]. Their incremental cost is offset by reduced requirements on the mobile terminals, and by the simplicity and efficiency of the radio protocols involved. The fixed relay stations are part of the cellular network infrastructure, therefore their deployment will be an integral part of the network planning, design and deployment process. It is necessary to establish strategies and methods for efficient deployment of fixed relay stations, such that the overall cost of the network is minimized. Efficient radio resource allocation to network elements is a critical part of the overall network cost optimization effort.

One of the open questions regarding the deployment of wireless networks using fixed relays is the optimal number of hops between the source and destination radio stations. It is important to be able to decide with reasonable accuracy in what conditions it is more advantageous to send a signal directly to destination (may be by increasing the allocated transmit power, bandwidth or time) or route the same signal over a number of relay stations, each using less resources compared to the replaced link. Due to their ambitious requirements, the 4G systems will most likely use solutions with at least two hops (whenever necessary), one of the hops being between the fixed relay and the mobile terminal. The remaining radio link going back to the base station, referred to as the "feeder", is comprised of a set of fixed relays interconnected through radio links, and can have one or more hops. The underlying scope of the following discussion is to determine the optimal number of hops of the feeder portion of the 4G wireless system.

In [2], the potential gains of a multi-hop link are analyzed based on the Shannon capacity of the radio channel. Assuming that the relays are equally placed along the multi-hop link and that the overhead increases linearly with the number of hops, it is concluded that a multi-hop link offers a positive system gain in specific radio conditions (low transmit power, large distance, large path loss exponent) only for a reduced number of hops (up to four).

In this paper we assume that the intermediate relays can be located anywhere, not only along straight lines at equal intervals. Also, we take a novel approach by analyzing the aggregate spectral efficiency of the multi-hop communication system. Spectral efficiency is important because the most significant challenge ahead of the 4G cellular systems is the provision of cost-effective quasi-ubiquitous coverage with very high data rates. A promising solution for surpassing this difficulty and extending the coverage of the classical cellular network is by using relays. The additional radio power introduced in the network by relays comes from the "wall plug" and should not be added to the radio resource costs. More significant than the relay consumed power is the effect of the relaying schemes over the aggregate end-to-end spectral efficiency. Higher overall spectral efficiency allows better use of the available spectrum license – the most expensive asset of the cellular operator.

II. System Description

We consider the multi-hop link R_0-R_n in the feeder part of a fixed relay network with an *n*-hop link, as shown in Fig. 1, where R_0 and R_n are the source and the recipient fixed relays, respectively. The message can be either sent directly from R_0 to R_n (single-hop operation in the feeder part), or the message can be sent via *n-1* intermediate fixed relays over *n* hops.

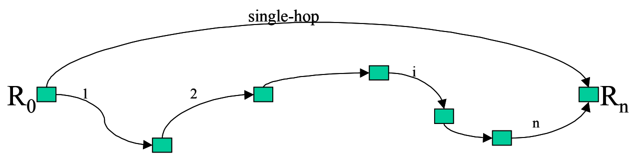

Figure 1. Multi-hop link topology.

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The relays are of the digital regenerative type. Each relay uses only the information received from its immediate predecessor in the chain, i.e., there is no diversity combining of received signals from multiple "uphill" transmitters. Since the relays are fixed, the topology of the system is considered known and non-dynamic, so very little overhead is needed for packet routing. As the overhead messaging related to multi-hop functionality will not be significant, it was not considered as a factor in the discussion below. As well, the processing delays in the relays have been considered to be much smaller compared with the transmission time of relayed data packets.

The system uses a time-slotted resource allocation scheme, where each radio link is assigned a channel in the frequency – time domain. Each intermediate radio link adopts an appropriate modulation scheme resulting in the best possible spectral efficiency based on the given signal-to-noise ratio (SNR) conditions.

We adopt the following assumptions as stated in [3]:

1. All n hops of the multi-hop link, as well as the single-hop R_0-R_n link, are allocated the same amount of bandwidth B (Hz), accessed in a time-division manner. The individual time required to pass a message over a hop i is

$$t_i = \frac{M}{B\eta_i}, \quad (1)$$

where M is the message size in bits and η_i is the spectral efficiency (in bits/sec/Hz) of the radio link over hop i.

2. The timeslots $\{t_i\}$ are considered orthogonal to each other. Also for simplicity but without loss of generality, we assume that all links operate on the same carrier frequency. Although the orthogonality condition seems conservative, this is indeed a realistic assumption; the number of hops in a wireless network is not expected to be excessive due to a number of practical reasons, and in such cases the multiple access interference may not allow channel reuse in a multihop chain. In conclusion, the total message transfer time (TMTT), T, required to pass a message of size M from R_0 to R_n is the sum of all intermediate timeslots:

$$T = \sum_{i=1}^{n} t_i = \frac{M}{B}\sum_{i=1}^{n}\frac{1}{\eta_i}. \quad (2)$$

3. Since the channels used by each hop in the multi-hop link are orthogonal, no co-channel interference is generated from within the multi-hop link itself. External interference affects all hops in the same way; without loss of generality, we assume it degrades the receiver thresholds of all relays by the same amount.

4. All radio links are part of the feeder system and have similar radio propagation parameters.

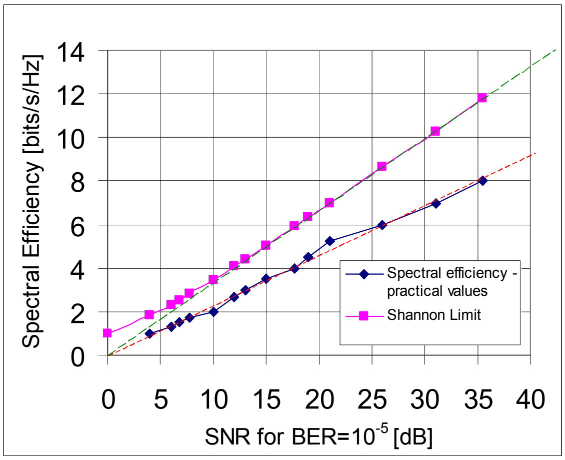

Figure 2. Spectral efficiency vs. SNR (The data for the practical spectral efficiency plot assume Bit Interleaved Coded Modulation [4]. The data were provided by Dr. Sirikiat Lek Ariyavisitakul.).

We observe that the spectral efficiency for a given link has an approximate linear dependency on the SNR measured in dB, as shown in Fig. 2.

We can approximate the spectral efficiency for a single hop, η, and for the hop i of the multi-hop link, η_i, as

$$\eta \cong K_1 \log_{10}\gamma, \quad (3)$$

$$\eta_i \cong K_1 \log_{10}\gamma_i, \quad (4)$$

where γ and γ_i are the SNR for the direct link R_0-R_n and link i, respectively, and K_1 is a constant of proportionality. The expressions (3) and (4) remain valid for other digital modulation schemes as long as the linear dependency of spectral efficiency on SNR is preserved as the one shown in Fig. 2 (straight line passing through origin).

Although the expressions (3) and (4) do resemble the Shannon capacity formula

$$C\,[\text{bits/sec/Hz}] = \log_2(1+\gamma) \cong 3.32\log_{10}(1+\gamma),$$

they are not identical. However, it can be observed in Fig. 2 that for high SNR values (more than 10 dB), the Shannon formula follows a similar progression law as in (3), albeit with a higher slope than that of the line of the practical spectral efficiency values.

Assuming that all transceivers at R_0, R_n, and the n-1 fixed relays are identical (in terms of transmit power, transmit and receive antenna gains, and receiver noise figure), we can express the mean SNR (excluding shadowing) for the single-hop case and for link i in the n-hop case as

$$\gamma = K_2 (D/d_0)^{-p} \quad (5)$$

$$\gamma_i = K_2 (d_i/d_0)^{-p} \quad (6)$$

respectively, where:

- D = distance R_0-R_n
- d_i = length of the hop i
- d_0 = reference close-in distance in radio propagation
- K_2 = a constant of proportionality
- p = path loss exponent

In general the constant K_2 captures the radio link system gains (or the link costs) in terms of radio resources:

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$$K_2 = \frac{P_T G_T G_R \lambda^2}{(4\pi d_0)^2 P_N}. \quad (7)$$

In (7), P_T is the transmit power, G_T and G_R are transmit and receive antenna gains, respectively, P_N is the noise power, and λ is the radio wavelength; d_0 was introduced earlier as the reference close in distance for radio propagation.

The total time required to pass the message over the multi-hop link (TMTT) can then be expressed, using (2), (4) and (6), as

$$T = \frac{M}{BK_1}\sum_{i=1}^{n}\frac{1}{\log_{10}\gamma_i} = \frac{M}{BK_1}\sum_{i=1}^{n}\frac{1}{\log_{10}\left(K_2\left(\frac{d_i}{d_0}\right)^{-p}\right)}. \quad (8)$$

III. Optimal Relay Locations

Improving the aggregate end-to-end spectral efficiency means reducing the message transfer time. We are looking for the optimal set $\{d_i\}$ which will minimize the value T in (8).

Theorem 1:

Consider a set of n realizations d_i, $i = 1, 2, \ldots, n$ of a real random variable, d, with mean $\overline{d_i}$.

a) If $\frac{d_0 K_2^{\frac{1}{p}}}{e^2} < d_i < d_0 K_2^{\frac{1}{p}}$, for any $i = 1, 2, \ldots, n$, then

$$\sum_{i=1}^{n}\frac{1}{\log_{10}\left(K_2\left(\frac{d_i}{d_0}\right)^{-p}\right)} \geq \frac{n}{\log_{10}\left(K_2\left(\frac{\overline{d_i}}{d_0}\right)^{-p}\right)}, \quad (9)$$

with equality if the random variable d is constant.

b) If $0 < d_i < d_0 K_2^{\frac{1}{p}}$, for any $i = 1, 2, \ldots, n$, then

$$\sum_{i=1}^{n}\frac{1}{\log_{10}\left(K_2\left(\frac{d_i}{d_0}\right)^{-p}\right)} \leq \frac{n}{\log_{10}\left(K_2\left(\frac{\overline{d_i}}{d_0}\right)^{-p}\right)}, \quad (10)$$

with equality if the random variable d is constant.

Proof:

We consider the function

$$f(d_i) = \frac{1}{\log_{10}\left(K_2\left(\frac{d_i}{d_0}\right)^{-p}\right)}. \quad (11)$$

By studying the sign of its second derivative, it can be shown that the function $f(d_i)$ is strictly convex for the interval

$$\frac{K_2^{\frac{1}{p}}}{e^2} < \frac{d_i}{d_0} < K_2^{\frac{1}{p}} \quad (12)$$

and strictly concave for

$$\frac{d_i}{d_0} < \frac{K_2^{\frac{1}{p}}}{e^2} \quad (13)$$

for $i = 1, 2, \ldots, n$.

For the interval where $f(d_i)$ is strictly convex, applying Jensen's inequality [5], we obtain

$$\sum_{i=1}^{n}\frac{1}{\log_{10}\left(K_2\left(\frac{d_i}{d_0}\right)^{-p}\right)} \geq \frac{n}{\log_{10}\left(K_2\left(\frac{\sum_{i=1}^{n} d_i}{nd_0}\right)^{-p}\right)} \quad (14)$$

with equality iff

$$d_1 = d_2 = \ldots = d_n. \quad (15)$$

Replacing $\overline{d}_i = \sum d_i / n$ in (14), we obtain equation (9), therefore part a) of Theorem 1 is proven.

If $f(d_i)$ is strictly concave, then, using Jensen's inequality we get

$$\sum_{i=1}^{n}\frac{1}{\log_{10}\left(K_2\left(\frac{d_i}{d_0}\right)^{-p}\right)} \leq \frac{n}{\log_{10}\left(K_2\left(\frac{\sum_{i=1}^{n} d_i}{nd_0}\right)^{-p}\right)}, \quad (16)$$

also with equality iff (15) is true.

Replacing $\overline{d}_i = \sum d_i / n$ in (16), we obtain equation (10), therefore also part b) of Theorem 1 is proven. □

The convexity and concavity intervals in (12) and (13) can also be expressed in SNR terms as

$$1 < \gamma_i < e^{2p} \quad (17)$$

$$\gamma_i > e^{2p}, \quad (18)$$

respectively, for $i = 1, 2, \ldots, n$. It is to be noted that the model adopted works only for SNR values larger than unity (i.e., $\frac{d_i}{d_0} > K_2^{\frac{1}{p}}$), in which case the spectral efficiency and the corresponding message transfer time remain positive.

We consider the special case of an n-hop link with "short" intermediate links, i.e., all intermediate hop lengths d_i satisfy the condition (13), and as a consequence all intermediate hop SNRs $\{\gamma_i\}$ satisfy (18). According to (10), an n-hop link respecting the conditions above would have a better aggregate spectral efficiency (smaller TMTT) compared to the special case of an n-hop link having all hops of the same length, equal with the mean hop length $\overline{d}_i = \sum d_i / n$.

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Further, in the special case when all relays are placed on the straight line R_0R_n, any configuration has a better aggregate spectral efficiency compared with the situation when the *n-1* relays are distributed along equal intervals on the line R_0R_n. In other words, if all relays are always placed on the straight line R_0R_n, evenly spaced relays achieve the worst performance (lower bound) in terms of spectral efficiency.

Next, considering the opposite case of an *n*-hop link with "long" intermediate links, we assume that all hop lengths $\{d_i\}$ respect the condition (12), therefore the intermediate SNRs $\{\gamma_i\}$ are as in (17). In this case, (9) stands true, and the minimum transfer time is be achieved when $d_i = \overline{d}_i = \sum d_i / n$, for all *i*.

Geometrically, the smallest value of the sum of all *n* individual hop lengths is reached when all relays are placed on the straight line R_0R_n, in which case, the sum is the actual distance from source to destination:

$$\min\left(\sum_{i=1}^{n} d_i\right) = D. \qquad (19)$$

As a result, the best possible geographical locations for the *n-1* intermediate fixed relays which would minimize the sum term in (8) and the TMTT *T*, are along equal intervals on the straight line R_0R_n. In such a case,

$$d_i = \frac{D}{n} \qquad \text{for } i = 1, 2, \ldots, n. \qquad (20)$$

We have reached the apparently peculiar conclusion that there is no unique optimal configuration for the locations of the *n-1* relays; rather, the optimal configuration (the number and location of relays) depend on the system parameters. For "long" intermediate hops, the configuration with evenly distributed relays is optimal, while for "short" intermediate hops, it is the worst.

In order to show this behavior graphically, we consider the example of a two-hop link as shown in Figure 3. We want to check if the relay location at equal distance from source and destination is optimal, and as such, in Figure 4, the ratio of TMTT for a two-hop link with hop lengths (d_1, d_2) and the TMTT for an imaginary comparison link with hop lengths ($D/2$, $D/2$), is plotted (where $D = d_1 + d_2$). The value of the ratio – below or above unity - indicates in which cases the "relay in the middle" scenario is optimal.

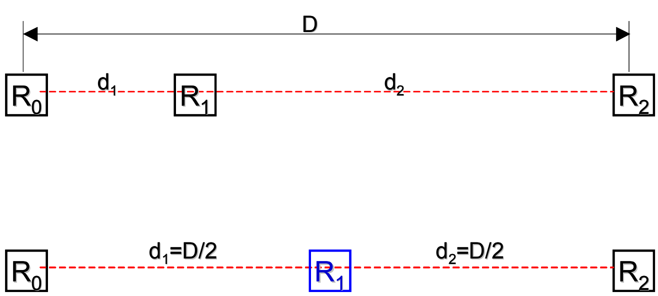

Figure 3. Two-hop link.

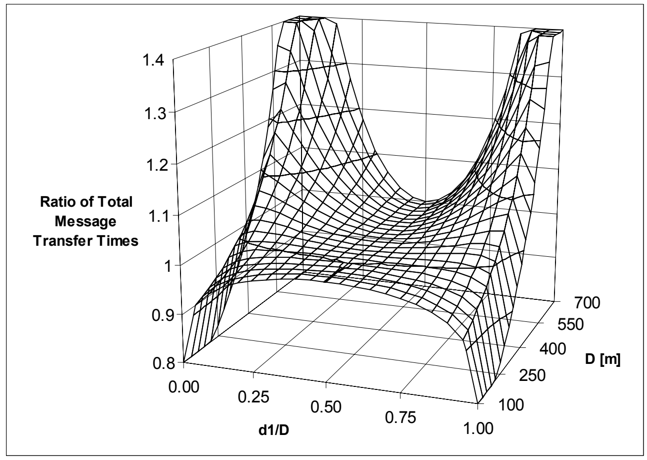

Figure 4. The ratio between the total message transfer times of a two-hop link and the link with the two hops of length $\overline{d}_i$.

Assuming the system parameters of P_T = 30 dBm, G_T = 10 dB, G_R = 10 dB, P_N = -95 dBm, d_0 = 2 m, λ = 0.06 m (carrier at 5 GHz) and p = 3.5, an intermediate hop *i* is "short" if

$$d_i < d_0 \frac{K_2^{\frac{1}{p}}}{e^2} \approx 119.6\,\text{m}. \qquad (21)$$

The distance $R_0R_2 = D$ determines whether the 2-hop link R_0R_2 has "short" intermediate hops or not (refer to Fig. 4). For distances R_0R_2 smaller than the value in (21) of about 120 m, both intermediate hops are always "short" and the TMTT is largest when the relay is placed in the middle of the segment R_0R_2. We can see from the plot that the best relay location in this case is as close as possible to either R_0 or R_2, which seems to suggest that for "short" intermediate hops the single-hop link outperforms the 2-hop link (within the framework of the initial assumptions, the 2-hop link with the relay placed at one end is equivalent to a single-hop link). For large distances R_0R_2, the situation changes to the opposite: the intermediate hops R_0R_1 and R_1R_2 are now "long", and the middle of the segment R_0R_2 is the optimal location for the relay; the use of the relay improves the overall performance compared with the single-hop.

Somewhere between these two extreme cases lies the curve where the TMTT for relay located at source or destination (equivalent with a single-hop link) is equal with the TMTT for the relay located at equal distance from link ends (2-hop link). To find out the threshold value when the 2-hop link is as efficient as the single-hop R_0R_2, we set the condition that the TMTT is the same for the single-hop and for the 2-hop link with the relay placed in the middle of the segment R_0R_2:

$$\frac{1}{\log_{10}\left(K_2\left(\frac{D}{d_0}\right)^{-p}\right)} = \frac{2}{\log_{10}\left(K_2\left(\frac{D}{2d_0}\right)^{-p}\right)}. \qquad (22)$$

Simplifying (22) we obtain

$$\frac{D}{d_0} = \frac{1}{2} K_2^{\frac{1}{p}}, \quad (23)$$

or, D = 442 m for the same system parameters as before. As seen in Figure 4, for R_0R_2 distances D = 442 m and larger, the best relay location is the middle of the segment R_0R_2.

In single hop SNR terms (23) can be expressed as

$$\gamma = 2^p \quad (24)$$

We can conclude that, for the link R_0R_2, if the SNR of the single hop is greater than 2^p, any 2-hop link would have a lower performance than the single hop. If the SNR is below 2^p, the best aggregate spectral efficiency is achieved by the 2-hop link with the relay placed in the middle of R_0R_2.

IV. MULTI-HOP CRITERION

Theorem 2 – The Multi-Hop Criterion

Consider a multi-hop link replacement of a single-hop link, where all hops have the same path-loss exponent p, and same message size M and bandwidth B are used in all hops. An n-hop replacement link which can achieve a better aggregate spectral efficiency than the direct link exists only if

$$\gamma < n^{\frac{p}{n-1}}, \quad (25)$$

where γ is the SNR of the direct link.

Proof:

With the message size M and bandwidth B being the same, the multi-hop link has a better aggregate spectral efficiency than the direct link if

$$\sum_{i=1}^{n} \frac{1}{\eta_i} < \frac{1}{\eta}. \quad (26)$$

The condition (26) simply states that in order for the multi-hop link to be more efficient, the time required to pass a message of a given size M from R_0 to R_n over the direct link must be longer than the time required for the same operation over the multi-hop link.

Using Theorem 1a) and (19), we can express the lower bound on the TMTT for the n-hop as

$$T \geq \frac{M}{BK_1} \frac{n}{\log_{10}\left(K_2 \left[\frac{D}{nd_0} \right]^{-p} \right)} \quad (27)$$

$$= \frac{M}{BK_1} \frac{n}{\log_{10}\left(n^p \gamma \right)}.$$

The expression (27) shows the smallest possible TMTT that can be achieved using n hops, given that (17) is true. Using (27), the inequality (26) can be rewritten as

$$\frac{n}{\log_{10}(n^p \gamma)} < \frac{1}{\log_{10} \gamma} \quad (28)$$

which can be simplified to the expression (25). □

If for a given single-hop link, the condition (25) is not met, an n-hop link replacement with a better overall spectral efficiency does not exist, no matter where the relays are located. The inequality (25) represents a quantitative criterion that can be used to decide in which situation a multi-hop link could be considered.

The "break-even" SNR values in (25) are plotted in Figure 5 for various values of the path loss exponent p. The plots show the variation in the threshold (or "break-even") SNR value at which a more efficient multi-hop alternative to the given single-hop link becomes feasible. We observe in Figure 5, for instance, that if the path loss exponent of all intermediate hops happens to be p = 3.6, then according to (25), there exists a possible 3-hop configuration (the 2 relays evenly distributed along the line R_0R_3) with a better aggregate spectral efficiency, as long as the single-hop link SNR is less than 8.6 dB.

Let us consider an n-hop with the n-1 relays placed at equal intervals on the straight line between the source and destination. Comparing the TMTT for a n-hop link versus an (n+1)-hop link, using a similar rationale as before, we find an inequality which can be used to determine the optimal number of hops from source to destination:

$$\gamma < \frac{(n+1)^{pn}}{n^{p(n+1)}}. \quad (29)$$

If the SNR of the single-hop link respects (29), the *(n+1)*-hop link will achieve a lower TMTT compared with the n-hop link, i.e., adding the $(n+1)^{th}$ hop would improve the aggregate end-to-end spectral efficiency. The expression (29) is plotted in Figure 6, for various values of the path-loss exponent p.

Plotting (29) for values of n =*1, 2, 3,* ...results in curves which delimit the SNR values above which n hops are optimal, and below which n+1 hops are optimal. This divides the plot area in regions where 1, 2, 3, … hops are optimal, as depicted in Figure 6 (shown up to 4 (or more)-hop region). This can be expressed analytically as

$$\frac{n^{p(n-1)}}{(n-1)^{pn}} < \gamma < \frac{(n+1)^{pn}}{n^{p(n+1)}}. \quad (30)$$

If (30) is true, then the optimal number of hops is n+1.

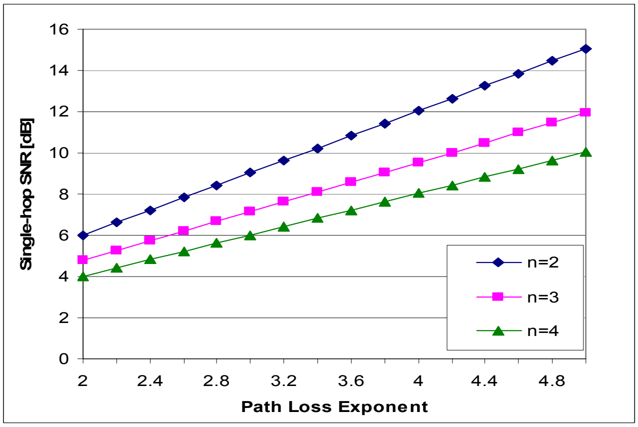

Figure 5. SNR values above which the single-hop has better spectral efficiency compared to any n-hop.

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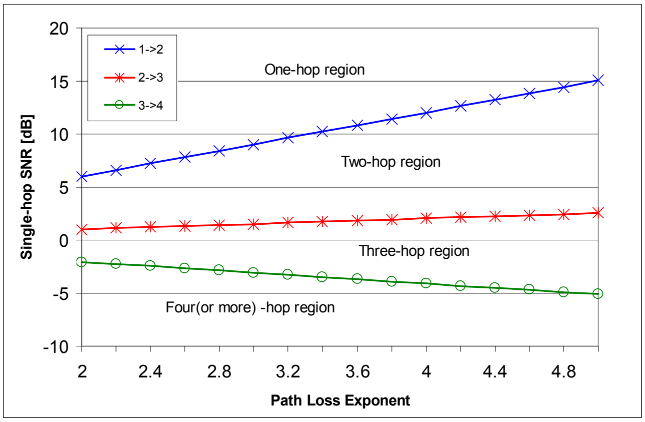

Figure 6. SNR values below which an *(n+1)*–hop is more efficient than an *n*-hop link.

Remarks:

1. The criterion developed above uses as the performance metric the consumption in the frequency-time plane for transferring messages from source to destination. The energy required for the transfer is not considered as a metric; that is, the additional power inserted in the system by intermediate relays is considered "free".

2. Since the developments in this paper are based on mean SNR values not including shadowing, these results are valid as a statistical average over a set of multi-hop links; the results above are not binding on a given particular realization of a multi-hop link.

3. As expected, in the case of $n=2$ (one fixed relay between R_0 and R_2), the inequality (25) becomes $\gamma < 2^p$, in concordance with (24). For example, if the propagation exponent has a value of 3, one relay placed in the ideal location (right in the middle of the link R_0-R_2) would be efficient only if the SNR of the link R_0-R_2 is less than 8 (9 dB). Furthermore, evaluating (25) for $n = 2$, and (29) for $n = 1$ (that is, comparing single-hop with two-hop) result in the same condition: $\gamma < 2^p$.

4. For the particular case of relays placed in straight line at equal intervals, (25) and (29)-(30), and therefore Fig.s 5 and 6, are complementary. For instance, we observe from (25) and Fig. 5 that, for $p = 3.6$, there exists a 2-hop solution better than the single-hop type when $\gamma <$ 10.8 dB; and there exists a 3-hop solution better than the single-hop type when $\gamma <$ 8.6 dB; and there exists a 4-hop solution better than the single-hop type when $\gamma <$ 7.2 dB. Based on these observations, one can deduce that the optimal number of hops is 2 when 8.6 dB $< \gamma <$ 10.8 dB. But, it is unclear whether the optimal number of hops is 2 or 3 when 7.2 dB $< \gamma <$ 8.6 dB. The answer of this question is obtained from (30) and Fig. 6: the 2-hop solution is indeed optimal when 1.8 dB $< \gamma <$ 10.8 dB.

5. The multi-hop criterion is in general applicable to multi-hop links with all individual hops having similar radio propagation characteristics, i.e., equal path loss exponents. In the special case when the mobile access link (the last hop of a 4G cellular link using fixed relays) has the same path loss exponent as the feeder system, the multi-hop criterion can be extended to cover the entire link between the base station and the mobile terminal.

6. Once again, (29) is valid only for statistical averages over sets of multi-hop links, with relays placed in straight line at equal intervals. For a given link, knowing the values for the single-hop SNR and the path loss exponent p, we can estimate the optimal number of hops achieving the best end-to-end aggregate spectral efficiency. However, in practice, the intermediate hops of the multi-hop link may have different path loss exponents, and additional random path losses due to shadowing, in which case the prediction given by (29) may not be accurate.

V. Conclusions

Using Jensen's inequality, we have shown that for relatively high SNR values, a single hop link has better spectral efficiency compared with an *n*-hop replacement; for relatively small SNR values, on the other hand, a more efficient *n*-hop link is possible, with the optimal locations of multi-hop digital relays being at equal intervals along the straight line between the source and destination. A novel quantitative criterion is developed, offering threshold mean SNR values below which an *n*-hop replacement should be considered over a single-hop link. Moreover, using the inequality (29), the optimal number of relays in a multi-hop link can be determined, under the assumptions that all links have the same path loss exponent and the relays are located at equal intervals. Additional research into the statistical distribution of spectral efficiencies for multi-hop links may bring further clarifications on the properties of cellular systems using fixed relays for multi-hop communications.

References

1. R. Pabst, B.H. Walke, D.C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D.D. Falconer, and G.P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio", *IEEE Communications Magazine*, vol. 42, no. 9, pp. 80-89, Sept. 2004.
2. M. Lott, M. Weckerle, and M. Siebert, "Scheduling in wireless multihop networks", *Proc. of International Symposium of Performance Evaluation of Computer and Telecommunication Systems (SPECTS'04),* San Jose, California, 25-29 July 2004.
3. H. Bolukbasi, H. Yanikomeroglu, D. Falconer, and S. Periyalwar, "Feasibility of providing high data coverage in cellular fixed relay networks", *World Wireless Research Forum meeting no. 12 (WWRF-12)*, 3-4 November 2004, Toronto, Canada.
4. G. Cair, G. Taricco, E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Info. Theory,* vol. 44, no. 3, pp. 927-946, May 1998.
5. T. M. Cover and J. A. Thomas, *Elements of Information Theory*, John Wiley and Sons, 1991.

