EUROPEAN TRANSACTIONS ON TELECOMMUNICATIONS
Eur. Trans. Telecomms. 2007; **18**:617–626
Published online 30 August 2007 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/ett.1231

Antenna combining for multi-antenna multi-relay channels†

Yijia Fan[1]*, Abdulkareem Adinoyi[2], John S. Thompson[1] and Halim Yanikomeroglu[2]

[1]*Institute for Digital Communications, University of Edinburgh, Edinburgh, EH9 3JL, UK*
[2]*Broadband Communications and Wireless Systems (BCWS) centre, Department of Systems and Computer Engineering, Carleton University, Ottawa, K1S 5B6, Canada*

SUMMARY

In this paper we analyse the performance of multiple relay channels when multiple antennas are deployed only at relays. We apply two antenna diversity techniques at relays, namely maximum ratio combining (MRC) on receive and transmit beamforming (TB). We show that for both decode-and-forward and amplify-and-forward relaying protocols, with K relays the network can be decomposed into K diversity channels each with a different channel gain, and that the signals can be effectively combined at the destination. We assume that the total number of antennas at all relays is fixed at N. With a reasonable power constraint at the relays, the network capacity will be lower bounded by that of N relay channels each with single antenna, and upper bounded by that of single relay channel with N antennas. Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

It is widely believed that *ad hoc* networking [1] or multi-hop cellular networks [2] are important new concepts for future generation wireless systems [3], where either mobile or fixed nodes (often referred to as relays) are used to help forward the information to the desired user. One advantage of these structures is that it is possible to unite multiple relays in the network as a 'virtual antenna array' to forward the information cooperatively, while appropriate combining at the destination realises diversity gain. The diversity achieved in this way is often named as *user cooperation diversity* or *cooperative diversity* [4], as it mimics the performance advantages of multiple-input multiple-output (MIMO) systems [5] in exploiting the spatial diversity of the relay channels. The performance limits of space–time codes, which can exploit cooperative diversity, are discussed in References [6–8] for single-antenna relay networks. For multiple-antenna relay channels where every terminal in the network can be deployed with multiple antennas, studies are mainly concentrated on spatial multiplexing systems References [9–11].

In this paper we exploit the spatial diversity of the relay channels in a different way from space–time coding based approach. We apply two kinds of antenna combining techniques at the relay, namely maximum ratio combining (MRC) [12] for reception and transmit beamforming (TB) [13] for transmission. Those techniques were often used in point-to-point single-input multiple-output (SIMO) or multiple-input single-output (MISO) wireless links, where either the transmitter or receiver is equipped with multiple antennas. It has been shown that MRC (TB) is able to achieve information theoretic upper bound of SIMO (MISO) systems [14]. In a relay context, we move the multiple antennas to the relays, while the source and the destination are only equipped with a single-antenna. Compared with the single-antenna relay network where every node is equipped with one antenna, our system model allows certain wired cooperation between the antennas at the relays, which is clearly a advantage. However, it will be shown that our schemes can also work effectively for single-antenna networks. More specifically, we will show that by applying the antenna combining techniques in the relay network, a network with K relays can be decomposed into

* Correspondence to: Yijia Fan, Institute for Digital Communications, University of Edinburgh, Edinburgh, EH9 3JL, UK. E-mail: y.fan@ed.ac.uk
†A previous edition of this paper has been presented in the 12th European Wireless Conference (EW 2006), Athens, Greece.

Accepted 1 December 2006

K diversity channels each with a different channel gain, and the signals from all K branches can be effectively combined at the destination. We derive the capacity bounds for this signal combining techniques and our analysis results can be applied to both ergodic capacity and outage capacity [15] performance. Our investigation is based on both the decode-and-forward (digital) relaying mode, where the relays decode, re-encode and re-transmit the signals, and the amplify-and-forward (analogue) mode, where the relays amplify and forward the signals without decoding the message.

The rest of this paper is organised as follows. In Section 2, the basic system model and assumptions are introduced. Section 3 introduces the antenna combining techniques. The capacity performance analysis are made in Section 4. Section 5 presents and discusses simulation results and finally, conclusions are drawn in Section 6.

2. SYSTEM MODEL

We consider a two-hop network model with one source, one destination and K relays. For simpler presentation we ignore the direct link between the source and destination. However, the extension of all the results to include the direct link is straightforward. We assume that the source and destination are deployed with single antennas, while relay k is deployed with m_k antennas; the total number of antennas at all relays is fixed to N. This can be expressed as:

$$\sum_{k=1}^{K} m_k = N \tag{1}$$

We restrict our discussion to the case where the channels are frequency-flat fading. The data transmission is over two times slots using two hops. In the first transmission time slot, the source broadcasts the signal to all the relay terminals. The input/output relation for the source to the kth relay is given by

$$\mathbf{r}_k = \sqrt{\eta}\mathbf{h}_k s + \mathbf{n}_k \tag{2}$$

where $\mathbf{r}_k$ is the $m_k \times 1$ receive signal vector, and η denotes the transmit power at the source. The scalar s is the unit mean power transmit signal and $\mathbf{n}_k$ is the $m_k \times 1$ complex circular additive white Gaussian noise vector at relay k with identity covariance matrix $\mathbf{I}_{m_k}$. The vector $\mathbf{h}_k$ is the $m_k \times 1$ channel transfer vector from the source to the kth relay. The entries of $\mathbf{h}_k$ are independent and identically distributed (i.i.d.) random variables. In the second hop, each relay processes its received signals and re-transmits them to the destination. The signal received at the destination can be written as:

$$y = \sum_{i=1}^{K} \mathbf{g}_k \mathbf{d}_k + n_\mathrm{d} \tag{3}$$

where the $1 \times m_k$ vector $\mathbf{g}_k$ is the channel vector from the kth relay to the destination, of which each entry is an i.i.d. random variable. The scalar n_d is the complex additive white Gaussian noise at the destination with unit variance. The $m_k \times 1$ vector $\mathbf{d}_k$ is the transmit signal vector at relay k, which should meet the total transmit power constraint:

$$\mathrm{E}\left[\|\mathbf{d}_k\|_F^2\right] \leqslant \frac{\eta m_k}{N} \tag{4}$$

where $\|\cdot\|_F$ denotes the Frobenius norm and $\mathrm{E}[\cdot]$ denotes the expectation. This power constraint means that the power is allocated at each relay in proportion to its number of antennas. For presentation simplicity we assume here that the total power at all relays is fixed to be η, i.e. the same as at the source. However, all the conclusions in the paper also hold when the total power at all relays is fixed to an arbitrary constant. We assume a coherent relay channel configuration context where the kth relay can obtain full knowledge of both backward channel vector $\mathbf{h}_k$ and forward channel vector $\mathbf{g}_k$. For fair comparison, we also assume that for each channel realisation, all the backward and forward channel coefficients for all N antennas remain the same regardless of the number of relays K. Figure 1 shows the system model.

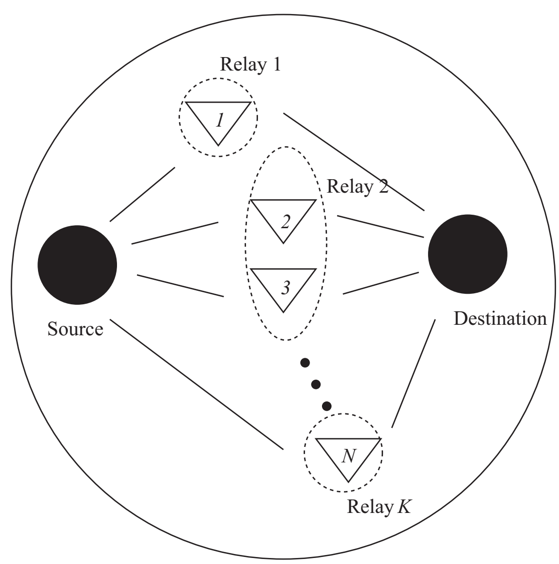


Figure 1. System model for a two-hop network: source and destination are each deployed with one antenna. A total of N antennas are deployed at K relays. For each channel realisation, both backward and forward channel coefficients for all N antennas remain the same regardless of the number of relays K.

DOI: 10.1002/ett

3. ANTENNA COMBINING TECHNIQUES IN RELAY CHANNELS

In this section we apply MRC and TB techniques to the system model described in Section 2. We discuss both decode-and-forward and amplify-and-forward relaying modes below.

3.1. Decode-and-forward relaying

We assume that each relay performs MRC of the received signals, by multiplying the received signal vector by the vector $\mathbf{h}_k^H/\|\mathbf{h}_k\|_F$, where $\mathbf{h}_k^H$ denotes the complex conjugate transpose of $\mathbf{h}_k$. The signal at the output of the relay receiver is given by

$$\tilde{r}_k = s\sqrt{\eta\sum_{i=1}^{m_k}|h_{i,k}|^2} + \frac{\sum_{i=1}^{m_k} h_{i,k}^* n_{i,k}}{\sqrt{\sum_{i=1}^{m_k}|h_{i,k}|^2}} \tag{5}$$

where $h_{i,k}$ denotes the ith antenna at relay k, and $n_{i,k}$ denotes the noise factor for ith receiver input branch. We denote $h_{i,k}^*$ the complex-conjugate of $h_{i,k}$. The signal-to-noise ratio (SNR) at the output of the receiver can be written as:

$$\rho_k^{m_k} = \eta\sum_{i=1}^{m_k}|h_{i,k}|^2 \tag{6}$$

After the relays decode the signals, each relay then performs TB of the decoded waveform. If we denote the transmitted signals as t_k with unit variance, the transmitted signal vector $\mathbf{d}_k$ for relay k can be written as

$$\mathbf{d}_k = \sqrt{\frac{\eta m_k}{N}}\frac{\mathbf{g}_k^H}{\|\mathbf{g}_k\|_F}t_k \tag{7}$$

The destination receiver simply detects the combined signals from all K relays. If we adjust the transmission data rate so that the signals are correctly decoded at all the relays (i.e. $t_k = s$), the output signal at the destination can be written as:

$$y = s\sum_{k=1}^{K}\sqrt{\frac{\eta m_k}{N}\sum_{i=1}^{m_k}|g_{i,k}|^2} + n_\mathrm{d} = s\sum_{k=1}^{K}\tilde{g}_k + n_\mathrm{d} \tag{8}$$

It can be seen from Equation (8) that by applying antenna diversity schemes at relays, the networks can be decomposed to K diversity channels each with channel gain $\tilde{g}_k$. The output SNR at the destination receiver can therefore be written as:

$$\rho_\mathrm{d}^{m_k} = \left(\sum_{k=1}^{K}\sqrt{\frac{\eta m_k}{N}\sum_{i=1}^{m_k}|g_{i,k}|^2}\right)^2 \tag{9}$$

When all the relays are deployed with a single antenna, there is no traditional MRC gain at the relays and the destination. However, the destination still observes a set of equal gain combined [16] amplitude signals from all relays.† Since we assume that the backward and forward channel coefficients for each antenna are kept the same for different values of K and m_i, the output SNR at the destination can be rewritten as:

$$\rho_\mathrm{d}^{1} = \frac{\eta}{N}\left(\sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|\right)^2 \tag{10}$$

when all the antennas are deployed in one relay (i.e. $K = 1$ and $m_1 = N$), the network can be separated into a SIMO (source-relay) and a MISO (relay-destination) channel. Using MRC and TB can achieve the information theoretic upper bound of each link. The SNR for this case can be rewritten as:

$$\rho_\mathrm{d}^{N} = \eta\sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|^2 \tag{11}$$

3.2. Amplify-and-forward relaying

For amplify-and-forward relaying, after each relay receiver performing MRC of the signal vector, it amplifies the signal (5) by a factor that can meet the power constraint (4). The amplifying factor can be computed as:

$$\gamma_k = \frac{\eta\frac{m_k}{N}}{\eta\sum_{i=1}^{m_k}|h_{i,k}|^2 + 1} \tag{12}$$

The transmitted signal t_k at each relay can be expressed by $t_k = \gamma_k\tilde{r}_k$. Note that unlike decode-and-forward mode, it now becomes a combination of the source signal and the noise at the relay. The relay then applies TB to t_k to form

† Unlike Reference [16], the equal gain combining weights for the relay channels are applied at the transmitter(s) instead of the receiver.

the transmit signal vector $\mathbf{d}_k$, which now can be expressed as:

$$\mathbf{d}_k = \frac{\mathbf{g}_k^H}{\|\mathbf{g}_k\|_F}\gamma_k \tilde{r}_k \tag{13}$$

The destination receiver receives the sum of the signals from all K relays and performs data detection. The output signal at the destination (3), after some modification, can be written as:

$$y = s\sum_{k=1}^{K}\sqrt{\eta\sum_{i=1}^{m_k}|g_{i,k}|^2\sum_{i=1}^{m_k}|h_{i,k}|^2\frac{\eta\frac{m_k}{N}}{\eta\sum_{i=1}^{m_k}|h_{i,k}|^2+1}} + \underbrace{\sum_{k=1}^{K}\frac{\sum_{i=1}^{m_k}h_{i,k}^{*}n_{i,k}}{\sqrt{\sum_{i=1}^{m_k}|h_{i,k}|^2}}\sqrt{\sum_{i=1}^{m_k}|g_{i,k}|^2\frac{\eta\frac{m_k}{N}}{\eta\sum_{i=1}^{m_k}|h_{i,k}|^2+1}}}_{n_{\mathrm{r}}} + n_{\mathrm{d}} \tag{14}$$

where we denote n_{r} as the equivalent noise generated from the relays. It can be seen from Equation (14) that compared with decode-and-forward mode, the signals can also be coherently combined at the destination, with a channel gain which takes into account the source to relay channel gains at the cost of additional noise n_{r}. Furthermore, we can observe from n_{r} that the noise generated at different relays *is not* coherently combined at the destination, though the signal can. Beamforming at the relays works only for the signal but not for the noise. Therefore, while the signal is enhanced by the beamforming, the noise generated at the relays is not. This implies that besides the equal gain combining gain, beamforming the signal from the different relays can offer an *additional coherent combining gain* for reducing the impact of the noise generated at different relays specially for the amplify-and-forward mode. Specifically, the SNR at the destination can be written as:

$$\rho_a^{m_k} = \frac{\left(\sum_{k=1}^{K}\sqrt{\eta\sum_{i=1}^{m_k}|g_{i,k}|^2\sum_{i=1}^{m_k}|h_{i,k}|^2\frac{\eta\frac{m_k}{N}}{\eta\sum_{i=1}^{m_k}|h_{i,k}|^2+1}}\right)^2}{\sum_{k=1}^{K}\sum_{i=1}^{m_k}|g_{i,k}|^2\frac{\eta\frac{m_k}{N}}{\eta\sum_{i=1}^{m_k}|h_{i,k}|^2+1}+1} \tag{15}$$

where we can clearly see an additional coherent combining gain of the signal power over the noise power generated at the relays.

Similar to the analysis for decode-and-forward mode, when all the relays are deployed with single antenna, the SNR at the destination can be rewritten as:

$$\rho_a^1 = \frac{\eta\left(\sum_{k=1}^{K}\sum_{i=1}^{m_k}|g_{i,k}|\,|h_{i,k}|\sqrt{\frac{\frac{\eta}{N}}{\eta|h_{i,k}|^2+1}}\right)^2}{\sum_{k=1}^{K}\sum_{i=1}^{m_k}|g_{i,k}|^2\frac{\frac{\eta}{N}}{\eta|h_{i,k}|^2+1}+1} \tag{16}$$

It can be seen that no maximal ratio combining gain can be obtained in this case. However, the additional coherent combining gain and equal gain combining gain are maximised as the number of relays is maximised. When all the antennas are deployed in one relay, the SNR can be rewritten as:

$$\rho_a^N = \frac{\eta\sum_{k=1}^{K}\sum_{i=1}^{m_k}|g_{i,k}|^2\sum_{k=1}^{K}\sum_{i=1}^{m_k}|h_{i,k}|^2\frac{\eta}{\eta\sum_{k=1}^{K}\sum_{i=1}^{m_k}|h_{i,k}|^2+1}}{\frac{\eta}{\eta\sum_{k=1}^{K}\sum_{i=1}^{m_k}|h_{i,k}|^2+1}\sum_{k=1}^{K}\sum_{i=1}^{m_k}|g_{i,k}|^2+1} \tag{17}$$

In this case there is no equal gain combining gain or additional coherent combining gain, as all the antennas belongs to one relay. However, the maximal ratio combining gain can be obtained due to the full cooperation of the antennas at the relay. It has been shown that in this scenario, using MRC and TB can achieve the information theoretic upper bound of the single relay channel [17] if the direct link is ignored.

4. CAPACITY BOUNDS

4.1. Decode-and-forward relaying

The network capacity for decode-and-forward relaying for each channel realisation can be written as:

$$C_D^{m_k} = \min\left(C_{\mathrm{r}}^{1,m_1}, C_{\mathrm{r}}^{2,m_2}, \ldots, C_{\mathrm{r}}^{K,m_k}, C_{\mathrm{d}}^{m_k}\right) \tag{18}$$

where $C_{\mathrm{r}}^{k,m_k} = 0.5\log_2(1+\rho_k^{m_k})$ denoting the Shannon capacity for the source to the kth relay channel, and $C_{\mathrm{d}}^{m_k} = 0.5\log_2(1+\rho_{\mathrm{d}}^{m_k})$ denoting the Shannon capacity for the relay to destination channels.[‡] The factor 0.5 denotes the half multiplexing factor compared with non-relay channels.

We firstly analyse channel capacity for the relays to destination link by bounding $\rho_{\mathrm{d}}^{m_k}$, i.e. the output SNR at the destination.

Lemma 1. *For any* m_k, $\rho_{\mathrm{d}}^{1} \leqslant \rho_{\mathrm{d}}^{m_k} \leqslant \rho_{\mathrm{d}}^{N}$.

Proof. See Appendix A. ■

From Lemma 1, we can see that

$$C_{\mathrm{d}}^{1} \leqslant C_{\mathrm{d}}^{m_k} \leqslant C_{\mathrm{d}}^{N} \tag{19}$$

where C_{d}^{1} denotes the capacity for the relays to destination channel when $K = N$, and C_{d}^{N} denotes the capacity for the relays to destination channel when $K = 1$. Now also considering the capacity for the source to relay links and extending the analysis to the whole network scenario, we have the following theorem:

Theorem 1. *If we denote the network capacity for* $K = N$ *as* C_D^1 *and for* $K = 1$ *as* C_D^N, *for any* m_k, $C_D^1 \leqslant C_D^{m_k} \leqslant C_D^N$.

Proof. Considering the SNR $\rho_k^{m_k}$ for the source to the kth relay link, if we denote it as ρ_n^1 for $K = N$ and ρ_1^N for $K = 1$, it can be shown that

$$\min\left(\rho_n^1\right) \leqslant \min\left(\rho_k^{m_k}\right) \leqslant \rho_1^N \tag{20}$$

Therefore, we have the following:

$$\min\left(C_{\mathrm{r}}^{1,1}, \ldots, C_{\mathrm{r}}^{N,1}\right) \leqslant \min\left(C_{\mathrm{r}}^{1,m_1}, \ldots, C_{r}^{K,m_k}\right) \leqslant C_{\mathrm{r}}^{1,N} \tag{21}$$

Combining Equations (21) and (19), we thus complete the proof. ■

From the above analysis we have shown that for the antenna combining techniques discussed in the paper, the network capacity will be lower bounded by that of N relay channels each with a single antenna, and upper bounded by that of a single relay channel with N antennas. This means that even there are more relays, the increased 'equal gain combining' gain at the destination cannot compensate for the loss of MRC gain at the relay and the destination when the numbers of antennas at each relay are reduced.

[‡] Here the SNR for the direct link should be included inside the log function when direct link is accounted for.

4.2. Amplify-and-forward relaying

The network capacity for amplify-and-forward relaying for each channel realisation can be written as:

$$C_A^{m_k} = 0.5 \log_2\left(1 + \rho_a^{m_k}\right) \tag{22}$$

The capacity analysis for the decode-and-forward mode cannot be directly extended to the amplify-and-forward mode, as they have different relaying mechanisms. In fact, the analysis for amplify-and-forward relaying is more difficult due to the following two reasons: (a) the source-relay link and the relay-destination link in amplify-and-forward mode cannot be considered separately, as decoding is not performed at the relays; (b) the impact of the noise component generated at the relays is complicated and play a vital role in the capacity performance.

However, we can conjecture the same conclusion as of the decode-and-forward mode by imagining an extreme case, i.e. a very high transmit power level. The performance of amplify-and-forward mode in this scenario mimics that of decode-and-forward mode, as the noise component generated at the relays becomes negligible. In fact, the same upper bound for the output SNR at the destination for amplify-and-forward mode can be made as in Lemma 1 for the decode-and-forward mode. This is stated in the following lemma:

Lemma 2. *For any* m_k, $\rho_a^{m_k} \leqslant \rho_a^N$.

Proof. See Appendix B. ■

Based on this lemma, we directly obtain the following theorem for the amplify-and-forward relaying mode:

Theorem 2. *If we denote the network capacity for* $K = 1$ *as* C_A^N, *for any* m_k, $C_A^{m_k} \leqslant C_A^N$.

The capacity lower bound, however, is difficult to obtain. The reason is that unlike decode-and-forward relaying, it is much more complicated to compare $\rho_a^{m_k}$ with ρ_a^1 due to the additional coherent combining gain among different relays for amplify-and-forward relaying as stated in Section 3. However, we can still give a comparison between the highest achievable capacity for both cases. In the following we derive two tight (achievable) upper bounds for both $\rho_a^{m_k}$ and ρ_a^1, and show that the upper bound for $\rho_a^{m_k}$ is strictly larger than that for ρ_a^1.

DOI: 10.1002/ett

Lemma 3. *We have the following upper bounds for $\rho_a^{m_k}$ and ρ_a^1.*

$$\rho_a^{m_k} \leqslant \rho_{\text{upper}}^{m_k} \triangleq \sum_{k=1}^{K} \frac{\eta \sum_{i=1}^{m_k} |g_{i,k}|^2 \sum_{i=1}^{m_k} |h_{i,k}|^2 \frac{\eta \frac{m_k}{N}}{\eta \sum_{i=1}^{m_k} |h_{i,k}|^2 + 1}}{\sum_{i=1}^{m_k} |g_{i,k}|^2 \frac{\eta \frac{m_k}{N}}{\eta \sum_{i=1}^{m_k} |h_{i,k}|^2 + 1} + \frac{1}{K}} \quad (23)$$

$$\rho_a^1 \leqslant \rho_{\text{upper}}^1 \triangleq \sum_{k=1}^{K} \frac{\left(\sum_{i=1}^{m_k} |g_{i,k}| \, |h_{i,k}| \sqrt{\frac{\frac{\eta}{N}}{\eta |h_{i,k}|^2 + 1}} \right)^2}{\sum_{i=1}^{m_k} |g_{i,k}|^2 \frac{\frac{\eta}{N}}{\eta |h_{i,k}|^2 + 1} + \frac{1}{K}} \quad (24)$$

Proof. See Appendix C. ■

Based on these bounds, we have the following identity:

Lemma 4. *For any m_k, $\rho_{\text{upper}}^{m_k} \geqslant \rho_{\text{upper}}^1$.*

Proof. See Appendix D. ■

We consequently has the following theorm regarding the capacity of the network:

Theorem 3. *If we denote the* maximal achievable *network capacity for $K = N$ as C_A^1, for any m_k, $C_A^1 \leqslant C_A^{m_k}$.*

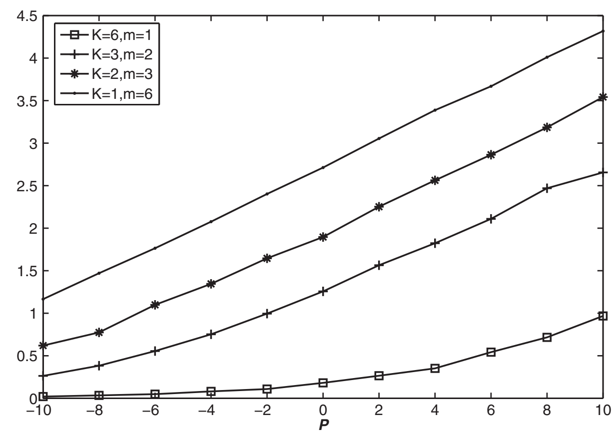
We note that for both analogue and digital relaying, the three capacity theorems can hold for *any* choice of fading distribution, as long as each $h_{i,k}$ ($g_{i,k}$) is i.i.d. For the simulations in the next section, we choose Rayleigh fading as an example to confirm the analysis.

5. SIMULATION RESULTS

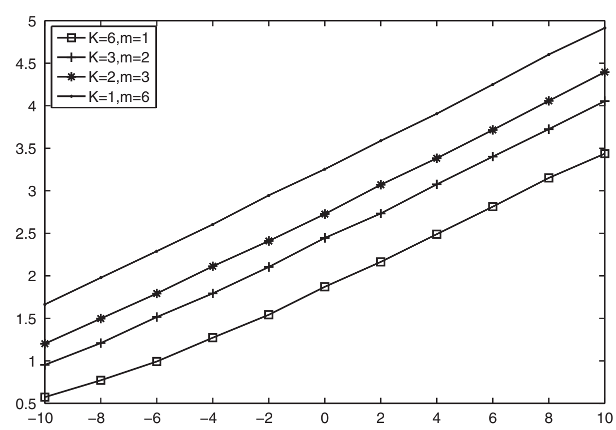
We consider both fast fading and slow fading scenarios. For fast fading, we calculate the ergodic capacity (in bits per channel use), which is the minimum of the average channel capacity for each link in the network. For slow fading, we calculate the 10% outage capacity. We define that an outage occurs whenever the transmission rate is above the channel capacity for the worst link in the network. We consider 1000 channel realisations for each value of η, denoted as P in the figures. We assume that the distance between source and destination is normalised. The relays are uniformly and randomly located in the middle region between the source and destination. Taking into account the pathloss and Rayleigh fading, each channel realisation can be expressed as:

$$h_{i,k} = \sqrt{0.5^{-4}}\tilde{h}_{i,k}, \quad g_{i,k} = \sqrt{0.5^{-4}}\tilde{g}_{i,k}$$

where $\sqrt{0.5^{-4}}$ denotes the pathloss (with exponent 4), the entries of $\tilde{h}_{i,k}$ ($\tilde{g}_{i,k}$) are i.i.d. complex Gaussian variables with zero mean and unit variance. We assume the total number of antennas at relays (N) is 6 and we also assume that all K relays have the same number of antennas m. Figures 2 and 3 show the capacity performance for digital relaying and analogue relaying. We can see that for different (K, m), the capacity is always upper bounded by (1, 6) and lower bounded by (6, 1). These results verify the analysis made in this paper. Furthermore, we can see through the

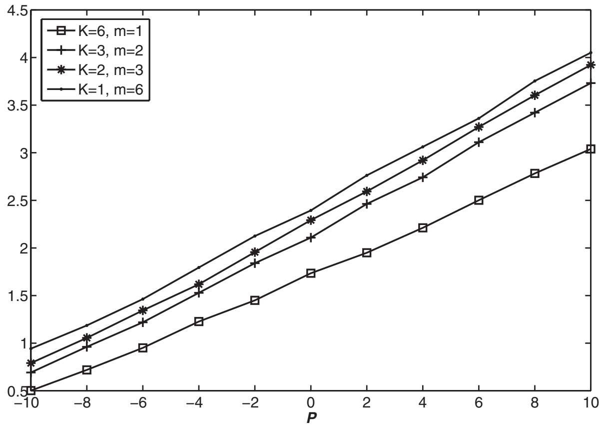


(a) 10% outage capacity

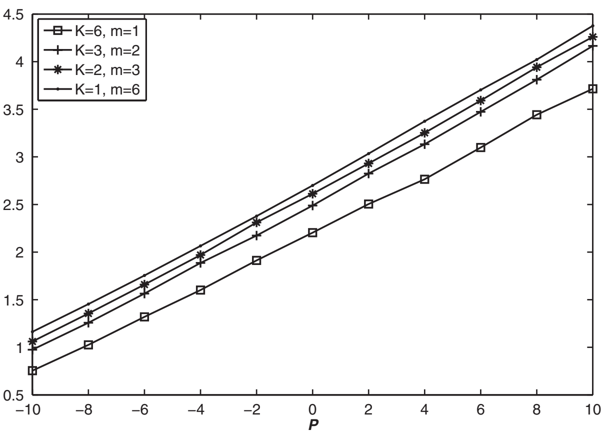


(b) Ergodic capacity

Figure 2. Capacity of relay channels using decode-and-forward relaying for different number of relays K, while each relay is deployed with m antennas. (a) 10% outage capacity, (b) ergodic capacity.



(a) 10% outage capacity



(b) Ergodic capacity

Figure 3. Capacity of relay channels using amplify-and-forward relaying for different number of relays K, while each relay is deployed with m antennas. (a) 10% outage capacity, (b) ergodic capacity.

simulation that larger m and small K might give larger benefit, since larger m allows more freedom of cooperation among the antennas at each relay. Therefore when m reaches N (K reduces to 1), full cooperation is made among all the antennas to give rise to the best performance.

Comparing the performance of digital relaying with that of analogue relaying, we can see that as K increases, the performance of digital relaying decays faster than that of analogue relaying. This is mainly because with more relays the capacity of digital relaying is constrained by the worst source-relay link in order for the signals to be correctly decoded at all relays. However, the capacity of analogue relaying takes into account the impact of all the K relay channels as decoding is only performed at the destination. Indeed one should note from Equation (18) that the performance advantage for digital relaying is significant only when $\min(C_{\mathrm{r}}^{k,m_k})$ is no less than the relay-destination link capacity. In this sense, deploying all the antennas on one relay turns out to be the optimal choice. This observation also confirms the suggestion in many existing papers, which argue that relays should be properly selected before being used for digital relaying (e.g. References [6, 7]).

6. CONCLUSIONS

In this paper we analyse the performance of multiple relay channels when multiple antennas are deployed only at relays. We apply antenna diversity techniques at the relays, which are known as MRC and TB. If we assume that the total number of antennas at all relays is fixed to N and the total transmit power at all relays is fixed to a constant, the network capacity will be lower bounded by that of N relay channels each with single antenna, and upper bounded by that of a single relay channel with N antennas.

The analysis in the paper also implies that given a certain amount of available antennas in the network, wired cooperation (i.e. all the antennas belong to one terminal) outperforms wireless cooperation (i.e. each antenna belongs to different terminals). We further note that the recently proposed fixed relay concept [2] in mesh networks allows the possibility to deploy a large number of antennas at the relay. This provides a good application for the antenna combining techniques discussed in the paper.

APPENDIX A: PROOF OF LEMMA 1

We first prove that $\rho_{\mathrm{d}}^{1} \leqslant \rho_{\mathrm{d}}^{m_k}$. We write the following

$$\sqrt{\rho_{\mathrm{d}}^{m_k}} - \sqrt{\rho_{\mathrm{d}}^{1}} = \sum_{k=1}^{K} \left(\underbrace{\sqrt{\frac{\eta m_k}{N} \sum_{i=1}^{m_k} \left|g_{i,k}\right|^2}}_{A_k} - \underbrace{\sqrt{\frac{\eta}{N}} \sum_{i=1}^{m_k} \left|g_{i,k}\right|}_{B_k} \right)$$

To compare A_k with B_k, we write

$$A_k^2 - B_k^2 = \frac{\eta}{N} \left(m_k \sum_{i=1}^{m_k} \left|g_{i,k}\right|^2 - \left(\sum_{i=1}^{m_k} \left|g_{i,k}\right| \right)^2 \right) \quad (25)$$

$$= \frac{\eta}{N}\left((m_k - 1)\sum_{i=1}^{m_k}\left|g_{i,k}\right|^2\right) - \frac{\eta}{N}\left(\sum_{i,j=1;i\neq j}^{m_k}\left|g_{i,k}\right|\left|g_{j,k}\right|\right) \quad (26)$$

Note that

$$(m_k - 1)\sum_{i=1}^{m_k}\left|g_{i,k}\right|^2 = \sum_{i=1}^{m_k}\sum_{j=1,j\neq i}^{m_k}\left|g_{j,k}\right|^2 = 0.5\sum_{i,j=1;i\neq j}^{m_k}\left(\left|g_{i,k}\right|^2 + \left|g_{j,k}\right|^2\right)$$

So Equation (26) can be further written as:

$$A_k^2 - B_k^2 = \frac{\eta}{2N}\sum_{i,j=1;i\neq j}^{m_k}\left(\left|g_{i,k}\right|^2 - 2\left|g_{i,k}\right|\left|g_{j,k}\right| + \left|g_{j,k}\right|^2\right) = \frac{\eta}{2N}\sum_{i,j=1;i\neq j}^{m_k}\left(|g_{i,k}| - |g_{j,k}|\right)^2 \geqslant 0$$

So $A_k \geqslant B_k$ and therefore $\rho_{\mathrm{d}}^1 \leqslant \rho_{\mathrm{d}}^{m_k}$.

Next we prove that $\rho_{\mathrm{d}}^N \geqslant \rho_{\mathrm{d}}^{m_i}$. For simplicity, we denote

$$a_k = \sum_{i=1}^{m_k}\left|g_{i,k}\right|^2 \quad (27)$$

in Equations (9) and (11). Then $\rho_{\mathrm{d}}^N - \rho_{\mathrm{d}}^{m_i}$ can be written as:

$$\rho_{\mathrm{d}}^N - \rho_{\mathrm{d}}^{m_i} = \frac{\eta}{N}\left(\sum_{k=1}^{K}(N - m_k)\,a_k - \sum_{i,j=1;i\neq j}^{K}\sqrt{m_i m_j a_i a_j}\right) \quad (28)$$

Noting the constraint (1) in Section 2, we have the following:

$$(N - m_k) = \sum_{i=1,i\neq k}^{K} m_i \quad (29)$$

Putting Equation (29) into (28), we then have:

$$\rho_{\mathrm{d}}^N - \rho_{\mathrm{d}}^{m_i} = \frac{\eta}{N}\left(\sum_{k=1}^{K}\sum_{i=1,i\neq k}^{K} m_i a_k - \sum_{i,j=1;i\neq j}^{K}\sqrt{m_i a_i m_j a_j}\right) \quad (30)$$

Note the following:

$$\sum_{k=1}^{K}\sum_{i=1,i\neq k}^{K} m_i a_k = \sum_{i,j=1;i\neq j}^{K}(m_i a_j) = 0.5\left(\sum_{i,j=1;i\neq j}^{K}(m_i a_j) + \sum_{i,j=1;i\neq j}^{K}(m_j a_i)\right)$$

We can further write Equation (30) as follows:

$$\rho_{\mathrm{d}}^N - \rho_{\mathrm{d}}^{m_i} = \frac{\eta}{2N}\sum_{i,j=1;i\neq j}^{K}(m_i a_j) - \frac{\eta}{N}\sum_{i,j=1;i\neq j}^{K}\sqrt{m_i a_j m_j a_i} + \frac{\eta}{2N}\sum_{i,j=1;i\neq j}^{K}(m_j a_i) = \frac{\eta}{2N}\sum_{i,j=1;i\neq j}^{K}\left(\sqrt{m_i a_j} - \sqrt{m_j a_i}\right)^2$$

Therefore $\rho_{\mathrm{d}}^N \geqslant \rho_{\mathrm{d}}^{m_i}$ and $\rho_{\mathrm{d}}^1 \leqslant \rho_{\mathrm{d}}^{m_k} \leqslant \rho_{\mathrm{d}}^N$.

APPENDIX B: PROOF OF LEMMA 2

To efficiently prove Lemma 2, we firstly introduce the following new lemma.

Lemma 5. *For any positive real numbers x_1, x_2, y_1, y_2, a, if*

$$x_1 \geqslant x_2 \text{ and } \frac{x_1}{y_1} \geqslant \frac{x_2}{y_2} \quad (31)$$

then

$$\frac{x_1}{y_1 + a} \geqslant \frac{x_2}{y_2 + a} \quad (32)$$

Proof. The proof is straightforward by showing $\frac{x_1}{y_1+a} - \frac{x_2}{y_2+a} \geqslant 0$. ■

Compare numerators of Equations (15) and (17), we have the following:

$$\left(\sum_{k=1}^{K}\sqrt{\eta\sum_{i=1}^{m_k}\left|g_{i,k}\right|^2\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2\frac{\eta\frac{m_k}{N}}{\eta\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}}\right)^2$$

$$\overset{(a)}{\leqslant}\sum_{k=1}^{K}\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2\frac{\eta}{\eta\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}\times\left(\sum_{k=1}^{K}\sqrt{\eta\sum_{i=1}^{m_k}\left|g_{i,k}\right|^2\frac{m_k}{N}}\right)^2$$

$$\overset{(b)}{\leqslant}\sum_{k=1}^{K}\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2\frac{\eta}{\eta\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}\times\eta\sum_{k=1}^{K}\sum_{i=1}^{m_k}\left|g_{i,k}\right|^2$$

where inequality (a) holds due to the fact that $\frac{x}{1+x}$ is monotonically increasing with x, inequality (b) can be found in the proof of Lemma 1 in Appendix A. From Lemma 3, we can now remove factor 1 in the denominators of Equations (15) and (17). After some modifications, it can be seen that to compare ρ_a^N and $\rho_a^{m_k}$ is to compare

$$\sum_{k=1}^{K}\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2$$

with

$$\frac{\left(\sum\limits_{k=1}^{K}\sqrt{\sum\limits_{i=1}^{m_k}\left|g_{i,k}\right|^2\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2\frac{\eta\frac{m_k}{N}}{\eta\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}}\right)^2}{\sum\limits_{k=1}^{K}\sum\limits_{i=1}^{m_k}\left|g_{i,k}\right|^2\frac{\eta\frac{m_k}{N}}{\eta\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}}$$

which is equivalent to comparing $\sum_{k=1}^{K} c_k \sum_{k=1}^{K} d_k$ with $(\sum_{k=1}^{K}\sqrt{c_k d_k})^2$, where

$$c_k=\sum_{i=1}^{m_k}\left|h_{i,k}\right|^2,\quad d_k=\sum_{i=1}^{m_k}\left|g_{i,k}\right|^2\frac{\eta\frac{m_k}{N}}{\eta\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}$$

By the Cauchy–Schwarz inequality, it is clear that

$$\sum_{k=1}^{K}c_k\sum_{k=1}^{K}d_k\geqslant\left(\sum_{k=1}^{K}\sqrt{c_k d_k}\right)^2 \tag{33}$$

the proof is thus completed.

APPENDIX C: PROOF OF LEMMA 3

The proof is straightforward if the following identity is noticed:

Lemma 6. *For positive real sequences $\{a_n\}$, $\{b_n\}$, the following inequality holds:*

$$\sum\frac{a_n}{b_n}\geqslant\frac{\left(\sum\sqrt{a_n}\right)^2}{\sum b_n} \tag{34}$$

with equality if $\{a_n\}$, $\{b_n\}$ are constant values that do not change with n.

Proof. The proof can be directly obtained by showing $\sum\frac{a_n}{b_n}-\frac{(\sum\sqrt{a_n})^2}{\sum b_n}\geqslant 0$. ■

APPENDIX D: PROOF OF LEMMA 4

The proof can be started by comparing each element in the summation of $\rho_{\text{upper}}^{m_k}$ and ρ_{upper}^{1}, i.e. by comparing

$$\text{SNR}_{m_k}^{m_k}\triangleq\frac{\eta\sum\limits_{i=1}^{m_k}\left|g_{i,k}\right|^2\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2\frac{\eta\frac{m_k}{N}}{\eta\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}}{\sum\limits_{i=1}^{m_k}\left|g_{i,k}\right|^2\frac{\eta\frac{m_k}{N}}{\eta\sum\limits_{i=1}^{m_k}\left|h_{i,k}\right|^2+1}+\frac{1}{K}}$$

DOI: 10.1002/ett

with

$$\mathrm{SNR}_{m_k}^1 \triangleq \frac{\left(\sum_{i=1}^{m_k} |g_{i,k}| \, |h_{i,k}| \sqrt{\frac{\frac{\eta}{N}}{\eta|h_{i,k}|^2+1}}\right)^2}{\sum_{i=1}^{m_k} |g_{i,k}|^2 \frac{\frac{\eta}{N}}{\eta|h_{i,k}|^2+1} + \frac{1}{K}} \tag{35}$$

It can be seen from the proof of Lemma 2 in Appendix B that $\mathrm{SNR}_{m_k}^{m_k} \geqslant \mathrm{SNR}_{m_k}^1$ by setting $K = m_k$ and $m_k = 1$. The proof is therefore completed.

REFERENCES

1. Goldsmith AJ, Wicker SB. Design challenges for energy-constrained ad hoc wireless networks Wireless Communications. *IEEE Personal Communications* 2002; **9**(4):8–27.
2. Pabst R, Walke BH, Schultz DC, Herbold P, Yanikomeroglu H, Mukherjee S, Viswanathan H, Lott M, Zirwas W, Dohler M, Aghvami H, Falconer D, Fettweis GP. Relay-based deployment concepts for wireless and mobile broadband radio. *IEEE Communications Magazine* 2004; **42**(9):80–89.
3. Frodigh M, Parkvall S, Roobol C, Johansson P, Larsson P. Future-generation wireless networks. *IEEE Personal Communications* 2001; **8**(5):10–17.
4. Laneman JN, Wornell GW, Tse DNC. An efficient protocol for realizing cooperative diversity in wireless networks. In *Proceedings of IEEE International Symposium on Information Theory*, Washington, DC, June 2001.
5. Gesbert D, Shafi M, Da-Shan S, Smith PJ, Naguib A. From theory to practice: an overview of MIMO space-time coded wireless systems. *IEEE Journal on Selected Areas in Communications* 2003; **21**(3):281–302.
6. Laneman JN, Tse DNC, Wornell GW. Cooperative diversity in wireless networks: efficient protocols and outage behavior. *IEEE Transactions on Information Theory* 2004; **50**(12):3062–3080.
7. Laneman JN, Wornell GW. Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks. *IEEE Transactions on Information Theory* 2003; **49**:2415–2425.
8. Rohit U Nabar, Helmut Bölcskei, and Felit W. Kneubühler. Fading relay channels: performance limits and space-time signal design. *IEEE Journal on Selected Areas in Communications* 2004; **22**(6):1099–1109.
9. Wang B, Zhang J, Host-Madsen A. On capacity of MIMO relay channel. *IEEE Transactions on Information Theory* 2005; **51**(1):29–43.
10. Bolcskei H, *et al*. Capacity scaling laws in MIMO relay networks. *IEEE Transactions on Wireless Communications* 2006; **5**(6):1433–1444.
11. Oyman O, Paulraj AJ. Energy efficiency in MIMO relay networks under processing cost. In *Conference on Information Science and Systems*, The Johns Hopkins University, March 16–18, 2005.
12. Proakis JG. *Digital Communications*, 4th edn. 2001.
13. Andersen JB. Antenna arrays in mobile communications: gain, diversity, and channel capacity. *IEEE Antennas Propagation Magazine* 2000; **42**(2):12–16.
14. Tse D, Viswanath P. *Fundamentals of Wireless Communications*. Cambridge University Press, 2005, Cambridge, UK.
15. Rhee W, Cioffi JM. On the capacity of multiuser wireless channels with multiple antennas. *IEEE Transactions on Information Theory* 2003; **49**(10):2580–2595.
16. Chen Y, Tellambura C. Performance analysis of L-Branch equal gain combiners in equally correlated Rayleigh fading channels. *IEEE Communications Letters* 2004; **8**(3):150–152.
17. Fan Y, Thompson JS. MIMO configurations for relay channels: theory and practice. *IEEE Transactions on Wireless Communications* (in press).

DOI: 10.1002/ett