Practical Capacity Calculation for Time-Hopping Ultra-wide Band Multiple-Access Communications

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***Abstract*— In this letter the practical capacity, known as the cutoff rate, of time-hopping (TH) ultra-wide band (UWB) communication system is evaluated for multiple-access channels. The cutoff rate can be used for determining various system trade-offs. For instance, it is shown in this letter that if synchronization problems would preclude high spreading factors, a suitable number of hops can be used instead to achieve the same performance. Moreover, it is demonstrated that the cutoff rate evaluated here can be a fast way of gaining insights into the multiuser capacity of TH-PPM UWB systems.**

***Index Terms*— Cutoff rate, ultra-wide band (UWB), time-hopping, multiple-access channel.**

I. INTRODUCTION

CONSIDERABLE interest has developed for time-hopping (TH) ultra-wide band (UWB) multiple-access communication systems [1]. This is due to UWB's appealing features; for instance, it does not require a sinusoidal carrier, it can highly resolve multipath, and it enjoys low probability of detection and interception. These features make UWB technology a promising option for high data rate communications.

The error exponent and cutoff rate are practical and important information-theoretic measures used extensively in the literature for comparing coding scheme performance [2] and constellation design [3]. These parameters set the bound on the performance and determine both the achievable rate and magnitude of the random-coding error exponent with practical modulation/coding schemes. This letter evaluates the cutoff rate for TH-PPM UWB multiple-access channels.

II. RANDOM CODING ERROR EXPONENT

The ensemble average probability of block decoding error using a maximum likelihood decoding is bounded by

$$P_e \leq \exp\left(-N_{blk}[E_0(P(x),\rho) - \rho R]\right), \tag{1}$$

where N_{blk} is the code block length, and R is the information rate per channel symbol forming the ensemble of (N_{blk}, R) block codes in which each alphabet is selected with probability $P(x)$. The argument $E(R,\rho) = [E_0(P(x),\rho) - \rho R]$ is known as the channel random coding error exponent [2] where ρ and $P(x)$ are chosen such that maximum exponent value is obtained, because (1) indicates that for some given codes with the same complexity (measured through N_{blk}) and same rate (R), the channel having the largest $E(R,\rho)$ value will result in the lowest error probability.

In this paper $\rho = 1$ is of interest and is referred to as the cutoff rate, R_0 (=$E_0(P(x),1)$), which can be expressed as [3]

$$R_0 = \max_{P_j}\left\{-\ln\left[\sum_j\sum_k P_jP_k\int_{\mathbf{r}}\sqrt{p(\mathbf{r}|j)p(\mathbf{r}|k)}d\mathbf{r}\right]\right\}, \tag{2}$$

where P_j is the a priori input probability, $p(\mathbf{r}|j)$ is probability density function of the output vector given that j-th signal was transmitted. If $\log_2$ is used instead of ln in (2), R_0 is in bits/transmitted waveform. Cutoff rate has been considered the practical capacity beyond which communication would be very expensive. Even though the recent experience with the near-capacity performance of turbo codes (TC) appears to threaten this belief, a substantial amount of price in terms of complexity and delay is still paid through long interleaver and iterative decoding of TC. In sequential decoding applications, cutoff rate remains a valuable parameter which provides insight complementary to that acquired by the study of capacity [4].

Let us consider a time hopping K-user UWB system employing M-ary PPM. A typical k^{th} user's received signal with perfect power control takes the form [1], [5]

$$s^{(k)}(t) = \sqrt{\frac{E_s}{N_s}}\sum_{j=0}^{N_s-1}p_{rx}(t - jT_f - c_j^{(k)}T_c - d_j^{(k)}), \tag{3}$$

then the total multiple-access received signal can be represented as

$$r(t) = s^{(1)}(t-\tau^{(1)}) + \sum_{k=2}^{K}s^{(k)}(t-\tau^{(k)}) + n(t), \tag{4}$$

where user 1 is the user of interest. In (3) and (4), $p_{rx}(t)$ is the basic pulse with a duration of T_p, T_f is the frame time, $T_s = N_sT_f$ is the symbol duration, $c_j^{(k)}$ is the hopping code, $d_j^{(k)}\epsilon\{\delta_1,\cdots,\delta_M\}$ represents the PPM time shift corresponding to the modulating data sequence of user k at hop j, $\tau^{(k)}$ represents time asynchronism, E_s is pulse (symbol) energy, and $n(t)$ is the additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$.

We assume that the receiver is in perfect synchronism with the user of interest so that the correlation receiver can be implemented. With this assumption, the receiver for the M-ary PPM scheme is composed of M filters matched to the template functions $\psi_i^{(1)}$, defined as

$$\psi_i^{(1)}(t) = p_{rx}(t - \delta_i - \tau^{(1)}), \qquad i = 1,2,\cdots,M. \tag{5}$$

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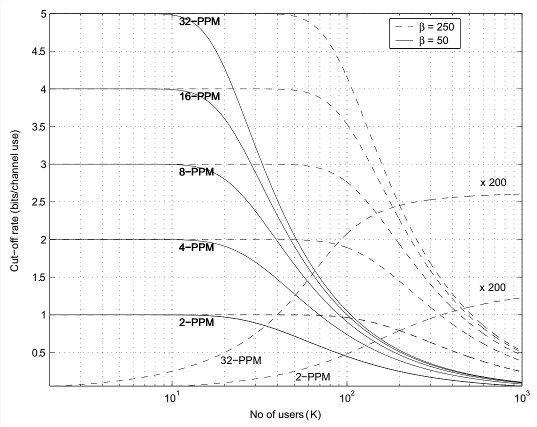

Fig. 1. Cutoff rates of M-ary PPM multiuser UWB for two different spreading factors at SNR = 20 dB in the absence of time hopping ($N_s = 1$).

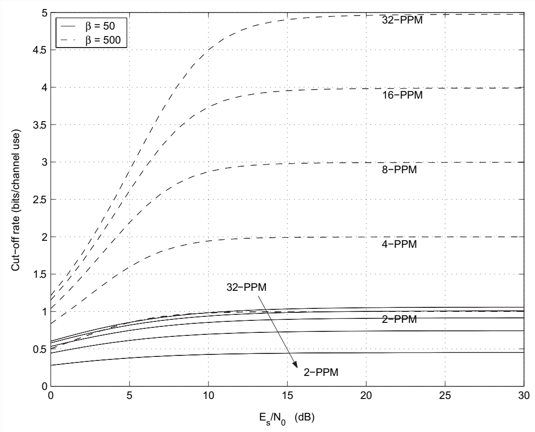

Fig. 2. Cutoff rates of M-ary PPM multiuser UWB for two different spreading factors for $K = 100$ in the absence of time hopping ($N_s = 1$).

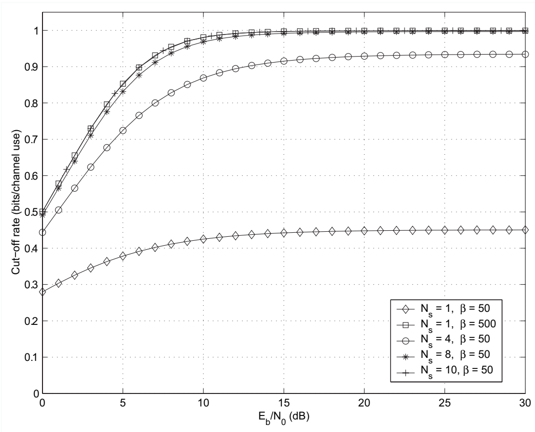

Fig. 3. Cutoff rates of TH 2-PPM multiuser UWB for different spreading and hopping factors for $K = 100$.

Each matched filter computes as output the following decision statistics

$$r_i = \sqrt{\frac{N_s}{E_s}} \sum_{j=0}^{N_s-1} \int_{jT_f}^{(j+1)T_f} r(t)\psi_i^{(1)}(t - jT_f - c_j^{(1)}T_c - \tau^{(1)})dt. \quad (6)$$

We can write $r_i = D_i + I_i + N_i$ where D_i contains the signal to be detected, I_i is the multiple-access interference (MAI) and N_i is the noise.[1] The variance of N_i, $\sigma_{N_i}^2$, is obtained as

$$\sigma_{N_i}^2 = \frac{N_s}{E_s} \sum_{j=0}^{N_s-1} \sum_{k=0}^{N_s-1} \delta_{kj}\sigma_n^2\Gamma(0) = N_s^2\Gamma(0)\sigma_n^2/E_s, \quad (7)$$

where $\sigma_n^2 = N_0/2$ and $\Gamma(\Delta)$ is the correlation of the basic pulse for a lag Δ. Similarly, $D_i = N_s\Gamma(0)$, and the MAI part is expressed as

$$I_i = \sum_{k=2}^{K} \sum_{j=0}^{N_s-1} \Gamma(\Delta_j^{(k)}), \quad (8)$$

where $\Delta_j^{(k)} = (c_j^{(1)} - c_j^{(k)})T_c + (d_j^{(k)} - d_j^{(1)}) + (\tau^{(k)} - \tau^{(1)})$ is a random time lag between users 1 and k in the j-th hop frame. We assume that all the time hopping (c_j's) are random, and hence, the monocycle time shift $c_j^{(k)}T_c$, and the time delays are i.i.d with uniform distribution over a frame interval. Since the UWB pulse duration $T_p << T_f$, each interfering pulse contributes to only a single correlation operation (i.e., MAI pulses fall within the same UWB frame). Therefore, $\Delta_j^{(k)}$ is uniformly distributed in the interval $[-T_f, T_f]$ [6]. For large values of $K \times N_s$, the probability density function (PDF) of I_i converges to a Gaussian distribution. Without loss of generality, we consider rectangular monocycle pulse, although, Gaussian and Rayleigh pulses fit easily in the analysis as well. Defining a spreading factor $\beta = T_f/T_p$, the variance of I_i can be expressed, with the help of [6], as

$$\sigma_{I_i}^2 = \sum_{k=2}^{K} \sum_{j=0}^{N_s-1} \text{var}[\Gamma(\Delta_j^{(k)})] = (K-1)N_s\left(\frac{1}{3\beta} - \frac{1}{4\beta^2}\right). \quad (9)$$

[1]The decision statistic for user 1, $r_i^{(1)}$, is written as r_i for notational convenience. The same simplified notation is used in the rest of the letter for other parameters related to user 1 as well, such as D_i, N_i and I_i.

It can be shown, by exploiting the orthogonality in the PPM M-ary signal set, that the distribution of the received signal $\mathbf{r}$ is given as

$$\begin{aligned} p(\mathbf{r}|\mathbf{x}_j) &= \left(\frac{1}{2\pi\sigma_{tot}^2}\right)^{M/2} \exp\left[-\frac{(r_j - m_p)^2}{2\sigma_{tot}^2}\right] \\ &\times \prod_{\substack{k=0 \\ k\neq j}}^{M-1} \exp\left[-\frac{(r_k - m_a)^2}{2\sigma_{tot}^2}\right] \\ &= p(r|j) \prod_{\substack{k=0 \\ k\neq j}}^{M-1} p(r|k) \end{aligned} \quad (10)$$

where, $\mathbf{x}_j = [x_0, \cdots, x_j, \cdots, x_{M-1}]$, is the transmitted signal and $\mathbf{r} = [r_0, \cdots, r_j, \cdots, r_{M-1}]$. In the above, $p(r|j)$ and $p(r|k)$ indicate the PDFs of the matched filter output when the desired signal is present and absent, respectively. The second PDF represents noise-only output. The parameter σ_{tot}^2 denotes the total noise variance, m_p and m_a represent slot signal strengths when signal is present and absent, respectively. Using (10) with (2) and $P_j = 1/M$, $0 \leq j \leq M-1$, the uniform input distribution that maximizes R_0 [7], the cut-off

TABLE I
CODE LENGTHS FOR PPM SIGNALING AT RATE 2.0 BITS/SYMBOL

	8-PPM		16-PPM		32-PPM	
Number of users	$P_e \leq 10^{-3}$	$P_e \leq 10^{-6}$	$P_e \leq 10^{-3}$	$P_e \leq 10^{-6}$	$P_e \leq 10^{-3}$	$P_e \leq 10^{-6}$
5	11	21	6	11	4	7
15	14	28	7	14	5	10
20	20	39	10	19	7	14
30	107	214	21	42	14	28

rate R_0 can be expressed as

$$R_0 = \max_{P_j} \left\{ \begin{array}{l} -\log_2 \left[\sum_{l=0}^{M-1} P_l^2 \left(\int_{-\infty}^{\infty} \sqrt{p(r|l)p(r|l)} dr \right)^2 \right. \\ \left. + \sum_{k=0}^{M-1} \sum_{\substack{j=0 \\ j \neq k}}^{M-1} P_j P_k \left(\int_{-\infty}^{\infty} \sqrt{p(r|j)p(r|k)} dr \right)^2 \right] \end{array} \right\}$$
$$= \log_2(M) - \log_2 \left[1 + (M-1) \exp\left(-\frac{[m_p - m_a]^2}{4\sigma_{tot}^2} \right) \right], \quad (11)$$

where $m_p = E\left[r_i|\text{given that the desired signal is present}\right] = N_s\Gamma[0]$, $m_a = E\left[r_i|\text{given that the desired signal is absent}\right] = E[I_i + N_i]$ = 0, and $\sigma_{tot}^2 = \sigma_{N_i}^2 + \sigma_{I_i}^2$. $E[\cdot]$ denotes the mean operator.

Finally, we observe that when the network becomes heavily loaded, the cutoff rate asymptotically approaches zero, but the aggregate rate $R_{0,agg}(= KR_0)$ converges to a nonzero constant; this is confirmed by the asymptotic aggregate cutoff rate expression derived as

$$R_{o,agg} = \lim_{K \longrightarrow \infty} K \log_2 \left[\frac{M}{1 + (M-1)\exp\left(-\frac{[m_p - m_a]^2}{4\sigma_{tot}^2} \right)} \right] \approx \frac{3\beta(M-1)(m_p - m_a)^2}{2N_s M \log_e(2)}. \quad (12)$$

III. NUMERICAL RESULTS

Fig. 1 shows the cutoff rate of the UWB system versus number of users and also shown is the asymptotic behaviour of the aggregate cutoff rate for 2- and 32-PPM. We observe that a certain maximum number of users can be accommodated in order to achieve the maximum R_0 (=$\log_2(M)$). It is further observed that using a higher spreading factor delays the fast drop in the cutoff rate with respect to the number of users.

Fig. 2 shows the cutoff rate as a function of SNR for a fixed number of users of $K = 100$. We observe that a heavily loaded channel results in an extremely low R_0 when the spreading factor is relatively low ($\beta = 50$). For example, 32-PPM could only operate at 1 bit/symbol even at large SNR when $\beta = 50$. Increasing the spreading factor yields a better performance (higher R_0), but this is not always desirable because of the potential synchronization problems. The impact of time hopping ($N_s = 4$, $N_s = 8$, and $N_s = 10$) on R_0 is shown in Fig. 3 for 2-PPM. It is observed that the capacity of TH system ($N_s = 4$) is approximately twice that of a non-hopping system ($N_s = 1$) for $\beta = 50$.

A. Coding Complexity Measure

Consider that a waveform with a rate of 2 bits/symbol is desired. A natural choice would have been 4-PPM, but Fig. 2 indicates that 4-PPM and 8-PPM require about 12 dB and 4 dB, respectively, for $\beta = 500$. Therefore, 8-PPM has a power saving advantage if a suitable coding scheme with certain complexity, in Shannon sense, can be found for a given error rate.

Table I shows the computed block lengths (complexity) required to satisfy a specific probability of error for schemes operating at 2 bits/symbol and 10 dB SNR per user. The results for $\beta = 50$ and $N_s = 1$ are given. We found that significant increase in code lengths is required in the presence of high amounts of MAI for 8-PPM as compared to 16-PPM and 32-PPM. Also, we observed that only a two-fold increase in the block length results in an error rate reduction from 10^{-3} to 10^{-6} for all the M-ary schemes. A further investigation of the actual coding scheme for the system described in this letter can be undertaken. The work on capacity limit achievable by Reed-Solomon (R-S) M-ary PPM in AWGN presented in [7] can complement this effort.

IV. CONCLUSION

This work evaluates the practical capacity, known as the cut-off rate, for TH-PPM adopted for UWB communication over multiple-access channels, without a need for numerical integration or Monte Carlo simulation. We have shown how the cutoff rate can be used in M-ary PPM UWB multiple-access communication systems for determining the system trade-offs. Moreover, cutoff rate evaluated in this letter can be a fast way of gaining insights into the multiuser capacity of TH-PPM UWB systems.

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