SINR Threshold Lower Bound for SINR-based Call Admission Control in CDMA Networks with Imperfect Power Control

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***Abstract*— Signal to interference plus noise ratio (SINR)-based call admission control schemes admit calls as long as SINR is higher than a threshold value ($SINR_{\mathrm{th}}$). Setting a relatively low $SINR_{\mathrm{th}}$ results in more users admitted into the system, and this in turn yields an increased outage probability (P_{out}). Hence, determining the lower bound of $SINR_{\mathrm{th}}$ ($SINR_{\mathrm{th\_lb}}$) is vital to keep P_{out} below a maximum value, $P_{\mathrm{out\_max}}$. In this letter, we derive $SINR_{\mathrm{th\_lb}}$ in the reverse link of CDMA systems with imperfect power control by finding the relationship between P_{out} (due to power control infeasibility and SINR fluctuation) and $SINR_{\mathrm{th}}$. Then, $SINR_{\mathrm{th\_lb}}$ is determined as the lowest $SINR_{\mathrm{th}}$ that keeps P_{out} below a certain $P_{\mathrm{out\_max}}$.**

***Index Terms*— Call admission control, *SINR*-based CAC, imperfect power control.**

I. Introduction

CALL admission control (CAC) is essential in CDMA networks to control the signal quality in terms of the signal to interference plus noise ratio (SINR). SINR-based CAC schemes that use the reverse link SINR as a criterion for admission have been proposed and shown to be effective in controlling the signal quality [1], [2]. In [1], [2] perfect power control is assumed such that received power at the base station (BS) is fixed regardless of the user's location or the channel condition (including shadowing and fast fading). In reality, however, a number of factors including power control (PC) command errors and response delays cause fluctuations in the received signal; it has been shown by simulation and from field measurements that the received SINR at BS is well approximated by the lognormal distribution [3], [4].

In [1], [2] the only constraint imposed on the SINR threshold value ($SINR_{\mathrm{th}}$) is $SINR_{\mathrm{th}} > SINR_{\min}$ where $SINR_{\min}$ is the minimum SINR level for acceptable signal quality. It has been shown in [5] that setting a high $SINR_{\mathrm{th}}$ might inflate the blocking probability, P_{b}, since a high $SINR_{\mathrm{th}}$ results in too many blocked calls. Therefore, an upper bound of $SINR_{\mathrm{th}}, SINR_{\mathrm{th\_ub}}$, has been derived in [5] in order to keep P_{b} less than a maximum value, $\mathrm{P_{b\_max}}$.

Although lowering $SINR_{\mathrm{th}}$ is desirable for better resource utilization as discussed above, this allows more users to be admitted and might render the PC infeasible if the number of admitted users per cell exceeds a certain limit. If PC turns out to be infeasible, outage probability ($P_{\mathrm{out}} = P(SINR < SINR_{\min})$) increases since SINR converges to a smaller level than the target value ($SINR_{\mathrm{trg}}$). Outage might also take place even if PC is feasible due to SINR fluctuation around the target value.

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TABLE I
Summary of different SINR terms.

$SINR_{\min}$	Minimum SINR value for acceptable signal quality
$SINR_{\mathrm{trg}}$	Target SINR of PC scheme
$SINR_{\mathrm{th}}$	SINR threshold value of call admission
$SINR_{\mathrm{th\_ub}}$	Upper bound of $SINR_{\mathrm{th}}$ to keep P_{b} below a maximum value
$SINR_{\mathrm{th\_lb}}$	Lower bound of $SINR_{\mathrm{th}}$ to keep P_{out} below a maximum value

*Whenever SINR is expressed in dB, a dB superscript is used.

A lower bound of $SINR_{\mathrm{th}}(SINR_{\mathrm{th\_lb}})$ that keeps the outage probability limited in the reverse link of a single-class of CDMA systems is derived in this letter. Table I lists the different SINR terms used throughout this paper. It is worth noting that CAC is performed based on the $SINR_{\mathrm{th}}$ value which is determined based on the P_{b} and P_{out} constraints.

The derivation of $SINR_{\mathrm{th\_lb}}$ is presented in Section II. Then, the results are shown in Section III. Finally, conclusions are discussed in Section IV.

II. Lower Bound of SINR Threshold

In order to find $SINR_{\mathrm{th\_lb}}$, we take the following approach. First, we determine the relationship between P_{out} and $SINR_{\mathrm{th}}$. Then, we find $SINR_{\mathrm{th\_lb}}$ as the lowest value of $SINR_{\mathrm{th}}$ that keeps P_{out} below the maximum acceptable value ($P_{\mathrm{out\_max}}$). P_{out} can be expressed as

$$P_{\mathrm{out}} = \sum_{N=1}^{\infty} P_{\mathrm{out|N}} P\left(N\right) \tag{1}$$

where $P_{\mathrm{out|N}} = P\left(SINR < SINR_{\min} \left|N\right.\right)$ is the outage probability given that the number of active users per cell is N and $P(N)$ is the probability that there are N active users per cell. When PC is imperfect, SINR can be represented as a lognormally-distributed random variable as discussed above.

Hence, $P_{\text{out}|\text{N}}$ can be expressed as

$$\begin{aligned} P_{\text{out}|\text{N}} &= P\left(SINR < SINR_{\min} \,|N\right) \\ &= P\left(SINR^{\text{dB}} < SINR^{\text{dB}}_{\min} \,|N\right) \\ &= 1 - Q\left(\frac{SINR^{\text{dB}}_{\min} - m}{\sigma}\right) \end{aligned} \tag{2}$$

where m and σ are the mean and the standard deviation of $SINR^{\text{dB}}$, respectively.

The condition of PC feasibility is that N is less than a maximum value $N_{\max}$ [6]. Therefore, as long as $N < N_{\max}$, PC is feasible, and hence, m converges to $SINR^{\text{dB}}_{\text{trg}}$ as shown in Fig. 1.a. On the other hand, at large values of N ($N > N_{\max}$), PC becomes infeasible and m converges to a smaller value than $SINR^{\text{dB}}_{\text{trg}}$ as depicted in Fig. 1.b. Using similar analysis to that given in [6], [7], it can be shown that $N_{\max}$ is related to the target SINR ($SINR_{\text{trg}}$) as follows

$$N_{\max} = \left\lfloor \left(\frac{1}{SINR_{\text{trg}}} - \frac{\eta_o W}{S}\right) \frac{1}{(1+f)} + 1 \right\rfloor \tag{3}$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x, η_o is the noise power spectral density, S is the target balanced received power level, W is the spreading bandwidth, and f is the ratio of the inter-cell interference to the intra-cell interference (no cap on the transmit power is assumed in this letter). Therefore, it can be shown that m can be expressed as

$$m = \begin{cases} SINR^{\text{dB}}_{\text{trg}} & N \le N_{\max} \\ 10\log_{10}\left(\frac{1}{(N-1)(1+f)+(\eta_o W/S)}\right) & N > N_{\max} \end{cases} \tag{4}$$

It is worth noting that σ is assumed to be constant and to depend only on the delay and the PC errors. It is apparent from (1)-(4) that $P_{\text{out}|\text{N}}$ has no dependence on N as long as PC is feasible. When PC becomes infeasible, $P_{\text{out}|\text{N}}$ increases monotonically with N as shown in Fig. 2.

We model N by a Poisson distribution as customary in the literature; then $P(N)$ in (1) can be given by

$$P(N) = \frac{\alpha^N}{N!}\exp(-\alpha), \tag{5}$$

where α is the mean value of the admitted traffic intensity in Erlang per cell, which in turn, is given by

$$\alpha = \Lambda(1 - P_{\text{b}}). \tag{6}$$

In (6), Λ is the average arriving traffic intensity in Erlang per cell, and P_{b} is a function of $SINR^{\text{dB}}_{\text{th}}$ as follows [5]:

$$P_{\text{b}} = 1 - Q\left(\frac{SINR^{\text{dB}}_{\text{th}} - m}{\sigma}\right). \tag{7}$$

From (1), (2), and (5)-(7), P_{out} can be expressed as a function of $SINR^{\text{dB}}_{\text{th}}$ as follows

$$\begin{aligned} P_{\text{out}} = \sum_{N=1}^{\infty}\Bigg\{&\left(1-Q\left(\frac{SINR^{\text{dB}}_{\min}-m}{\sigma}\right)\right) \\ &\times \frac{\left(\Lambda Q\left(\frac{SINR^{\text{dB}}_{\text{th}}-m}{\sigma}\right)\right)^N}{N!}\exp\left(-\Lambda Q\left(\frac{SINR^{\text{dB}}_{\text{th}}-m}{\sigma}\right)\right)\Bigg\} \end{aligned} \tag{8}$$

Now, $SINR^{\text{dB}}_{\text{th\_lb}}$ can be obtained by substituting $P_{\text{out\_max}}$ for P_{out} in (8) and then by solving for $SINR^{\text{dB}}_{\text{th\_lb}}$. Since a closed form expression for $SINR^{\text{dB}}_{\text{th\_lb}}$ cannot be obtained, $SINR^{\text{dB}}_{\text{th\_lb}}$ has to be determined by solving (8) numerically.

An approximate closed form expression for $SINR^{\text{dB}}_{\text{th\_lb}}$ can be obtained by approximating N as a Gaussian random variable and $P_{\text{out}|\text{N}}$ as a step function as shown in Fig. 2; in this case, m is made equal to $10\log_{10}\left(\frac{1}{(N_{\text{tr}}-1)(1+f)+(\eta_o W/S)}\right)$ for $N > N_{\max}$ where N_{tr} is the transition value of the step function as shown in Fig. 2. Thus, (8) can be modified to

$$\begin{aligned} P_{\text{out}} &= \mu P(N \le N_{\text{tr}}) + P(N > N_{\text{tr}}) \\ &= \mu + (1-\mu) Q\left(\frac{N_{\text{tr}} - E(N)}{\sqrt{Var(N)}}\right) \end{aligned} \tag{9}$$

where $\mu = P_{\text{out}|\text{N}<\text{N}_{\max}}$, and $E(N)$ and $Var(N)$ are the mean and variance of N, respectively. Since N is originally modeled as a Poisson random variable, it is valid to assume that both $E(N)$ and $Var(N)$ are equal to α. From (6) & (7), $E(N)$ and $Var(N)$ can be expressed as

$$E(N) = Var(N) = \alpha = \Lambda(1 - P_b) = \Lambda Q\left(\frac{SINR^{\text{dB}}_{\text{th}} - m}{\sigma}\right) \tag{10}$$

Let us define Ψ as the inverse Q-function such that

$$y = Q(x) \Leftrightarrow x = \Psi(y) \tag{11}$$

By rewriting (9) using (10) and (11), it can be shown that

$$\Psi\left(\frac{P_{\text{out}} - \mu}{1-\mu}\right) = \frac{N_{\text{tr}} - \Lambda Q\left(\frac{SINR^{\text{dB}}_{\text{th}} - m}{\sigma}\right)}{\sqrt{\Lambda Q\left(\frac{SINR^{\text{dB}}_{\text{th}} - m}{\sigma}\right)}} \tag{12}$$

Then, solving for Q yields

$$Q\left(\frac{SINR^{\text{dB}}_{\text{th}} - m}{\sigma}\right) = \frac{\Psi^2\left(\frac{P_{\text{out}}-\mu}{1-\mu}\right) + 2N_{\text{tr}} \pm \sqrt{\left(\Psi^2\left(\frac{P_{\text{out}}-\mu}{1-\mu}\right) + 2N_{\text{tr}}\right)^2 - 4N_{\text{tr}}^2}}{2\Lambda} \tag{13}$$

The solution with the positive sign before the square root in (13) is not considered since a lower bound is sought here. Finally, the lower bound $SINR^{\text{dB}}_{\text{th\_lb}}$ can be obtained by equating P_{out} to $P_{\text{out\_max}}$ in (13) and solving for $SINR^{\text{dB}}_{\text{th}}$

$$\begin{aligned} SINR^{\text{dB}}_{\text{th\_lb}} = m + \\ \sigma\Psi\Bigg(&\frac{\Psi^2\left(\frac{P_{\text{out\_max}}-\mu}{1-\mu}\right) + 2N_{\text{tr}}}{2\Lambda} \\ &- \frac{\sqrt{\left(\Psi^2\left(\frac{P_{\text{out\_max}}-\mu}{1-\mu}\right) + 2N_{\text{tr}}\right)^2 - 4N_{\text{tr}}^2}}{2\Lambda}\Bigg) \end{aligned} \tag{14}$$

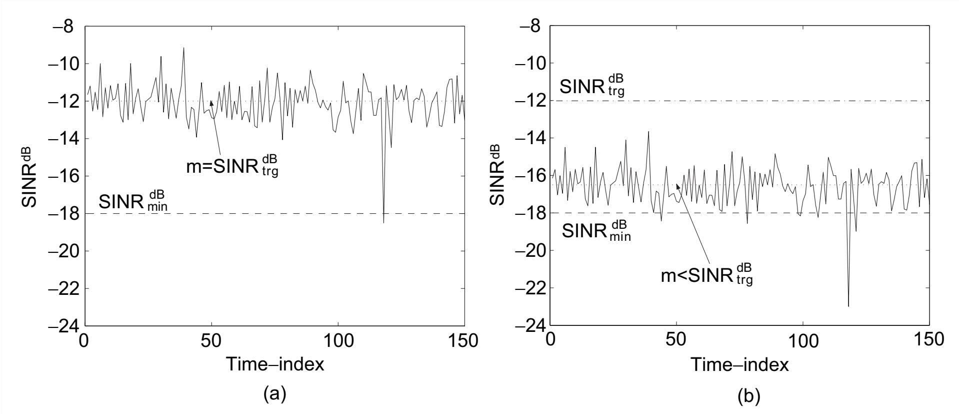

Fig. 1. SINR fluctuation around m for a) feasible PC and b) infeasible PC.

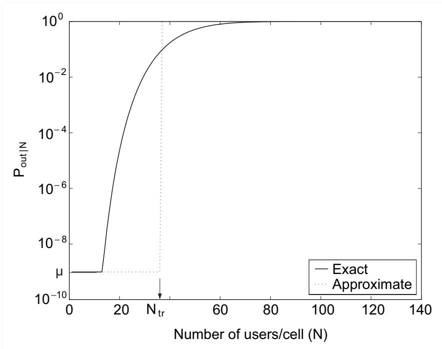

Fig. 2. $P_{\mathrm{out|N}}$ dependence on N.

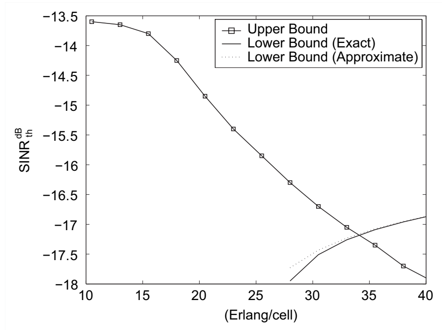

Fig. 3. Lower and upper bounds of $SINR_{\mathrm{th}}^{\mathrm{dB}}$.

III. Results

Fig. 3 shows $SINR_{\mathrm{th\_lb}}^{\mathrm{dB}}$ (exact from (8) and approximate from (14)) versus arrival traffic intensity in Erlang per cell (Λ) at $P_{\mathrm{b\_max}}$ = 5%, $P_{\mathrm{out\_max}}$ = 1%, $SINR_{\mathrm{trg}}^{\mathrm{dB}}$ = -12 dB, $SINR_{\mathrm{min}}^{\mathrm{dB}}$ = -18 dB, N_{tr} = 36, $\eta_o W/S$ = 0.01, f = 0.3 and σ = 1 dB. The value of N_{tr} is chosen to minimize the error of the approximate solution with respect to the exact one, while the value of the term $\eta_o W/S$ is chosen to represent an interference-limited system. Fig. 3 also shows the upper bound ($SINR_{\mathrm{th\_ub}}^{\mathrm{dB}}$) derived in [8] by modifying that given in [5]. It is evident that the approximate lower bound is very close to the exact lower bound. At light traffic conditions ($\Lambda < 27$ Erlang/cell), there is no lower bound as P_{out} is very small (much less than $P_{\mathrm{out\_max}}$) at these relatively low traffic values. At higher traffic values ($\Lambda > 27$ Erlang/cell), $SINR_{\mathrm{th\_lb}}^{\mathrm{dB}}$ increases monotonically with Λ. However, when Λ exceeds 34 Erlang/cell, $SINR_{\mathrm{th\_lb}}^{\mathrm{dB}}$ turns out to be higher than $SINR_{\mathrm{th\_ub}}^{\mathrm{dB}}$ as both requirements ($P_{\mathrm{b}} < P_{\mathrm{b\_max}}$ & $P_{\mathrm{out}} < P_{\mathrm{out\_max}}$) cannot be achieved simultaneously at these high traffic values. Hence, at higher traffic values, at least one of these two constraints has to be relaxed. For Λ between 27 and 34 Erlang/cell, it can be seen that both requirements can be met and $SINR_{\mathrm{th}}^{\mathrm{dB}}$ is bounded by $SINR_{\mathrm{th\_lb}}^{\mathrm{dB}}$ and $SINR_{\mathrm{th\_ub}}^{\mathrm{dB}}$ ($SINR_{\mathrm{th\_lb}}^{\mathrm{dB}} < SINR_{\mathrm{th}}^{\mathrm{dB}} < SINR_{\mathrm{th\_ub}}^{\mathrm{dB}}$).

IV. Conclusions

A lower bound of $SINR_{\mathrm{th}}$ has been derived in order to limit P_{out}. $SINR_{\mathrm{th\_lb}}$ is obtained by determining the relationship between P_{out} and $SINR_{\mathrm{th}}$ and then finding the lowest value of $SINR_{\mathrm{th}}$ that keeps P_{out} below $P_{\mathrm{out\_max}}$. It is assumed that outage occurs due to PC infeasibility and $SINR$ fluctuation. It has been shown that $SINR_{\mathrm{th\_lb}}$ is important for choosing the appropriate value of $SINR_{\mathrm{th}}$. Also, it has been demonstrated that $SINR_{\mathrm{th\_lb}}$ gives an indication of the system capacity calculation taking the quality of service (QoS) constraints into account. Finding lower bounds of $SINR_{\mathrm{th}}$ in multi-class CDMA systems is considered for future work.

References

[1] Z. Liu and M. El Zarki, "SIR-based call admission control for DS-CDMA cellular systems," *IEEE J. Sel. Areas Commun.*, vol. 12, pp. 638-644, May 1994.

[2] I.-M. Kim, B.-C. Shin, and D. Lee, "SIR-based call admission control by intercell interference prediction for DS-CDMA systems," *IEEE Commun. Lett.*, vol. 4, pp. 29-31, Jan. 2000.

[3] A. Viterbi and A. Viterbi, "Erlang capacity of a power controlled CDMA system," *IEEE J. Sel. Areas Commun.*, vol. 11, pp. 892-900, Aug. 1993.

[4] S. Ariyavisitakul and L. Chang, "Signal and interference statistics of a CDMA system with feedback power control," *IEEE Trans. Commun.*, vol. 41, pp. 1626-1634, Nov. 1993.

[5] D. Kim, "On upper bounds of SIR-based call admission threshold in power-controlled DS-CDMA mobile systems," *IEEE Commun. Lett.*, vol. 6, pp. 13-15, Jan. 2002.

[6] A. Sampath, P. Kumar, and J. Holtzman, "Power control and resource management for a multimedia CDMA wireless system," *IEEE International Symposium on Personal, Indoor, and Mobile Communications*(*PIMRC*), 1995, pp. 21-25.

[7] C. Hemsi and P. Jeszensky, "Comparison of alternatives for capacity increase in multiple-rate dual-class DS/CDMA systems," *Proc. IEEE Personal, Indoor, and Mobile Radio Conference* (*PIMRC*), 2000, pp. 824-828.

[8] M. Ahmed and H. Yanikomeroglu, "Modified upper bound of SINR threshold in SINR-based call admission control with imperfect power control in CDMA systems," to be submitted.