Power Allocation Optimization in Selective DF Relaying With Different Modulation Levels in the Presence of Imperfect Channel Estimations

Hamza Umit Sokun, Akram Bin Sediq, Salama Ikki, and Halim Yanikomeroglu

***Abstract*—In this paper, we focus on the systematic optimization of power allocation in a selective relaying scheme with imperfect channel estimation in a multi-relay decode-and-forward cooperative network in which the relays have the flexibility to employ a modulation level different than that of the source, whenever there is a merit. First, we obtain an asymptotic expression for the error probability in the presence of Gaussian imperfect channel estimations. Then, by using this expression, we derive the power allocation scheme that minimizes the asymptotic error probability. We show that the power allocation problem can suitably be presented as a geometric programming problem that is solved by an efficient convex programming technique. Finally, the derivations are confirmed through Monte Carlo simulations.**

***Index Terms*—Cooperative communication, channel estimation error, selective relaying, selection combining.**

I. Introduction

RELAY-BASED cooperative communication have attracted substantial attention in the last 15 years towards enabling reliable high speed data transmission. Fixed relaying has already been adopted in IEEE 802.16m and 3GPP LTE-Advanced (release 10) standards to enhance coverage and service quality for cell edge users. Cooperative relaying with terminals is expected to be a native technology in 5G standards.

Although there already exists a substantial body of literature on cooperative diversity, results are mostly restricted to the case when source and relays employ the same modulation levels. There are only a limited number of publications that have examined the employment of different modulation levels by the source and the relay terminals [1]–[7] in decode-and-forward (DF) relaying networks (the same modulation level at the source and relays is a special case of this more generic scenario). Note that such employment is not applicable for amplify-and-forward (AF) relaying networks since in AF relaying, the relay only amplifies the received signal before retransmitting it. Some of these studies have focused on the techniques in the form of post-detection combining, such as Chase Combining and Packet Selection Combining [1], [2]. On the other hand, some of them have dealt with pre-detection combining due to simplicity. However, the pre-detection combining of signals with different modulation levels is a rather new problem in the literature of cooperative networks. In [4]–[6], the pre-detection combining of signals with different modulation levels by considering selection combining in a single-relay scenario is studied. The link selection in [4]–[6] is performed based on minimizing the bit-error-rate (BER). Due to different error resilience of the signals with different modulation levels, selecting the signal with the highest SNR won't be optimal. The theoretical analysis in [4]–[6] is further extended to the multi-relay selective DF networks in [7]. However, a common assumption in the aforementioned works is the availability of perfect channel state information (CSI) at the receiver.

In this paper, we build on the recent work published in [7], and extend the analysis to include the impact of imperfect CSI and the power allocation (PA) optimization. We discuss the PA problem by considering the minimization of the asymptotic BER expression as the optimization criterion. We show that the corresponding optimization problem can be formulated as a geometric programming (GP) problem, which can be solved efficiently using convex optimization techniques [8]. Finally, we analyse the impact of imperfect CSI on the performance.

II. System and Channel Models

A multi-relay system implementing selective DF relaying is considered. The system consists of a source (S), a destination (D), and a number of cooperating nodes $(R_i, i = 1, \ldots, L)$. All devices are equipped with single antennas, and they operate in the half-duplex mode. The channel coefficients of the $S \to D$ link, the $S \to R_i$ link and the $R_i \to D$ link are denoted by h, f_i, and g_i, respectively, which are constant over a transmission block, and independent from block to block. All links are modelled as independent non-identically distributed Rayleigh fading channel, i.e., they are zero-mean complex Gaussian random variables with variance Ω_k, where $k \in \{h, f_i, g_i\}$. It is also assumed that the additive white Gaussian noise (AWGN) terms of all links have zero-mean and equal variance (N_0).

Similar to [4], [6], [7], the modulation levels at all nodes are assumed to be known and BER-based selection combining scheme in which biased (weighted) SNR values are utilized for link selection is considered. In the first phase, the source broadcasts an N-bit packet to the destination and the relays by using M_S-QAM. Then, each relay decodes the received packet from the source and performs a cyclic redundancy codes (CRC) check. Only those relays that correctly detect the packet are allowed to forward that packet; these relays form the decoding set $(\mathcal{C})$. In case of the set of $\mathcal{C}$ is empty, the destination will only utilize the transmission from the source. In the second phase, the destination makes a decision based on the biased SNRs of the relays in $\mathcal{C}$, $\{M_{R_i} - \text{QAM} | i \in \mathcal{C}\}$, and the source to the destination in a way to minimize BER. If the link from the source is selected, then the destination relies on the source's packet and the relays do not forward the packet. Otherwise, the

Manuscript received September 5, 2014; revised January 31, 2015; accepted February 13, 2015. Date of publication March 4, 2015; date of current version May 7, 2015. This work is supported in part by Huawei Canada Co., Ltd., and in part by the Ontario Ministry of Economic Development and Innovation's ORF-RE (Ontario Research Fund-Research Excellence) Program. The associate editor coordinating the review of this paper and approving it for publication was D. W. K. Ng.

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Digital Object Identifier 10.1109/LCOMM.2015.2408596

selected relay transmits the packet and the destination performs detection based on the selected link only.

To detect the transmitted bits, the channel f_i must be estimated at the i^{th} relay. The channel estimation error is defined as $e_{f_i} = f_i - \tilde{f}_i$, where e_{f_i} is assumed to be a complex Gaussian random variable with zero-mean and variance $\sigma^2_{e_{f_i}}$. The estimation error e_{f_i} and the channel estimate $\tilde{f}_i$ are assumed to be mutually independent; this is a valid assumption for minimum mean square error estimation in which the estimate and error are orthogonal. Thus, the channel estimate $\tilde{f}_i$ is also Gaussian distributed with variance $\Omega_{\tilde{f}_i} = \Omega_{f_i} - \sigma^2_{e_{f_i}}$. Similarly, the channel estimates $\tilde{h}$ and $\tilde{g}_i$ are Gaussian distributed with variances $\Omega_{\tilde{h}} = \Omega_h - \sigma^2_{e_h}$ and $\Omega_{\tilde{g}_i} = \Omega_{g_i} - \sigma^2_{e_{g_i}}$, respectively. Note that $\sigma^2_{e_h}$, $\sigma^2_{e_{f_i}}$, and $\sigma^2_{e_{g_i}}$ are parameters that capture the quality of channel estimation and can be appropriately chosen depending on the channel dynamics and estimation schemes.

Let the received signal of j^{th} symbol of $x(j)$ be $y_{SR_i}(j)$ at the i-th relay within the first phase. To decode the received symbol, the relay estimates the corresponding the $S \to R_i$ channel and then decodes the received signal using a maximum likelihood decoder. The received signal at the relay can be written as

$$\begin{aligned}\tilde{x}(j) &= \left(\sqrt{P_S}\tilde{f}_i^*/N_0\right)\left(\sqrt{P_S}(\tilde{f}_i + e_{f_i})x(j) + n_{SR_i}(j)\right)\\ &= P_S|\tilde{f}_i|^2 x(j)/N_0 + \sqrt{P_S}\tilde{f}_i^*\Big(\sqrt{P_S}e_{f_i}x(j)\\ &\qquad + n_{SR_i}(j)\Big)/N_0, \qquad (1)\end{aligned}$$

where $\{\cdot\}^*$ and $\tilde{x}$ denote the complex conjugate and the soft decision of the variable x, respectively; also, P_S is the transmitting power at the source. Following the procedure presented in [9] and [10], the received signal can be decomposed into the signal part given as $P_S|\tilde{f}_i|^2/N_0$ and the noise part given as $\sqrt{P_S}\tilde{f}_i^*(\sqrt{P_S}e_{f_i} + n_{SR_i}(j))/N_0$. Therefore, the instantaneous effective SNR (i.e., incorporating the effect of the channel estimation) at the i-th relay within the first phase is given as $\gamma^{\text{eff}}_{SR_i} = \frac{P_S|\tilde{f}_i|^2/N_0}{P_S\sigma^2_{e_{f_i}}/N_0+1} = \frac{\gamma_{SR_i}}{P_S\sigma^2_{e_{f_i}}/N_0+1}$, where γ_{SR_i} is the estimated received SNR. The average effective SNR at the i-th relay can be further written as $\bar{\gamma}^{\text{eff}}_{SR_i} = \mathbb{E}(\gamma^{\text{eff}}_{SR_i}) = \frac{P_S\Omega_{\tilde{f}_i}}{P_S\sigma^2_{e_{f_i}}+N_0} = \frac{P_S(\Omega_{f_i}-\sigma^2_{e_{f_i}})}{P_S\sigma^2_{e_{f_i}}+N_0}$, where $\mathbb{E}(\cdot)$ denotes the expectation operator. Note that the average estimated SNR at the i-th relay is $\bar{\gamma}_{SR_i} = P_S\Omega_{\tilde{f}_i}/N_0$.

Similarly, the average effective SNR between the source and the destination is given by $\bar{\gamma}^{\text{eff}}_{SD} = \frac{P_S(\Omega_h-\sigma^2_{e_h})}{P_S\sigma^2_{e_h}+N_0}$, and the average effective SNR between the i-th relay and the destination within the second phase can be written as $\bar{\gamma}^{\text{eff}}_{R_iD} = \frac{P_{R_i}(\Omega_{g_i}-\sigma^2_{e_{g_i}})}{P_{R_i}\sigma^2_{e_{g_i}}+N_0}$, where P_{R_i} is the transmitting power at the i^{th} relay. Note that $\gamma^{\text{eff}}_{SR_i}$, γ^{eff}_{SD} and $\gamma^{\text{eff}}_{R_iD}$ are independent and exponentially distributed with mean, respectively, given as $\bar{\gamma}^{\text{eff}}_{SR_i}$, $\bar{\gamma}^{\text{eff}}_{SD}$, and $\bar{\gamma}^{\text{eff}}_{R_iD}$.

III. Power Allocation

In this section, we will first show the impact of imperfect CSI and then, discuss how performance degradation due to imperfect CSI can be mitigated by PA optimization.

A. Performance Analysis of Asymptotic BER With Channel Estimation Error

Following the steps towards developing [7, eq. 27], the asymptotic BER performance in the presence of imperfect estimation can be derived as (2), shown at the bottom of the page. The definition of variables used in (2) is given in Table I. However, differently from [7], the packet error rate (PER) in block fading is obtained following a similar procedure adopted in [11]; using binomial theorem and Prony approximation. The asymptotic PER is shown at the bottom of the page.

To make the analysis tractable, we consider the asymptotic BER expression for the two-relay scenario and assume that

$$\begin{aligned}\mathsf{P(e)} \overset{\mathsf{SNR}\to\infty}{\approx} &\left(\prod_{i=1}^{L}\frac{T\left(P_S\sigma^2_{e_{f_i}}+N_0\right)}{P_S\Omega_{\tilde{f}_i}}\right)\frac{c_{M_S}\left(P_S\sigma^2_{e_h}+N_0\right)}{4d^2_{M_S}P_S\Omega_{\tilde{h}}} + \sum_{r=1}^{L}\sum_{m=1}^{|P_r(\mathcal{S}_{all})|}\left(\prod_{t_o\notin P_{r,m}(\mathcal{S}_{all})}\frac{T\left(P_S\sigma^2_{e_{f_{t_o}}}+N_0\right)}{P_S\Omega_{\tilde{f}_{t_o}}}\right)\\ &\times\left[\left[\prod_{i=1}^{|P_{r,m}(\mathcal{S}_{all})|}\frac{\rho_i^{-1}\left(P_{R_i}\sigma^2_{e_{g_i}}+N_0\right)}{P_{R_i}\Omega_{\tilde{g}_i}}\right]\frac{\left(P_S\sigma^2_{e_h}+N_0\right)c_{M_s}\Gamma\left(|P_{r,m}(\mathcal{S}_{all})|+1.5\right)}{2\sqrt{\pi}P_S\Omega_{\tilde{h}}\left(1+|P_{r,m}(\mathcal{S}_{all})|\right)\left(d^2_{M_s}\right)^{|P_{r,m}(\mathcal{S}_{all})|+1}}\right.\\ &\left.+\sum_{i=1}^{|P_{r,m}(\mathcal{S}_{all})|}\left[\prod_{\substack{j=1\\ j\neq i}}^{|P_{r,m}(\mathcal{S}_{all})|}\frac{\beta_{ij}\left(P_{R_j}\sigma^2_{e_{g_j}}+N_0\right)}{P_{R_j}\Omega_{\tilde{g}_j}}\right]\frac{\left(P_S\sigma^2_{e_h}+N_0\right)\left(P_{R_i}\sigma^2_{e_{g_i}}+N_0\right)\rho_i c_{M_i}\Gamma\left(|P_{r,m}(\mathcal{S}_{all})|+1.5\right)}{2\sqrt{\pi}P_S\Omega_{\tilde{h}}P_{R_i}\Omega_{\tilde{g}_i}\left(1+|P_{r,m}(\mathcal{S}_{all})|\right)\left(d^2_{M_i}\right)^{|P_{r,m}(\mathcal{S}_{all})|+1}}\right] \qquad (2)\end{aligned}$$

$$\mathsf{PER}_{SR_i} \overset{\mathsf{SNR}\to\infty}{\approx} \frac{1}{\bar{\gamma}^{\text{eff}}_{SR_i}}\sum_{w=1}^{N/log_2(M_0)}\sum_{z=0}^{w}\binom{N/log_2(M_0)}{w}\binom{w}{z}\frac{(-1)^{(w+1)}\left(log_2(M_0)c_{M_S}\right)^w A_1^{w-z}A_2^z}{2d^2_{M_S}\left(a_1(w-z)+a_2 z\right)} \triangleq \frac{T}{\bar{\gamma}^{\text{eff}}_{SR_i}},$$

where $A_1 = 0.204$, $A_2 = 0.105$, $a_1 = 1.504$ and $a_2 = 1.024$ (3)

TABLE I
NOTATIONS DEFINITION

- $|P_r(S_{all})|$ represents the cardinality of $P_r(S_{all})$.
- $P_r(S_{all})$ is the r-th element power set of S_{all}.
- $P_{r,m}(S_{all})$ is the m-th element of $P_r(S_{all})$, i.e., $P_r(S_{all}) = \{P_{r,1}(\mathcal{S}_{all}), P_{r,2}(\mathcal{S}_{all}), ..., P_{r,|P_r(\mathcal{S}_{all})|}(\mathcal{S}_{all})\}$.
- S_{all} is the set of all relays indexes, i.e., $S_{all} = \{1, \ ..., L\}$.
- $\rho_i = d^2_{M_{R_i}}/d^2_{M_S}$ is a biasing factor between the source and the i^{th} relay.
- $\beta_{ij} = d^2_{M_{R_i}}/d^2_{M_{R_j}}$ is a biasing factor among the i^{th} relay and the j^{th} relay.
- $(c_{M_i}, \ d_{M_i}) = \begin{cases} (1, \ 1), & M_i=2 \\ \left(\frac{2-2/\sqrt{M_i}}{\log_2\sqrt{M_i}}, \sqrt{\frac{3}{2(M_i-1)}}\right), & M_i \geq 4. \end{cases}$
- N is the number of bits in a transmission packet.

$P_S = P_{R_i} = P$, $P/N_0 = \mathsf{SNR}$, and $\sigma^2_{e_{f_i}} = \sigma^2_{e_{g_i}} = \sigma^2_{e_h} = \sigma^2_e$. Then, the asymptotic error probability can be written as

$$\begin{aligned}\mathsf{P(e)} \overset{\mathsf{SNR}\to\infty}{\approx} &\Delta_1 \left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{f}_1}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{f}_2}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{h}}}\right)\\ &+\Delta_2 \left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{f}_1}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{g}_2}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{h}}}\right)\\ &+\Delta_3 \left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{f}_2}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{g}_1}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{h}}}\right)\\ &+\Delta_4 \left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{g}_1}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{g}_2}}\right)\left(\frac{N_0+P\sigma_e^2}{P\Omega_{\tilde{h}}}\right),\end{aligned} \tag{4}$$

where Δ_1, Δ_2, Δ_3 and Δ_4 are given at the next page in (5), shown at the bottom of the page.

If the estimation error is independent from SNR and assumed to be fixed, then, for a considerably high SNR, (4) can be further simplified as

$$\mathsf{P(e)} \overset{\mathsf{SNR}\to\infty}{\approx} \frac{\sigma_e^6\Delta_1}{\Omega_{\tilde{f}_1}\Omega_{\tilde{f}_2}\Omega_{\tilde{h}}} + \frac{\sigma_e^6\Delta_2}{\Omega_{\tilde{f}_1}\Omega_{\tilde{g}_2}\Omega_{\tilde{h}}} + \frac{\sigma_e^6\Delta_3}{\Omega_{\tilde{f}_2}\Omega_{\tilde{g}_1}\Omega_{\tilde{h}}} + \frac{\sigma_e^6\Delta_4}{\Omega_{\tilde{g}_1}\Omega_{\tilde{g}_2}\Omega_{\tilde{h}}}. \tag{6}$$

It can be seen that the expression isn't a function of SNR. Even the channel is assumed as noiseless, the system cannot provide better performance than the error floor given by (6).

B. Performance Enhancement by PA Optimization

This section discusses the PA optimization problem that minimizes the asymptotic BER expression as the optimization criterion. As mentioned before, due to imperfect channel estimation, the asymptotic BER will reach a fixed level, i.e., an error floor. Since optimization doesn't have any effect on this constant term, we only investigate the PA problem for a case where the asymptotic BER is greater than the error floor. The PA problem can be stated as

$$\min_{P_S, P_R} \quad \mathsf{P(e)}_{(\mathsf{SNR}\to\infty)} \tag{7a}$$

$$\text{subject to} \quad P_S + P_R \leq P_T, \tag{7b}$$

$$0 < P_S \leq P_S^{\max}, \tag{7c}$$

$$0 < P_R \leq P_R^{\max}, \tag{7d}$$

where P_T is the total transmit power, and $P_S^{\max}$ and $P_R^{\max}$ are individual power constraints at the source and the selected best relay, respectively. In practice, source and relays may have their own power constraints due to the individual power supplies. However, since the interference is not considered in the system model, each transmitting nodes will tend to transmit with the highest power to minimize the objective function. Therefore, to make the formulation more realistic and prevent excessive power consumption, total power constraint is also included. Each individual power constraint is smaller than the total power constraint whereas the total power constraint is smaller than the sum of the individual separate power constraints, i.e., $P_T < P_S^{\max} + P_R^{\max}$.

For mathematical tractability, we first consider the simplest case of a single relay with perfect CSI, i.e., $\sigma^2_{e_{f_1}} = \sigma^2_{e_{g_1}} = \sigma^2_{e_h} = 0$, by neglecting the last two constraints for the moment, and obtain a closed-form solution for PA. The asymptotic BER expression for one relay case is

$$\mathsf{P(e)} \overset{\mathsf{SNR}\to\infty}{\approx} \Lambda_1/P_S^2 + \Lambda_2/(P_S P_R), \tag{8}$$

where the constant terms (Λ_1 and Λ_2) are defined as $\Lambda_1 = \frac{N_0^2 T c_{M_S}}{4d^2_{M_S}\Omega_h\Omega_{f_1}}$, and $\Lambda_2 = \frac{N_0^2(\Gamma(2.5)/4\sqrt{\pi})(\rho^{-1}c_{M_s}d^4_{M_R}+\rho c_{M_R}d^4_{M_s})}{d^4_{M_R}d^4_{M_s}\Omega_h\Omega_{g_1}}$.

The Lagrangian of the optimization problem is

$$L(\Lambda_1, \Lambda_2, \lambda) = \frac{\Lambda_1}{P_S^2} + \frac{\Lambda_2}{P_S P_R} + \lambda(P_S + P_R - P_T), \tag{9}$$

where $\lambda \geq 0$. Direct computation of the Hessian of the objective function shows that the problem is convex for $P_S > 0$ and $P_R > 0$. Moreover, it can be readily verified that Slater condition is satisfied since there exists a strictly feasible solution. Since the objective and the feasible set are convex and the Slater condition is satisfied, then the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality. By solving the KKT conditions, the optimal power allocated to relay can be expressed as a fraction of the total power $P_R^* = \frac{\sqrt{\Lambda_2^2+8\Lambda_1\Lambda_2}-3\Lambda_2}{4(\Lambda_1-\Lambda_2)}P_T$, and $P_S^* = P_T - P_R^*$.

Then, we consider the PA problem for a multi-relay network with the constraints (7c) and (7d), in the presence of imperfect

$$\Delta_1 = \frac{T^2 c_{M_S}}{4d^2_{M_S}}, \Delta_2 = \frac{\Gamma(2.5)T(\rho_2^{-1}c_{M_S}d^4_{M_{R_2}} + \rho_2 c_{M_{R_2}}d^4_{M_S})}{4\sqrt{\pi}d^4_{M_{R_2}}d^4_{M_S}}, \Delta_3 = \frac{\Gamma(2.5)T\left(\rho_1^{-1}c_{M_S}d^4_{M_{R_1}} + \rho_1 c_{M_{R_1}}d^4_{M_S}\right)}{4\sqrt{\pi}d^4_{M_{R_1}}d^4_{M_S}},$$

$$\Delta_4 = \frac{\Gamma(3.5)\left(\rho_1^{-1}\rho_2^{-1}c_{M_S}d^6_{M_{R_2}}d^6_{M_{R_1}}\right)}{6\sqrt{\pi}d^6_{M_{R_1}}d^6_{M_{R_2}}d^6_{M_S}} + \frac{\Gamma(3.5)\left(\rho_1\beta_{12}c_{M_{R_1}}d^6_{M_S}d^6_{M_{R_2}} + \rho_2\beta_{21}c_{M_{R_2}}d^6_{M_S}d^6_{M_{R_1}}\right)}{6\sqrt{\pi}d^6_{M_{R_1}}d^6_{M_{R_2}}d^6_{M_S}} \tag{5}$$

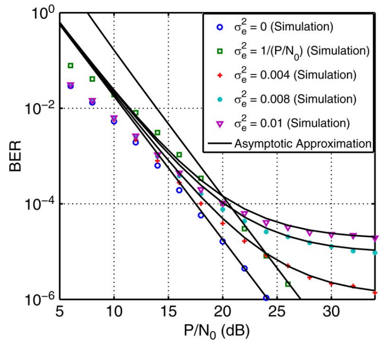

Fig. 1. BER performance for $L = 2$ scenario ($3\bar{\gamma}_{SR_1} = \bar{\gamma}_{SR_2} = 1.5\bar{\gamma}_{R_1D} = 3\bar{\gamma}_{R_2D} = 15\bar{\gamma}_{SD}$ with $16M_S = 4M_{R_1} = M_{R_2} = 64$), assuming $P_S = P_{R_1} = P_{R_2} = P$ and $N = 96$ bits.

CSI, and obtain the following compact form of (2):

$$\mathsf{P(e)} \overset{\mathsf{SNR}\to\infty}{\approx} \sum_{j=1}^{2^L(2^{L+1}-1)} \Lambda_j P_S^{\chi^j} P_R^{\ell_1^j} \ldots P_R^{\ell_L^j}, \tag{10}$$

where $(\ell_1^j, \ldots, \ell_L^j) \in \{-1, 0\}$, $\chi^j \in \{-(L+1), -L, \ldots, 0\}$, and the coefficients Λ_j are independent of P_S and P_R.

The optimization problem for PA can be given as

$$\min_{P_S, P_R} \sum_{j=1}^{2^L(2^{L+1}-1)} \Lambda_j P_S^{\chi^j} P_R^{\ell_1^j} \ldots P_R^{\ell_L^j} \tag{11a}$$

$$\text{subject to} \quad P_S + P_R \leq P_T, \tag{11b}$$

$$0 < P_S \leq P_S^{\max}, \tag{11c}$$

$$0 < P_R \leq P_R^{\max}, \tag{11d}$$

where the objective function and the inequality constraints are posynomial functions. We can show that (11) is a GP type. Hence, it can be transformed into convex optimization problem by a change of variables, and solved efficiently using interior-point method, with a computational complexity of $\mathcal{O}((11 + 2^{L+1}(2^{L+1} - 1))^{3.5})$ [8], [12].

IV. Numerical Analysis and Discussions

Fig. 1 shows the error probability performance considering various estimation error variances. For fixed estimation error variances, we observe that there are error floors at high SNRs. Error floors start to be seen around 2×10^{-5}, 1×10^{-5} and 1.3×10^{-6} for $\sigma_e^2 = 0.01$, $\sigma_e^2 = 0.008$ and $\sigma_e^2 = 0.004$, respectively. On the other hand, when error variances are inversely proportional to SNR, the slope of the performance curves and, hence, the diversity order remain the same for both perfect and imperfect CSI. This is because the diversity order does not decrease as the SNR increases. It is also worth noting that the coding gain is worse than that for the perfect CSI case.

In Fig. 2, we show the effect PA on the system performance using the CVX package with SeDuMi solver [13]. We observe that the proposed power allocation (PPA) algorithm outperforms the equal power allocation (EPA) (i.e., $P_S = P_{R_1} = P_{R_2} = P_T/3$) algorithm for both Scenario 1 and Scenario 2, and the improvement coming with PA optimization can change according to the given scenario. In the existence of estimation error, the performance gap between PPA and EPA algorithms is widened inmedium SNRs and gets smaller in high SNRs. Because PPA algorithm reaches a fixed level in high SNR, it is losing its effect on the BER performance.

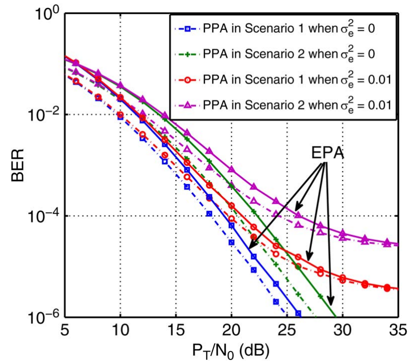

Fig. 2. Simulation results of PPA versus EPA for two different $L = 2$ scenarios ($5\bar{\gamma}_{SR_1} = 7.5\bar{\gamma}_{SR_2} = \bar{\gamma}_{R_1D} = 1.5\bar{\gamma}_{R_2D} = 30\bar{\gamma}_{SD}$ (Scenario 1) and $\bar{\gamma}_{SR_1} = 1.5\bar{\gamma}_{SR_2} = 5\bar{\gamma}_{R_1D} = 7.5\bar{\gamma}_{R_2D} = 30\bar{\gamma}_{SD}$ (Scenario 2) with $16M_S = 4M_{R_1} = M_{R_2} = 64$), assuming $P_S^{\max} = P_R^{\max} = 0.8P_T$ and $N = 96$ bits.

V. Conclusion

In this paper, we have derived a closed-form asymptotic BER expression in the presence of channel estimation error when different modulation levels are employed by source and relays. Based on the derived expression, the power allocation by using geometric programming is performed to enhance the performance. We have shown that the proposed power allocation scheme have superior performance over the equal power allocation scheme. As future works, we plan to investigate the performance of the system along with outdated CSI and two-way relaying.

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