Analytic Modeling of SIR in Cellular Networks With Heterogeneous Traffic

Meisam Mirahsan, Halim Yanikomeroglu, Gamini Senarath, and Ngọc-Dũng Đào

***Abstract*— We derive closed-form expressions for reasonably tight lower bounds and upper bounds for the signal-to-interference ratio (SIR), and consequently, the coverage probabilities, in wireless cellular networks with heterogeneous spatial traffic distribution, especially in the cases where the locations of the users and base stations are correlated. The traffic model used in this letter is tractable, adjustable, and more realistic compared with the existing spatial traffic models in the literature. We show that the SIR follows an exponentially modified Gaussian distribution in heterogeneous scenarios.**

***Index Terms*— Analytic modeling, wireless networks, heterogeneous spatial UE distribution, coverage, SIR.**

I. Introduction

In FUTURE 5G HetNets, the small cell BSs are to be deployed (often by customers) in locations where the users are likely to occur; therefore, most traffic hot-spots are envisioned to be equipped with a base station (BS). As a result, the locations of the user equipment (UEs) and BSs are expected to be correlated; i.e., UEs are presumed to be biased towards BSs, rather than being uniformly distributed. However, the most common approach in the recent literature in the spatial modeling of wireless cellular networks is the use of two independent homogeneous Poisson point processes (PPPs) for the locations of UEs and BSs [1], mainly, for the sake of analytical tractability. Recently, in wireless modeling literature, e.g., [2] and [3], heterogeneous spatial traffic modeling in cellular networks has been investigated. However, to the best of the authors' knowledge, there is no analytical study.

The contribution of this letter is to study the statistical characteristics, i.e., the probability distribution function, of the signal-to-interference ratio (SIR) in the downlink of wireless cellular networks with non-uniform spatial traffic distribution. We use a smoothly adjustable model for modeling heterogeneous spatial traffic distribution in wireless cellular networks. In the proposed model, the correlation between the UEs and BSs is tunable and the distribution of UEs is adjustable between two extreme cases, i.e., the uniform PPP distribution and the totally correlated scenarios (Section II). Consequently, closed-form expressions for the lower-bounds and the upper-bounds of the coverage (and outage) probabilities are drawn (Section III). It is shown via simulation that the bounds are reasonably tight and these bounds can be used as estimators of the network performance metrics (Section IV). Some remarks and future work directions are given in Section V.

II. Proposed Non-Uniform UE Modeling

We study a simplified model of the downlink of a wireless cellular network which is comprised of macro-BSs with omni-directional antennas. The BSs are distributed by a uniform PPP Φ_B with density λ_B. The transmit power P_t at all BSs is fixed and the distance dependent attenuation between a typical UE and a BS b located d_b meters away is denoted by $L_b = 10\alpha \log_{10}(d_b)$, where $\alpha > 2$ is the path-loss exponent. The multi-path fading is not considered in this study. The shadow fading in the channel (in the logarithmic domain), h_b, is modeled by an i.i.d. Gaussian distribution with 0-dB mean and σ-dB standard deviation, i.e., $h_b \sim N(0, \sigma)$. Hence, assuming that the antenna gains are included in the transmission power, the received power at a typical UE from BS b_i is $P_{b_i} = P_t - L_{b_i} + h_{b_i}$. We consider a dense network which is interference-limited; therefore, in this letter, we assume that the noise power is negligible. We denote the BS that serves the UE as BS s; then, the interference and the aggregate SIR can be written as,

$$I_{agg} = \sum_{b_i \in \Phi_B \setminus s} P_{b_i} \tag{1}$$

and

$$\gamma = \frac{P_s}{I_{agg}}, \tag{2}$$

respectively. So, in logarithmic domain, SIR can be stated as

$$\gamma\,[\mathrm{dB}] = P_s[\mathrm{dBm}] - I_{agg}[\mathrm{dBm}]. \tag{3}$$

We assume that each UE is associated with the closest BS. The spectral efficiency (SE) can be stated as $\eta = \log_2(1+\gamma)$ using the Shannon's formula. UE temporal traffic model is assumed to be full-buffer and best-effort.

Starting with a uniform PPP, we move every UE closer to its serving BS. The distances from the UE to its serving BS (BS_1 or s) and to its second closest (dominant interferer) BS (BS_2 or I) are denoted as R_1 (or d_s) and R_2 (or d_I), respectively. After moving the UE closer to BS_1, the new distances are denoted as R'_1 and R'_2. Figure 1 shows a sample scenario. Since we start with the PPP, any move of the already dropped UEs towards their serving BSs would increase the correlation to the BSs and also the clustering effect in distribution. For the

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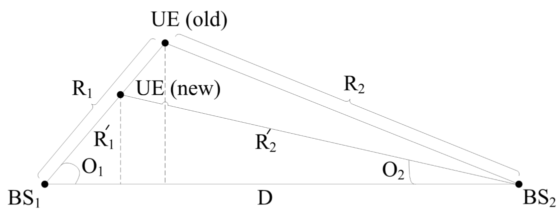


Fig. 1. Starting with a PPP, every UE is moved closer to its serving BS.

reasons that will become clear soon, in our model, we move the UE to the new location so that

$$\frac{R_1'}{R_2'} = \left(\frac{R_1}{R_2}\right)^n, \tag{4}$$

where n determines the correlation between UEs and BSs. Since $R_1 \leqslant R_2$, for $n > 1$ the UE moves towards the serving BS resulting in a more clustered and BS-correlated pattern, and for $0 < n < 1$ the UE moves towards the cell-edges. The $n = 1$ case represents the traditional uniform PPP distribution. In Appendix A, we explain how we can calculate the new location of UEs and move the UEs in order to generate the desired patterns. Figure 2 shows some sample patterns with associated n values in a wireless cellular network.

III. Analytical Modeling

For a homogeneous PPP, the SIR and an upper-bound on it can be expressed as

$$\begin{aligned}\gamma &= P_s - I_{agg} \leqslant \hat{\gamma} = P_s - P_I \\ &= P_t - L_s + h_s - (P_t - L_I + h_I),\end{aligned} \tag{5}$$

where $\hat{\gamma}$ is an upper-bound on the SIR, $h_s \sim N(0,\sigma)$, and $h_I \sim N(0,\sigma)$. In (5) we use the fact that the aggregate interference power I_{agg} is always greater than the power of the dominant interferer alone P_I. So, the SIR can be upper-bounded as

$$\gamma \leqslant \hat{\gamma} = L_s - L_I + h, \tag{6}$$

where $h \sim N(0,\sigma')$ is the summation of two independent Gaussian variables and $\sigma' = \sqrt{2}\sigma$. Using the path-loss equation in Section II, $\hat{\gamma}$ is formulated as

$$\begin{aligned}\hat{\gamma} &= 10\log_{10}(d_s^{-\alpha}) - 10\log_{10}(d_I^{-\alpha}) + h \\ &= 10 \cdot \alpha \cdot \left(\frac{\log(d_I)}{\log(10)} - \frac{\log(d_s)}{\log(10)}\right) + h.\end{aligned} \tag{7}$$

The upper-bound, $\hat{\gamma}$, can be simplified as

$$\begin{aligned}\hat{\gamma} &= \frac{10}{\log(10)} \cdot \alpha \cdot \log\left(\frac{d_I}{d_s}\right) + h \\ &= -\frac{10}{\log(10)} \cdot \alpha \cdot \log\left(\frac{d_s}{d_I}\right) + h.\end{aligned} \tag{8}$$

Substituting d_s/d_I with R we have

$$\hat{\gamma} = -\frac{10}{\log(10)} \cdot \alpha \cdot \log(R) + h. \tag{9}$$

Since $0 \leqslant d_s \leqslant d_I$, $R \in [0,1]$.

Lemma 1: In PPP scenarios (where BSs and UEs are distributed by independent PPPs), R has the following cumulative distribution function (the case of $R \geqslant 1$ is investigated in [4]):

$$F_R(r) = p(R \leqslant r) = r^2. \tag{10}$$

Proof: See **Appendix B.**

Using Lemma 1 and the inverse transform theorem [5], it can be shown that the random variable $U = R^2$ has uniform distribution. Substituting R with $U^{0.5}$, $\hat{\gamma}$ is expressed as

$$\begin{aligned}\hat{\gamma} &= -\frac{10}{\log(10)} \cdot \alpha \cdot \log(U^{0.5}) + h \\ &= -\frac{10}{2\log(10)} \cdot \alpha \cdot \log(U) + h\end{aligned} \tag{11}$$

where U has standard uniform distribution. Further, again from the inverse transform theorem, we know that the negative logarithm function of a standard uniform random variable has exponential distribution with parameter 1. So, substituting $X = -\log(U)$,

$$\hat{\gamma} = \frac{10}{2\log(10)} \cdot \alpha \cdot X + h, \tag{12}$$

where $X \sim \exp(1)$.

Finally, the SIR is estimated as

$$\begin{aligned}\hat{\gamma} &\sim \exp\left(\frac{10}{2\log(10)} \cdot \alpha\right) + N(0,\sigma') \\ &\sim \mathrm{EMG}\left(0, \sigma', \frac{10}{2\log(10)} \cdot \alpha\right),\end{aligned} \tag{13}$$

where EMG is the well-known exponentially modified Gaussian random variable.

Having the statistical distribution of SIR, we can calculate the coverage probability with a threshold θ as

$$P_c(\theta) = p(\hat{\gamma} > \theta) = 1 - F_{\hat{\gamma}}(\theta). \tag{14}$$

Now we can consider a non-uniform UE pattern which is achieved by applying the modeling approach explained in Section II. In this pattern, starting from a PPP pattern, every UE is moved so (4) holds. Therefore, for each UE, the following holds:

$$R' = \frac{R_1'}{R_2'} = R^n. \tag{15}$$

So, following the same procedure as PPP, we can substitute the new R' variable with the n-th power of the old R leading to the following statement:

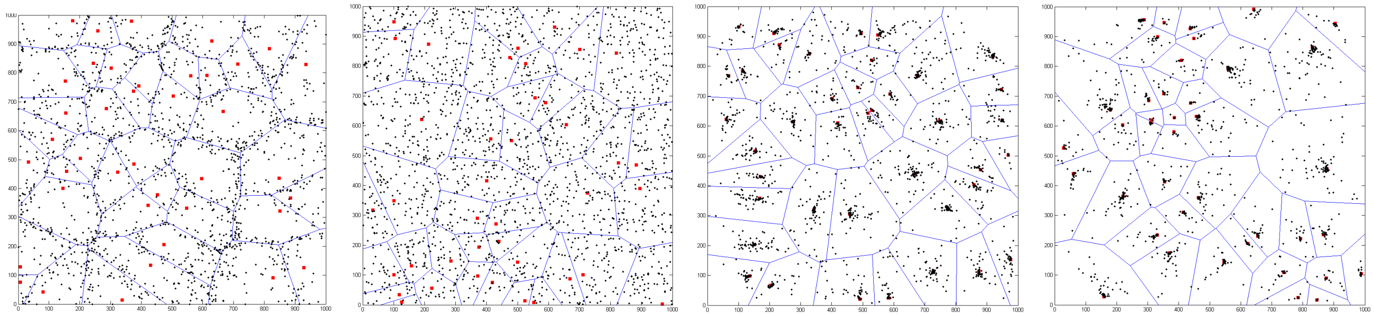
$$\begin{aligned}\hat{\gamma} &= -\frac{10}{\log(10)} \cdot \alpha \cdot \log(R^n) + h \\ &= -\frac{10}{\log(10)} \cdot \alpha \cdot n \cdot \log(R) + h.\end{aligned} \tag{16}$$

Then, substituting R with $U^{0.5}$, we get

$$\hat{\gamma} = -\frac{10}{2\log(10)} \cdot \alpha \cdot n \cdot \log(U) + h, \tag{17}$$

where U has standard uniform distribution. So,

$$\hat{\gamma} = \frac{10}{2\log(10)} \cdot \alpha \cdot n \cdot X + h, \tag{18}$$



(a) (b) (c) (d)

Fig. 2. The red squares denote the BSs and the blue lines represent their associated Voronoi tessellation. Small black dots denote the UEs. (a) $n = 0.5$. (b) $n = 1$ (i.e., PPP). (c) $n = 5$. (d) $n = 10$.

where $X \sim \exp(1)$. Finally, we have

$$\hat{\gamma} \sim \exp(\frac{10}{2\log(10)} \cdot \alpha \cdot n) + N(0, \sigma') \\ \sim \text{EMG}(0, \sigma', \frac{10}{2\log(10)} \cdot \alpha \cdot n). \quad (19)$$

In modeling wireless cellular networks with PPP distribution of BSs, for the sake of tractability, the number of BS points which are distributed in the field is assumed to be infinite. However, the real number of BSs is normally limited. Therefore, the amount of interference in infinite modeling is over-estimated. So,

$$\gamma = P_s - I_{agg} \geqslant \check{\gamma} = P_s - I_\infty, \quad (20)$$

where I_∞ is the interference power when there are infinitely many BSs and $\check{\gamma}$ is a lower-bound on the SIR.

In order to achieve a closed-form expression for $\check{\gamma}$, in this letter, we assume that the aggregate interference in an infinite network can be stated as a coefficient multiplied by the strongest (dominant) interference, i.e.,

$$I_\infty = \beta \cdot P_I, \quad (21)$$

where β is a coefficient and P_I is the power of the dominant interfering BS. The coefficient β can be calculated as the expected value of the ratio of the total interference to the strongest interference. So,

$$\beta = \mathbb{E}\Big(\frac{P_\infty}{P_I}\Big) = \mathbb{E}\Big(\frac{P_I + P_2 + P_3 + \ldots}{P_I}\Big) \\ = \mathbb{E}\Big(\frac{d_I^{-\alpha} + d_2^{-\alpha} + d_3^{-\alpha} + \ldots}{d_I^{-\alpha}}\Big), \quad (22)$$

where P_i is the interference power from the i-th strongest interferer and d_i is the distance from the typical UE to the i-th interferer. Therefore,

$$\beta = \mathbb{E}\Big(1 + \Big(\frac{d_I}{d_2}\Big)^\alpha + \Big(\frac{d_I}{d_3}\Big)^\alpha + \Big(\frac{d_I}{d_4}\Big)^\alpha + \ldots\Big). \quad (23)$$

It is shown in [6] that in PPP scenarios, the second power of the distance to k_{th} neighbor has Erlang distribution with mean $k \cdot \lambda$ where λ is the density of the PPP process. So,

$$\beta = \mathbb{E}\Big(1 + \Big(\frac{2}{3}\Big)^{\alpha/2} + \Big(\frac{2}{4}\Big)^{\alpha/2} + \Big(\frac{2}{5}\Big)^{\alpha/2} + \ldots\Big), \quad (24)$$

which can be expressed as

$$\beta = \mathbb{E}\Big(2^{\alpha/2}\Big(\Big(\frac{1}{2}\Big)^{\alpha/2} + \Big(\frac{1}{3}\Big)^{\alpha/2} + \Big(\frac{1}{4}\Big)^{\alpha/2} + \ldots\Big)\Big). \quad (25)$$

Using the Riemann Zeta Function, β can be written as

$$\beta \approx 2^{\alpha/2}(\zeta(\alpha/2) - 1). \quad (26)$$

Therefore, SIR in homogeneous scenarios can be bounded as follows:

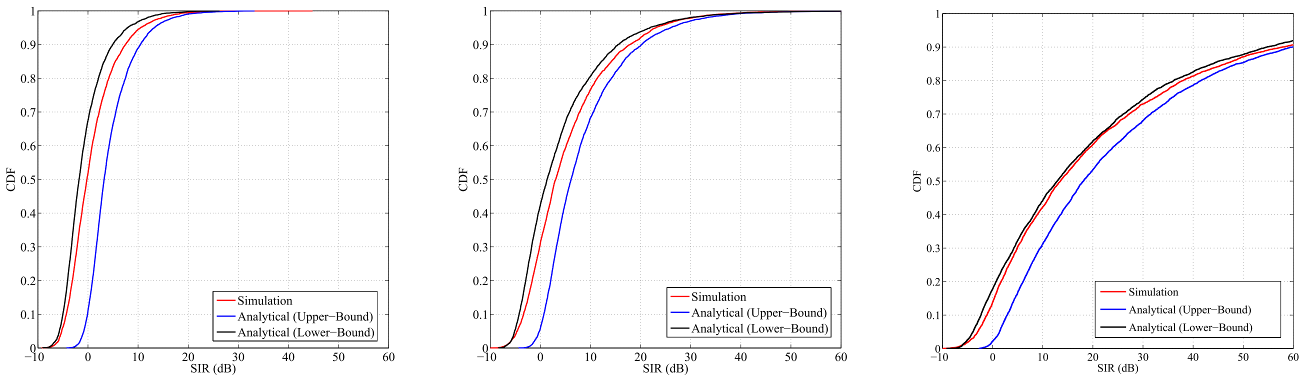
$$\check{\gamma} \sim \text{EMG}(0, \sigma', \frac{10}{2\log(10)} \cdot \alpha) - 10 \cdot \log_{10}(2^{\alpha/2}(\zeta(\alpha/2) - 1)) \\ \leqslant \gamma \leqslant \hat{\gamma} \sim \text{EMG}(0, \sigma', \frac{10}{2\log(10)} \cdot \alpha). \quad (27)$$

Using the same procedure which we followed in deriving the upper-bound in heterogeneous cases, we can derive a lower-bound estimate for SIR in cellular networks with heterogeneous UE distribution. Therefore, the lower-bound and upper-bound on the SIR can be expressed as

$$\check{\gamma} \sim \text{EMG}(0, \sigma', \frac{10}{2\log(10)} \cdot n \cdot \alpha) - 10 \cdot \log_{10}(2^{\alpha/2}(\zeta(\alpha/2) - 1)) \\ \leqslant \gamma \leqslant \hat{\gamma} \sim \text{EMG}(0, \sigma', \frac{10}{2\log(10)} \cdot n \cdot \alpha). \quad (28)$$

IV. Simulation Results

This section presents the simulation setup and results. In our simulations, the spatial density of UEs and BSs is $10^{-3}/\text{m}^2$ and $5.0 \times 10^{-5}/\text{m}^2$, respectively. The total BS transmit power is 30 dBm and path-loss exponent is 4. First, we captured the CDF of SIR in various scenarios with different n values. Figure 3 shows the results. As seen in Fig. 3, as n increases, the distribution of SIR tends to higher values. Moreover, it is illustrated that the lower-bound and the upper-bounds presented in Section III are very tight, i.e., in the worst case scenarios the difference between the analytical bounds and the simulation results is around 5 dB. The next performance metric is the mean SIR over the entire network. Mean UE SIR for different n values is presented in Fig. 4. It is shown that the relation between the n value and the mean SIR is a linear relation, as suggested by (18).



(a) (b) (c)

Fig. 3. The CDF of SIR is illustrated. With increase in n value, the distribution of SIR tends to higher values. (a) $n = 0.5$. (b) $n = 1$ (i.e., PPP). (c) $n = 3$.

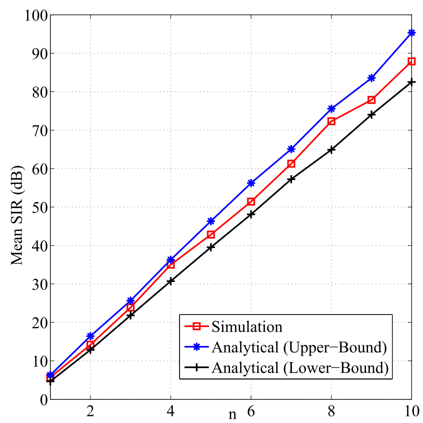


Fig. 4. Mean UE SIR for different n values is shown.

V. Remarks and Future Work

In this letter, we derived closed-form analytical models for SIR and coverage in cellular networks with heterogeneous traffic. This work can be extended in many directions. First, other modeling approaches might be presented which result in more accurate expressions. Moreover, closed-form expressions for other metrics such as UE rates could be derived.

Appendix A
The Movement of the UEs to the New Locations

All parameters in Figure 1 except R_1', R_2' and O_2 are known. We define

$$A = \left(\frac{R_1}{R_2}\right)^n. \tag{29}$$

Then, equation (4) can be stated as

$$R_1' = AR_2'. \tag{30}$$

In Fig. 1, D can be calculated as follows:

$$D = R_1' \cos(O_1) + R_2' \cos(O_2). \tag{31}$$

On the other hand, based on the "Law of Sines in Triangles", we have

$$\frac{\sin(O_1)}{\sin(O_2)} = \frac{R_1'}{R_2'}. \tag{32}$$

So, combining (31) and (32), D is stated as

$$\begin{aligned} D &= R_1' \cos(O_1) + R_2' \sqrt{1 - \sin(O_2)^2} \\ &= R_1' \cos(O_1) + R_2' \sqrt{1 - \left(\frac{R_1'}{R_2'} \sin(O_1)\right)^2}. \end{aligned} \tag{33}$$

From (30) and (33), we obtain the following:

$$R_1' = \frac{D}{\cos(O_1) + \sqrt{A^{-2} - \sin(O_1)^2}}. \tag{34}$$

Appendix B
Proof of Lemma 1

The joint pdf of R_1 and R_2 is given in [6] (30) as follows:

$$f_{R_1,R_2}(r_1, r_2) = e^{-\lambda\pi r_2^2}(2\lambda\pi)^2 r_1 r_2. \tag{35}$$

Using this density, the CDF of R can be expressed as

$$F_R(r) = p(R \leqslant r) = p\left(\frac{R_1}{R_2} \leqslant r\right) = p\left(\frac{R_2}{R_1} \geqslant r^{-1}\right), \tag{36}$$

which can be calculated as

$$F_R(r) = \int_{r_2=0}^{\infty} \int_{r_1=0}^{r r_2} f_{R_1,R_2}(r_1, r_2) dr_1 dr_2 = r^2; \tag{37}$$

this completes the proof. ■

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