BCWS Lab

Broadband Communications and
Wireless Systems Laboratory
Seminar Series

Theory of Transmit Power Control in Wireless Multiple-Access Systems

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Outline

1. Interference in Wireless Communications
2. Some Results From Linear Algebra
3. Power Control in FDMA/TDMA Systems
    - SIR-balancing
    - Centralized Algorithms
    - Cooperative Algorithms
    - Distributed Algorithms
4. Forward-Link versus Reverse-Link Formulation
5. Power Control in CDMA Systems
6. Power Control in CDMA Macrodiversity Systems
7. Introduction to Radio Resource Management in CDMA Multimedia Systems
8. Concluding Remarks
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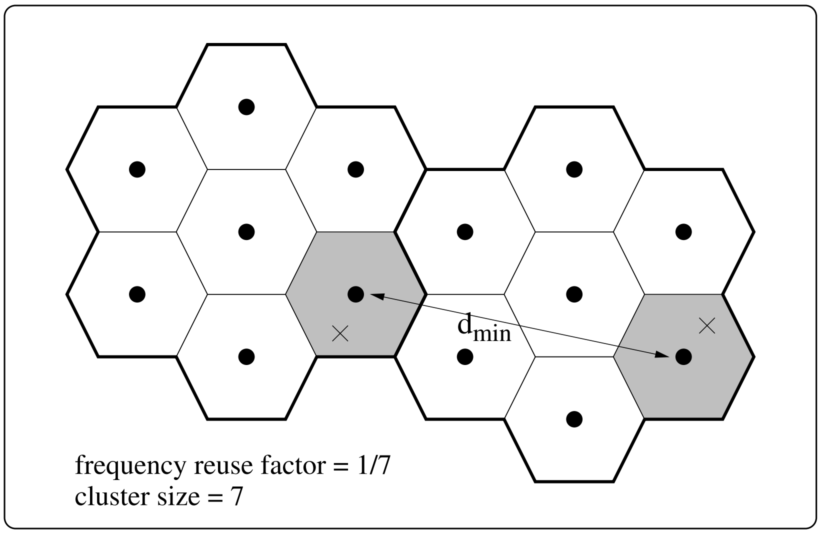
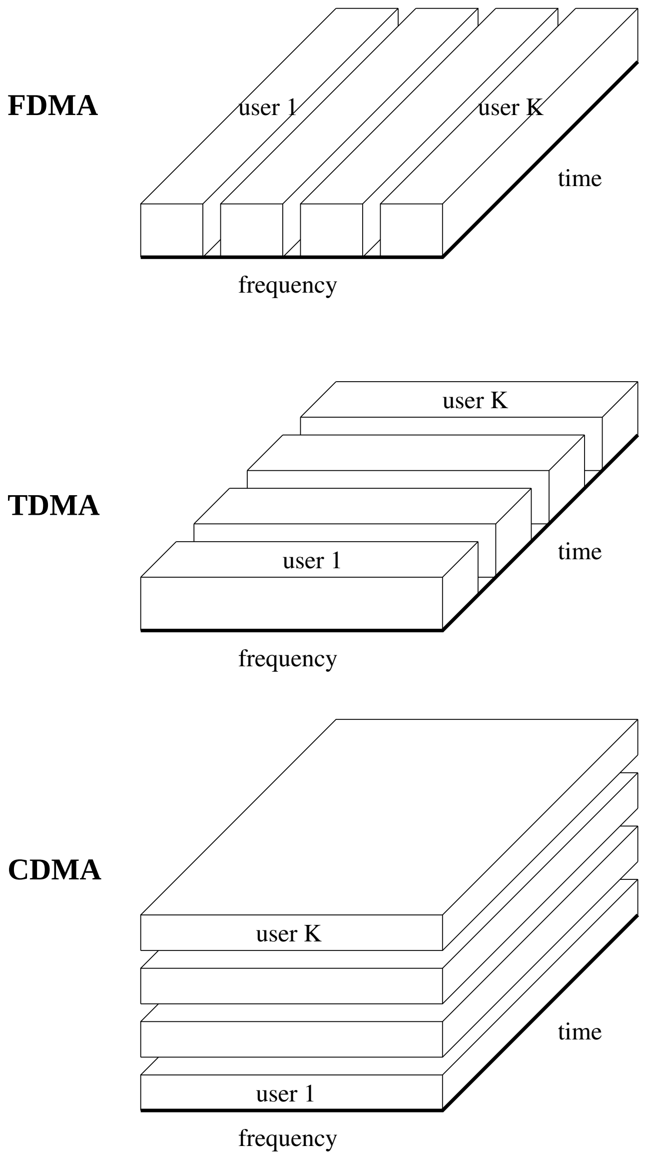
1. Interference in Wireless Communications

- **Types of Interference:**
  - overlay interference
  - multipath interference
  - adjacent-channel interference
  - cochannel interference
- **Cochannel Interference**
  - cellular systems: reuse $\longrightarrow$ cochannel interference

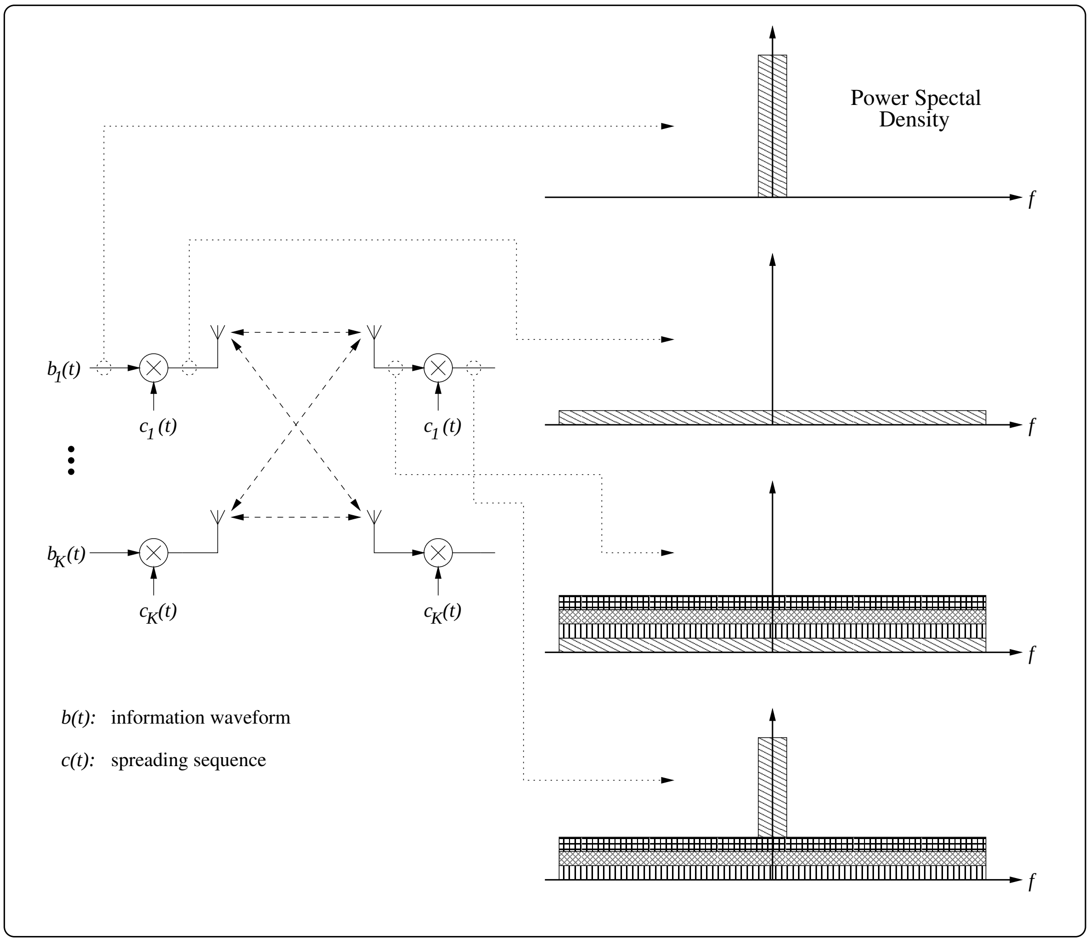


  - performance and capacity: conflicting goals

Multiple Access Techniques



Spread Spectrum (CDMA)



- Processing gain: $N = \frac{\text{spread spectrum bandwidth}}{\text{transmission rate}}$

  $\rightarrow$ protection against multiple access (cochannel) interference

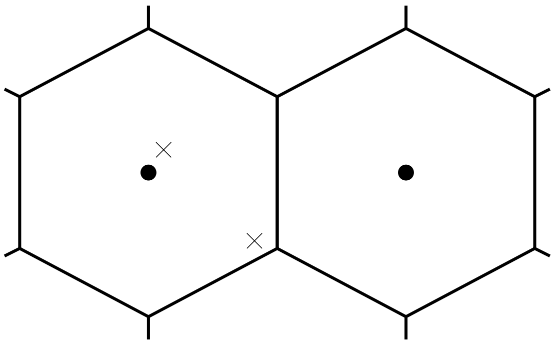
Interference in FDMA/TDMA and CDMA Systems

- Cochannel Interference
  - intercell interference
  - intracell interference
- **FDMA** and **TDMA**: slots are disjoint (orthogonal)
  - $\rightarrow$ no intracell interference
- **CDMA**: all the available BW is utilized in each cell
  - $\rightarrow$ frequency reuse factor = 1 (cluster size = 1)
  - $\rightarrow$ all users interfere with one another
  - $\rightarrow$ intracell and intercell interference
  - $\rightarrow$ $\boxed{\text{interference management: crucial}}$
- **Cochannel Interference Management (Mitigation)**

  rejection, cancellation, toleration, regulation
  - Antenna Structures
    - smart antennas
    - distributed antennas
  - Multiuser Detection
  - Base & Channel Assignment
  - Channel Coding, Spreading
  - $\boxed{\text{Transmit Power Control}}$

Power Control Problem

- Propagation effects:
  - near-far problem
  - multipath fading
  - shadowing



$$\text{SIR} = N \frac{\text{signal power}}{\text{interference power}}$$

SIR: signal-to-interference ratio

- $\boxed{\text{Reverse-link power control}}$
- Forward-link power control
- **Remark**: PC $\rightarrow$ $*$ extended battery life
  $*$ reduced biological concerns

2. Some Results from Linear Algebra

- **Eigenvalue and Eigenvector**

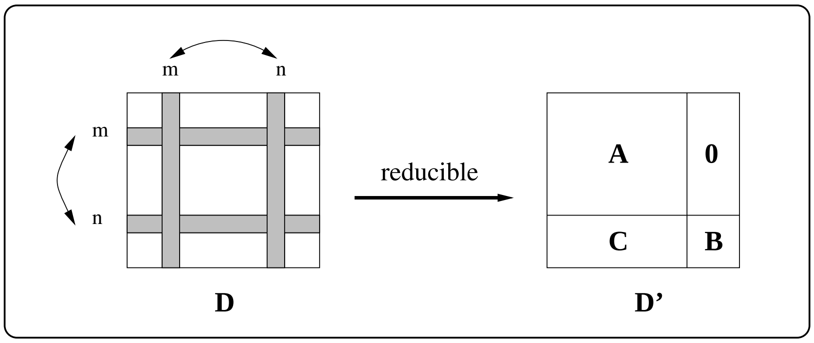
  * Let $\mathbf{Z}$ be an $L \times L$ square matrix.

  * $\{\lambda_i\}_{i=1}^{L}$: eigenvalues of $\mathbf{Z}$ are the roots of the characteristic equation $|\lambda \mathbf{I} - \mathbf{Z}| = 0$.

  * $\{\mathbf{P}_i\}_{i=1}^{L}$: eigenvectors of $\mathbf{Z}$ are the $L \times 1$ vectors that satisfy $\lambda_i \mathbf{P}_i = \mathbf{Z}\mathbf{P}_i$.

  * An eigenvector can arbitrarily be scaled.

- **Frobenius-Perron Theorem**



  * Let $\mathbf{Z}_{L \times L}$ be an irreducible, nonnegative matrix with $\{\lambda_i\}_{i=1}^{L}$, then

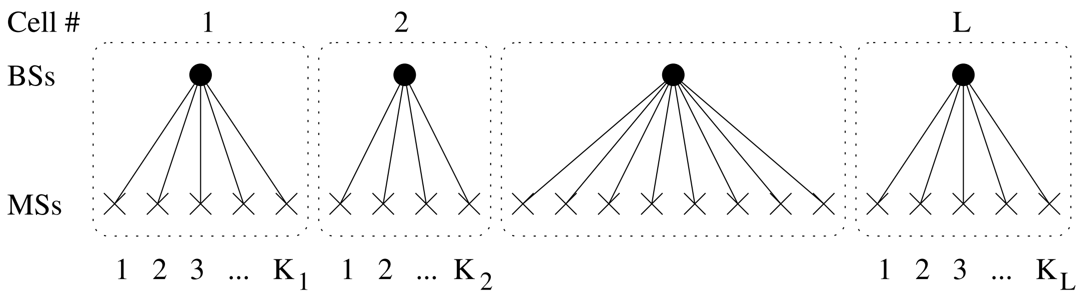
$$\exists\, \lambda^* = \max_i \{|\lambda_i|\} \text{ (with multiplicity 1) } : \ \mathbf{P}^* > \mathbf{0}$$

  * Also, $$\min_i \sum_j Z_{ij} \ \leq \ \lambda^* \ \leq \ \max_i \sum_j Z_{ij}$$

System Description

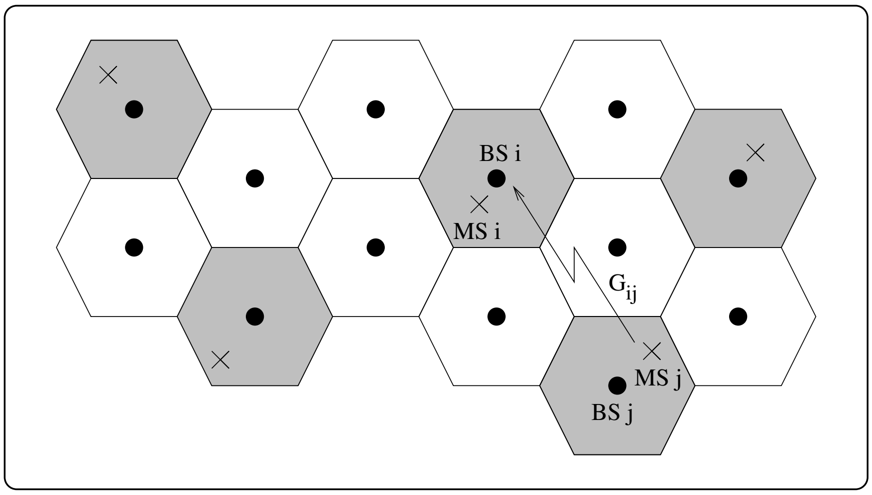
- L: number of cells = number of BSs
- K_i: number of MSs in cell i
- K: total number of MSs $\rightarrow$ $K = K_1 + K_2 + \ldots + K_L$
- Single-service systems

  $\rightarrow$ same SIR and rate requirement for all MSs.
- No macrodiversity



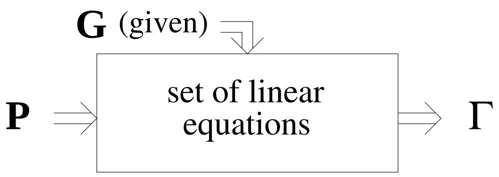
- Interference-limited systems (thermal noise omitted)

3. Power Control in FDMA/TDMA Systems



- M: # of cells (MSs) in a particular cochannel set
  Remark: $M \leq L \ll K$
- BS i is assigned to MS i.
- G_{ij}: radio link gain between BS i and MS j
- $\mathbf{G} = [G_{ij}]_{M \times M}$: link gain matrix (random process)
- P_i: transmit power of MS i
- $\mathbf{P} = [P_i]_{M \times 1}$: transmit power vector for MSs

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1j} & \cdots & G_{1M} \\ G_{21} & G_{22} & \cdots & G_{2j} & \cdots & G_{2M} \\ \vdots & & \vdots & & \vdots & \\ G_{i1} & G_{i2} & \cdots & G_{ij} & \cdots & G_{iM} \\ \vdots & & \vdots & & \vdots & \\ G_{M1} & G_{M2} & \cdots & G_{Mj} & \cdots & G_{MM} \end{bmatrix}$$



$$\Gamma_i = \frac{P_i G_{ii}}{\sum_{j=1}^{M} P_j G_{ij} - P_i G_{ii}}, \; \forall i$$

- Γ_i: SIR for MS i

Straightforward PC Algorithms:

- <u>Constant Tx Power</u> (no PC): $P_i = c, \; \forall i$

$$\Gamma_i = \frac{c\,G_{ii}}{\sum_{j=1}^{M} c\,G_{ij} - c\,G_{ii}} = \frac{G_{ii}}{\sum_{j=1}^{M} G_{ij} - G_{ii}}$$

  * $\Gamma_i \neq \Gamma_j$

- <u>Constant Rx Power:</u> $P_i G_{ii} = c, \; \forall i$

$$\Gamma_i = \frac{c}{\sum_{j=1}^{M} P_j G_{ij} - c} = \frac{c}{\sum_{j=1}^{M} c\,\frac{G_{ij}}{G_{jj}} - c} = \frac{1}{\sum_{j=1}^{M} \frac{G_{ij}}{G_{jj}} - 1}$$

  * $G_{ij}/G_{jj} \;\propto\; (d_{jj}/d_{ij})^n \ll 1, \;\; j \neq i$
  * $\Gamma_i \neq \Gamma_j$
  * Capacity(Constant Rx Power) $\simeq$
    $2\times$ Capacity(Constant Tx Power)

- Can we do better?

SIR-Balanced PC in FDMA/TDMA Systems

$$\gamma : \text{achievable} \mid \exists\, \mathbf{P} > \mathbf{0} : \Gamma_i \geq \gamma, \forall i$$

$$\gamma^* = \max_{\mathbf{P}>\mathbf{0}} \{\gamma\}$$

PC Algorithm	a) Try to achieve $(\mathbf{P}^*, \gamma^*)$, b) Develop a strategy for the case $\gamma^* < \gamma_o$ (threshold)

a) Obtaining $\mathbf{P}^*, \gamma^*$ [Aein_73]

Normalized link gain matrix $\mathbf{Z} = [Z_{ij}]_{M \times M}$, where $Z_{ij} = G_{ij}/G_{ii}$. Then,

$$\Gamma_i = \frac{P_i}{\sum_{j=1}^{M} P_j Z_{ij} - P_i}$$

$$\frac{1+\Gamma_i}{\Gamma_i} P_i = \sum_{j=1}^{M} P_j Z_{ij}$$

$$\frac{1+\Gamma_i}{\Gamma_i} \mathbf{P} = \mathbf{Z}\mathbf{P}$$

- SIR-balancing: set $\Gamma_i = \Gamma$, $\forall i$

$$\lambda \mathbf{P} = \mathbf{Z}\mathbf{P}, \text{ where } \lambda = \frac{1+\Gamma}{\Gamma}$$

Eigenvalue problem $\rightarrow \begin{cases} \lambda : \text{an eigenvalue of } \mathbf{Z} \\ \mathbf{P} : \text{corresponding eigenvector} \end{cases}$

- Remark: not all solutions are physically realizable!

Constraints: $\lambda > 1 \left(\Gamma = \frac{1}{\lambda - 1} > 0\right)$, $\mathbf{P} > \mathbf{0}$

- $\mathbf{Z}_{M\times M}$: irreducible, nonnegative

Frobenius-Perron Theorem $\rightarrow$

$$\exists\, \lambda^* = \max_i \{|\lambda_i|\} \text{ (with multiplicity 1) } : \mathbf{P}^* > \mathbf{0}$$

$$Z_{ii} = 1 \;\rightarrow\; \lambda^* > 1 \;\rightarrow\; \gamma^* = \frac{1}{\lambda^* - 1} > 0$$

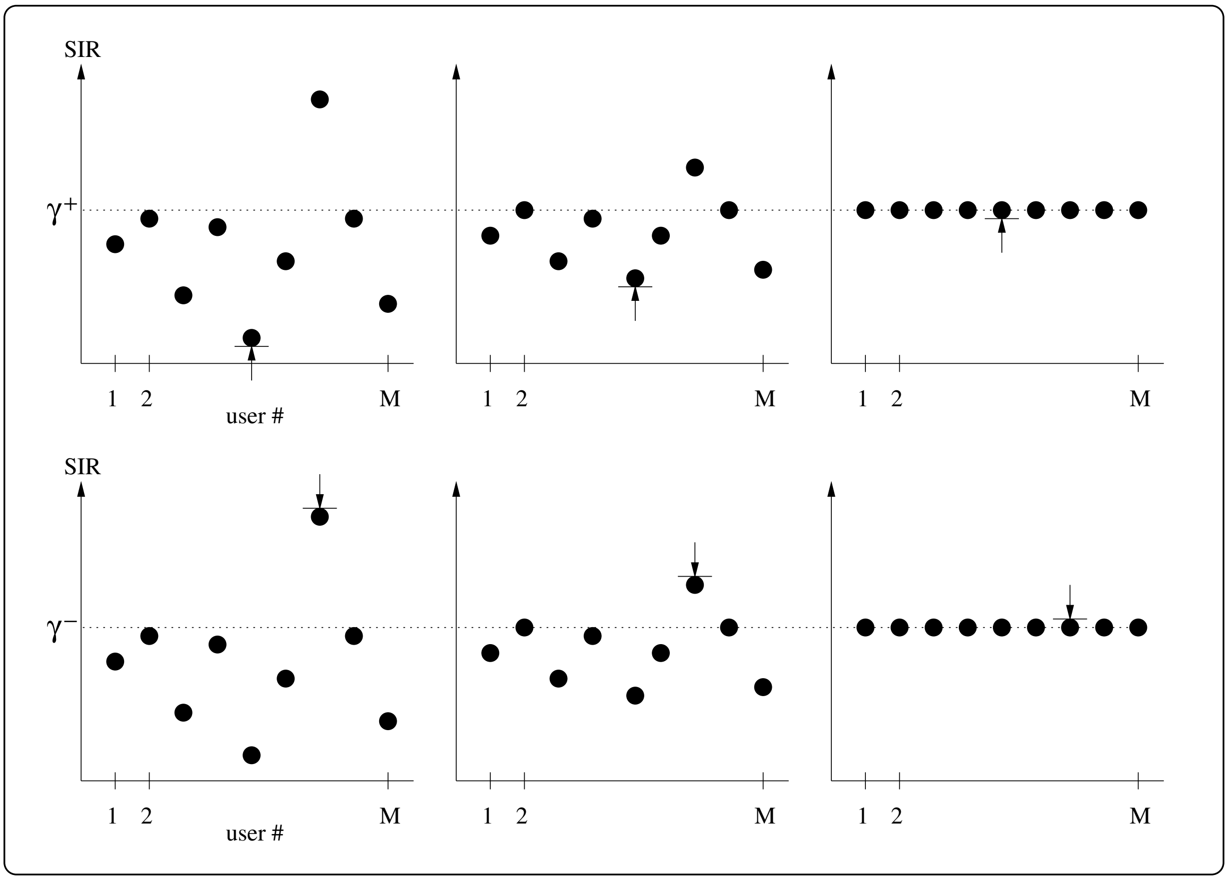
- SIR-balancing guarantees a solution to the PC problem: $\gamma^* = \frac{1}{\lambda^* - 1}$, $\mathbf{P}^*$

• Can an unbalanced case yield a higher γ ?

Define: $\gamma^+ = \max_{\mathbf{P}>\mathbf{0}} \min_{1\le i\le M} \{\Gamma_i\}$, $\gamma^- = \min_{\mathbf{P}>\mathbf{0}} \max_{1\le i\le M} \{\Gamma_i\}$

Then, $\gamma^+ = \gamma^- = \gamma^* = \frac{1}{\lambda^* - 1}$

[Grandhi.Vijayan.Goodman.Zander_93]



• SIR-balanced system yields the largest achievable γ.

• Remark: largest achievable SIR is related to the spectral properties of $\mathbf{Z}$.

PC Algorithm	$\begin{cases} \text{a) Try to achieve } (\mathbf{P}^*, \gamma^*), \\ \text{b) Develop a strategy for the case} \\ \quad \gamma^* < \gamma_o \end{cases}$

b) MS (Cell) Removal [Zander_92a]

• Straightforward SIR-balancing, without MS (cell) removal, may cause all links drop below the SIR threshold.

• Suppose $\mathbf{P}^* \rightarrow \gamma^* < \gamma_o$

Form $\bar{\mathbf{P}}^* : \begin{cases} \bar{P}_i^* = P_i^*, & i \neq k \\ \bar{P}_k^* = 0 \end{cases}$, i.e., turn off MS k.

$\bar{\Gamma}_k = 0$

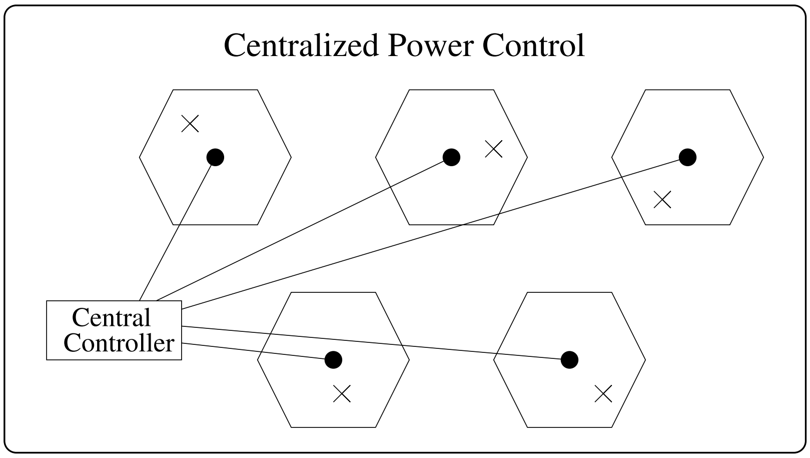
$$\bar{\Gamma}_i = \frac{\bar{P}_i}{\sum_{i=1}^{M} \bar{P}_j Z_{ij} - \bar{P}_i} = \frac{P_i^*}{\sum_{i=1}^{M} P_j^* Z_{ij} - P_i^* - P_k^* Z_{ik}} > \gamma^*, i \neq k$$

• Remark: the above system is not balanced (by balancing it even a higher SIR level can be achieved for all MSs)

• Turn off MS kth $\equiv$ form $\bar{\mathbf{Z}}$ by removing the kth row and column of $\mathbf{Z}$. Then, SIR balance $\rightarrow \bar{\gamma}^* > \gamma^*$

PC Algorithm Types			
Structure	Centralized	Cooperative	Distributed
Link Info	extensive	limited	very limited

Centralized algorithm: instantaneously controls the entire vector **P**.



• BSs report radio link information to a central controller. The controller then distributes the power control decisions throughout the network.

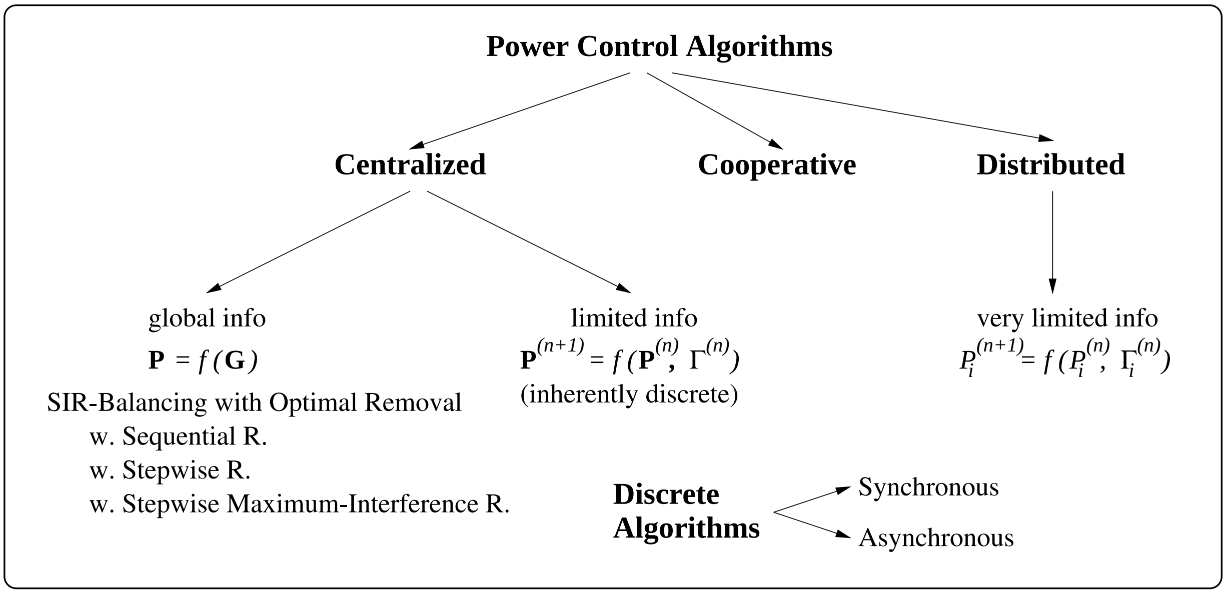
• logical limit: global information, i.e., instantaneous access to the entire link gain matrix **G** ($\rightarrow$ optimum algorithm possible)

$*$ a central controller required
$*$ significant control signal exchange
$*$ delay introduced
$*$ high complexity
$\rightarrow$ sets the performance upper bound
but, difficult to implement

- limited radio link information
  - $\rightarrow$ discrete PC algorithms
    - $\rightarrow$ power evolution (adaptation)

Cooperative algorithm: limited data exchange (information flow) among BSs.

Distributed algorithm: each MS (or BS) controls its own tx power based on only limited knowledge about $\mathbf{G}$.



Perf(best centralized algo) $\geq$ Perf(best cooperative algo)
$\geq$ Perf(best distributed algo)

Centralized PC Algorithms

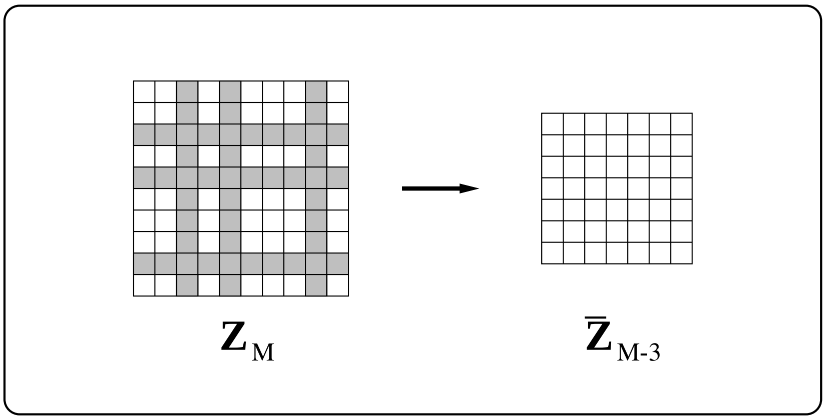
Optimum MS (Cell) Removal Strategy: Find the largest square submatrix $\bar{\mathbf{Z}}$, for which γ_o is achievable, by removing as few MSs (cells) as possible.

Optimum PC Algorithm (Brute-Force Implementation) [Zander_92a]

1. SIR-balance the system $\mathbf{Z}_M$.
   If $\gamma_M^* \geq \gamma_o$, then use $\mathbf{P}_M^*$ and stop; otherwise,

2. Set $m = 1$.
   While $m < M$,

   Find the submatrix $\bar{\mathbf{Z}}_{M-m}$ that will yield the highest achievable SIR: $\bar{\gamma}_{M-m}^*$.
   If $\bar{\gamma}_{M-m}^* \geq \gamma_o$, then use $\bar{\mathbf{P}}_{M-m}^*$ and stop; otherwise set $m = m + 1$.



• Eventually, $M-1$ MSs (cells) will be removed $\rightarrow$ no interference left $\rightarrow$ algorithm works.

• Construct smaller and smaller balanced systems by removing MSs. Find the largest square submatrix of **Z** with an eigenvalue

$$\lambda^* < \lambda_o = \frac{1+\gamma_o}{\gamma_o}$$

• Straightforward but tedious method (NP-complete?)

• The number of eigenvalue computations in the worst case:

$$\binom{M}{1} + \binom{M}{2} + \cdots + \binom{M}{M-2} \simeq 2^M$$

$\rightarrow$ exponential complexity

Sequential Removal Algorithm: Remove one cell at a time until γ_o is achieved.

• The number of eigenvalue computations in the worst case:

$$M + (M-1) + \cdots + 2 \simeq \frac{M^2}{2}$$

Stepwise Removal Algorithm (SRA): [Zander_92a]
Remove MS (cell) k for which the max of the row and column sums is maximized.

1. SIR-balance the system $\mathbf{Z}_M$.
   If $\gamma^*_M \geq \gamma_o$, then use $\mathbf{P}^*_M$ and stop; otherwise,

2. Set $m = 0$ and $\bar{\mathbf{Z}}_M = \mathbf{Z}_M$.
   While $m < M$,

   form submatrix $\bar{\mathbf{Z}}_{M-(m+1)}$ from $\bar{\mathbf{Z}}_{M-m}$ by removing MS k for which

   $$\max\{r_k = \sum_{j=1}^{M-m} \bar{Z}_{kj},\;\; r_k^T = \sum_{j=1}^{M-m} \bar{Z}_{jk}\} \geq \max\{r_i, r_i^T\},$$

   $\forall i$. If $\bar{\gamma}^*_{M-(m+1)} \geq \gamma_o$, then use $\bar{\mathbf{P}}^*_{M-(m+1)}$ and stop; otherwise set $m = m + 1$.

$\rightarrow$ linear complexity in eigenvalue computation $\surd$

• The SRA seeks to maximize the lower bound for γ^*

• Remark: SRA uses full knowledge of the link gain matrix in order to calculate its eigenvalues

• Outage(SRA with cluster size = 3) $>$ Outage(Fix tx power with cluster size = 13) $\rightarrow$ capacity gain of 4

Stepwise Maximum-Interference Removal Algorithm (SMIRA)

[Lee.Lin.Su_95]

Tx power is also taken into account in removal process.

1. SIR-balance the system $\mathbf{Z}_M$.
   If $\gamma_M^* \geq \gamma_o$, then use $\mathbf{P}_M^*$ and stop; otherwise,

2. Set $m = 0$, $\bar{\mathbf{Z}}_M = \mathbf{Z}_M$, and $\bar{\mathbf{P}}_M = \mathbf{P}_M$.
   While $m < M$,

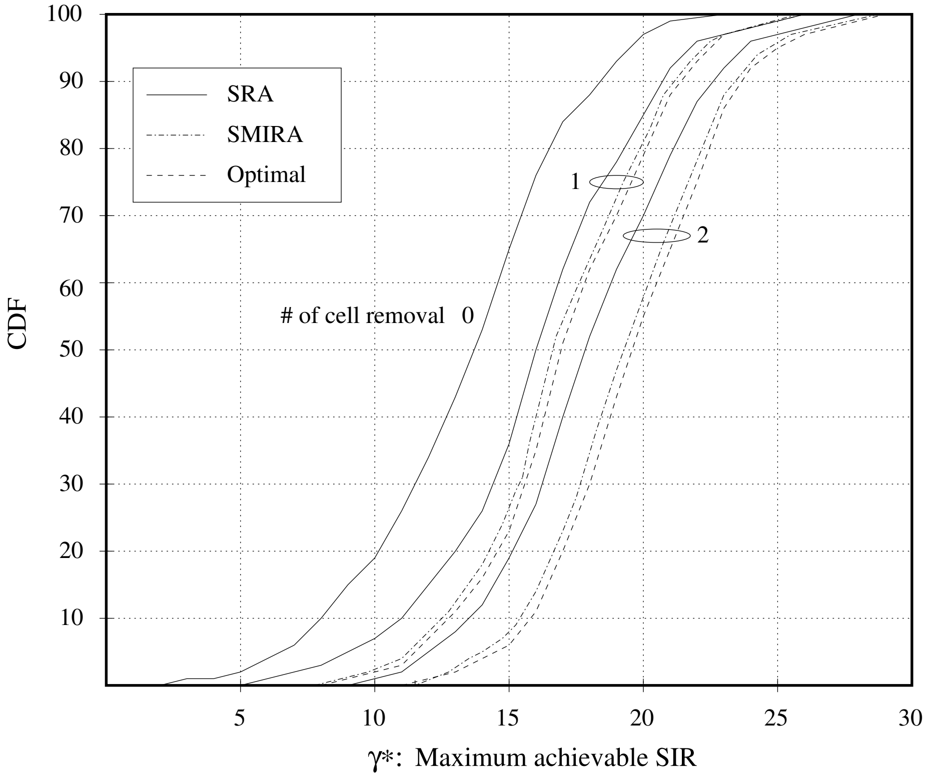
   form submatrix $\bar{\mathbf{Z}}_{M-(m+1)}$ from $\bar{\mathbf{Z}}_{M-m}$ by removing MS l for which

   $$\max\{r_l = \sum_{j=1}^{M-m} \bar{P}_j \bar{Z}_{lj},\ \ r_l^T = \sum_{j=1}^{M-m} \bar{P}_l \bar{Z}_{jl}\} \geq \max\{r_i, r_i^T\},$$

   $\forall i$. If $\bar{\gamma}_{M-(m+1)}^* \geq \gamma_o$, then use $\bar{\mathbf{P}}_{M-(m+1)}^*$ and stop; otherwise set $m = m + 1$.

Performance Comparison

[Lee.Lin.Su_95]: 19 cochannel cells with 7-cell cluster

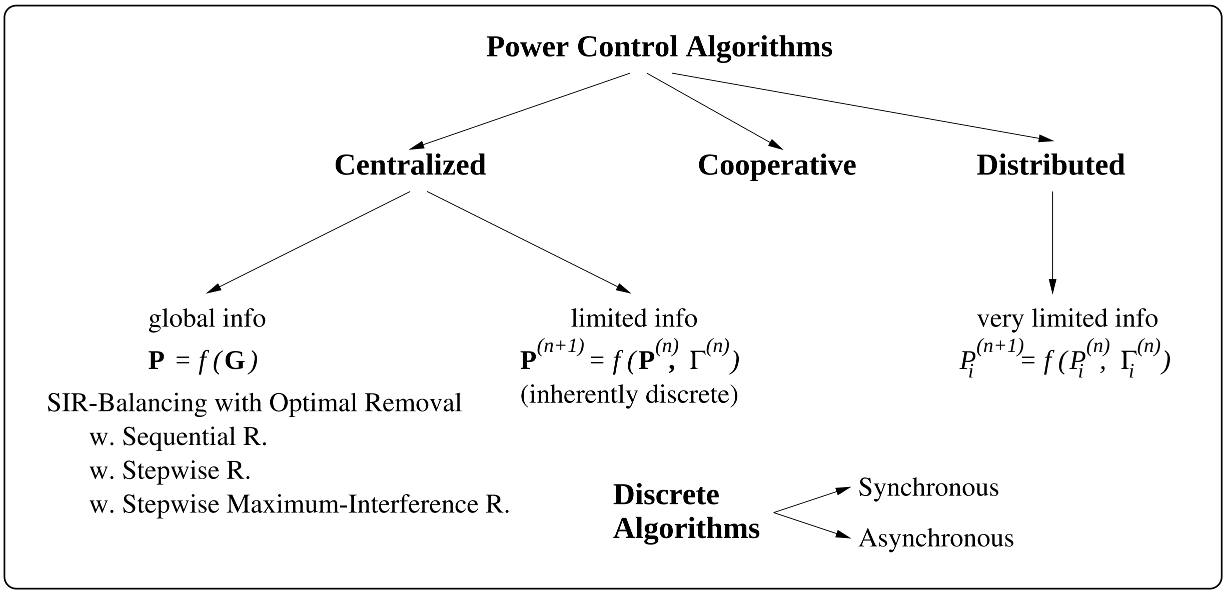


Ensemble mean of γ^*

# of MSs rmvd	Optimal	SMIRA	SRA
1	17.29	17.02	16.16
2	19.91	19.58	18.24

Percentage of optimum removals

# of MSs rmvd	Optimal	SMIRA	SRA
1	100	64.8	44
2	100	48.8	22.2



Distributed PC Algorithms

Distributed SIR-Balancing I: [Zander_92b]

$$\mathbf{P}^{(0)} = \mathbf{P}_o, \ \ \mathbf{P}_o > \mathbf{0}$$

$$P_i^{(n+1)} = \beta P_i^{(n)} \left(1 + \frac{1}{\Gamma_i^{(n)}} \right), \ \ \beta > 0$$

• From numerical analysis: power method for finding the dominant eigenvalue and its corresponding eigenvector.

• Let $\{\lambda\}_i$ be ordered: $\lambda^* = \lambda_1 > |\lambda_2| \geq \cdots \geq |\lambda_M|$. Speed of convergence depends on how much larger λ_1 is than $|\lambda_2|$.

• error $\propto |\lambda_2/\lambda_1|^n$

• For a high achievable SIR, λ^* is close to 1.

Note that trace $(\mathbf{Z}) = \sum_i^M \lambda_i = M$, since $Z_{ii} = 1$, $\forall i$.

For a high achievable SIR, $\lambda^* \simeq 1$. So, $\sum_2^M \lambda_i \simeq M-1$.

But since $|\lambda_i| < \lambda^*, \forall i \neq 1 \; \rightarrow \; |\lambda_i| \simeq 1, \forall i$.

$\rightarrow$ slow convergence

• the choice of $\mathbf{P}_o$ is not critical.

• calculating β may not be possible in a completely distributed system

Distributed SIR-Balancing II:

[Grandhi.Vijayan.Goodman_92]

$$P_i^{(n+1)} = \beta \frac{P_i^{(n)}}{\Gamma_i^{(n)}} = \beta I_i^{(n)}, \;\; \beta > 0$$

where I_i is the normalized interference power received by MS i.

$\rightarrow$ faster convergence compared to algo I.

A Class of Distributed SIR-Balancing Algorithms

[Lee.Lin.Su_95]

$$\mathbf{P}^{(0)} = \mathbf{P}_o, \;\; \mathbf{P}_o > \mathbf{0}$$

$$P_i^{(n+1)} = \beta P_i^{(n)} \left((1-\alpha) + \frac{1}{\Gamma_i^{(n)}} \right),$$

where $\beta > 0$, and $-\infty < \alpha \leq 1$.

• $\alpha = 0$ and $\alpha = 1$ correspond, respectively, to algorithms I and II.

• $\alpha \to -\infty$: fix tx power (no power control)

• Numerical results: $\alpha = 1$ yields fastest convergence $\to$ best outage for a given number of iteration steps

• Numerical results: Let $|\lambda_1 - \alpha| \geq |\lambda_2 - \alpha| \geq \cdots \geq |\lambda_M - \alpha|$. Speed of convergence depends on how much larger $|\lambda_1 - \alpha|$ is than $|\lambda_2 - \alpha|$.

• Theoretical explanation ?

Generalized Limited-Information Stepwise Removal Algorithm (LI-SRA)

[Zander_92b], [Lee.Lin.Su_95]

1. Set $\mathbf{P} = 1$. If $\Gamma_i^{(0)} > \gamma_o$, $\forall i$, stop; else,
2. Operate a distributed balancing algorithm for at most N steps. If for $n < N, \Gamma_i^{(n)} > \gamma_o$, $\forall i$, stop; else,
3. Remove MS i with the smallest $\Gamma_i^{(0)}$. Go to step 1.

• LI-SRA uses the SIR measurements of the wanted links

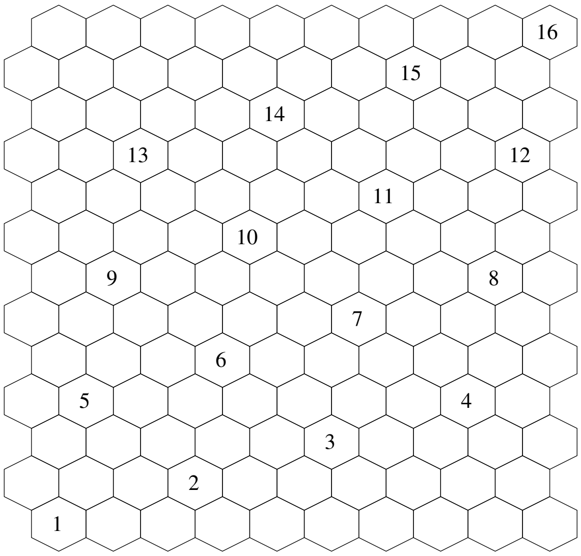
• In LI-SRA, the removal procedure requires the collection of data from the BSs in order to compare the SIR values in the different cells.
  → straightforward procedure in a global network control scheme.

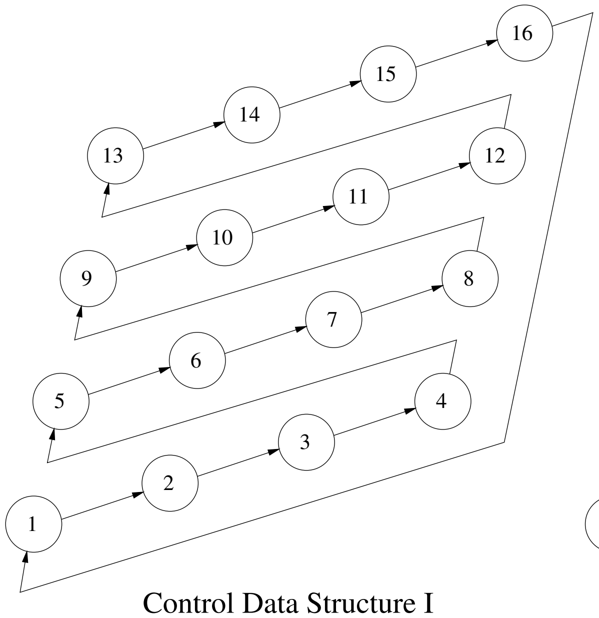
• LI-SRA is rather insensitive to SIR estimation errors

• Capacity improvements comparable to that provided by the SRA algorithm.

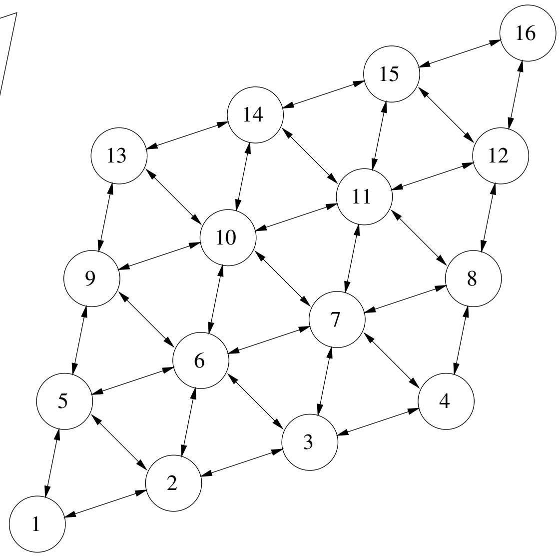
Cooperative PC Algorithms

[Sung.Wong_99]





Control Data Structure I

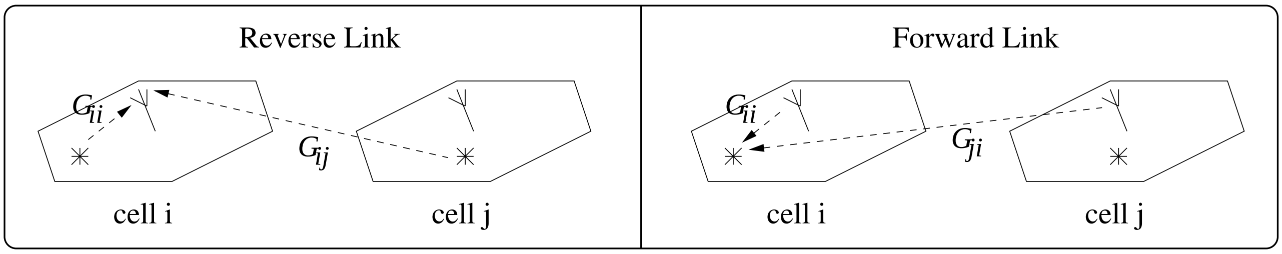


Control Data Structure II

4. Relation Between Forward and Reverse Link

Power Control Formulation

[Nettleton.Alavi_83], [Zander.Frodigh_94], [Wu_99]



$$\Gamma_{i,RL} = \frac{G_{ii}P_i}{\sum_{j=1}^{M} G_{ij}P_j - G_{ii}P_i} = \frac{P_i}{\sum_{j=1}^{M} \frac{G_{ij}}{G_{ii}}P_j - P_i},$$

$$\Gamma_{i,FL} = \frac{G_{ii}P_i}{\sum_{j=1}^{M} G_{ji}P_j - G_{ii}P_i} = \frac{P_i}{\sum_{j=1}^{M} \frac{G_{ji}}{G_{ii}}P_j - P_i},$$

with $Z_{ij} = \frac{G_{ij}}{G_{ii}}$ and $W_{ij} = \frac{G_{ji}}{G_{ii}}$.

Since, $Z_{ji} = \frac{G_{ji}}{G_{jj}} \quad \rightarrow \quad W_{ij} = Z_{ji}\frac{G_{jj}}{G_{ii}}$.

• Therefore, $\mathbf{W} \neq \mathbf{Z}^T$, but they are related; indeed,

$$|\mathbf{Z} - \lambda\mathbf{I}| = |\mathbf{W} - \mu\mathbf{I}|$$

$\rightarrow$ **Z** and **W** have the same characteristic equations and thus identical eigenvalues: $\lambda_i = \mu_i, \forall i \quad \rightarrow \quad \lambda^* = \mu^*$.

$\rightarrow$ The maximum achievable SIR in the forward and reverse links are identical, i.e., $\gamma^*_{\text{reverse}} = \gamma^*_{\text{forward}}$.

Remark: the corresponding eigenvectors (the tx powers) will in general be different.

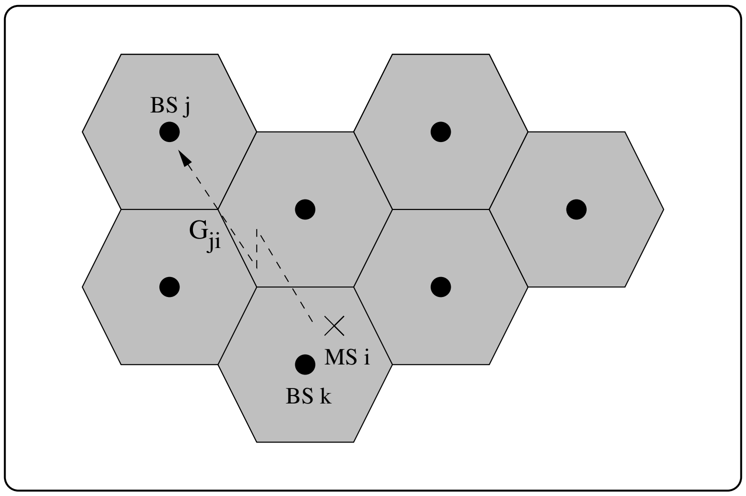
• Whenever a certain SIR level is achieved in one direction, the tx power in the other direction can always be adjusted to achieve at least the same SIR value.

• For a given γ_o, the optimal removal set of the forward and reverse links are always the same when the optimal removal algorithm is executed independently on either direction.

$\rightarrow$ The optimal removal action performed in one direction can always be directly performed on the other direction to achieve the optimum performance in both directions.

5. Power Control in CDMA Systems

- Both intercell and intracell interference
- Cluster size = 1



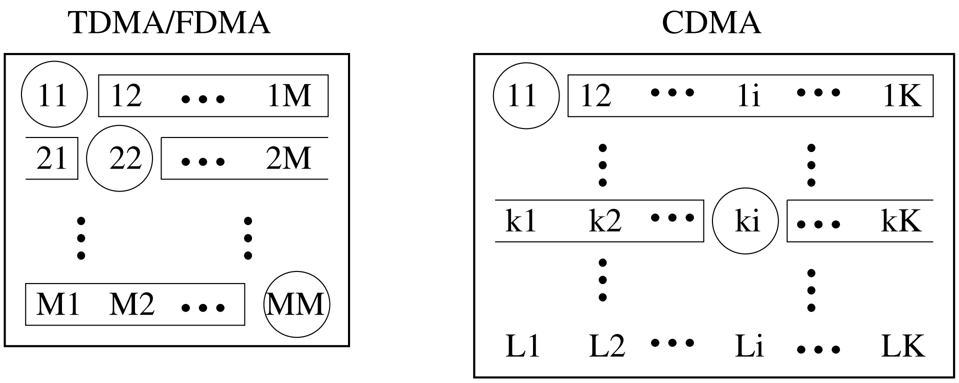
- **System Description**
  - $*$ L BSs and K MSs
  - $*$ G_{ij}: radio link gain between BS i and MS j
  - $*$ $\mathbf{G} = [G_{ij}]_{L\times K}$: link gain matrix
  - $*$ $\mathbf{P} = [P_i]_{K\times 1}$: transmit power vector for MSs
  - $*$ Γ_i: SIR for MS i before despreading

• Assume MS i is assigned to BS k, then

$$\Gamma_i = \frac{G_{ki}P_i}{\displaystyle\sum_{j=1}^{L} G_{kj}P_j - G_{ki}P_i}, \quad i \in \{1, 2, \ldots, K\}$$

• Can the PC problem be transformed into the eigenvalue/eigenfunction formulation?

• ? Define a normalized link gain matrix $\mathbf{Z} = [Z_{ki}]_{L\times K}$, with $Z_{ki} = G_{kj}/G_{ki}$ $\rightarrow$ 3-D matrix needed



$$\frac{1+\Gamma}{\Gamma}\mathbf{P}_{K\times 1} \neq \mathbf{Z}_{L\times K}\mathbf{P}_{K\times 1}$$

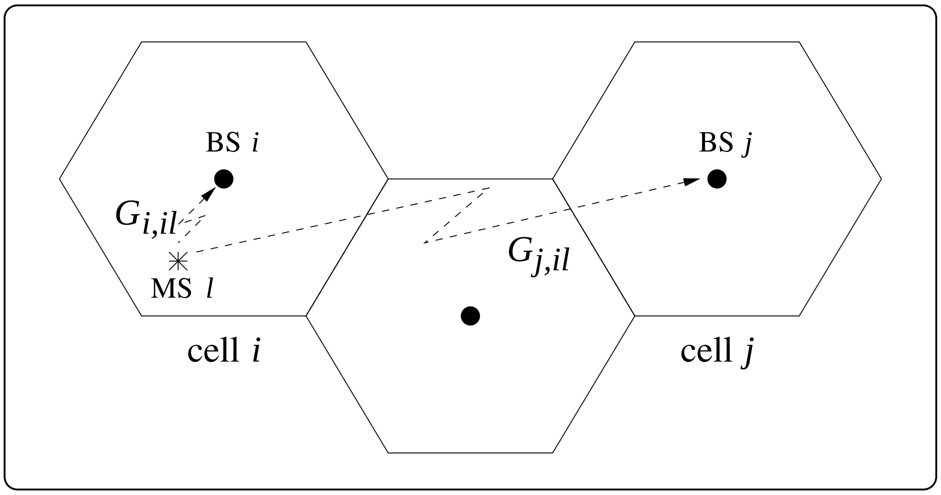
$\rightarrow$ Staightforward eigenvalue formulation is not possible since **Z** is not a square matrix !

Eigenvalue Formulation in CDMA Systems

Method I

[Nettleton_80], [Alavi.Nettleton_82], [Nettleton.Alavi_83]

- SIR-balancing through power-balancing
  - $*$ L cells with K_i users ($i \in \{1, 2, \ldots, L\}$) in each cell
    $\rightarrow$ use double indices for MSs
  - $*$ P_{jk}: tx power of MS k in cell j
  - $*$ $G_{i,jk}$: link gain between BS i and MS k in cell j



- SIR for MS l in cell i (reverse link):

$$\Gamma_{il} = \frac{G_{i,il} P_{il}}{\sum_{j=1}^{L} \sum_{k=1}^{K_j} G_{i,jk} P_{jk} - G_{i,il} P_{il}}$$

$\tilde{P}_{i,ik}$: rx power at BS i from MS k in cell i

- Apply power balancing within a cell

$\rightarrow$ $\tilde{P}_{i,ik} = P_{ik}G_{i,ik} = \tilde{P}_i, \;\; \forall ik$ where $k \in \{1, 2, \ldots, K_i\}$

$$\Gamma_{il} = \frac{\tilde{P}_i}{\sum_{j=1}^{L} \sum_{k=1}^{K_j} P_{jk} G_{i,jk} - \tilde{P}_i} = \Gamma_i, \;\; \forall l \in \{1, 2, \ldots, K_i\}$$

$\rightarrow$ Intracell SIR-balancing achieved through intracell power-balancing

- Since $P_{jk} = \tilde{P}_j / G_{j,jk}$,

$$\Gamma_i = \frac{\tilde{P}_i}{\sum_{j=1}^{L} \tilde{P}_j \sum_{k=1}^{K_j} \frac{G_{i,jk}}{G_{j,jk}} - \tilde{P}_i}, \;\; i \in \{1, 2, \ldots, L\}$$

Define $$a_{ij} = \sum_{k=1}^{K_j} \frac{G_{i,jk}}{G_{j,jk}}$$

a_{ij}: the total normalized interference from the MSs in cell j to BS i

$$\Gamma_i = \frac{\tilde{P}_i}{\sum_{j=1}^{L} a_{ij}\tilde{P}_j \; - \; \tilde{P}_i}, \quad i \in \{1, 2, \ldots, L\}$$

$\rightarrow$ Set of L linear equations

- Intercell SIR-balancing $\rightarrow$ $\Gamma_i = \Gamma, \ \forall i$

  Define the normalized link gain matrix $\mathbf{A} = [a_{ij}]_{L \times L}$

$$\rightarrow \frac{1+\Gamma}{\Gamma}\tilde{\mathbf{P}} = \mathbf{A}\tilde{\mathbf{P}}$$

- Frobenius-Perron Theorem $\rightarrow$ $\gamma^* = \frac{1}{\lambda^* - 1}, \ \tilde{\mathbf{P}}^*$

- Finally, obtain $P^*_{jk} = \frac{\tilde{P}^*_j}{G_{j,jk}}, \ \forall k \in \{1, 2, \ldots, K_j\}$

- <u>Remark</u>: although the PC problem is converted into an eigenvalue/eigenvector formulation, matrix **A** is not suitable for MS removal algorithms.

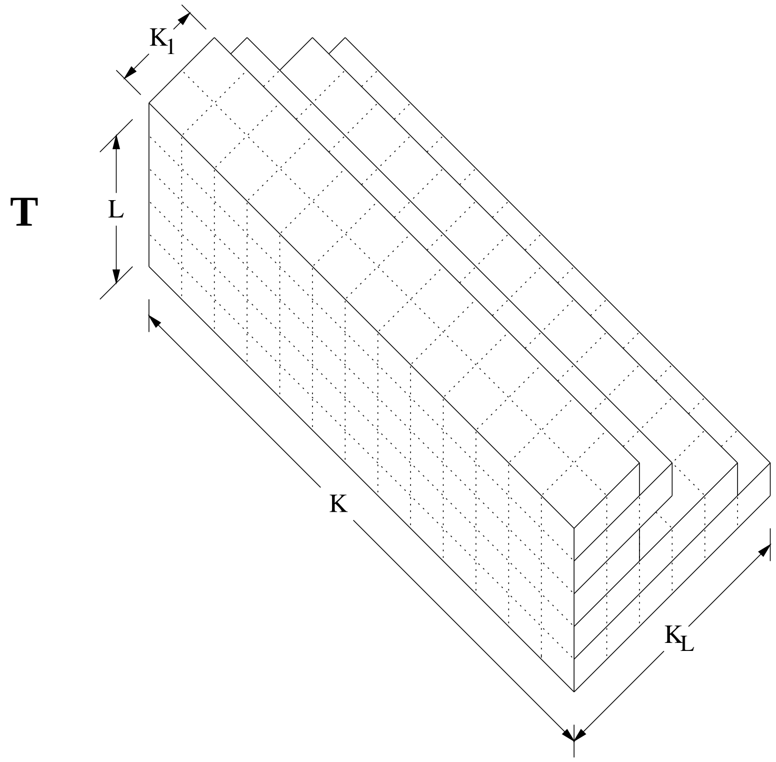
Eigenvalue Formulation in CDMA Systems

Method II [Wu_99]

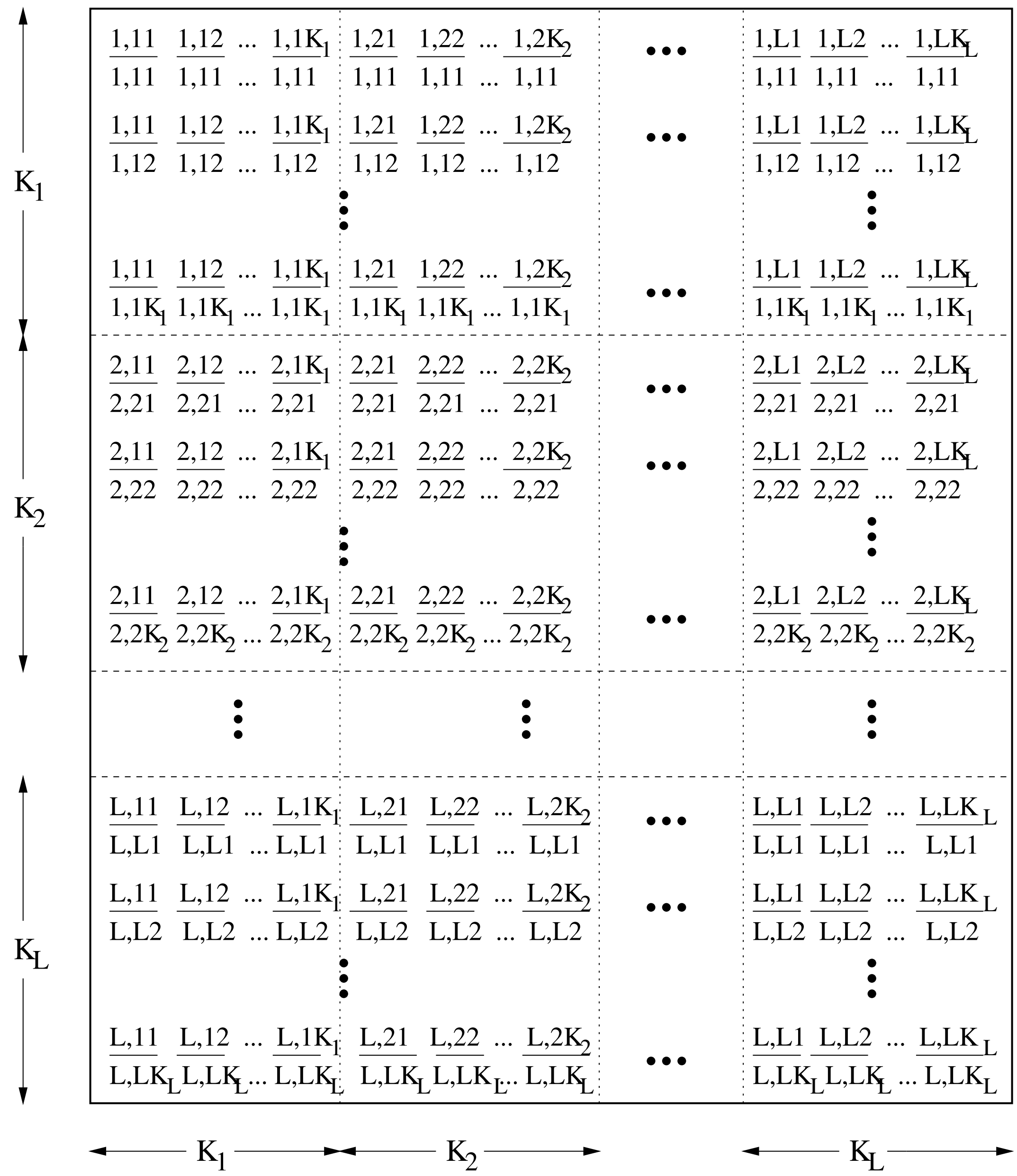
• Generate a 3-D link gain matrix with the 3rd dimension dependent on the 1st dimension index: $\mathbf{T}_{L \times K \times K_i}$.

• Then, convert the 3-D $\mathbf{T}$ matrix into a 2-D $\mathbf{U}_{K \times K}$ matrix by unfolding the 3rd dimension onto the 1st dimension.

Note that $K = K_1 + K_2 + \ldots + K_L$



- Normalize $\mathbf{U}_{K\times K}$ to get $\mathbf{Y}_{K\times K}$:



Stepwise Removal Algorithm:

1. SIR-balance the system $\mathbf{Y}_K$.
   If $\gamma_K^* \geq \gamma_o$, then use $\mathbf{P}_K^*$ and stop;
   otherwise, set $m = 1$.
2. Form $\bar{\mathbf{Y}}_{K-m}$ matrix by removing the MS ij for which the achievable SIR of the remaining system, $\bar{\gamma}_{K-m}^*$, is maximized.
   If $\bar{\gamma}_{K-m}^* \geq \gamma_o$, then use the eigenvector $\bar{\mathbf{P}}_{K-m}^*$ and stop; otherwise set $m = m + 1$ and goto step 2.

• The global removal algorithms defined for the FDMA/ TDMA systems

  * Optimal Removal
  * Sequential Removal
  * Stepwise Removal
  * Stepwise Maximum-Interference Removal
  * ...

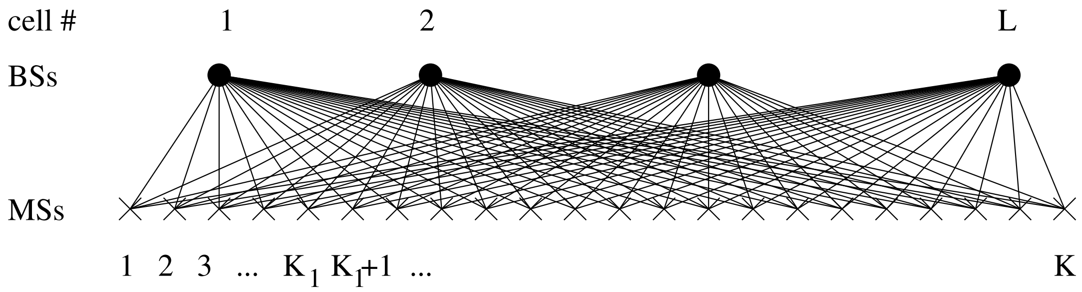
also apply to the CDMA systems.

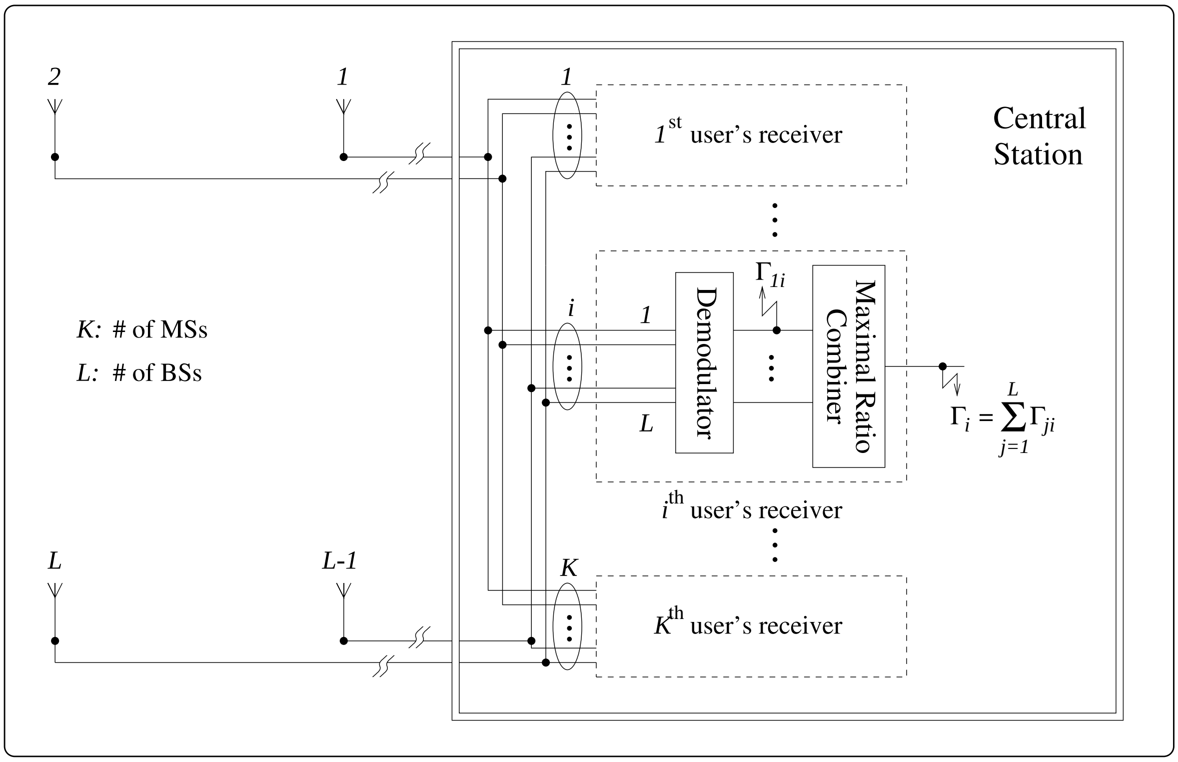
• In the case of limited knowledge of the link gain matrix, the distributed SIR-balancing and the related removal algorithms applies as well.

• Remark: $M \leq L \ll K \;\; \rightarrow$ required computation for the global optimum removal algorithm is tremendous.

6. PC in CDMA Macrodiversity Systems

- Logical limit of macrodiversity (MD): all MSs communicate with all BSs $\rightarrow$ not a cellular structure
- single index for MSs: $1 \rightarrow K$





- Γ_{ji}: SIR at the jth finger of the combiner for MS i, that is, the SIR contribution from BS j

$$\Gamma_{i,\mathrm{MD}} = \sum_{j=1}^{L} \Gamma_{ji} = \sum_{j=1}^{L} \frac{G_{ji}P_i}{\left(\sum_{k=1}^{K} G_{jk}P_k\right) - G_{ji}P_i}, \quad \forall i$$

- Obtain γ^* and $\mathbf{P}^*$

SIR-Balancing in Macrodiversity Systems

$$\Gamma_{i,\mathrm{MD}} = \sum_{j=1}^{L} \frac{G_{ji}P_i}{\left(\sum_{k=1}^{K} G_{jk}P_k\right) - G_{ji}P_i} = \gamma, \quad \forall i$$

- nonlinear set of equations

  $\rightarrow$ eigenvalue method does not apply

  $\rightarrow$ no closed-form solution exists

  $\rightarrow$ solve iteratively

[Yanikomeroglu.Sousa_98]

- Assume a centralized structure with global knowledge

Centralized Iterative Solution Algorithm:

- K eqn.s, $K+1$ unknowns $\Rightarrow$ fix P_1, find $\mathbf{P}$ with respect to P_1
- Arrange equations as follows:

$$\mathcal{E}_1: \quad \gamma = \sum_{j=1}^{L} \frac{G_{j1}P_1}{\left(\sum_{k=1}^{K} G_{jk}P_k\right) - G_{j1}P_1},$$

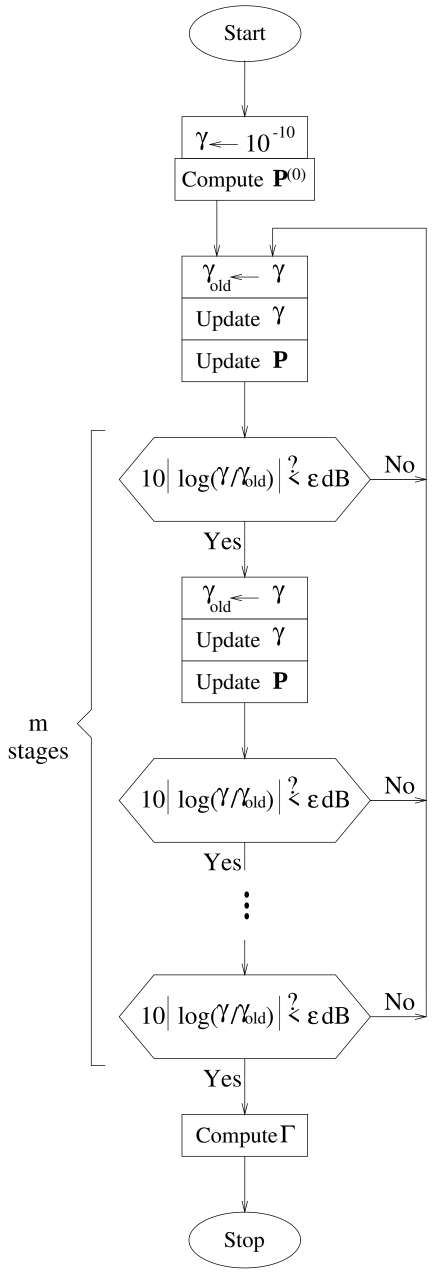
$$\mathcal{E}_i: \quad P_i = \frac{\gamma}{\sum_{j=1}^{L} \frac{G_{ji}}{\left(\sum_{k=1}^{K} G_{jk}P_k\right) - G_{ji}P_i}}, \quad i = 2, 3, \ldots, K.$$

- Compute

$$\begin{aligned}
\mathbf{P}^{(0)} &= \left\{P_i^{(0)}\right\} = \{(\sum_{j=1}^{K} G_{ji})^{-1}\}, \quad i = 1, 2, \ldots, K, \\
\gamma^{(n+1)} &= \mathcal{E}_1\left(\mathbf{P}^{(n)}\right), \\
P_1^{(n+1)} &= P_1^{(n)}, \\
P_i^{(n+1)} &= \mathcal{E}_i\left(\gamma^{(n+1)}, \mathbf{P}^{(n)} - \left\{P_i^{(n)}\right\}\right), \quad i = 2, 3, \ldots, K.
\end{aligned}$$

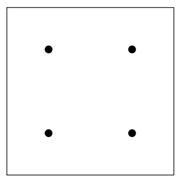
Multi-Stage Termination Criterion

Flow Chart



Simulation Parameters

square cell



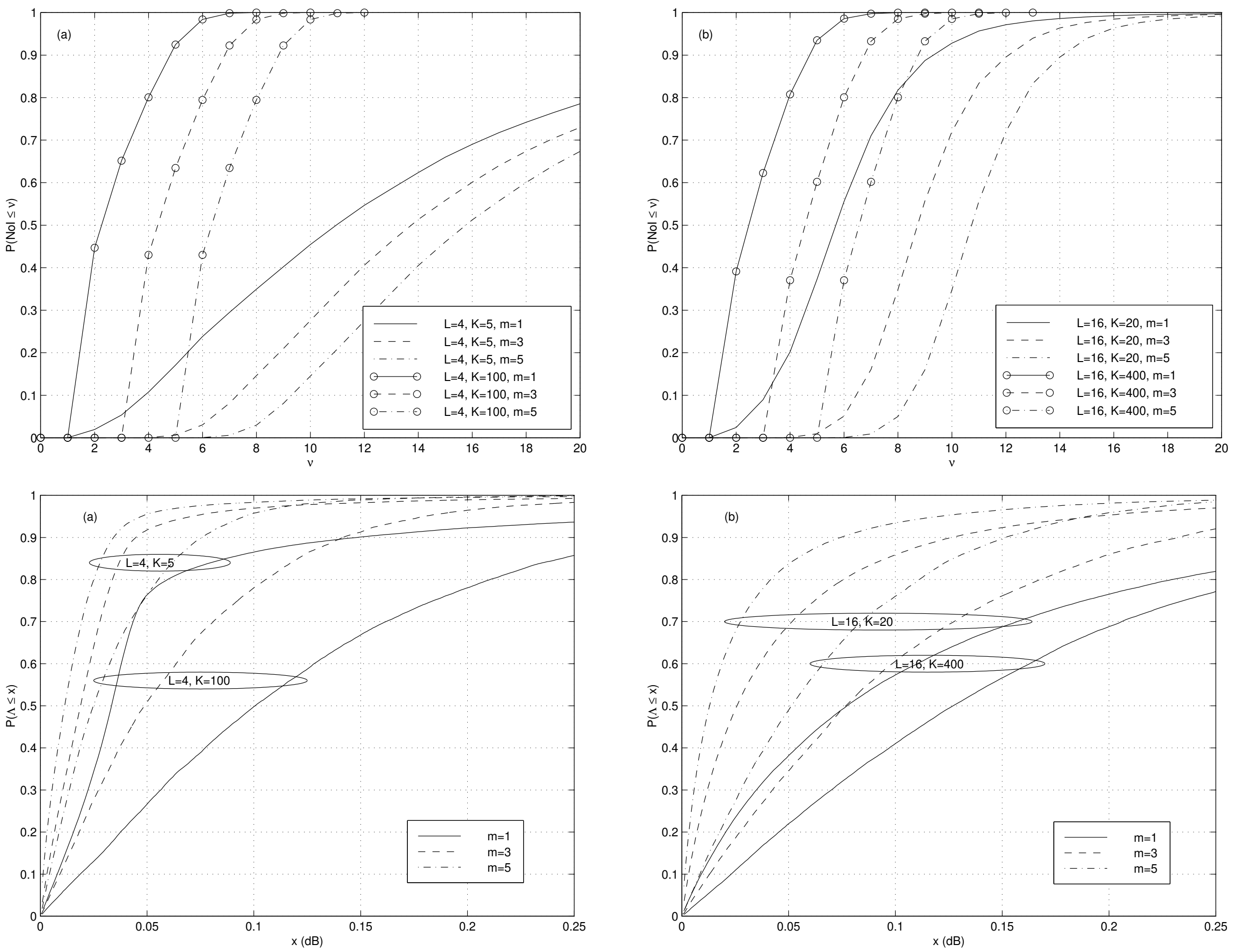
N=128

distance-power-law coefficient = 4

log-normal shadow fading with 0-dB mean 8-dB standard deviation

- Iterative solution always converges
- Error: $\Lambda_i^{(n_t)} = 10 \left| \log \left(\Gamma_i^{(n_t)} / \gamma^* \right) \right|$ dB, where
  n_t: number of iterations (NoI), and $\gamma^* = \lim_{n\to\infty} \gamma^{(n)}$.

Number of Iterations (NoI) and Error (Λ) Statistics



- [Hanly_96] gives a distributed iterative algorithm

7. Introduction to Radio Resource Management in CDMA Multimedia Systems

[Sampath.Kumar.Holtzman_95]

- Reverse link of a single-cell CDMA system with K users

W: spread-spectrum bandwidth

G_i: link gain between MS i and the BS ($\mathbf{G}$)

Γ_i: SIR of MS i (Γ)
γ_i: minimum required SIR for MS i (γ)

P_i: tx power of MS i ($\mathbf{P}$)
p_i: maximum allowable tx power for MS i ($\mathbf{p}$)

R_i: information rate of MS i ($\mathbf{R}$)
r_i: minimum required information rate for MS i ($\mathbf{r}$)

N_o: one-sided PSD of the background AWGN

$$\Gamma_i = \frac{W}{R_i} \frac{G_i P_i}{\sum_{j \neq i} G_j P_j + N_o W}, \quad \forall i.$$

• **Goal**: find **P** and **R** that will satisfy the QoS, power, and rate constraints

$$\Gamma_i \geq \gamma_i, \quad 0 < P_i \leq p_i, \quad R_i \geq r_i, \quad \forall i.$$

• The systems is **feasible** if there exists a solution.

• If there is more than one solution, choose the best **P** and **R** $\rightarrow$ optimization

---

$$\min_{\mathbf{R},\mathbf{P}} \sum P_i \quad \text{or} \quad \max_{\mathbf{R},\mathbf{P}} \sum R_i$$

$$\text{subject to} \qquad \mathbf{\Gamma} \geq \gamma, \quad 0 < \mathbf{P} \leq \mathbf{p}, \quad \mathbf{R} \geq \mathbf{r}$$

• Base assignment and mobility can also be incorporated

• Constrained optimization problem
$\rightarrow$ linear/nonlinear programming, game theory

Admission Control (Feasibility Condition)

- Feasibility condition $\rightarrow$ maximum throughput
- Assume no power constraints
- Single-cell systems [Hanly_93], [Yun.Messerschmitt_94]:

$$\sum_{i=1}^{K} \frac{r_i \gamma_i}{W + r_i \gamma_i} < 1$$

- Macrodiversity system with L BSs and $W \rightarrow \infty$ (approximate) [Hanly_96]:

$$\sum_{i=1}^{K} r_i \gamma_i < LW$$

- Macrodiversity system with L BSs, and $W \rightarrow \infty$ (exact) [Yanikomeroglu]:

$$\sum_{i=1}^{K} \frac{r_i \gamma_i}{LW + r_i \gamma_i} < 1$$

- Macrodiversity system with L BSs, and finite W: open question ([Yanikomeroglu.Sousa_99], [Yanikomeroglu.-Sousa_00])

8. Concluding Remarks

- **Transmit power control**: efficient tool in reducing cochannel interference.
  * PC can be used in any wireless system where cochannel interference exists (mobile, fixed [LMDS], FDMA, TDMA, CDMA).
  * The returns depend on the severeness of the cochannel interference in the system.

- PC is closely related to other interference management techniques:
  * distributed antennas
  * dynamic channel assignment
  * base assignment
  * ...

- **Hot Topics**:

  * Joint power-rate-QoS-handoff-... control in multimedia communications

  * Computationally efficient and fast-converging suboptimal algorithms for

    - macrodiversity and
    - limited-information (distributed and cooperative) systems with power constraints

8. References

- Halim Yanikomeroglu, “The exact admission criterion in CDMA macrodiversity systems”, in preparation for publication.

- Halim Yanikomeroglu and Elvino S. Sousa, “Effects of correlated interference on the potential linear antenna gain in CDMA macrodiversity systems”, submitted to IEEE International Conference on Communications (ICC’00), 18-22 June 2000, New Orleans, Louisiana, USA.

- Halim Yanikomeroglu and Elvino S. Sousa, “Correlated interference analysis in CDMA multi-antenna systems”, Proc. International Conference on Communications (ICC’99), pp. 23-28, 6-10 June 1999, Vancouver, BC, Canada.

- C. W. Sung and W. S. Wong, “The convergence of an asyncronous cooperative algorithm for distributed power control in cellular systems”, *IEEE Trans. Veh. Technol.*, vol. 48, no. 2, pp. 563-570, Mar. 1999.

- Q. Wu, “Performance of optimum transmitter power control in CDMA cellular mobile systems”, *IEEE Trans. Veh. Technol.*, vol. 48, no. 2, pp. 571-575, Mar. 1999.

- R. Rezaiifar and J. M. Holtzman, “Proof of convergence for the distributed optimal rate assignment algorithm”, in *Proc. Vehicular Technol. Conf.*, v. 3, pp. 1841-1845, 1999.

• S. Ramakrishna and J. M. Holtzman, “A scheme for throughput maximization in a dual-class CDMA system”, IEEE JSEC, V. 16, no. 6, pp. 830-844, August 1998.

• H. Yanikomeroglu and E. S. Sousa, ”SIR-balanced macro power control for CDMA sectorized distributed antenna systems”, in *Proc. 9th IEEE Int. Symp. PIMRC*, Boston, USA, 1998.

• N. Bambos, “Towards power-sensitive network architectures in wireless communications: concepts, issues, and design aspects”, *IEEE Personal Commun.*, pp. 50-59, June 1998.

• M. Soleimanipour, W. Zhuang, and G. H. Freeman, “Modelling and resource allocation in wireless multimedia CDMA systems”, in *Proc. Vehicular Technol. Conf.*, pp. 1279-1283, 1998.

• M. Soleimanipour, G. H. Freeman, and W. Zhuang, “An algorithm for maximal resource utilization in wireless multimedia CDMA communications”, in *Proc. Vehicular Technol. Conf.*, pp. 2594-2598, 1998.

• S. V. Hanly, “Capacity and power control in spread spectrum macrodiversity radio networks”, *IEEE Trans. Commun.*, pp. 247-256, Feb. 1996.

• T.-H. Lee, J.-C. Lin, and Y. T. Su, “Downlink power control algorithms for cellular radio systems”, *IEEE Trans. Veh. Technol.*, vol. 44, no. 1, pp. 89-94, Feb. 1995.

- A. Sampath, P. S. Kumar, and J. M. Holtzman, "Power control and resource management for a multimedia CDMA wireless system", in *Proc. 6th IEEE Int. Symp. PIMRC*, Toronto, Canada, 1995.
- L. Yun and D. Messerschmitt, "Power control for variable QoS on a CDMA channel", *Proc. MILCOM*, 1994.
- J. Zander and M. Frodigh, "Comment on "Performance of optimum transmitter power control in cellular radio systems"", *IEEE Trans. Veh. Technol.*, vol. 43, no. 3, p. 636, Aug. 1994.
- S. V. Hanly, *Information Capacity of Radio Networks*, Ph.D. thesis, University of Cambridge, 1993.
- S. A. Grandhi, R. Vijayan, D. J. Goodman, and J. Zander, "Centralized power control in cellular radio systems", *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 466-468, Nov. 1993.
- S. A. Grandhi, R. Vijayan, and D. J. Goodman, "A distributed algorithm for power control in cellular radio systems", in *Proc. 30th Annual Allerton Conf. Commun., Control, Computing*, Monticello, Illinois, 1992.
- J. Zander, "Distributed cochannel interference control in cellular radio systems", *IEEE Trans. Veh. Technol.*, vol. 41, no. 3, pp. 305-311, Aug. 1992.
- J. Zander, "Performance of optimum transmitter power control in cellular radio systems", *IEEE Trans. Veh. Technol.*, vol. 41, no. 1, pp. 57-62, Feb. 1992.

• R. W. Nettleton and H. Alavi, ‘Power control for a spread spectrum cellular in mobile radio system”, in *Proc. IEEE VTC*, 1983.

• H. Alavi and R. W. Nettleton, “Downstream power control for a spread spectrum cellular in mobile radio system”, in *Proc. IEEE Globecom*, 1982.

• R. W. Nettleton, “Traffic theory and interference management for a spread spectrum cellular mobile radio system”, in *Proc. IEEE ICC*, 1980.

• J. M. Aein, “Power balancing in systems employing frequency reuse”, *COMSAT Technical Review*, vol. 3, no. 2, pp. 277-299, Fall 1973.