CARLETON UNIVERSITY
Department of Systems and Computer Engineering

94.565 | **Advanced Digital Communications** | **Winter 1999**

Wednesday, February 17, 1999 **Midterm** in-class, closed-book, duration: 110 min.s

The full mark is **70** points. You can make any reasonable assumption provided that it is stated clearly.

1) (5 points) Signals $s_1(t)$ and $s_2(t)$ are shown in Fig. 1. Draw an $s_3(t)$ which is orthogonal to both $s_1(t)$ and $s_2(t)$. (*Hint:* this question can be solved by inspection.)

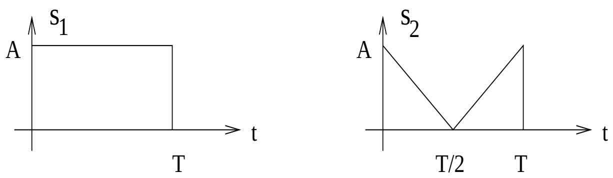

Figure 1:

2) (5 points) A (14,8) code is designed as follows: from the 8 message bits incoming to the encoder $(m_0, \cdots, m_7)$, the first and the last 4 bits are used to generate two separate (7,4) Hamming code blocks; then these two blocks are put one after the other to form a single (14,8) linear block code:

$\boxed{m_0 m_1 m_2 m_3 m_4 m_5 m_6 m_7} \quad \Longrightarrow \quad \boxed{\boxed{b_0 b_1 b_2 m_0 m_1 m_2 m_3}\boxed{b_3 b_4 b_5 m_4 m_5 m_6 m_7}}$

Find t for this (14,8) code. (*Hint:* this question can be solved by inspection.)

3) (10 points) In a (7,3) linear block code, the message bits m_0, m_1, m_2 and the parity bits $b_0, \cdots, b_3$ are related as follows:

$$b_0 = m_0 + m_1, \quad b_1 = m_0 + m_2, \quad b_2 = m_1 + m_2, \quad b_3 = m_0 + m_1 + m_2.$$

The code is designed in a non-systematic form such that the bit positions in a code vector are as follows: $\mathbf{c} = m_0 b_0 b_1 m_1 b_2 m_2 b_3$.

(a) Find **G** and **H** matrices (in non-systematic form).

(b) What is d_{min}?

(c) If this code is used for only error-detection, how many errors in a block can be detected?

4) (10 points) Consider a fading wireless channel where linear block codes are used. It is given that the bit rate after coding is 10 Kbps. Also, it is statistically known that the probability of occurence of fades with duration greater than 0.5 msec is negligible. During a fade, the received SNR is so weak that a burst of errors occur.

(a) If it is given that a (15,11) Hamming code with $t = 1$ is used, design an interleaver (with minimal dimensions) that will fix the burst error problem. Briefly explain how your interleaver works.

(b) Repeat part (a) for a (15,7) BCH code with $t = 2$ (assume that the bit rate after coding is still 10 Kbps).

5) (15 points) Consider the rate-1/2, $K = 3$, convolutional code with function generators $g_1 = [110]$ and $g_2 = [011]$.

(a) Construct the trellis diagram.

(b) Find d_{free}.

(c) This convolutional code is used for transmission over an AWGN channel with hard-decision decoding. The output of the demodulator detector is (1011101001...). Using Viterbi algorithm, find the transmitted sequence.

6) (15 points) Consider the TCM scheme shown in Fig. 2. The trellis diagram for the rate-1/2, $K = 2$, convolutional coder within the TCM scheme is also given in Fig. 2.

(a) Draw the two-state trellis corresponding to the entire TCM scheme, indicating parallel paths and the corresponding outputs.

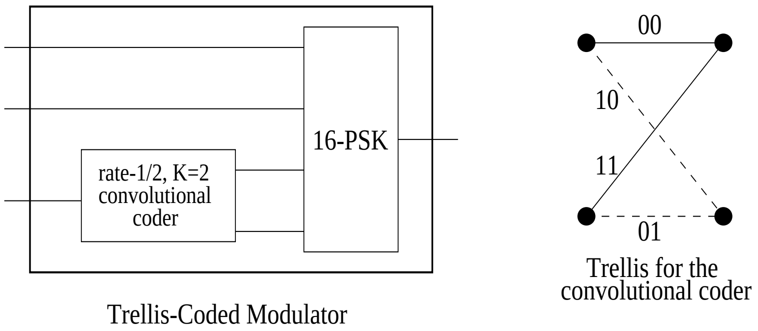

Figure 2: The TCM scheme

(b) Choosing the right signal from the 16-PSK constellation constitutes an important part of the TCM scheme. For this purpose "mapping by set partitioning" is used, as described in the lectures. A 16-PSK constellation constructed in this way is shown in Fig. 3, where only some of the signals are identified. Complete this figure, keeping the distance properties in mind. (The answer to this question in not unique.)

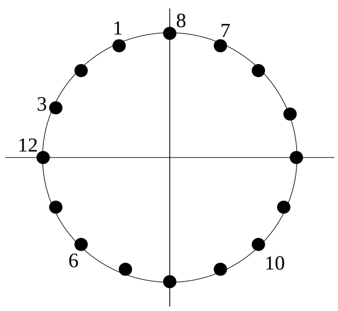

Figure 3: 16-PSK constellation

7) (10 points) A message consists 16-bits. The channel error probability is determined by the received E_b/N_0 of 7 dB, where E_b/N_0 is the information bit energy per noise spectral dansity. Assume that the information rate must be the same for all choices of coding or lack of coding. Also assume that noncoherent BFSK modulation is used over an AWGN channel (for this modulation scheme, $p = (1/2)\exp(-\mathrm{SNR}/2)$). A message error probability, P_M, of 10^{-2} is required.

(a) What is P_M if no error control coding is employed?

(b) If a (31,16) BCH code with error-correcting capability of $t = 3$ is used to encode the message, what is the probability that the decoded message will be in error? Will this code do the job?

(c) Repeat part (b) for a (63,16) BCH code with $t = 11$.