CARLETON UNIVERSITY
Department of Systems and Computer Engineering

94.565 | **Advanced Digital Communications** | **Winter 1999**

Assignment #1

Given: Monday, January 25, 1999 **Due:** Monday, February 15, 1999 (hard-deadline)

46 points (+15 points bonus)

Part I. (20 points) From text (*Digital Communications*, J. Proakis, 3rd ed., 1995).
Answer 5 of the following questions (each question has a weight of 4 points): 8.1, 8.2, 8.4(a), 8.7, 8.8, 8.12, 8.13, 8.14(a)(b), 8.15, 8.16.

Part II. (25 points) From *Digital Communications*, B. Sklar, 1988.
Questions 5.21, 5.22, 5.25, and 5.27 (modified), which are reprinted below. First three question have weights of 5 points each, and the last one has a weight of 10 points.

5.21. A (15,5) cyclic code has a generator polynomial as follows:

$$g(x) = 1 + x + x^2 + x^5 + x^8 + x^{10}$$

(a) Draw a diagram of an encoder for this code.
(b) Find the code polynomial (in systematic form) for the message $m(x) = 1 + x^2 + x^4$.
(c) Is $v(x) = 1 + x^4 + x^6 + x^8 + x^{14}$ a code polynomial in this system? Justify your answer.

5.22. Consider the (15,11) cyclic code generated by $g(x) = 1 + x + x^4$.
(a) Devise a feedback register encoder and decoder for this code.
(b) Illustrate the encoding procedure with the message vector 11001101011 by listing the states of the regiser (the rightmost bit is the earliest bit).
(c) Repeat part (b) for the decoding procedure.

5.25. Information from a source is organized in 36-bit messages that are to be transmitted over an AWGN channel using noncoherently detected BFSK modulation.
(a) If no error control coding is used, compute the E_b/N_0 required to provide a message error probability of 10^{-3}.
(b) Consider the use of a (127,36) linear block code (minimum distance is 31) in the transmission of messages. Compute the coding gain for this code for a message error probability of 10^{-3}. (*Hint*: The coding gain is defined as the difference between the E_b/N_0 required without coding and the E_b/N_0 required with coding.)

5.27. A message consists of English text (assume that each word in the message contains six letters). Each letter is encoded using the 7-bit ASCII character code. Thus, each word of text consists of a 42-bit sequence. The channel error probablity is determined by a received E_b/N_0 of 12 dB, where E_b/N_0 is the information bit energy per noise spectral density. Assume that the information rate must be the same for all choices of coding or

lack of coding. Also assume that noncoherent BFSK modulation is used over an AWGN channel.
(a) What is the probability that a word will be received in error if no error control coding is employed?
(b) If a form of repetition code is used such that each letter in each word is repeated three times, and at the receiver, majority voting is used to decode the message, what is the probability that a decoded word will be in error?
(c) If a (126,42) BCH code with error-correcting capability of of $t = 14$ is used to encode each 42-bit word, what is the probability that a decoded word will be in error?
(d) Discuss the relative error performance capabilities of the above coding schemes.

Part III. Miscellaneous.

1. (10 points) Linear and nonlinear block codes with maximum rate.
(a) Construct a $(n, 3)$ linear block code which can correct all 1 and 2 errors in a block of n (that is, $t = 2$).
(b) What is the minimal value of n, n_{min}, for the requirements given in part (a)? Develope a qualitative algorithm for the search of n_{min} (versus "trial-and-error" or "brute-force" methods).
(c) Explain, using your algorithm, how you will find n_{min} for the $(n, 4)$ code with $t = 2$ (you do not have to find n_{min}, just explain qualitatively how you will obtain n_{min}).
(d) Find the minimal n for a *nonlinear* block code of $(n, 3)$ with $t = 2$. Note that intuitively, $n_{min,nonlinear} \leq n_{min,linear}$.

2. (Bonus question, 15 points) p_B versus E_b/N_0 plots.
Plot p_B versus E_b/N_0 curves for (15,11) Hamming, (7,4) Hamming, (3,1) repetation, and (5,1) repetation codes, along with the curve for the uncoded case. Assume coherent BPSK modulation and hard-decision decoding (p_B and E_b/N_0 are the probability of bit error and the information bit energy per noise spectral density, respectively). What is the coding gain at $p_B = 10^{-5}$? Comment on the position of the coded curves with respect to the uncoded one. You can use a package like MatLab. Submit the script that you used to generate the above curves with this assignment.

Part IV. (1 point) Howlong did it take to do this assignment? Did you find the assignment relevant to the topics covered in the class?