CARLETON UNIVERSITY
Department of Systems and Computer Engineering

94.565 | **Advanced Digital Communications** | **Winter 1999**

Examiner: Professor Halim Yanıkömeroğlu

Wednesday, April 21, 1999 **Final Exam** in-class, closed-book, duration: 3.5 hours

The full mark is **100** points. Attempt all the questions.
You can make any reasonable assumption provided that it is stated clearly.

Question 1 (18.5 points) **Partial Response Signalling and PSD**

Consider the partial response signalling (correlative coding) scheme shown in Fig. 1, where a_k denotes the equally likely, independent, data sequence taking values $\pm A$.

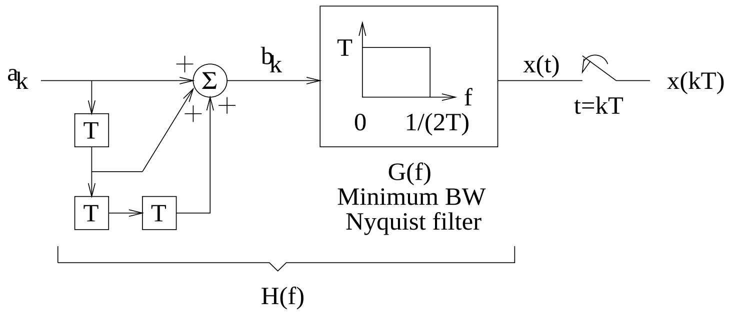

Figure 1: A partial response signalling (correlative coding) scheme.

(a) (4.5 pts) Sketch $h(t)$ indicating its values at the multiples of T.

(b) (4.5 pts) Find the autocorrelation function for b_k.

(c) (5 pts) Find the power spectral density of $x(t)$, $S_X(f)$. What is the DC power of $x(t)$?

Note that for $y(t) = \sum_m I_m p(t - mT)$,

$$S_Y(f) = \frac{1}{T}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_I(n) e^{-j2\pi f n T}$$

where $R_I(n) = E[I_m I_{m-n}]$.

(d) (4.5 pts) Find the discrete probability density function for $x(kT)$.

Question 2 (9 points) **Adaptive Equalization**

In this question, you are not required to write any equations. Give the answers in a qualitative manner.

(a) (4 pts) What is the main difference between the steepest descent (gradient) and least-mean-square (stochastic gradient) algorithms?

(b) (5 pts) Briefly compare the convergence speed and complexity of the least-mean-square and recursive least-squares algorithms (justify your comparison).

Question 3 (25.5 points) **Multipath Channels and Equalization**

The equivalent complex lowpass model of a communication system is given in Fig. 2, where $z(t)$ represents zero-mean AWGN. Assume that $g(t)$ has duration T and unit energy,

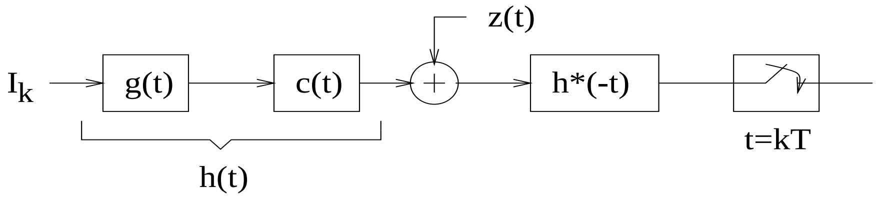

Figure 2: The equivalent complex lowpass model.

and the channel impulse response is $c(t) = a\delta(t) + b\delta(t-T)$, where a and b are complex coefficients.

(a) (4.5 pts) Find $x_k \triangleq h(t) * h^*(-t) \mid_{t=KT}$

(b) (4.5 pts) Factorize $X(z)$ (obtained from the results of part (a)) into the form

$$X(z) = (\alpha z^{-1} + \beta)(\gamma z + \delta).$$

Find $\alpha, \beta, \gamma, \delta$ in terms of a and b. Give two different possible solutions.

(c) (7.5 pts) Now, consider a channel where $a = 0.8$ and $b = 0.6$. Find the discrete whitening filter. Show the block diagrams of the *equivalent discrete-time white noise filter model*, $F(z)$, and the whitening filter, in terms of delay elements, summers and multipliers.

(d) (7.5 pts) Suppose the channel given in part c) is to be equalized by a DFE having two-tap feedforward filter (c_{-1}, c_0) and a one-tap feedback filter (c_1). The $\{c_i\}$ are optimized using the MSE criterion. Write the set of linear equations, in matrix form, that determine the optimum coefficients. For $N_0 \ll 1$, obtain $\{c_i\}_{-1}^{1}$.

(e) (1.5 pts) If the previous decisions are correct in the above DFE, can this equalizer completely eliminate the ISI?

Note: For a DFE optimized under MSE criterion which has $K_1 + 1$ taps in its feedforward section and K_2 taps in its feedback section, the $\{c_i\}_{-K_1}^{K_2}$ are given as

$$\sum_{j=-K_1}^{0} \psi_{lj} c_j = f_{-l}^*, \quad l = -K_1, \ldots, -1, 0$$

where $$\psi_{lj} = \sum_{m=0}^{-l} f_m^* f_{m+l-j} + N_0 \delta_{lj}, \quad l, j = -K_1, \ldots, -1, 0$$

and

$$c_k = -\sum_{j=-K_1}^{0} c_j f_{k-j}, \quad k = 1, 2, \ldots, K_2.$$

Question 4 (15.5 points) **CDMA**

In a direct-sequence CDMA system with K users, the probability of bit error for user j, P_{ej}, is given as

$$P_{ej} = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_j}{N_0 + \frac{2}{3N}\sum_{i=1,2,\ldots,K,i\neq j} E_i}}\right),$$

where $N_0/2$ is the power spectral density of the background AWGN, and N is the processing gain defined as the ratio of bit duration over chip duration. Both the bit and chip pulses have rectangular shapes. The bit rate is 10 Kbps, and the carrier frequency is 1 GHz. At least 24 users are to be accommodated in the system. In the calculations for the below parts, you can omit the effects of the background noise.

(a) (6 pts) Consider the case where there is perfect power control; that is, all users' signals are received with the same energy level. Find the minimum N that will satisfy a P_e requirement of 10^{-5} for each user. (For $\frac{1}{2}\text{erfc}(\sqrt{x}) = 10^{-5}$, $x = 9.6$ dB.) If the bandwidth is defined as the null-to-null bandwidth, find the required spread spectrum bandwidth.

(b) (4.5 pts) Now, consider a case where the power control is imperfect: half of the users have a received energy level which is twice that of the other half of the users; i.e., $E_1 = E_2 = \ldots = E_{12} = 2E_{13} = 2E_{14} = \ldots = 2E_{24}$. Determine the signal-to-interference ratio for each user, by using the value of N obtained in part a).

(c) (5 pts) We observe from part b) that, due to imperfect power control, for $K = 24$ some of the users are unable to meet the required P_e of 10^{-5}. For the case of imperfect power control described in part b) (i.e., $E_1 = E_2 = \ldots = E_{K/2} = 2E_{K/2+1} = 2E_{K/2+2} = \ldots = 2E_K$) find the system capacity; i.e., find the value of K (assuming K is even) for which all the K users meet the required P_e of 10^{-5}. Use the value of N obtained in part a).

Question 5 (7.5 points) **CDMA versus Coding**

We have discussed in the lectures that in an (n, k) linear block code scheme, if the bandwidth expansion is too high (that is, if k/n is much less than unity), the coding gain turns out to be negative (i.e., coding incurs a performance loss)! An example is the (5,1) repetition code. This is due to the fact that for such cases the bit energy after coding reduces significantly; although some error patterns are corrected as a result of coding, this does not compansate the extra errors made in detecting the coded bits at the receiver.

Using the above argument, your manager is claiming that since there is a significant bandwidth expansion in any CDMA scheme (note that processing gain corresponds to bandwidth expansion, and it is typically around 100), the performance would be much lower compared to a non-CDMA type. S/he attributes this performance loss to the fact that after spreading the chip energy is much lower than the non-spreaded bit energy. S/he, therefore, concludes that the performance of a CDMA scheme is inversely proportional to its processing gain.

What is the validity of your manager's claim (justify your answer)?

Question 6 (13.5 points) **Signal Space**

A signal $x(t)$ is to be approximated by a linear combination of a complete set of orthonormal functions, $\{\phi_i(t)\}_{i=1}^N$, as follows:

$$\hat{x}(t) = \sum_{i=1}^{N} a_i \phi_i(t).$$

The error due to this approximation is

$$\varepsilon(t) = x(t) - \hat{x}(t).$$

(a) (6 pts) From the first principles, derive the set of coefficients that minimizes $\|\varepsilon(t)\|^2$ ($\|.\|$ denotes the norm).

(b) (3 pts) How would you modify your answer to part a) if $\{\phi_i(t)\}_{i=1}^N$ is an *orthogonal* set (but not orthonormal)?

(c) (4.5 pts) How would you modify your answer to part a) if $\{\phi_i(t)\}_{i=1}^N$ is a *linearly independent* set (but not orthonormal)? Based on your answer, discuss why it is more convenient to use an orthonormal set rather than a linearly independent set.

Question 7 (10.5 points) **Error Control Coding**

- *Claim #1:* The minimum distance of a linear block code, d_{min}, is the smallest Hamming weight of the nonzero codeword.
- *Claim #2:* A block code with d_{min} can detect upto $d_{min} - 1$ errors in a block.

Consider the $(k+1, k)$ odd parity check code. For a k-bit message vector, the single parity bit is appended in such a way that the number of 1's in the codeword is an odd number.

- *Claim #3:* The above described odd parity check code can detect 1 error in a block of $k + 1$ bits.

(a) (6 pts) What is the truth value of the above claims? Justify your answer.

(b) (4.5 pts) Someone makes the following reasoning: Since the all-zero message vector yields a codeword with the parity bit 1, according to claim #1, $d_{min} = 1$. Then, according to claim #2, this code has no error detection capability. However, according to claim #3, it should be detecting single errors. How do you resolve this dilemma. Is the above reasoning incorrect, or the confusion is due to incorrect claims, or both? Justify your answer.
