CARLETON UNIVERSITY
Department of Systems and Computer Engineering

94.460A | **Digital Communications** | **Fall 1999**

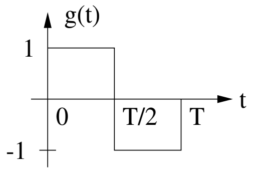
Examiner: Professor Halim Yanıkömeroğlu

Thursday, November 11, 1999 | **Midterm Exam II** | closed-book, 55 min.s

The full mark is **50** points. Attempt all the questions.

Question 1 (19 points) **Miscellaneous Short Questions**

(a) (4 pts)



Draw the matched-filter impulse response, h(t), corresponding to the transmitted pulse g(t) shown on the left.

(b) (3 pts) The bandwidth of a baseband channel allows a maximum symbol rate of $R_s = 5$ Ksymbols/sec. Since the required transmission rate is $R_b = 14$ Kbits/sec, M-ary PAM is decided to be used. Find M.

(c) (4 pts) In a baseband communication system, the equivalent channel bandwidth is $W = 1$ MHz. If raised-cosine pulses with 75% excess bandwidth are used, find the maximum symbol rate that would not cause any ISI.

(d) (4 pts) It was shown in the lecture notes that probability of error for binary NRZ and OOK schemes (in baseband AWGN channels and with matched-filter receivers) are $P_{e,NRZ} = \frac{1}{2}\,\mathrm{erfc}(\sqrt{\frac{E_{b,NRZ}}{N_o}})$ and $P_{e,OOK} = \frac{1}{2}\,\mathrm{erfc}(\frac{1}{2}\sqrt{\frac{E_{b,OOK}}{N_o}})$, respectively, where E_b denotes the bit energy. In order to have $P_{e,NRZ} = P_{e,OOK}$, how many dB's more bit energy is required for OOK scheme in comparison to NRZ type?

(e) (4 pts) A digital communications system, which is limited only by additive background noise (no ISI), is used for the transfer of data files. Assume that the receiver is designed to maximize the SNR (optimal). Although the transmission is performed at the maximum power level, the received SNR is not high enough to achieve an adequate bit error rate. What could be done to enhance the performance (that is, to increase the SNR)?

Question 2 (15 points) **Error Performance of Matched-Filter Receiver**

A baseband binary NRZ scheme is used in an AWGN channel (no ISI). The background noise is **not** zero-mean. At the receiver, the decision variable at the output of the matched-filter is $y = \pm E + n$ (depending on whether a 0 or a 1 is transmitted), where E denotes the energy of the transmitted pulse, n denotes a random variable due to noise. Assume that n is a Gaussian rv with mean μ and variance σ_n^2, that is $n : \mathrm{G}(\mu; \sigma_n^2)$.

(a) (6 pts) Find $f_Y(y|0)$ and $f_Y(y|1)$.

(b) (3 pts) Sketch $f_Y(y|0)$ and $f_Y(y|1)$ together [note that you may sketch these pdf's even if you could not have done part (a)].

(c) (2 pts) Assuming that 0's and 1's are equally likely, find the threshold level, λ, for the threshold detector.

(d) (4 pts) Write the equations for P_{miss} and $P_{false-alarm}$ in terms of integrals with proper limits (do not evaluate them).

Question 3 (16 points) **Power Spectral Density**

In a digital signaling scheme $x(t) = \sum_k b_k p(t - kT)$, the encoding rule is follows: $b_k = -A/3$ when the information bit is 0, and $b_k = 2A/3$ when it is 1. It is known a priori that 0's and 1's are independent with probabilities $P(0) = 3/4$ and $P(1) = 1/4$.

(a) (6 pts) Find $\overline{b_k}$, $\overline{b_k^2}$, and σ_b^2.

(b) (4 pts) Plot the discrete autocorrelation function $R_b(m) = \mathrm{E}(b_k b_{k+m})$.

(c) (6 pts) The PSD expression is given as

$$S_x(f) = \frac{\sigma_b^2}{T}|P(f)|^2 + \frac{\overline{b_k}^2}{T^2} \sum_{m=-\infty}^{\infty} |P(\frac{m}{T})|^2 \delta(f - \frac{m}{T}).$$

If $p(t)$ is a raised-cosine pulse with 50% excess bandwidth, sketch $S_x(f)$.
