CARLETON UNIVERSITY
Department of Systems and Computer Engineering

94.460 | **Digital Communications** | **Fall 1999**

Examiner: Professor Halim Yanıkömeroḡlu

Thursday, October 7, 1999 | **Midterm Exam I** | closed-book, no calculators, 50 min.s

The full mark is **25** points. Attempt all the questions.

Question 1 (10 points) **Sampling Theory**

Consider the following signal: $x(t) = \sum_{n=1}^{3} \cos(2\pi n 60 t)$.

(a) (2 pts) What is the Nyquist sampling rate for $x(t)$?

(b) (5 pts) $x_\delta(t)$ is the sampled version of $x(t)$ with the sampling rate $f_s = 200$ Hz. In analytical terms,

$$x_\delta(t) = \sum_{m=-\infty}^{\infty} x(mT_s)\delta(t - mT_s), \qquad T_s = 1/f_s.$$

Sketch the spectrum of $x_\delta(t)$ in the range $(-200, 200)$ Hz.

(c) (3 pts) $x'(t)$ is generated from $x_\delta(t)$ by passing it through an ideal reconstruction filter (low-pass filter). Which frequency tones will $x'(t)$ have?

Question 2 (5 points) **Time-Division Multiplexing**

• A number of voice signals are to be time-division multiplexed. The highest frequency component of each signal is 4 KHz.

• The signals are sampled at the Nyquist sampling rate, and then passed through a non-uniform, 128-level, midrise, μ-law quantizer with $\mu = 255$.

• The resulting digital data is multiplexed and sent through a high-speed link.

If the transmit capacity of the link is 2 Mbps, what the maximum the number of signals that can be multiplexed (omit any control bits)?

Question 3 (10 points) **Statistics of a Random Process**

Consider the random process

$$X(t) = A\cos(2\pi f_c t + \theta),$$

where f_c and θ are constants and A is a uniformly distributed random variable over the interval $(0, \alpha)$.

(a) (2 pts) Find $\mathrm{E}[X(t)]$.

(b) (5 pts) Find the autocorrelation $R_X(t_i, t_j) = \mathrm{E}[X(t_i)X(t_j)]$.

(**Help**: $\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$.)

(c) (2 pts) Is the process $X(t)$ wide-sense stationary? Justify.

(d) (1 pts) Is the process $X(t)$ stationary in the strict sense? Justify.
