



# Mount Ontake Landslide Simulation by the Cellular Automata Model SCIDDICA-3

S. Di Gregorio<sup>1</sup>, R. Rongo<sup>1</sup>, C. Siciliano<sup>2</sup>, M. Sorriso-Valvo<sup>3</sup>, W. Spataro<sup>1</sup>

<sup>1</sup>Department of Mathematics, University of Calabria, I-87036 Arcavacata di Rende, Italy

<sup>2</sup>Sodalia, via del Brennero, 364, I 38100 Trento, Italy

<sup>3</sup>CNR IRPI, via Verdi 248, I 87030 Roges di Rende, Italy

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**Abstract.** Cellular Automata (CA), a paradigm of parallel computing, represent an alternative to differential equations and are used for modelling and simulating very complex phenomena; CA models have been developed by our research group for the simulation of landslides. We present SCIDDICA-3, our most efficient model, a two-dimensional CA model together with the simulation results of the Mount Ontake (Japan) debris avalanche which occurred in 1984. Landslides are viewed as a dynamic system based exclusively on local interactions with discrete time and space, where space is represented by square cells, whose specifications (states) describe physical and chemical characteristics (friction, viscosity, altitude, debris thickness, etc.) of the corresponding portion of space. At the time  $t=0$ , cells are in states which describe initial conditions; the CA evolves then changing the state of all cells simultaneously at discrete times. Input for each cell is given by the states in the adjacent cells; the outflow computation from the cells gives the evolution of the phenomenon. The comparison between the real and simulated event is satisfying within limits to forecast the surface covered by debris.

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## 1 Introduction

A flow of debris and water mixture is physically described by fluid-dynamics equations and may range rheologically from approximately Newtonian liquids to brittle solids by water loss (Johnson, 1973).

Forecasting the development of a debris flow can involve almost insuperable difficulties or rough results, when the simulations are based on differential equation solution methods, since it is extremely arduous to solve the governing flow equations (e.g., the Navier-Stokes equations for viscous fluids) without making substantial simplifications.

Many approaches have been tried in order to overcome these difficulties:

Scheidegger (1973), Hsü (1975), and Davies (1982) proposed different and conflicting empirical methods in order to individuate the maximal landslide elongation

Barca et al. designed three-dimensional Cellular Automata (CA) models (1986, 1987), but computational high complexity and costs did not permit to apply the model to the simulation except in few cases of small landslides.

Sassa (1988) introduced a numerical method for simplified solutions of the debris flow equation and applied it to the Mount Ontake landslide with a rough phenomenon description.

Di Gregorio et alii (1994) developed a two-dimensional CA model, validated on the Mount Ontake landslide simulation with a good approximation level.

Deangeli, Giani and Segre (1994), Segre and Deangeli (1995) presented a three-dimensional numerical model, based on CA, for debris flows and validated it on the Mount XiKou landslide, capturing its main characteristics.

This paper describes a significant evolution of the CA model developed by Di Gregorio et alii (1994, 1995) for landslides; the new CA model is presented together with its validation, given by the simulation of the Mount Ontake landslide.

## 2 A Cellular Automata model for landslide

The following CA model for debris flow can be seen as a two-dimensional plane, partitioned into square cells of uniform size, each one embedding an identical finite automaton, i.e. the elementary automaton (ea). Each cell represents a portion of surface, whose altitude and physical characteristics of the debris column laid on it, are described by the ea states. The state evolution at constant time intervals (the CA clock) depends on the states of the neighbouring cells, according to a transition function, which simulates the

Correspondence to : Salvatore Di Gregorio

physical processes of the present debris flow. At the beginning of time (step 0) the states of the cells are specified, defining the CA initial configuration and at each next step the transition function is applied to all the ea contemporaneously, changing the configuration and obtaining the evolution of the CA.

Of course the value of the time interval, corresponding to a step of the CA and determining the clock of the CA, must be fixed according to the characteristics of the phenomenon and the cell size.

The time interval does not have to be too large so that the debris flow may go in that time interval beyond the neighbours because of its speed, but it can be chosen small as one likes. Usually it is derived subsequently to the first arrangements and proofs of the simulation.

The step effects also some parameters of the transition function, e.g. the larger the step of the CA, the larger must be the parameter describing the potential loss given by friction. Such a model is given by

$$A_{\text{debris}} = (R_2, X, S, \sigma)$$

where:

- $R_2 = \{(x, y) | x, y \in N, 1 \leq x \leq l_x, 1 \leq y \leq l_y\}$  is the set of points with integer coordinates in the finite region, where the phenomenon evolves.  $N$  is the set of natural numbers.
- The set  $X = \{(0,0), (0,1), (0,-1), (1,0), (-1,0)\}$  identifies the geometrical pattern of the cells which influence the cell state change; they are respectively the cell itself and the "north", "south", "east" and "west" cells.
- The finite set  $S$  of states of the ea is:

$$S = S_a \times S_t \times S_f \times S_r \times S_w \times S_s$$

$S_a$  is correlated to the altitude of the cell;

$S_t$  is correlated to the debris thickness of the cell;

$S_r$  is correlated to the "run up" height of the cell debris;

$S_f$  is correlated to the debris flows toward the only four neighbourhood directions.

$S_w$  is correlated to the water content in the debris of the cell;

$S_s$  is correlated to the critical value of debris thickness, above which the spooning effect occurs.

The elements of  $S_a$  are integers, which express the value of the altitude in dm; the elements of  $S_t$  are integers, which represent the debris quantity inside the cell, expressed as debris thickness in dm; the elements of  $S_f$  are integers, which express the outflow rate as debris thickness in dm; the elements of  $S_r$  are integers, which express the "run up" height in dm; at last the elements of  $S_w$  are integers, which express the water content.

•  $\sigma: S^5 \rightarrow S$  is the deterministic state transition for the cells in  $R_2$ . An outline of the main characteristics of  $\sigma$  is given in the next section.

At the beginning of time we specify the states of the cells in  $R_2$ , defining the initial configuration of the CA; the initial values of the substates  $S_t$  are zero everywhere, except the area of moving mass, determined by such values;  $S_a$  represents the

morphology, to which the moving mass has been subtracted; the initial values of the substates  $S_s$  are determined by the soil features in the cell; the initial values of the substates  $S_r$  are equal to  $S_t$ ; of course initial values of substate  $S_f$  are zero. At each next step the function  $\sigma$  is applied to all cells in  $R_2$ , so the configuration is changed in the time and the evolution of the  $A_{\text{debris}}$  is obtained.

The  $A_{\text{debris}}$  states are very numerous compared with the states usually involved in CA simulations, which deal mainly with microscopic phenomena and rarely with mesoscopic ones; landslides are macroscopic events, but they can be described with a finite number of states at any approximation: of course the number of possible altitudes and debris thicknesses is large, but finite; in such a condition the local response of each cell, i.e., the ea transition function, cannot be efficiently managed by a simple look-up table, but in a more complex way by calculating a suitable function  $\sigma$ , later illustrated.

$\sigma$  could look as a finite-difference approximation method because of its computational characters, but there is a diversity: here principles of qualitative physics are adopted in a context of acentrism, so solutions of the Navier-Stokes equation for the debris are not looked for, and some parameters are introduced without a direct correspondence in physics, while others are managed in an orthodox way.

### 3. Main characteristics of $\sigma$

The main mechanisms of the transition function concern computing the "debris outflows" from the single cell toward the cells with common sides and updating the cell water content; they are illustrated by the following algorithms written in a Pascal-like language, which calculate the values of the cell substates at time  $t+1$  according to the values of neighbouring cells substates at time  $t$ . Indexes 0, 1, 2, 3, 4 are used for the cell itself (the central cell) and neighbours "north", "east", "west", "south" respectively.

#### 3.1 $S_a$

The cell altitude (alt) at the step  $t+1$  (new\_alt) is increased by the thickness (th) of debris and once the debris water content drops below the motion liquid limit (liquid\_limit), the movement is almost blocked; at the same time the debris thickness of the cell is reset to zero.

The motion liquid limit is a critical water content that for clay-rich material is between the Atterberg liquid and the plastic limits of the clay fraction of the debris. For the whole real debris, the average real water content can be much lower than the Atterberg plastic limit, since only the water content in the active layer is important. The ratio between real average water content and motion liquid limit could be defined through comparison between simulation and physical models. In the case under study, the motion liquid limit has been set to

10, the same as the real water content of the debris [Sassa, 1988].

The type of superficial soil can be such to be eroded when the weight of the debris is larger than a critical value ( $sp\_crit\_value$ ), giving rise to a spooning effect.

When the debris thickness is enough large and there is no solidification, the cell altitude is decreased by the spooning effect (spooning) proportionally to the difference between the debris thickness and the spooning critical value; contemporaneously the thickness is increased. The following Pascal-like procedure explains better the statement. Note that this procedure may update sometime also the thickness substrate.

```

procedure new_altitude;
begin
  new_alt := alt;
  if th > sp_crit_value then
    begin
      spooning := (th - sp_crit_value) * k;
      new_alt := alt - spooning;
      th := th + spooning;
    end;
  if water_content_liquid_limit then
    begin
      new_alt := new_alt + th;
      th := 0;
    end;
end;

```

Fig. 1 Pascal-like procedure updating the substate  $S_a$

It is worth pointing out that the mass conservation is guaranteed if we consider the debris solidification and the contribution to the debris mass; consequently the altitude is modified.

### 3.2 $S_t$

The debris thickness at step  $t+1$  ( $new\_th$ ) is trivially given by the thickness at step  $t$  ( $th$ ) adding debris inflows from the neighbour cells ( $in\_th[i]$ ) and subtracting debris outflows to the neighbour cells ( $out\_th[i]$ ). Inflows and outflows are expressed in terms of thickness.

Note that the debris thickness may be also updated by water loss (§ 3.5).

### 3.3 $S_r$

Considering the high speed of debris flow, we may not neglect the kinetic energy as in the case of the classical problem of slope stability.

In order to account the transformation of kinetic energy in potential one, we must consider the maximum height that debris in a cell is able to overcome, starting from the soil.

Of course the "run up" height at the initial step is equal to the debris thickness.

The new run up height is determined considering the maximum jump of the debris inflows from the neighbours; the jump is given by the potential difference ( $pot\_diff$ ) between the central cell and a neighbour  $i$ ; the potential ( $pot[i]$ ) is given by the sum of altitude and the run up height.

So, an additional height must be added to debris thickness in order to obtain the run up height (Fig. 2).

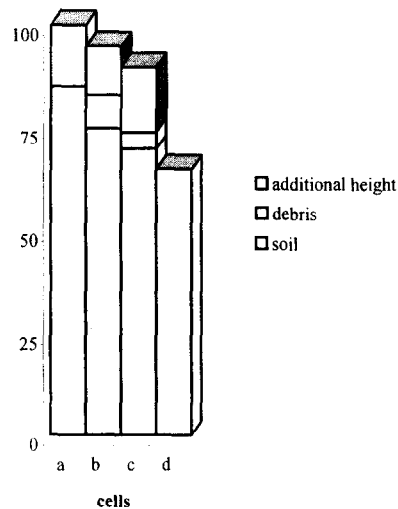


Fig. 2 A row of cells

Since we do not have a conservative system, a loss of potential ( $pot\_loss$ ) is considered for each step of computation. Such a loss, depending on the type of debris and also on the clock of the CA, is considered constant for simplicity and cannot lower the value of the run up height below the value of the debris thickness ( $th$ ). The following Pascal-like procedure (Fig. 3) explains better the statement.

```

procedure run_up_height;.....
begin
  new_run_up := run_up;
  for i := 1 to 4 do
    if in_th[i] > 0 then
      begin
        pot_diff := pot[i] - pot[0];
        if pot_diff > new_run_up then
          new_run_up := pot_diff;
      end;
  new_run_up := new_run_up - pot_loss;
  if new_run_up < th then
    new_run_up := th;
end;

```

Fig. 3 Pascal-like procedure updating the substate  $S_r$

In Fig. 2 a row of four cells (three-dimensional view) is given with the representation of altitude (soil), debris thickness (debris) and run up height (debris thickness + additional height); "pot" is given by the soil altitude + debris thickness + additional height.

### 3.4 Sf

Because of the soil friction and internal friction, only a portion of the debris in the cell can be distributed. We assume that flow can occur between the cell and the  $i$ -th neighbour only if the following condition is verified:

$$\tan \theta_{0,i} > (\gamma - \gamma_w) / \gamma \tan \phi \quad (1)$$

where  $\theta_{0,i}$  is the local slope angle between the cell and the  $i$ -th neighbour,  $\gamma$  (gamma) is the weight of the volume unit of the debris,  $\gamma_w$  (gamma\_w) is the weight of the volume unit of water,  $\phi$  (phi) is the friction angle.

The expression (1) results from the equilibrium condition of an infinite slope. The problem consists in establishing the value of the friction angle  $\phi$ . According to Sassa (1988), we identify  $\phi$  in the expression (1) with the apparent friction angle  $\phi_a$ , which is determined by:

$$\tan \phi_a > (\sigma_n - u) / \sigma_n \tan \phi_m = (1 - r_u) \tan \phi_m \quad (2)$$

where  $\phi_m$  is the internal friction angle during motion,  $\sigma_n$  is the normal stress,  $u$  is the pore pressure and  $r_u = u / \sigma_n$  is the pore pressure ratio.

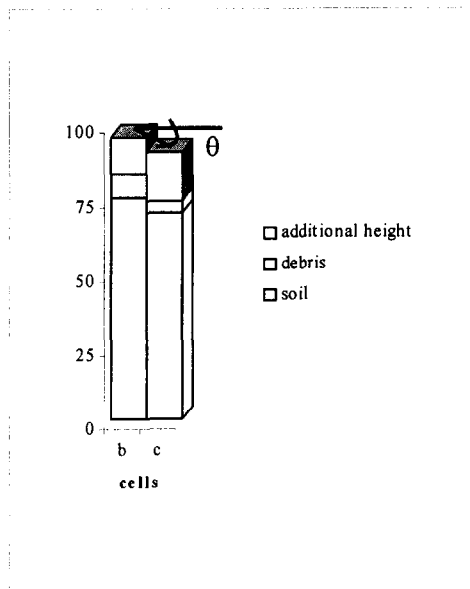


Fig. 4 Angle  $\theta$  between two cells

Typically the internal friction angle during motion is a few

degrees smaller ( $4^\circ$ - $5^\circ$ ) than the static friction angle.

The distribution of the debris in the cell ( $th[0]$ ) towards a neighbour is permitted when the expression (1) is satisfied: we first calculate the sum "pot\_sum" of potential differences, related to the cell itself and its neighbours; then, for each neighbour, we determine the tangent " $\tan\_theta[i]$ " of the angle between the cell and a neighbour  $i$  (Fig. 4).

The debris can flow toward a neighbour  $i$  if:

1)  $\tan\_theta[i]$  is equal or greater than the friction coefficient "friction\_coeff",

2) the sum of potential "pot\_sum" isn't null.

The possible debris flow "poss\_out\_th[i]" toward  $i$  is given by the potential difference  $pot[0] - pot[i]$  divided by "pot\_sum".

"step\_path" is the path covered by the debris in a CA step, when the "path" from a cell to a neighbour is greater than "step\_path" (because of the potential slope), only a "portion" of debris flow ( $out\_th[i]$ ) can reach the neighbour in each CA step; "side" is the cell side length. The following Pascal-like procedure (Fig. 5) explains better the statement.

The main difference between SCIDDICA-3 and the previous models (Di Gregorio *et alii* 1994, 1995) stays in the procedure debris\_distribution.

The threshold permitting debris flows toward the neighbours is given in the previous models by a parameter "adherence", empirically derived, concerning the central cell. A necessary condition (but not always a sufficient condition) in order to have debris flows from the central cell to neighbours is a threshold (adherence) value, smaller than the debris thickness in the central cell. Debris flows can be allowed by the previous rule in particular conditions with low values of potential differences, sometime giving rise to approximation errors.

```

procedure debris_distribution;
begin
  pot_sum := 0;
  friction_coeff := (gamma-gamma_w)/gamma*tan(phi);
  for i:=1 to 4 do
    begin
      tan_theta[i] := (pot[0]-pot[i])/side;
      if (tan_theta[i] >= friction_coeff) then
        pot_sum := pot_sum+pot[0]-pot[i];
      end
    end
  for i:=1 to 4 do
    begin
      if (tan_theta[i] >= friction_coeff) and (pot_sum < 0) then
        begin
          poss_out_th[i] := th[0]*(pot[0]-pot[i])/pot_sum;
          path := side+pot[0]-pot[i];
          portion := step_path / path;
          out_th[i] := poss_out_th[i]*portion;
        end
      else
        out_th[i] := 0;
      end;
    end;
  end;
end;

```

Fig. 5 Pascal-like procedure updating the substate Sf

This problem is solved with the new model SCIDDICA-3,

where the threshold is represented by the coefficient of the friction, derived by the physical characteristics of the debris. The necessary condition (but not always a sufficient condition), in order to have debris flows from the central cell to a neighbour, is that the threshold results smaller than the tangent of the slope angle between the two cell potentials, as previously illustrated.

### 3.5 $S_W$

Changes in cell water content are modelled as a two-step process to describe changes due to different water content debris mixing into the cell and to water loss at the ground-debris surface.

To account for water transported by debris motion, the first step considers inflows and outflows and simplifies the real situation by averaging the water content of the residual debris in the cell and the incoming flows of its neighbours.

The second step then estimates the water content drop due to losses at the surface proportional to an empirical parameter  $p$ , depending on the type of debris.

The debris thickness is also reduced according to the water loss and to debris type.

The fate of lost water is not followed, because it is supposed to be not more involved in the phenomenon.

### 3.6 $S_S$

The value of the spooning state depends on the soil in the cell and from the previous spooning effects; layers of different types of soil with different spooning values can be considered during the evolution of the phenomenon.

## 4 Implementation

Several versions of the CA model SCIDDICA (Smart Computational Innovative methoDs for Debris flow simulation with Cellular Automata, it must be read "she'dd'ekah"), based on  $A_{\text{debris}}$ , had been developed on different computer types, from PC to a parallel network of 32 transputers (S. Di Gregorio et alii, 1995).

The last version was written in the Pascal programming language and was implemented on a Macintosh Power PC 7100/66, where the simulation of the Mount Ontake landslide was performed, which will be described in the next chapter.

In this case the CA space is based on a matrix of  $224 \times 70$  cells and the landslide full simulation takes less than eight hours.

It is worth pointing out that SCIDDICA can be runned on the parallel CA environment CAMEL with high performance. This is very important in order to simulate landslide with particular accuracy (S. Di Gregorio et alii, 1996).

## 5 The Ontake Volcano Landslide Simulation.

The Mount Ontake landslide was chosen as a study case because of the large base of information and data before, during and after the event in comparison with other cases to our knowledge (in Fig. 6 the morphology before the event occurrence is shown); another reason was the possibility to compare our results directly with the results of the method developed by Sassa (1988).

### 5.1 Brief description of the event

In 1984, an earthquake triggered a  $3.6 \times 10^7$  cu.m. landslide on the slopes of the Mount Ontake Volcano, Japan (Fig.13); it moved along the Denjo river at about 20-26 m/s, with a jump of 1625 m (Sassa, 1988). The landslide initiated its movement as a translational slide (Varnes, 1978); the debris immediately broke down and the movement continued as a debris flow of huge size, which flowed into the Denjo River for a runout of ca. 13 km.

Due to the rugged morphology of the volcanic flanks, the movement of the mass was actually very complex; soon after breaking down the debris hit against the opposite slope of a tributary valley of the Denjo River, and a small part of it overtopped the slope ridge. The most of the debris flowed downstream and at a distance of ca. 5.5 km from the scar, reaching a sharp bent of the river. Due to its great momentum, part of the flowing mass climbed up the external wall of the bent, overtopped the ridge and flowed down in the siding valley. As the two streams joined at a confluence 2 km downstream, the two branches of the landslide rejoined at this confluence, and continued flowing into the valley to the final rest.

The phenomenon was surprisingly fast and long-reaching with respect to the volume of the moving mass. Sassa (1988) claims for the effect of sudden undrained loading of soil by the flowing mass, and actually it is difficult to find any other physically logic explanation.

Those landslide particularities justify that the debris water content is considered constant and the spooning effect is negligible; so the model is simplified (no transition for  $S_W$  which is considered constant and for  $S_S=0$ ) and the management of too many parameters is avoided in the validation phase.

### 5.2 Simulation results

The simulation step 0 initialises  $A_{\text{debris}}$  (Fig. 6), where  $S_a$  contains morphology data to which the moving mass is subtracted, whose data are represented in  $S_t$ ; the cell edge is 500 dm; while the time step is approximately 0.1 s.

The simulation step 250 (Fig. 7) shows that the landslide has moved away and has been enlarged more than the real event.

The simulation step 600 (Fig. 8) shows how the debris flow canalises correctly into the river bed.

The simulation step 1500 (Fig. 9) shows the debris flow before breaking into two branches.

The simulation step 3250 (Fig. 10) shows the confluence of the two branches, it occurs at the right point, respecting the right temporal succession. The width of the two branches is smaller than in the real event.

The simulation step 6000 (Fig. 11) shows that the landslide extension is almost defined.

The simulation step 12000 (Fig. 12) gives the final shape of the simulated landslide; a comparison between the real event (Fig. 13) and the simulation evidences a substantial agreement in the landslide development.

The debris path is well individuated, but the simulated flow widths are at the beginning larger while, later, narrower since the initial cohesion is not considered (all the debris is detached simultaneously); furthermore the deposit thicknesses are 15% smaller from the river bed area on.

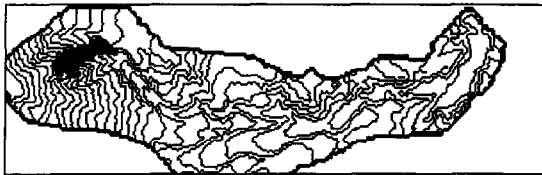


Fig. 6. Step 0 of the simulation

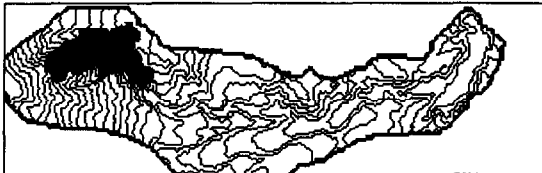


Fig. 7. Step 250 of the simulation

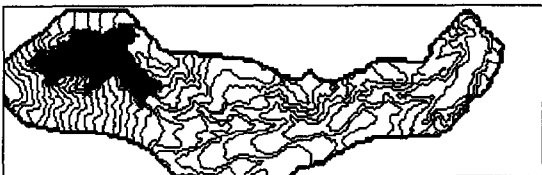


Fig. 8. Step 600 of the simulation

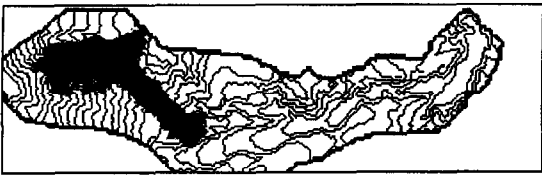


Fig. 9. Step 1500 of the simulation

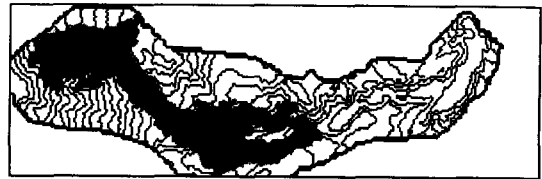


Fig. 10. Step 3250 of the simulation



Fig. 11. Step 6000 of the simulation

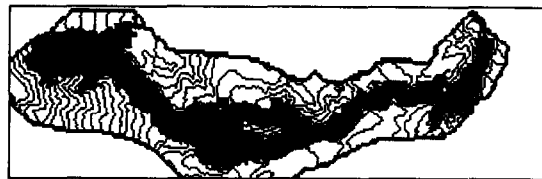


Fig. 12. Step 12000 of the simulation

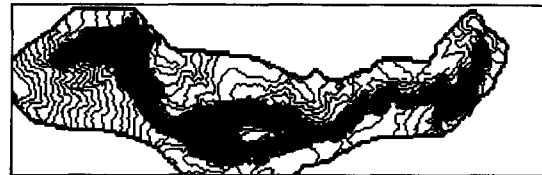


Fig. 13. The real event

## 8 Conclusions

The model represents, to our opinion, a foundation stage for CA applications to the debris flows; some model aspects need to be tested because the exceptional landslide characteristics did not permit to validate the water drop mechanism, furthermore the spooning effect was not considered.

The cell edge length was chosen too large, so there is a lack of important details, which influences the general simulation, e.g. in the initial phase the cohesion problem would be less evident with smaller cells.

The model can be considered to work well in this phase, but more tests, involving all the model aspects, are necessary with more data detail and smaller cells; more precision would be gained with an hexagonal tessellation; the algorithm must be refined in order to describe better the cohesion effects.

Some applications could be given by this type of simulation; the most interesting ones concern the fields of

intervention against the landslide risk:

- a) forecasting of the area interested by the landslide considering various initial water contents in order to locate potential risk areas and to permit the creation of microzonal maps of risk;
- b) the possibility to precisely follow the progress of an event and predict its evolution;
- c) the verification of the possible effects of human intervention on real or simulated flows in stream deviation. Introduction of data, which represent alterations of the original conditions or of the present ones (e.g., simulation of the construction of a canal or embankment), is possible during the simulation. The program can check the effects on the flow and allows for corrections where necessary.

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