SYSC 5104 – Assignment #2
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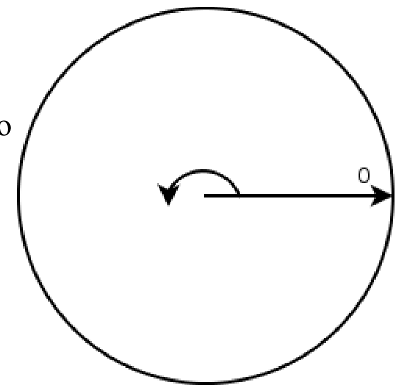
Problem Description

The Billiard Ball Machine (http://www.mirekw.com/ca/rullex_marg.html) has a very specific meaning in the field of cellular automata, and uses the Margolus neighbourhood. The Margolus partitioning scheme requires some cleverness to use with CellDEVS, and the particular scheme used in the reference cannot represent movements on anything other than a 45 degree diagonal. However, a billiard table in general is a system that can be well represented by a cellular system. There is a 'bouncing ball' CellDEVS model in the samples library, but it to seems to be limited to movements on a 45 degree angle. The model described herein attempts to model a wider range of ball behaviour, and some possible extensions will be discussed.

Each cell is either empty or contains a ball, represented on a two-dimensional plane as the direction it is about to move (1 to 4 for North to West, clockwise) and an angle (between 0 and 360) representing its overall direction. A von Neumann neighbourhood is used for most ball movement, as diagonal moves between squares are not necessary, although a 2-plane Moore neighbourhood is actually used to support the behaviour of the pockets.



As the space is broken down into squares, a ball must use a sequence of lateral and vertical moves to represent diagonal movement (e.g. if we want to move towards the top left we must sometimes move to the top, and sometimes to the left, in a proportion related to the angle of motion). Using the tan function we can determine how often we should pick one or the other direction, and generate a random number to pick one. For instance, if want to travel at 60 degrees toward the top left (measured from straight left), we should move to the top tan(60) = 1.732 times as often as to the left. Normalizing, if a random number is generated between 0 and 1 we should move up if it is greater than 0.634, and to the left if it less. If the cells are at a fine enough granularity the motion should approximate actual diagonal motion. Two balls can be in the system at once, and rules are generated for handling collisions.

The borders are unwrapped, with special behaviour used to have the balls ricochet appropriately. Corners will be treated as pockets, and a ball that reaches the corner will disappear from the system. As they disappear new balls are introduced with a randomized westerly direction from the head of the table.

Cell Model Specification

Formally, the model is specified as follows:

$$\text{Ball Zone} = < \text{I}, X, Y, \text{Xlist}, \text{Ylist}, \eta, \text{N}, \{m, n, o\}, \text{C}, \text{B}, \text{Z}, \text{select}>$$

- Xlist – {(8,28,0)}
- Ylist – ∅
- $\eta = 19$
- I = < P^X, P^Y>, with P^X = { <X(8,28,0), binary>}; P^Y = { ∅ };
- N = (0,0,0),(-1,0,0),(0,-1,0),(1,0,0),(0,1,0),(0,0,1),(-1,0,1),(0,-1,1),(1,0,1),(0,1,1),(0,0,-1),(-1,0,-1),(0,-1,-1),(1,0,-1),(0,1,-1),(1,1,1),(1,-1,1),(-1,1,1),(-1,-1,1)
- X = BallPresence{1, 2, 3, 4}, CollisionPresence{12, 13, 14, 23, 24, 34}, WallPresence{-1, -2, -3, -4}, NewBall{400}
- Y = BallPresence{1, 2, 3, 4}, CollisionPresence{12, 13, 14, 23, 24, 34}, WallPresence{-1, -2, -3, -4}, AnglePresence{1..360}, NewBall{400}
- m = 16, n = 32, o = 2
- B = {∅} (Border is not wrapped, but rules are the same)
- C = {C_{ijk} / {i ∈ [1,14], j ∈ [1,30], k ∈ [0,0]} ∪ {i ∈ [1,14], j ∈ [0,0], k ∈ [0,0]} ∪ {i ∈ [0,0], j ∈ [1,30], k ∈ [0,0]} ∪ {i ∈ [1,14], j ∈ [31,31], k ∈ [0,0]}} ∪ {i ∈ [15,15], j ∈ [1,30], k ∈ [0,0]}}
- Z =

$P_{ijk}\,{}^{Y}{}_{1} \rightarrow P_{i,j-1,k}\,{}^{X}{}_{1}$

$P_{ijk}\,{}^{Y}{}_{2} \rightarrow P_{i+1,j,k}\,{}^{X}{}_{2}$

$P_{ijk}\,{}^{Y}{}_{3} \rightarrow P_{i,j+1,k}\,{}^{X}{}_{3}$

$P_{ijk}\,{}^{Y}{}_{4} \rightarrow P_{i-1,j,k}\,{}^{X}{}_{4}$

$P_{ijk}\,{}^{Y}{}_{5} \rightarrow P_{ijk}\,{}^{X}{}_{5}$

$P_{ijk}\,{}^{Y}{}_{6} \rightarrow P_{i+1,j+1,k+1}\,{}^{X}{}_{6}$

$P_{ijk}\,{}^{Y}{}_{7} \rightarrow P_{i+1,j-1,k+1}\,{}^{X}{}_{7}$

$P_{ijk}\,{}^{Y}{}_{8} \rightarrow P_{i-1,j+1,k+1}\,{}^{X}{}_{8}$

$P_{ijk}\,{}^{Y}{}_{9} \rightarrow P_{i-1,j-1,k+1}\,{}^{X}{}_{9}$

$P_{ijk}\,{}^{Y}{}_{10} \rightarrow P_{i,j-1,k+1}\,{}^{X}{}_{10}$

$P_{ijk}\,{}^{Y}{}_{11} \rightarrow P_{i+1,j,k+1}\,{}^{X}{}_{11}$

$P_{i,j+1,k}\,{}^{Y}{}_{1} \rightarrow P_{ijk}\,{}^{X}{}_{1}$

$P_{i-1,j,k}\,{}^{Y}{}_{2} \rightarrow P_{ijk}\,{}^{X}{}_{2}$

$P_{i,j-1,k}\,{}^{Y}{}_{3} \rightarrow P_{ijk}\,{}^{X}{}_{3}$

$P_{i+1,j,k}\,{}^{Y}{}_{4} \rightarrow P_{ijk}\,{}^{X}{}_{4}$

$P_{ijk}\,{}^{Y}{}_{5} \rightarrow P_{ijk}\,{}^{X}{}_{5}$

$P_{i+1,j+1,k+1}\,{}^{Y}{}_{6} \rightarrow P_{ijk+1}\,{}^{X}{}_{6}$

$P_{i+1,j-1,k+1}\,{}^{Y}{}_{7} \rightarrow P_{ijk+1}\,{}^{X}{}_{7}$

$P_{i-1,j+1,k+1}\,{}^{Y}{}_{8} \rightarrow P_{ijk+1}\,{}^{X}{}_{8}$

$P_{i-1,j-1,k+1}\,{}^{Y}{}_{9} \rightarrow P_{ijk+1}\,{}^{X}{}_{9}$

$P_{i,j+1,k}\,{}^{Y}{}_{10} \rightarrow P_{ijk+1}\,{}^{X}{}_{10}$

$P_{i-1,j,k}\,{}^{Y}{}_{11} \rightarrow P_{ijk+1}\,{}^{X}{}_{11}$

$P_{ijk}\ Y_{12} \rightarrow P_{i,j+1,k+1}\ X_{12}$ $\quad$ $P_{i,j-1,k}\ Y_{12} \rightarrow P_{ijk+1}\ X_{12}$
$P_{ijk}\ Y_{13} \rightarrow P_{i-1,j,k+1}\ X_{13}$ $\quad$ $P_{i+1,j,k}\ Y_{13} \rightarrow P_{ijk+1}\ X_{13}$
$Pijk\ Y_{14} \rightarrow Pijk{+}1\ X_{14}$ $\quad$ $Pijk\ Y_{14} \rightarrow Pijk{+}1\ X_{14}$
$P_{ijk}\ Y_{15} \rightarrow P_{i,j-1,k-1}\ X_{15}$ $\quad$ $P_{i,j+1,k}\ Y_{15} \rightarrow P_{ijk-1}\ X_{15}$
$P_{ijk}\ Y_{16} \rightarrow P_{i+1,j,k-1}\ X_{16}$ $\quad$ $P_{i-1,j,k}\ Y_{16} \rightarrow P_{ijk-1}\ X_{16}$
$P_{ijk}\ Y_{17} \rightarrow P_{i,j+1,k-1}\ X_{17}$ $\quad$ $P_{i,j-1,k}\ Y_{17} \rightarrow P_{ijk-1}\ X_{17}$
$P_{ijk}\ Y_{18} \rightarrow P_{i-1,j,k-1}\ X_{18}$ $\quad$ $P_{i+1,j,k}\ Y_{18} \rightarrow P_{ijk-1}\ X_{18}$
$Pijk\ Y_{19} \rightarrow Pijk{-}1\ X_{19}$ $\quad$ $Pijk\ Y_{19} \rightarrow Pijk{-}1\ X_{19}$

- Select = (0,0,0),(-1,0,0),(0,-1,0),(1,0,0),(0,1,0),(0,0,1),(-1,0,1),(0,-1,1),(1,0,1),(0,1,1),(0,0,-1),(-1,0,-1),(0,-1,-1),(1,0,-1),(0,1,-1),(1,1,1),(1,-1,1),(-1,1,1),(-1,-1,1)

Angle Zone = < I, *X*, *Y*, Xlist, Ylist, η, N, {m, n, o}, C, B, Z, select>

- Xlist – ∅
- Ylist – ∅
- η = 19
- I = < P^x, P^y>, with $P^x = \{ \varnothing \}$; $P^y = \{ \varnothing \}$;
- N = (0,0,0),(-1,0,0),(0,-1,0),(1,0,0),(0,1,0),(0,0,1),(-1,0,1),(0,-1,1),(1,0,1),(0,1,1),(0,0,-1),(-1,0,-1),(0,-1,-1),(1,0,-1),(0,1,-1),(1,1,1),(1,-1,1),(-1,1,1),(-1,-1,1)
- X = BallPresence{1, 2, 3, 4}, CollisionPresence{12, 13, 14, 23, 24, 34}, WallPresence{-1, -2, -3, -4}, NewBall{400}
- Y = AnglePresence{1..360}
- m = 16, n = 32, o = 2
- B = {∅} (Border is not wrapped, but rules are the same)
- C = $\{C_{ijk}$ / $\{i \in [1,14], j \in [1,30], k \in [0,0]\} \cup \{i \in [1,14], j \in [0,0], k \in [1,1]\} \cup \{i \in [0,0], j \in [1,30], k \in [1,1]\} \cup \{i \in [1,14], j \in [31,31], k \in [1,1]\}\} \cup \{i \in [15,15], j \in [1,30], k \in [1,1]\}\}$
- Z =

$P_{ijk}\ Y_{1} \rightarrow P_{i,j-1,k}\ X_{1}$ $\quad$ $P_{i,j+1,k}\ Y_{1} \rightarrow P_{ijk}\ X_{1}$
$P_{ijk}\ Y_{2} \rightarrow P_{i+1,j,k}\ X_{2}$ $\quad$ $P_{i-1,j,k}\ Y_{2} \rightarrow P_{ijk}\ X_{2}$
$P_{ijk}\ Y_{3} \rightarrow P_{i,j+1,k}\ X_{3}$ $\quad$ $P_{i,j-1,k}\ Y_{3} \rightarrow P_{ijk}\ X_{3}$
$P_{ijk}\ Y_{4} \rightarrow P_{i-1,j,k}\ X_{4}$ $\quad$ $P_{i+1,j,k}\ Y_{4} \rightarrow P_{ijk}\ X_{4}$

$P_{ijk}\ Y_5 \rightarrow P_{ijk}\ X_5$	$P_{ijk}\ Y_5 \rightarrow P_{ijk}\ X_5$
$P_{ijk}\ Y_6 \rightarrow P_{i+1,j+1,k+1}\ X_6$	$P_{i+1,j+1,k+1}\ Y_6 \rightarrow P_{ijk+1}\ X_6$
$P_{ijk}\ Y_7 \rightarrow P_{i+1,j-1,k+1}\ X_7$	$P_{i+1,j-1,k+1}\ Y_7 \rightarrow P_{ijk+1}\ X_7$
$P_{ijk}\ Y_8 \rightarrow P_{i-1,j+1,k+1}\ X_8$	$P_{i-1,j+1,k+1}\ Y_8 \rightarrow P_{ijk+1}\ X_8$
$P_{ijk}\ Y_9 \rightarrow P_{i-1,j-1,k+1}\ X_9$	$P_{i-1,j-1,k+1}\ Y_9 \rightarrow P_{ijk+1}\ X_9$
$P_{ijk}\ Y_{10} \rightarrow P_{i,j-1,k+1}\ X_{10}$	$P_{i,j+1,k}\ Y_{10} \rightarrow P_{ijk+1}\ X_{10}$
$P_{ijk}\ Y_{11} \rightarrow P_{i+1,j,k+1}\ X_{11}$	$P_{i-1,j,k}\ Y_{11} \rightarrow P_{ijk+1}\ X_{11}$
$P_{ijk}\ Y_{12} \rightarrow P_{i,j+1,k+1}\ X_{12}$	$P_{i,j-1,k}\ Y_{12} \rightarrow P_{ijk+1}\ X_{12}$
$P_{ijk}\ Y_{13} \rightarrow P_{i-1,j,k+1}\ X_{13}$	$P_{i+1,j,k}\ Y_{13} \rightarrow P_{ijk+1}\ X_{13}$
$Pijk\ Y_{14} \rightarrow Pijk+1\ X_{14}$	$Pijk\ Y_{14} \rightarrow Pijk+1\ X_{14}$
$P_{ijk}\ Y_{15} \rightarrow P_{i,j-1,k-1}\ X_{15}$	$P_{i,j+1,k}\ Y_{15} \rightarrow P_{ijk-1}\ X_{15}$
$P_{ijk}\ Y_{16} \rightarrow P_{i+1,j,k-1}\ X_{16}$	$P_{i-1,j,k}\ Y_{16} \rightarrow P_{ijk-1}\ X_{16}$
$P_{ijk}\ Y_{17} \rightarrow P_{i,j+1,k-1}\ X_{17}$	$P_{i,j-1,k}\ Y_{17} \rightarrow P_{ijk-1}\ X_{17}$
$P_{ijk}\ Y_{18} \rightarrow P_{i-1,j,k-1}\ X_{18}$	$P_{i+1,j,k}\ Y_{18} \rightarrow P_{ijk-1}\ X_{18}$
$Pijk\ Y_{19} \rightarrow Pijk-1\ X_{19}$	$Pijk\ Y_{19} \rightarrow Pijk-1\ X_{19}$

- Select = (0,0,0),(-1,0,0),(0,-1,0),(1,0,0),(0,1,0),(0,0,1),(-1,0,1),(0,-1,1),(1,0,1),(0,1,1),(0,0,-1),(-1,0,-1),(0,-1,-1),(1,0,-1),(0,1,-1),(1,1,1),(1,-1,1),(-1,1,1),(-1,-1,1)

Pocket Zone = < I, *X*, *Y*, Xlist, Ylist, η, N, {m, n, o}, C, B, Z, select>

- Xlist – ∅
- Ylist – {(0,0,0),(15,0,0),(0,31,0),(15,31,0)}
- η = 19
- I = < P^x, P^y>, with P^x = { ∅ }; P^y = { <Y(0,0,0), binary>, <Y(15,0,0), binary>, <Y(0,31,0), binary>, <Y(15,31,0), binary>};
- N = (0,0,0),(-1,0,0),(0,-1,0),(1,0,0),(0,1,0),(0,0,1),(-1,0,1),(0,-1,1),(1,0,1),(0,1,1),(0,0,-1),(-1,0,-1),(0,-1,-1),(1,0,-1),(0,1,-1),(1,1,1),(1,-1,1),(-1,1,1),(-1,-1,1)
- X = AnglePresence{1..360}, PocketStates{-5,-6}
- Y = PocketStates{-5,-6}, NewBall{1}
- m = 1, n = 1, o = 2
- B = {∅} (Border is not wrapped, but rules are the same)
- C = {(0,0,0),(15,0,0),(0,31,0),(15,31,0),(0,0,1),(15,0,1),(0,31,1),(15,31,1)}
- Z =

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$P_{ijk}\ Y_1 \rightarrow P_{i,j-1,k}\ X_1$ | $P_{i,j+1,k}\ Y_1 \rightarrow P_{ijk}\ X_1$
$P_{ijk}\ Y_2 \rightarrow P_{i+1,j,k}\ X_2$ | $P_{i-1,j,k}\ Y_2 \rightarrow P_{ijk}\ X_2$
$P_{ijk}\ Y_3 \rightarrow P_{i,j+1,k}\ X_3$ | $P_{i,j-1,k}\ Y_3 \rightarrow P_{ijk}\ X_3$
$P_{ijk}\ Y_4 \rightarrow P_{i-1,j,k}\ X_4$ | $P_{i+1,j,k}\ Y_4 \rightarrow P_{ijk}\ X_4$
$P_{ijk}\ Y_5 \rightarrow P_{ijk}\ X_5$ | $P_{ijk}\ Y_5 \rightarrow P_{ijk}\ X_5$
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$P_{ijk}\ Y_8 \rightarrow P_{i-1,j+1,k+1}\ X_8$ | $P_{i-1,j+1,k+1}\ Y_8 \rightarrow P_{ijk+1}\ X_8$
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$Pijk\ Y_{14} \rightarrow Pijk+1\ X_{14}$ | $Pijk\ Y_{14} \rightarrow Pijk+1\ X_{14}$
$P_{ijk}\ Y_{15} \rightarrow P_{i,j-1,k-1}\ X_{15}$ | $P_{i,j+1,k}\ Y_{15} \rightarrow P_{ijk-1}\ X_{15}$
$P_{ijk}\ Y_{16} \rightarrow P_{i+1,j,k-1}\ X_{16}$ | $P_{i-1,j,k}\ Y_{16} \rightarrow P_{ijk-1}\ X_{16}$
$P_{ijk}\ Y_{17} \rightarrow P_{i,j+1,k-1}\ X_{17}$ | $P_{i,j-1,k}\ Y_{17} \rightarrow P_{ijk-1}\ X_{17}$
$P_{ijk}\ Y_{18} \rightarrow P_{i-1,j,k-1}\ X_{18}$ | $P_{i+1,j,k}\ Y_{18} \rightarrow P_{ijk-1}\ X_{18}$
$Pijk\ Y_{19} \rightarrow Pijk-1\ X_{19}$ | $Pijk\ Y_{19} \rightarrow Pijk-1\ X_{19}$

- Select = (0,0,0),(-1,0,0),(0,-1,0),(1,0,0),(0,1,0),(0,0,1),(-1,0,1),(0,-1,1),(1,0,1),(0,1,1),(0,0,-1),(-1,0,-1),(0,-1,-1),(1,0,-1),(0,1,-1),(1,1,1),(1,-1,1),(-1,1,1),(-1,-1,1)

State Representation in CellDevs

Ball Directions:

- 1 – North
- 2 – East
- 3 – South
- 4 – West

Ball Collisions:

- 12 – Balls resulting north and east
- 13 – Balls resulting north and south
- 14 – Balls resulting north and west
- 23 – Balls resulting east and south
- 24 – Balls resulting east and west
- 34 – Balls resulting south and west

Bumpers:

- -1 North bumper
- -2 East bumper
- -3 South bumper
- -4 West bumper

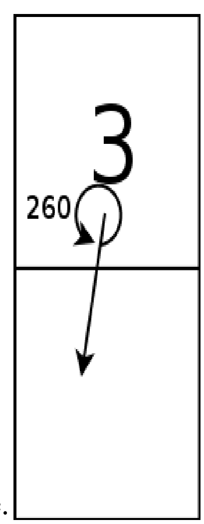
Angles:

- 1..360

Pockets:

- -5 Pocket in normal state
- -6 Temporary value on ball sinking to ensure correct updating of drawlog

Ball Behaviour



In order to ensure consistent results the direction a ball is going to leave the cell must be computed on entry, rather than exit. Otherwise the two cells that may receive the ball would calculate different random values, and both or neither may receive the ball. In normal behaviour a cell checks if a ball is about to enter it (say, from the north). If so it reads the angle θ (say 260) from the upper plane, and compares a random value between 0 and 1 to the result of $1/(1 + \tan\theta)$, after moving the angle to the current quadrant. Based on the result it is decided whether the next move will again be to the south (more likely in this case, as 260 is closer to 270 than 180) or to the west. This becomes the new value of the cell. On the upper plane the angle is merely moved into position above the new value.

Upon collision with a bumper two things must happen – the angle must be translated according to basic laws of reflection and the ball must exit the cell in the direction opposite to its entry. The ball and angle are updated in place and exit on the next cycle.

Collisions are handled in two cases. If a ball is trying to directly enter a cell with a ball already in it, it reflects off using the same rules as for collision with a bumper. If two balls try to enter the same cell on the same cycle the cell enters a temporary collision state. This collision state is the concatenation of the opposites of the two entry states (e.g. if balls enter from the north and west, it will be east (2) concatenated with south (3). On the next update balls will leave the cell in those two directions with randomized angles in that quadrant.

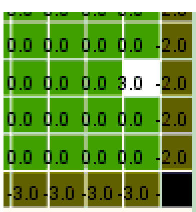
The last case of interest is falling into a corner pocket. When a corner cell detects an angle in the table cell diagonally adjacent to it, it checks to see if it is moving toward its quadrant. If so, it sends a message to a cell near the head of the table (8,28,0). A new ball is generated with a random angle toward the west. At the same time the table cell from which the ball sinks detects the same condition and resets its state to 0.

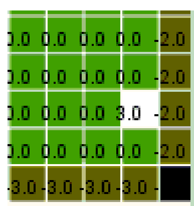
Test Runs

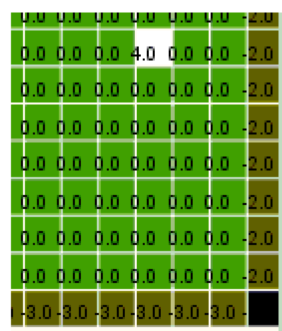
See the read.me file for directions on running the different tests.

Run 1 – Entering a Pocket

This first test run (Pool1.bat) shows a ball heading toward a pocket, disappearing, and finally reappearing at the head line with a westward heading.

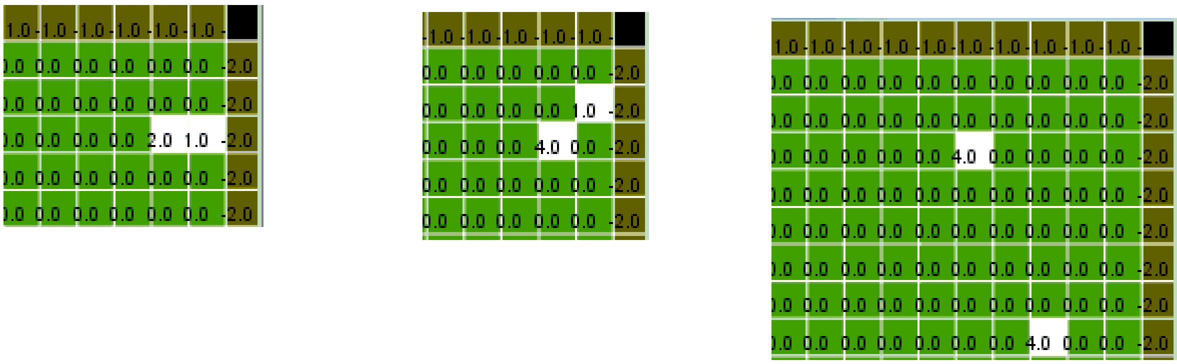






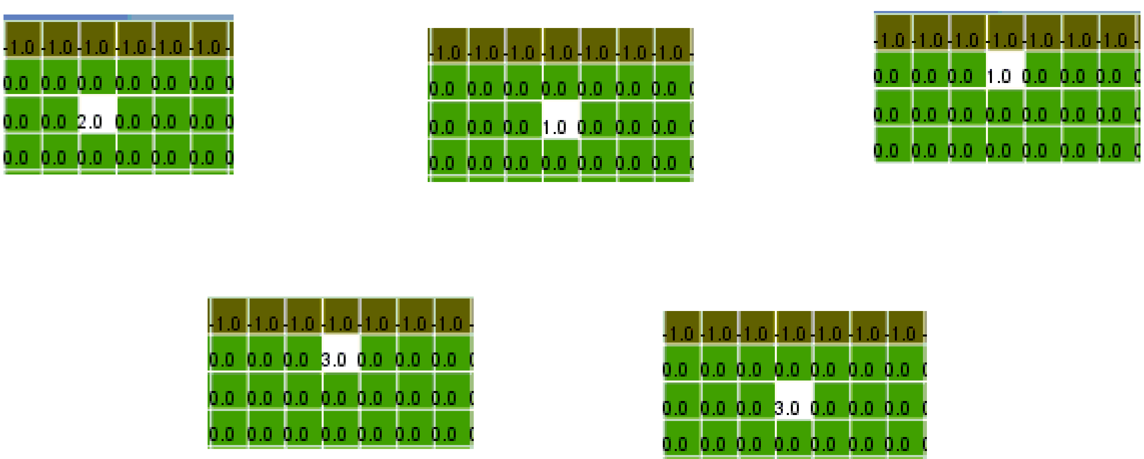
Run 2 – Collision Near a Pocket

In this example (Pool2.bat), an eastward (direction 2) ball ricochets off of a northward (direction 1) ball near the north-east pocket. It reverses it direction, while the other ball falls into the pocket and appears at the head of the table.



Run 3 – Standard Bumper Play

Pool3.bat takes a more detailed look at a simple bumper collision. A ball approaching at a sharp angle (89 degrees) takes one step eastward before driving northward into the bumper. It switches its direction to south, and its angle to 271 degrees, and starts moving southward.



Run 4 – An East-West Collision

Executing Pool4.bat shows an immediate collision at position (5,4) between an east and a west facing ball. They correctly rebound off of each other, and collide again between 3 and 4 seconds into the simulation.