The Application of Cellular Automata to the Consumer's Theory: Simulating a Duopolistic Market

Sobei H. Oda[1], Kouhei Iyori[2], Miura Ken[3], and Kanji Ueda[4]

[1] Faculty of Economics, Kyoto Sangyo University, Motoyama, Kamigamo, Kita-ku, Kyoto 603-8501, Japan (oda@cc.kyoto-su.ac.jp).
[2] Graduate School of Science and Technology, Kobe University, 1-1 Rokkodai-cho, Nada-ku, Kobe 657-8501, Japan (iyori@mi-2.mech.kobe-u.ac.jp).
[3] Graduate School of Science and Technology, Kobe University, 1-1 Rokkodai-cho, Nada-ku, Kobe 657-8501, Japan (miuraken@mbox.kyoto-inet.or.jp).
[4] Faculty of Engineering, Kobe University, 1-1 Rokkodai-cho, Nada-ku, Kobe 657-8501, Japan (ueda@mech.kobe-u.ac.jp).

Abstract. This paper presents a cellular automata model of a duopolistic market with consumers' leraning and network externalities. The model produces various dynamics of the market. In particular, if the user cost can locally be different, it generates such rich dynamics that aggregate models could not explain. The results of simulations also suggest that the long-run consequence of duopolistic competition may crucially depends on the initial condition.

1 Introduction

When you buy an application, you may probably take account of how long you have used it and how many of others will use it. Even if an application with higher performance is available, you may hesitate to change it from the one you are familiar with, suspecting understandably that mastering a new application may require considerable time and effort. You may however abandon the use of your favorite application if increasingly many of your friends and colleagues use another one, fearing naturally that adhering to it may make it difficult to exchange data and programs with them. We should like to examine such markets where consumers consider these things: consumers' learning by doing and network externality, which plays an important role in the so-called information-oriented society.

This paper, which describes a duopolistic market with network externality and consumers' learning by doing, is a generalisation of our previous one (Oda et al [3]) does, which examines the dynamics of a monopolistic market with network externality. In fact both are cellular automata models; what is new in the present work are only the existence of a rivalling product and the dependence of the consumers' reservation prices for products on their past purchasing behaviour. Although consumers do not like move around, their behaviour is influenced by

X. Yao et al. (Eds.): SEAL'98, LNCS 1585, pp. 454–461, 1999.

their experience so that our model has become similar to the CA+Agent models of (Epstein and Axtell [1]).[1]

We shall explain our model in Section 2 and mention a few results of its simulation in Section 3. Owing to the introduction of consumer's learning and a rivalling product, the dynamics of the market has become much more complex: in addition to the drastic change of the final equilibrium by a small change in the initial condition, it is often observed that the market goes on changing in a complicated — not simple but not random — manner.

2 The model

Let us assume the following.

1. There are M^2 consumers in a closed society. Every consumers has a personal computer for which two operating systems are available. To use an OS, each consumer must make a new or renewal contract at its supplier at the beginning of every week. We designate $X(m,n,t) = 1$ if Consumer m ($m = (m_1, m_2)$, $1 \leq m_1 \leq M$ and $1 \leq m_2 \leq M$) contracts with the supplier of OSn ($n = 1$ or 2) for Week t ($t = 0, 1, 2, \ldots$) and $X(m,n,t) = 1$ if he or she does not.

2. The utility which Consumer m obtains from using his or her computer for Week t is

$$U(m,t) = \max_{n \in (1,2)} \left(X(m,n,t)U(m,n,t) + \alpha X(m,3-n,t)U(m,3-n,t) \right) \quad (1)$$

where α is a constant while $U(m,n,t)$ represents Consumer m's utility from using OSn alone. Here $0 \leq \alpha < 1$ is assumed because using two operating systems does not brings in twice as much utility as using one.

3. Consumer's utility from using an OS consists of three terms:

$$U(m,n,t) = U_{min} + \theta(U_{max} - U_{min})L(m,n,t) + (1-\theta)(U_{max} - U_{min})N(m,n,t) \quad (2)$$

where U_{min}, U_{max} and θ are given constants ($0 \leq U_{min} \leq U_{max}$ and $0 \leq \theta \leq 1$). Here the first term of (2) stands for the basic utility that a beginner can readily obtain from standing alone computer usage.

4. The second term of (2) represents the effect of consumers' learning by doing: one can obtain more utility from the same OS as he or she uses it longer. Here

[1] The other new point is that each consumer's neighbourhood is probabilistically determined according to the method developed by (Markus and Hess [2]). Yet it does not seem to make significant effects at least in the simulations mentioned in this paper.

$L(m,n,t)$ stands for the skill for using OSn that Consumer m has acquired till time t (the beginning of Week t), which is defined as

$$L(m,n,t)\begin{cases} = L(m,n,0) & \text{for } t = 0 \\ = \lambda\sum_{k=1}^{t}(1-\lambda)^k \max(X(m,n,t-k), \beta X(m,3-n,t-k)) & \text{for } t \geq 1 \end{cases} \tag{3}$$

where λ and β are given constants $(0 \leq \lambda \leq 1)$ while $L(m,n,0)$ are all given as a part of the initial condition $(0 \leq L(m,n,0) \leq 1)$. Here λ stands for the speed of skill depreciation, while β represents the substitutability between the two operating systems: the increase of the skill for using an OS by using the other OS alone is 100β percent of its increase by using the OS. Note that the second term of (2) is regarded as the product of the degree of skill accumulation $L(m,n,t)$ and its absolute weight on the total consumer's utility $\theta(U_{max} - U_{min})$, because $0 \leq L(m,n,t) \leq 1$, $\lim_{t\to\infty} L(m,n,t) = 1$ if $X(m,n,0) = X(m,n,1) = \ldots = 1$, and $\lim_{t\to\infty} L(m,n,t) = 0$ if $X(m,n,0) = X(m,n,1) = \ldots = 0$.

5. The third term of (2) stands for the effect of network externality, which is determined by

$$N(m,n,t) = \frac{\sum_{i\in\Omega(m)} \max(X(i,n,t), \gamma X(i,3-n,t))}{|\Omega(m)|}. \tag{4}$$

Here γ is a given constant $(0 \leq \gamma \leq 1)$; $\Omega(m)$ represents the set of Consumer m's neighbours:

$$\Omega(m) = \{\text{Consumer } i |\ \text{dis}(i,m) < R\} \tag{5}$$

where R is a given constants $(1 < R)$; $|\Omega|$ stands for the number of Consumer m's neighbours. It is tacitly assumed that in our model cells (consumers) are arranged so that those who exchange more information are nearer. That is to say, in our terms neighbours are not those who live in neighbourhood but those who share the same interest.

We can find some similarities in the second and the third term of (2). First, since $0 \leq N(m,n,t) \leq 1$, we can regard the third term as the product of the degree of network externality $N(m,n,t)$ and its absolute weight on the total consumer's utility $(1-\theta)(U_{max} - U_{min})$. Secondly, β and γ play a similar role: β is smaller if consumers can use both operating systems in a more similar way, while γ is smaller if users of different operating systems can more easily exchange data and programs. Thirdly, $\Omega(m)$ corresponds to λ: the former sets the contemporary boundary to network externality while the latter limits the benefit from past experience.

6. Consumers follow a simple adoptive behaviour: they calculate $N(m,n,t)$ on the supposition that $X(i,n,t) = X(i,n,t-1)$ for all $i \in \Omega(m)$. In other words, at time t Consumer m expects the following utility for Week t:

$$\hat{N}(m,n,t)\begin{cases} = \hat{N}(m,n,0) & \text{for} t = 0 \\ = \frac{\sum_{i\in\Omega(m)} max(X(i,n,t-1),\gamma X(i,3-n,t-1))}{|\Omega(m)|} & \text{for} t \geq 0 \end{cases} \tag{6}$$

where $\hat{N}(m,n,0)$ are given as the other part of the initial condition ($0 \leq \hat{N}(m,n,0) \leq 1$). We also define $\hat{U}(m,n,t)$ by replacing $N(m,n,t)$ with $\hat{N}(m,n,t)$ in (2) and $\hat{U}(m,t)$ by replacing $U(m,n,t)$ with $\hat{U}(m,n,t)$ in (1). Here we have explained how consumers expect their weekly utility $\hat{U}(m,t)$ at the beginning of each week.

7. Consumer's cost for using a computer is given by

$$C(m,t) = X(m,1,t)P_1 + X(m,2,t)P_2 \tag{7}$$

where P_n represents cost for using OSn. In the next section it will be assumed to be constant for Examples 1, 2 and 3 of the next section while it will be regarded as

$$P(m,n,t) = Q(n,t) + R(m,n,t) \tag{8}$$

for Examples 4, 5, 6 and 7. Here $Q(n,t)$ stands for the rental fee for using OSn while $R(m,n,t)$ represents the consumer m's fees for using OSn applications. The former decreases as the total number of the users of the OS increases, while the latter decreases as the number of the consumer m's neighbours who use the OS:

$$Q(n,t) = Q_{n\min} + (Q_{n\max} - Q_{n\min})x(n,t) \tag{9}$$

$$R(m,n,t) = R_{n\min} + (R_{n\max} - R_{n\min})y(m,n,t) \tag{10}$$

$$x(n,t) = rZ(n,t-1) + \frac{x(n,t-1)}{1+r} \tag{11}$$

$$y(m,n,t) = rW(m,n,t-1) + \frac{y(m,n,t-1)}{1+r} \tag{12}$$

$$Z(n,t-1) = \frac{\sum_{\text{all } m} X(m,n,t-1)}{M^2} \tag{13}$$

$$W(m,n,t-1) = \frac{\sum_{l\in\Omega(m)} X(l,n,t-1)}{|\Omega(m)|}. \tag{14}$$

Here $Q_{n\max}$, $Q_{n\min}$, $R_{n\max}$, $R_{n\min}$ and r are all given positive constants.

8. At time t Consumer m calculates

$$\hat{V}(m,n,t) = \hat{U}(m,t) - C(m,t) \tag{15}$$

for all the four possible combinations of $X(m,1,t)$ and $X(m,2,t)$: $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$, and chooses the combination that maximises $\hat{V}(m,n,t)$ as $(X(m,1,t), (m,2,t))$.

3 Simulations

Let us show some results of simulations for the following value of parameters and the set of the initial condition: $M = 50$, $R = 2$, $U_{min} = 0.2$, $U_{max} = 0.4$, $\alpha = \beta = \gamma = 0$, $\lambda = 0.5$ and $L(m,n,0) = 0.5$ for all m and n. In the following figures, black points, gray points and white points represent OS1 users, OS2 users and non-users respectively (no consumer use both operating systems simultaneously in the following examples).

3.1 Examples 1, 2 and 3

These are cases where P_n are given constants ($P_1 = P_2 = 0.25$). Since all the parameters are common to both operating systems, their technical performance is the same. Fail or success totally depends on the distribution of the initial users.

Example $k(1 \leq k \leq 3)$ is more advantageous for OS2 unconditionally than Example $k-1$. Because the initial distribution of the OS1 users are common to the three examples, while initial OS2 users are chosen so that an OS2 initial user in Example $k(1 \leq k \leq 3)$ is an OS2 initial user in Example $k-1$. In addition, the initial condition for every example is chosen so that all consumers will user an OS if the initial users of the other OS does not exist.

The long-run consequence of competition is quite understandable: in Example 1 OS1 dominates the whole markets; in Example 2 OS1 and OS2 shares the market; in Example 3 OS2 monopolises the market.

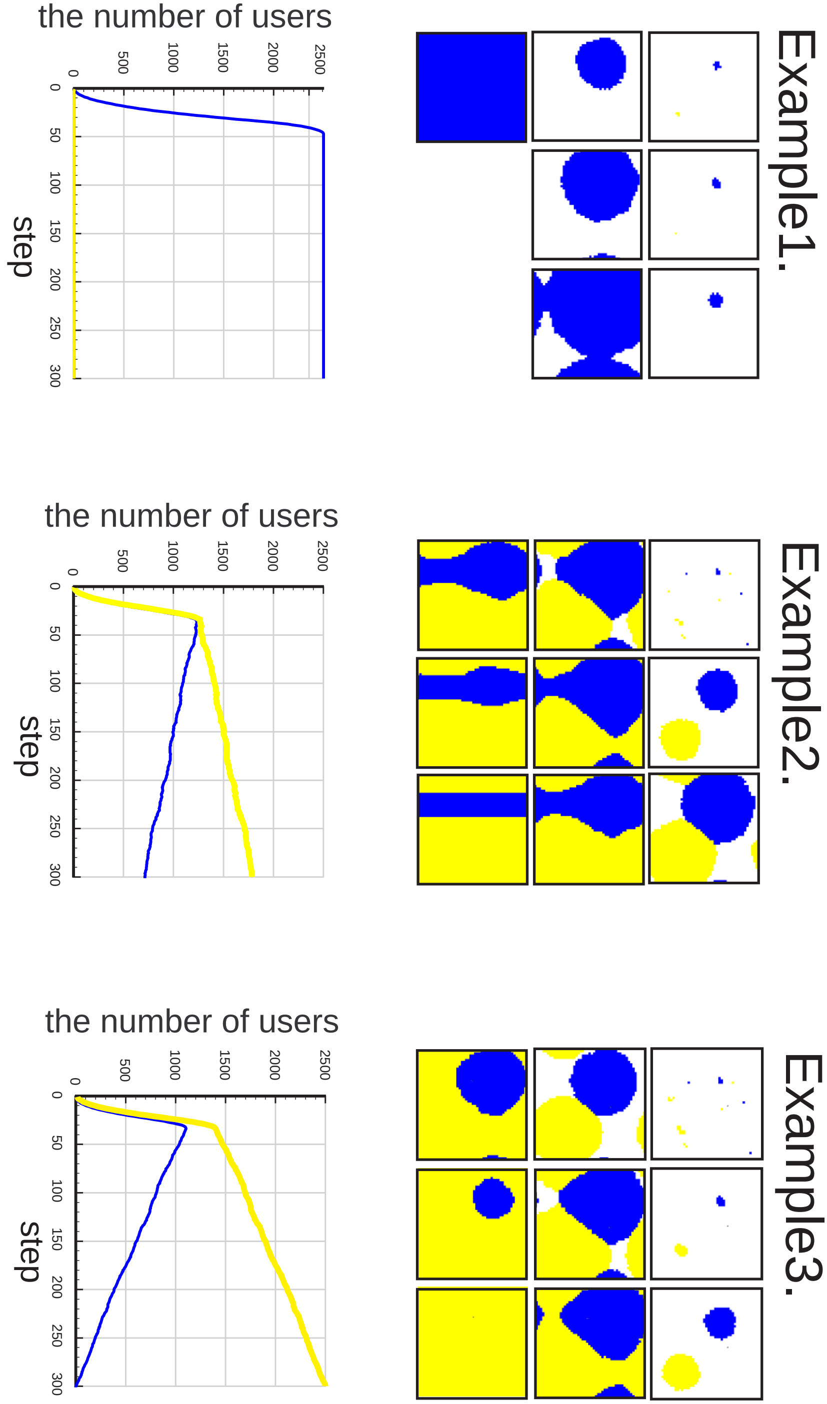
Example 3 seems noteworthy. OS1 users and OS2 users rapidly increase almost at the same rate till every consumer uses either product, but then the former gradually decrease and disappear in the end. Yet neither products' properties nor consumers' behaviour has changed when the market is saturated. Both the rapid diffusion of OS1 and its fade-out are explained by the same value of parameters and the same utility functions.

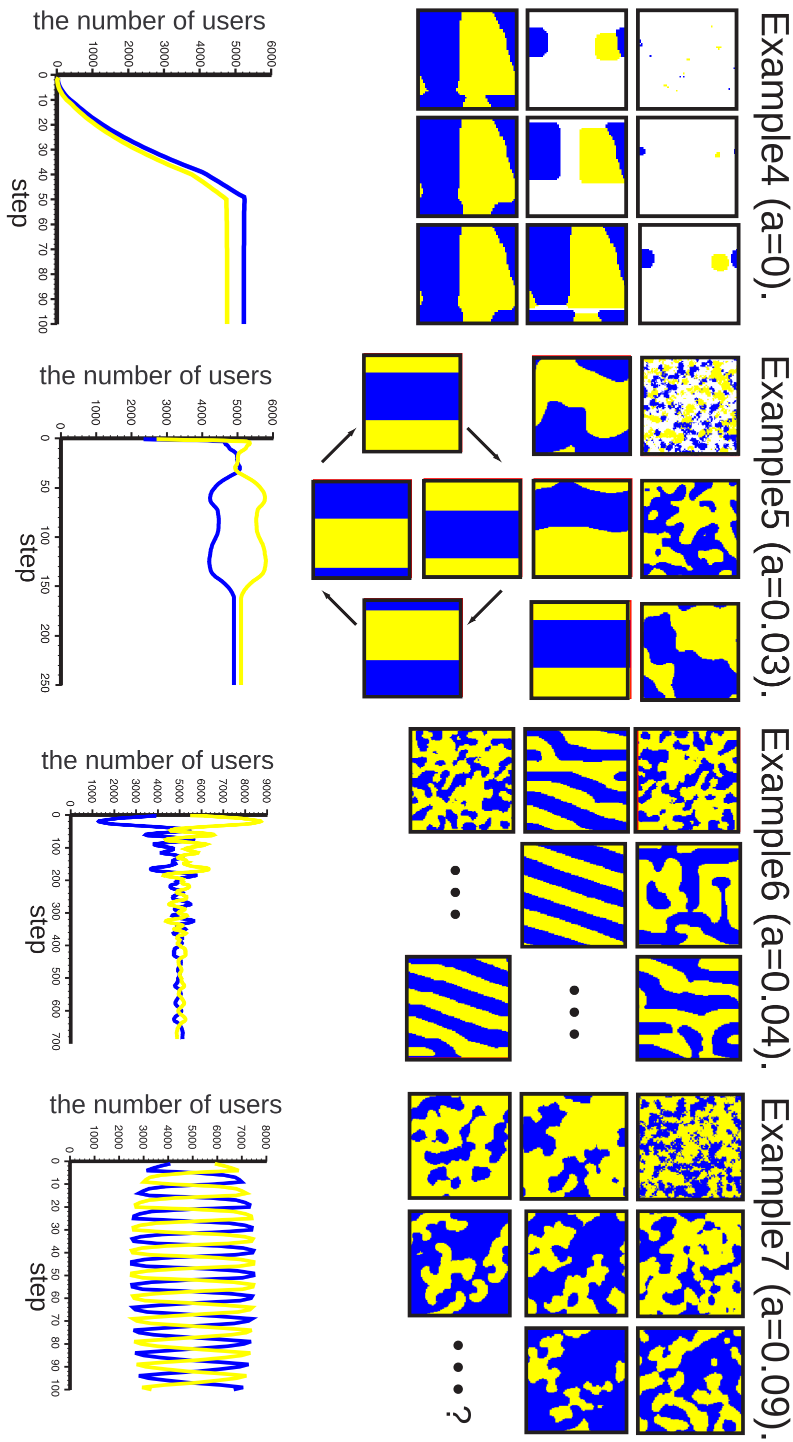
3.2 Examples 4, 5, 6 and 7

Let us examine cases where the price of the same product may locally be different. Let us assume that $R_{n\max} = R + a$ and that $R_{n\min} = R - a$. Since $R_{n\max} - R_{n\min} = 2a$, the greater a is, the larger P_i can locally differ. Figures 4, 5, 6 and 7 show cases where a equals 0, 0,03, 0.04 and 0.09 respectively (all the other parameters and the initial condition are common to all the four cases).

The dynamics of the distribution of the users of the two OSs (the black-gray patterns in the figures) qualitatively changes according as a increases. For $a = 0$, the black-gray patterns are finally fixed (every consumer goes on using either OS in the long run). For $a = 0.03$, the black and gray belts move rightwards for ever(every consumer continue to change the product he or she buys cyclically). For $a = 0.04$, the dynamics is most interesting: after 300 weeks, winding black and gray stripes (from upper right to lower left) emerge and continue to move (from upper right to lower left) with their shapes changing for 1000 weeks. Then the shifting diagonal stripes suddenly disappear; then no steady changing patterns can apparently be seen even approximately for thousands weeks; then again suddenly the shifting diagonal stripes appear.

For $a = 0.09$, the black-gray patterns change unsteadily at least as long as the simulation is observed. In short, the larger a is, the more difficult it is to predict each consumer's behaviour in the long run.


Example1.
Example2.
Example3.
the number of users
step
0
500
1000
1500
2000
2500
0
50
100
150
200
250
300



Example4 (a=0).
Example5 (a=0.03).
Example6 (a=0.04).
Example7 (a=0.09).
the number of users
step
the number of users
step
the number of users
step
the number of users
step

4 Concluding Remarks

Though having made a small number of simulations for very limited combinations of parameters and the initial condition, we have found both results of simulations which could and could not be explained by some aggregate model. This suggests that the cellular automata model of oligopolistic market may have much richer dynamics than the aggregate model describes. We hope that our model and simulations could contribute to connecting individual consumers' interaction and the dynamics of aggregate values in oligopolistic markets.

This project "Methodology of Emergent Synthesis"(JSPS-RFTF96P00702) has been supported by the Research for the Future Program of the Japan Society for the Promotion of Science.

References

1. Epstein, J.M. and Axtell, R.: Growing Artificial Society; Social Science From the Bottom Up. Brooking Institution Press (1996)
2. Markus, M. and Hess, B.: Isotropic cellular automaton for modeling excitable media. Nature **347** (1990) no.6288, 56-58
3. S.H. Oda, K. Miura and K. Ueda.: The application of cellular automata to network externality in consumer's theory: a generalisation of life game. *Artificial Life Five* in Langton, C.G. and Shimohara, K. (ed). MIT Press (1997) 473-480