Enriching the formalism of coloured Petri nets for modelling alternative structural configurations of a discrete event system: disjunctive CPN

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Abstract: Discrete event systems (DESs) provide an approximate approach for dealing with certain types of real systems. DES has been extensively and successfully used in modelling and simulation of technological systems and processes. In this field, coloured Petri nets (CPNs) have arisen as a practical formalism for modelling discrete event systems, widely used thanks to the easiness of their application, as well as the compactness of the resulting models, and the availability of computer software, ready to be used for modelling, simulation, theoretical analysis, as well as performance evaluation. This paper derives from the CPN, a formalism focused on modelling discrete event systems with alternative structural configurations. That is to say, systems with freedom degrees in their structure, which should be solved by decision-makers, deducing the best configuration among a set of alternative structures.

Keywords: disjunctive coloured Petri nets; alternative structural configurations; manufacturing facility; design; decision support systems; discrete event systems; DESs.

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1 Introduction

Simulation has led to a wide range of methodologies for the acquisition of knowledge related to the models and, hence, the real systems and processes they represent (Longo and Nicoletti, 2015; Nicoletti et al., 2015). It is very common that the models of systems and processes of technological interest present freedom degrees whose configuration can profit from optimal behaviours (Mota and Piera, 2011). Simulation permits to calculate the ability of a potential solution as a solution for the configuration of the DES (Music et al., 2008; Narciso et al., 2010; Wainer, 2016).

Certain freedom degrees relate to the structure of the model, hence, there are different structural alternative configurations for the system to be modelled. It is in this area that the disjunctive coloured Petri nets (CPNs) find their place. Conventional CPN are not specifically suitable for modelling this kind of systems. Disjunctive CPNs can profit from the properties of compacting the model of the system by folding similar subnets in applications, such as the design of discrete event systems (DESs), where the structure presents freedom degrees that need to be solved by adequate decisions (Latorre-Biel and Jiménez-Macías, 2012).

In this approach, simulation can be used for calculating a quality parameter for every potential solution of the problem of finding a suitable structural configuration for the modelled system or process (Latorre et al., 2013b, 2014c; Music, 2009).

CPNs is a well-known formalism (Piera and Music, 2011), based on the paradigm of the Petri nets (Silva, 1993), for the development of models of DESs with subsystems presenting structural similarities (Jensen and Kristensen, 2009; Jimenez and Perez, 2004). The folding of these common structures, described by means of ordinary or generalised Petri nets, leads to compact and easy-to-understand models. This is a main advantage of CPNs, which makes them very popular among practitioners (Xiao and Ming, 2011; Zaitsev and Shmeleva, 2011; Jimenez et al., 2014; Piera et al., 2004).

The folding process mentioned in the previous paragraph transfers the redundant information of repeated subsystems into attributes of the tokens. The values of the attributes of a given coloured token allow knowing through which one of the original subsystems it belongs, even though in the coloured model there is a single subsystem, equivalent to a set of them in the original model (David and Alla, 2005).

Dealing with models of DESs, such as those attainable by means of the formalism of the CPNs, allows developing processes of structural analysis, and specially performance evaluation, simulation, and optimisation for decision making (Mújica et al., 2010). Petri nets are not the only formalism able to cope with these tasks, as it can be seen in Bruzzone and Longo (2010) and Longo et al. (2013), however, they have been applied with success in many applications: generalised Petri nets (Latorre and Jiménez, 2013a, 2013b; Latorre et al., 2013a), as well as CPNs (Piera and Music, 2011; Zaitsev and Shmeleva, 2011; Piera et al., 2004).

The disjunctive CPNs can be considered as an extension of the CPNs, which make them able to cope with the modelling of a DES with alternative structural configurations (Latorre et al., 2014a; Latorre et al., 2010a, 2010b). This modelling may be very useful for the task of designing a DES, where some freedom degrees in the structure of the system in process of being designed lead to a set of alternative configurations for the structural freedom degrees (Latorre and Jiménez, 2012). This formalism has been proven very useful for obtaining compact models for decision making (Latorre et al., 2014b).

This paper presents the disjunctive CPNs, provides some of their characteristics, as well as an algorithm to construct a model of a DES based on the mentioned formalism, and explains an example for illustrating its applicability.

2 Disjunctive CPNs

In the present section, some definitions are provided as preparation for the introduction of the disjunctive CPNs.

As it has been explained before, the main improvement of this formalism with respect to the CPN is the ability to intuitively model alternative structural configurations, which are very common in the design of DESs.

A common and very natural way to represent the alternative structures of the Petri net model of a given DES consists of developing an independent model for every structure. This approach leads to a much less compact model than a disjunctive CPN and can be implemented by a set of Petri nets called alternative Petri nets, which verify the following property.

Definition 1: Mutually exclusive evolution.

Given two different Petri nets R and R'. They are said to have mutually exclusive evolutions if it is verified:

1 If $\mathbf{m}(R) \neq \mathbf{m}_0(R) \Rightarrow \mathbf{m}(R') = \mathbf{m}_0(R')$

2 If $\mathbf{m}(R') \neq \mathbf{m}_0(R') \Rightarrow \mathbf{m}(R) = \mathbf{m}_0(R)$.

□

Based on the previous property, it is possible to define a set of alternative Petri nets in the following way.

Definition 2: Set of alternative Petri nets.

Given a set of Petri nets $S_R = \{R_1, \ldots, R_n\}$, S_R is said to be a set of alternative Petri nets if $n > 1$ and $\forall i,\ j \in \mathbb{N}^*$ such that $i \neq j$, $1 \leq i, j \leq n$, it is verified that

1 R_i and R_j have mutually exclusive evolution.

2 $\mathbf{W}(R) \neq \mathbf{W}(R')$. That is to say, the structure of the Petri nets, represented by their incidence matrices, is different.

R_i is called the i^{th} alternative Petri net of S_R.

□

The definition of alternative Petri nets provides an intuitive, but non-efficient, way to represent a DES with alternative structural configurations. This approach consists of detailing the different and exclusive models that arise when the model of the system with freedom degrees is associated to every alternative configuration.

A possible way to abstract the concept of alternative structural configurations consists of considering a set of exclusive entities. They can be defined as stated in the following definition, based on the concept of alternative Petri nets.

Definition 3: Monotypic set of exclusive entities.

Given a DES D, a monotypic set of exclusive entities associated to D is a set $S_x = \{X_1, \ldots, X_n\}$, which verifies that

1 The elements of S_x are exclusive, that is to say, only one of them can be chosen as a consequence of a decision.

2 $\forall i,\ j \in \mathbb{N}^*,\ i \neq j$ and $1 \leq i, j \leq n$ it is verified that $X_i \neq X_j$.

3 The elements of S_x are of the same type.

4 $\exists f: S_x \rightarrow S_R$ such that

4.a $S_R = \{R_1, \ldots, R_n\}$ is a set of alternative Petri nets, feasible models of D.

4.b f is a bijection $\Rightarrow \forall X_i \in S_x\ \exists!\ f(X_i) = R_i \in S_R$ such that R_i is a feasible model for D and $\forall R_i \in S_R\ \exists!\ f^{-1}(R_i) = X_i \in S_x$.

□

In order to adapt the formalism of the CPNs by including a mechanism able to represent a set of exclusive entities, several solutions can be found.

One of this solutions is a set of Boolean choice colours, as stated in the following definition.

Definition 4: Set of Boolean choice colours.

$S_C = \{c_1, c_2, \ldots, c_n \mid c_i$ is Boolean and $\exists!\ c_i =$ true, $i \in \mathbb{N}^*$, $1 \leq i \leq n \wedge c_j =$ false $\forall j \neq i,\ j \in \mathbb{N}^*,\ 1 \leq j \leq n\ \}$, and the assignment $c_i =$ true is the result of a decision.

□

In CPM ML language, a standard in the description of CPN models, the set of Boolean choice variables would lead to the following sentences.

First, the colour set for a single Boolean choice variable is defined:

```
colset CHOICE = bool;
```

In a disjunctive CPN, it is expected that a token bears the complete information that corresponds to a decision, since a given decision implies the possibility for the evolution of a Petri net to mark certain places and not others. The complete information of a decision requires the knowledge of the value associated to every Boolean choice variable, hence, the choice colour set of an token is a n-tuple of so many Boolean values as the cardinality of the set of Boolean choice colours S_C.

```
colset DECISION = product CHOICE*CHOICE
*...*CHOICE;
```

where the colour set choice appears |SC| times.

If we consider an example where |SC| = 5, and the decision that makes c2 = 1, a marking in a place, where there were 3 tokens would be:

```
3`(false, true,false, false, false)
```

As it can be seen, this way to represent a decision is not very compact and does not seem to be the most efficient way to do it in case that |SC| is a large number.

For this reason, another way to represent a set of excusive entities by means of the colour of a token is explained in the following definition.

Definition 5: Natural choice colour.

A natural choice colour is a pair formed by a natural number $c \in C \subseteq \mathbb{N}^*$ and the set C, (c, C) where the actual value of c is assigned as a result of a decision.

□

In CPM ML language, the natural choice colour would lead to the following sentences.

First, the colour set for a single Boolean choice variable is defined:

```
colset DECISION = int with 1..3;
```

As it has been explained before, in a disjunctive CPN it is expected that a token bears the complete information that corresponds to a decision. In this case, the complete information of a decision requires the knowledge of the value associated to the variable c.

If we consider the previous example where $|S_C| = 5$, and the decision that makes $c = 2$, a marking in a place, where there were three tokens would be:

```
3`2
```

As it can be seen, this way to represent a decision is more compact than the one based in a set of Boolean choice variables.

In Latorre et al. (2014b), it has been proven that the conditions of the definition of monotypic set of exclusive entities are verified for any set of Boolean choice variables or any natural choice colour.

One interesting property of the disjunctive CPN is described in the following.

Definition 6: Monochrome choice marking.

Let $R = \langle N, \mathbf{m}_0 \rangle$ be a CPN system.

Let us consider a feasible marking **m** of R, reached from the initial marking $\mathbf{m}_0$ when the sequence of transitions $\sigma(R)$ is fired.

Let S_C be a set of Boolean choice colours such that $|S_C| = n$.

Let (c, C) be a natural choice colour.

If every token of **m** verifies that c is constant and $\forall\ c_i \in S_C$, c_i is constant, then the marking m of the Petri net system R is said to be a monochrome choice marking.

□

This property can be interpreted as the fact that a decision on the structure of a Petri net model with freedom degrees should be kept constant during its evolution. In other words, once a decision has been made, the monochrome choice marking guarantees that the evolution of the system is coherent with this decision and there is not a mixture between different alternative structural configurations, which would lead a false set of reachable states.

From the previous statement, it might seem that the model of a system with changing structure could not be represented by a disjunctive CPN. However, this situation can be considered taking into account that the evolution of the system as a sequence of decisions, where any decision is kept constant in the period, when the structure of the real DES remains constant.

Furthermore, the property of the monochrome choice marking does not constrain the number of different colours a disjunctive Petri net can associate to its different tokens, simultaneously. In fact, a disjunctive CPN can have tokens with different non-choice colours, that is to say, colours not related to the choice of an exclusive entity of the model.

Once the previous considerations have been done, it is possible to provide with a definition for a disjunctive CPN.

Definition 4: Disjunctive CPN

A disjunctive CPN $R = \langle N, \mathbf{m}_0 \rangle$ is a 12-tuple

$$CPN = \langle P, T, F, \mathbf{m}_0, \Sigma, V, c, g, e, i, S_\alpha, S_{val\alpha} \rangle,$$

where

1 P is a finite set of places.

2 T is a finite set of transitions T such that $P \cap T = \emptyset$.

3 $F \subseteq P \times T \cup T \times P$ is a set of directed arcs.

4 $\mathbf{m}_0$ is the initial marking that is a monochrome choice marking.

5 Σ is a finite set of non-empty colour sets, such that verifies one of the following two conditions:

  5a $\exists S_C$ set of Boolean choice variables such that $S_C \in \Sigma$.

  5b $\exists (c, C)$ a natural choice colour such that $C \in \Sigma$.

6 V is a finite set of typed variables such that type$[v] \in \Sigma$ for all variables $v \in V$.

7 $c: P \rightarrow \Sigma$ is a colour set function that assigns a colour set to each place.

8 $g: T \rightarrow EXPR_V$ is a guard function that assigns a guard to each transition t such that type$[g(t)]$ = Boolean.

9 $e: F \rightarrow EXPR_V$ is an arc expression function that assigns an arc expression to each arc a such that type$[e(a)] = c(p)_{MS}$, where p is the place connected to the arc a.

10 S_α is a set of undefined parameters.

11 $S_{val\alpha}$ is a set of feasible combination of values for the undefined parameters.

And it is verified that $\forall\ \mathbf{m} \in rs(N, \mathbf{m}_0)$, **m** is a monochrome choice marking and every token of **m** verifies that c is constant and $\forall c_i \in S_C$, c_i is constant.

□

As a result of this section, a formalism based on the CPN, the disjunctive CPNs, is stated. This formalism can be used directly to construct the model of a system with alternative structural configurations. This is the case, for example, of a DES in process of being designed, when a decision on its final structure should already be made. In fact, the resulting model might be much more compact than an equivalent set of alternative Petri nets, since there is the possibility that the different structures present shared subsystems, which can be folded, and hence compacted, in the resulting model based on a disjunctive CPN.

The following section describes an algorithm to construct a disjunctive CPN model of a DES with alternative structural configurations.

3 Modelling algorithm

In order to produce a disjunctive CPN model for a DES with alternative structural configurations, it is very convenient to use a systematic methodology. The following algorithm is proposed for the case of a DES with alternative structural configurations, where every configuration is modelled by means of a low-level Petri net (non-coloured) and the resulting set of alternative Petri nets presents a certain number of shared subnets.

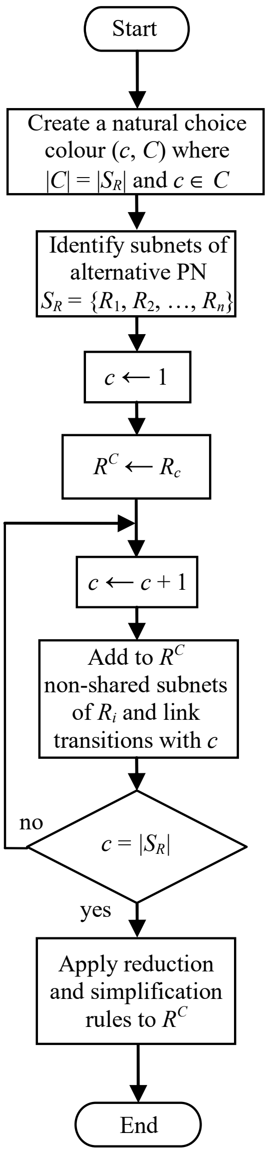
In the aforementioned algorithm, an iterative procedure is described for constructing a single disjunctive CPN.

Figure 1 and Figure 2 provide two algorithms to obtain the model of a DES with alternative structural configurations using the formalism of the disjunctive Petri nets. The first algorithm is prepared to cope with a disjunctive Petri net, where the exclusive entities are represented by means of a set of choice colours, while the second algorithm is referred to as the definition of a natural choice colour instead.

For the first algorithm, let us consider, $S_R = \{R_1, R_2, \ldots, R_n\}$, a set of n alternative Petri nets, where $n \in N$, and N is the set of natural numbers, and let us create a set of choice colours $S_C = \{c_1, c_2, \ldots, c_n\}$, such as $|S_C| = |S_R|$. As a consequence of having the same cardinality both sets, it is possible to define a bijection between them and, hence, to associate one and only one choice colour from S_C to every alternative Petri net from S_R. As a general rule it will be associated c_i to R_i, where $1 \le i \le n = |S_R| = |S_C|$.

For the second algorithm, let us consider the set of n alternative Petri nets, and let us create a natural choice colour (c, C), such as $C \subseteq N^*$, $|C| = |S_R|$ and $c \in C$. Analogously, since both sets present the same cardinality both sets, it is possible to define a bijection between them and, hence, to associate one and only one value from C to every alternative Petri net from S_R. As a general rule it will be associated $c = i$ to R_i, where $1 \le i \le n = |S_R| = |C|$.

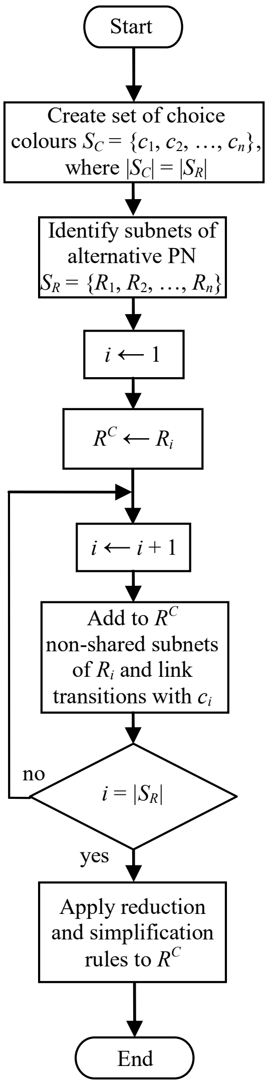
Figure 1 Algorithm for constructing a disjunctive CPN from a set of alternative Petri nets using a natural choice colour



The first of the stages consists of decomposing the different alternative Petri nets into subnets and link transitions. A fast and usually effective way of coping with this task consists of considering as subnets the Petri net models of physical subsystems present in the manufacturing facility.

As a second stage, it is envisaged assigning the first alternative Petri net R_1 to the disjunctive CPN R^C. In this process, the link transitions are associated to a guard function that consists of the choice variable corresponding to the first alternative Petri net. Furthermore, the initial marking, conditioned by the mentioned choice colour, will be composed exclusively by tokens of this choice colour.

Figure 2 Algorithm for constructing a disjunctive CPN from a set of alternative Petri nets using a set of choice colours



The following step in building up the disjunctive CPN will be adding the subnets of the alternative Petri net R_2 not contained by R_1, also called subnets not shared by R_2. Afterwards, the link transitions, with guard functions consisting of the choice colour associated to R_2 should be added to R^C.

This last step should be repeated so many times as alternative Petri nets have not been considered yet, in fact $|S_R| - 2$.

The resulting Petri net will be a disjunctive CPN. The appellative 'coloured' is due to the fact that the tokens may have attributes or colours and the adjective 'disjunctive' is explained because the set of colours includes a subset of choice colours, which is a set of exclusive entities.

In the following section, an example of application of this algorithm will be applied to the process of design of a manufacturing facility.

4 Example of application

In the case study described in this section, it will be illustrated how the modelling process of a DES with alternative structural configurations can be developed by the use of a disjunctive CPN.

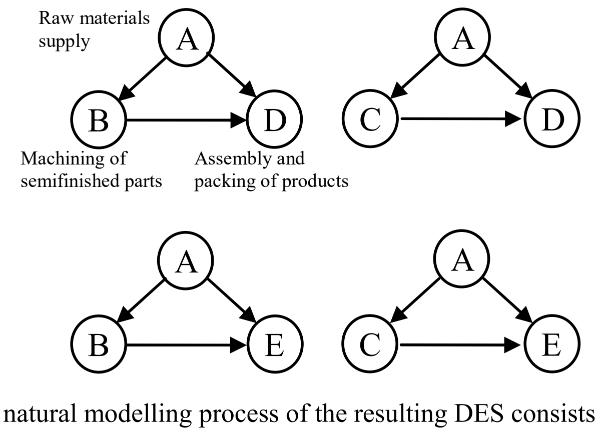
The system that will be considered is a manufacturing line in process of being designed, which will be composed of three stages: the raw materials supply, the machining of the semi-finished parts and the assembly and packing of the resulting products.

The raw materials reception system has already been chosen by the decision makers involved in the design process and will be called subsystem 'A'. Moreover, the machining process can be implemented by means of two alternative subsystems, offered by two different suppliers, which will be called 'B' and 'C', respectively. In the same way, the assembly and packing cell can be built up by means of other two alternative subsystems, called 'D' and 'E'.

The design of the resulting manufacturing facility will require choosing a single solution from the pool of four alternative systems obtained by the different combinations of the alternative subsystems.

The mentioned four solutions have been represented in a simplified way in Figure 3, where the different subsystems are depicted by labelled circles, while the arrows inform about material flow in the manufacturing process.

Figure 3 Alternative configurations for the DES in process of being designed



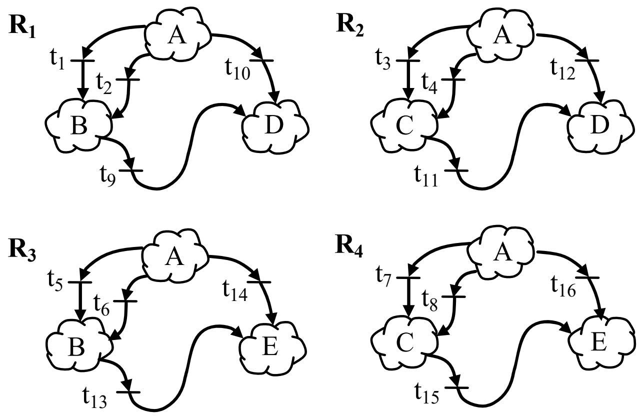
A natural modelling process of the resulting DES consists of obtaining a Petri net for every one of the alternative structural configurations of the system. This resulting system, modelled in the form of a set of alternative Petri nets, may be inefficient for tasks such as performance evaluation and optimisation, since there is usually redundant information that can be removed.

As a result of this modelling process, it is possible to obtain four different alternative Petri nets, represented in a simplified way by means of subnets, depicted by clouds, and link transitions between some of them.

The four alternative Petri net models, $S_R = \{R_1, R_2, R_3, R_4\}$, have been represented in Figure 4.

As it can be seen in Figure 4, the subnets represented in the alternative Petri nets, which correspond to real subsystems in the DES in process of being designed, are shared by different nets. For example, subnet 'A' is shared by the four alternative Petri nets, while subnet 'B' is shared by R_1 and R_3.

Figure 4 Simplified representation of the four alternative Petri nets decomposed into subnets and link transitions



For this reason, the set of four models represented in Figure 4 include redundant information. This redundant information arises because the subnets shared by several alternative Petri nets. This redundant information may reduce the computational performance of a decision making algorithm implemented to cope with the decision making in the design process of the manufacturing facility.

One way to remove the redundant information of the nets is by using CPN, where the attributes or colours of the tokens will avoid losing information when this removal is applied.

However, two considerations should be made before dealing with this modelling process. First of all, a conventional CPN is not appropriate for modelling a DES with alternative structural configurations. For this use, a disjunctive CPN is much more adequate, since in its definition it is included a subset of choice colours, which is a set of exclusive entities, as the set of alternative Petri nets is.

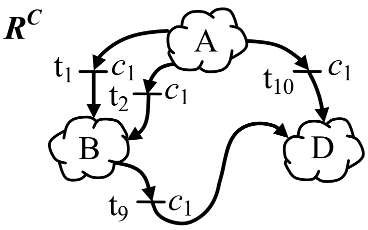
Secondly, a folding of shared subnets, while a certain number of them are present and also a given number of Petri nets are taken into account may be complicated without a clear and simple methodology.

This example will illustrate the application of a technique, described in the previous section, able to cope with this problem.

The starting point of the application of the algorithm for constructing a disjunctive CPN from the original DES is a decomposition of the alternative Petri nets into subnets and link transitions as it has been shown, in a simplified way, in Figure 4. The criterion followed for achieving the mentioned decomposition is to consider as subnets, the models of the subsystems present in the manufacturing system: the raw materials supply, the machining centres, and the assembly and packing system.

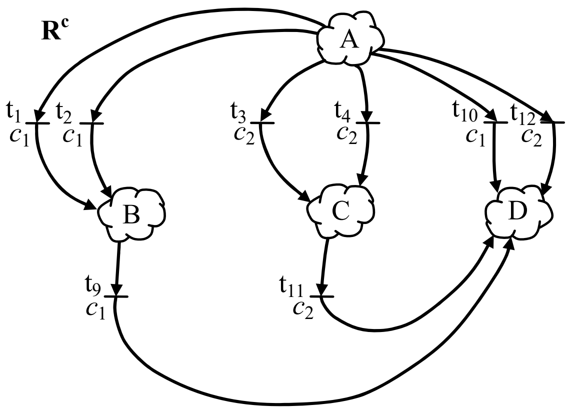
The following step in the application of the algorithm is to consider, in a first iteration, the first alternative Petri net as the disjunctive CPN that will be constructed. A guard function, which corresponds to the choice colour related to the first alternative Petri net, c_1, is associated to the link transitions {t_1, t_2, t_9, t_{10}} of R_1. In Figure 5, it is possible to see the result.

Figure 5 Simplified representation of the disjunctive CPN after the application of the first step of the algorithm



As a second step in the construction of the disjunctive CPN, as model of the DES consists of including in R^c the subnets of R_2 that are not present in R^c.

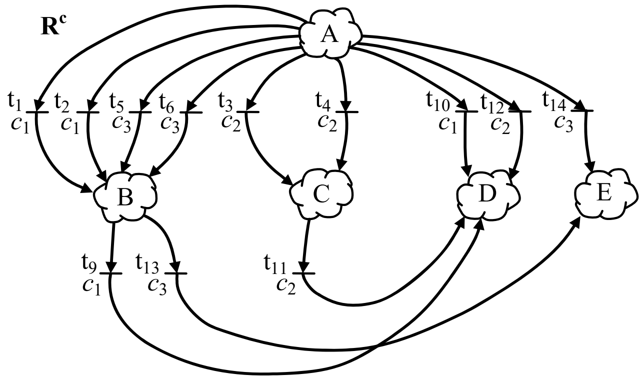
Figure 6 Simplified representation of the disjunctive CPN after the application of the second step of the algorithm



At this early stage in the application of the construction algorithm is the same as saying that the subnets to be included in R^c should be present in R_2 but not in R_1. In particular, the subnets in which, by decision of the modeller, the alternative Petri net R_2 has been decomposed are {A, C, D}. It is a fact that {A, D} are shared by R_1 and R_2, however, {C} belongs to R_2 but not to R_1; hence, it should be included in R^c as well as all the link transitions of R_2, which are {t_3, t_4, t_{11}, t_{12}}. In Figure 6, it can be seen the result of the application of this step of the construction algorithm.

The third step in the application of the algorithm consists of including in the disjunctive CPN, the subnets of R_3 that do not belong to R^c so far, that is to say {E}. Moreover, all the link transitions of R_3 should also be included: {t_5, t_6, t_{13}, t_{14}}. The result of these operations can be found in Figure 7.

Figure 7 Simplified representation of the disjunctive CPN after the application of the third step of the algorithm

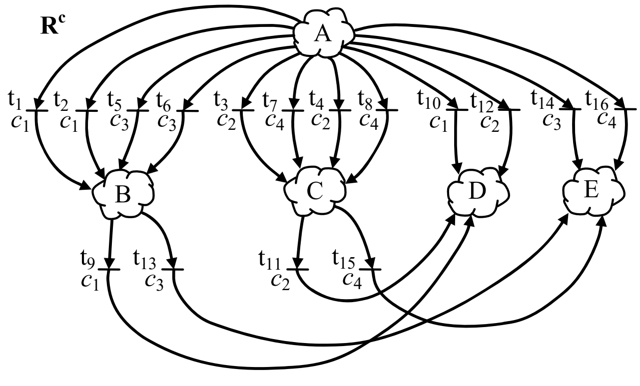


As fourth step in the application of the algorithm, the alternative Petri net R_4 has to be considered. All the subnets in which R_4 has been decomposed already belong to R^c so far. For this reason, the application of this step only implies adding to R^c the link transitions of R_4; in other words, the transitions of R_4 that does not belong to any of the subnets in which this alternative Petri net has been decomposed.

The result of this fourth step of the algorithm has been represented in Figure 8.

The last step in the application of the algorithm consists of simplifying the last model obtained from the development of the previous steps in order to try to limit the number of link transitions with the purpose of reduce the size of the model.

Figure 8 Simplified representation of the disjunctive CPN after the application of the fourth step of the algorithm



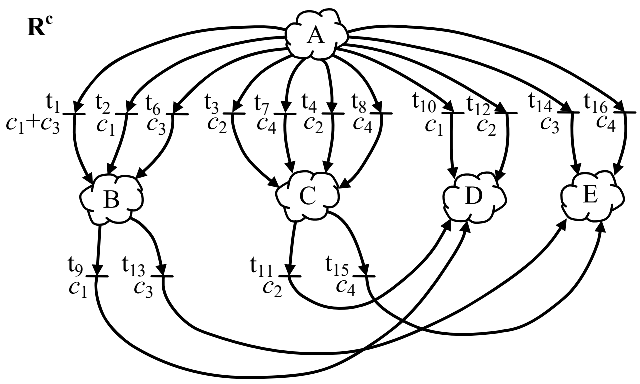
Even though the details of every subnet of R^c are not shown in this paper, since they are not essential for illustrating the construction of a disjunctive CPN, it is possible to state that in the example that has been considered the transitions {t_1, t_5} are quasi identical. This fact means that they present input and output arcs of the same weight from and to the same places. The only difference between quasi-identical transitions is the guard functions associated to them. In the case of t_1, the guard function is the choice colour c1, while in the case of t_5, the guard function is c_3.

Two or more quasi-identical transitions can be reduced into a single one by creating a guard function that combines by means of the logic operator 'or' the guard functions of the quasi-identical transitions. In the case of $\{t_1, t_5\}$, the new transition will be called t_1 and the associated guard function will be $c_1 + c_3$.

The result of the application of the reduction rule to the couple of quasi-identical transitions $\{t_1, t_5\}$ can be found in Figure 9.

It is possible to continue applying the reduction rule mentioned in the previous paragraphs. In doing so, it is possible to identify the following couples of quasi-identical transitions: $\{t_2, t_6\}$, $\{t_3, t_7\}$, $\{t_4, t_8\}$, $\{t_1, t_5\}$, $\{t_{10}, t_{12}\}$, and $\{t_{14}, t_{16}\}$.

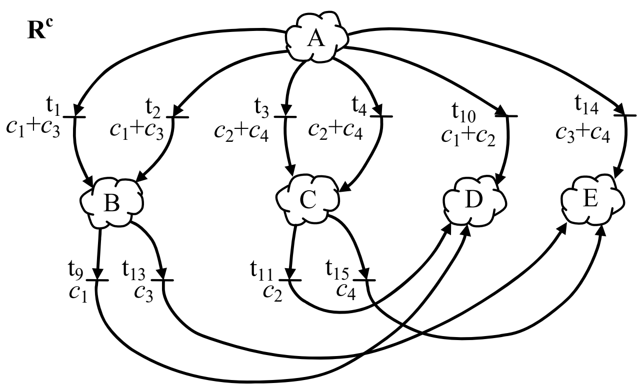
Figure 9 Simplified representation of the disjunctive CPN after the application of a reduction rule to the quasi-identical transitions $\{t_1, t_5\}$



The result in the application of the reduction rule to the couples of quasi-identical transitions has been represented in Figure 10. This resulting disjunctive CPN can be compared with the one presented in Figure 8, where no reduction rule has been applied yet.

Furthermore, it is also possible to compare the disjunctive CPN of Figure 10 with the set of four alternative Petri nets depicted in Figure 4.

Figure 10 Simplified representation of the disjunctive CPN after the application of a reduction rule to all the quasi-identical transitions of R^c



As a complement to the previous description, it is possible to quantify the reduction rate in size of the model of the manufacturing system in process of being designed by using the disjunctive CPN R^c instead of the original set of alternative Petri nets $\{R_1, R_2, R_3, R_4\}$.

The reduction rate, under the circumstances described in the previous paragraph, can be calculated as a size ratio as indicated in the following:

$$\text{Size ratio} = \text{size of } R^c \text{ / size of } \{R_1, R_2, R_3, R_4\}$$

In order to calculate this parameter, as a first step, the numerator and denominator of the size ratio will be determined.

First of all, the denominator of the size ratio is taken into account:

$$\text{Size of } \{R_1, R_2, R_3, R_4\} = \text{size of } R_1 + \text{size of } R_2 + \text{size of } R_3 + \text{size of } R_4$$

Let us call $\mathbf{nr}(R_i)$ and $\mathbf{nc}(R_i)$ the operators for calculating the number of rows and columns of the incidence matrix of the Petri net R_i, respectively.

$$\text{Size of } R_i = \mathbf{nr}(R_i) \cdot \mathbf{nc}(R_i) = r_i \cdot c_i, \text{ where } i \in \{1, 2, 3, 4\}$$

Furthermore,

$$\mathbf{nr}(R_i) = \sum \mathbf{nr}(R_j), \text{ where } R_j \in \{R_A, R_B, R_C, R_D, R_E\} \text{ and } i \in \{1, 2, 3, 4\}$$

and

$$\mathbf{nc}(R_i) = 4 + \sum \mathbf{nc}(R_j), \text{ where } R_j \in \{R_A, R_B, R_C, R_D, R_E\} \text{ and } i \in \{1, 2, 3, 4\}$$

Notice that the subnets R_j that should be considered in the calculation of the number of rows and columns of R_i are the subnets that belong to R_i, as detailed in Figure 4.

Moreover, the number 4 in the calculation of the number of columns of R_i correspond to the transitions that link the different subnets in R_i, as it can be seen in Figure 5.

As a result, the different calculations can be detailed more, as the following examples show:

$$\mathbf{nr}(R_1) = \mathbf{nr}(R_A) + \mathbf{nr}(R_B) + \mathbf{nr}(R_D)$$
$$\mathbf{nc}(R_1) = 4 + \mathbf{nc}(R_A) + \mathbf{nc}(R_B) + \mathbf{nc}(R_D)$$

$$\mathbf{nr}(R_1) = \mathbf{nr}(R_A) + \mathbf{nr}(R_C) + \mathbf{nr}(R_D)$$
$$\mathbf{nc}(R_1) = 4 + \mathbf{nc}(R_A) + \mathbf{nc}(R_D) + \mathbf{nc}(R_D) + 4$$

and so on.

On the other hand, the numerator of the size ratio can be calculated as detailed in the following:

$$\text{Size of } R^c = \mathbf{nr}(R_1) \cdot \mathbf{nc}(R^c) = r_c \cdot c_c$$

where

$$\mathbf{nr}(R^c) = \mathbf{nr}(R_A) + \mathbf{nr}(R_B) + \mathbf{nr}(R_C) + \mathbf{nr}(R_D) + \mathbf{nr}(R_E)$$
$$\mathbf{nc}(R^c) = 8 + \mathbf{nc}(R_A) + \mathbf{nc}(R_B) + \mathbf{nc}(R_D) + \mathbf{nc}(R_D) + \mathbf{nc}(R_E)$$

Now that the calculation of the size rate has been detailed it is possible to discuss the quantitative results of the transformation of the model of the system, from the original set of alternative Petri nets to the final disjunctive compound Petri nets.

As a reasonable assumption, let us consider a case where the sizes of the shared subnets R_B to R_E are similar, while the size of R_A, that represents the raw materials supply of the manufacturing facility, might be different.

The second assumption that is being considered is that the number of rows is the same as the number of columns for every subnet.

Under both assumptions the next steps can be developed:

$$\mathbf{nr}(R_j) = x, \mathbf{nc}(R_j) = x \; \forall \; R_j \in \{R_B, R_C, R_D, R_E\}$$

and x is the number of rows of R_j, which is the same as its number of columns.

Moreover,

$$\text{Size of } R_j = x \cdot x = x^2 \text{ if } R_j \in \{R_B, R_C, R_D, R_E\}$$

$$\mathbf{nr}(R_A) = y, \mathbf{nc}(R_A) = y$$

As a consequence,

$$\text{Size of } R_A = y \cdot y = y^2$$

Hence,

$$\mathbf{nr}(R^c) = y + 4x$$
$$\mathbf{nc}(R^c) = 8 + y + 4x$$

Moreover,

$$\text{Size of } R^c = \mathbf{nr}(R^c) \cdot \mathbf{nc}(R^c) = (y + 4x) \cdot (8 + y + 4x) = y^2 + 8y + 8xy + 32x + 16x^2$$

On the other hand,

$$\mathbf{nr}(R_i) = \sum \mathbf{nr}(R_j) = y + x + x = y + 2x$$

$$\mathbf{nc}(R_i) = 4 + \sum \mathbf{nc}(R_j) = 4 + y + 2x$$

$$\text{Size of } R_i = \mathbf{nr}(R_i) \cdot \mathbf{nc}(R_i) = (y + 2x) \cdot (4 + y + 2x) = y^2 + 4y + 4xy + 8x + 4x^2$$

$$\text{Size of } \{R_1, R_2, R_3, R_4\} = 4 \cdot \text{Size of } R_i = 4 \cdot (y^2 + 4y + 4xy + 8x + 4x^2) = 4y^2 + 16y + 16xy + 32x + 16x^2$$

Now, it is possible to find an expression for the size ratio of this case study:

$$\text{Size ratio} = (y^2 + 8y + 8xy + 32x + 16x^2) / (4y^2 + 16y + 16xy + 32x + 16x^2)$$

Let us consider some particular cases.

If $y = x = 50$, then the size ratio is 0.698, meaning that the amount of redundant information in the set of alternative Petri nets, which is not included in the equivalent disjunctive CPN is

$$100 \cdot (1 - \text{size ratio}) = 30.2\%$$

Another interesting case is $y = 100$ and $x = 50$. In this case, the size ratio is 0.566 and the redundant information removed is 43.4%.

If $y = 50$ and $x = 10$ the size ratio is 0.425 and the redundant information removed is 57.5%.

Finally, if $y = 100$ and $x = 10$ the size ratio is 0.348 and the redundant information removed is 65.2%.

As it can be seen, the largest are the dimensions of R_A in relation with the size of the rest of subnets, the smallest is the size ratio, the largest the amount of removed information, and the most advantageous is the use of the disjunctive CPNs instead of a set of alternative Petri net.

The specific and non-approximated data for the original manufacturing facility that has led to these considerations are the following:

$$\text{Size ratio} = 0.681$$

$$\text{Amount of removed redundant information} = 31.9\%.$$

In this case study, the sizes of the incidence matrices of the different subnets are all similar and around 60 rows and 60 columns. These dimensions fit approximately with the first quantitative example that has been analysed, where $x = 50$, $y = 50$, and the size ratio = 0.698.

5 Conclusions

In this paper, an extension of the CPNs, the disjunctive CPNs, has been introduced as a way to construct models of DESs with alternative structural configurations.

Furthermore, an algorithm, describing the steps to be followed for constructing such model, is also detailed, as well as an application example, where a manufacturing facility in process of being designed has been modelled.

As a result, it can be said that the formalism of the disjunctive CPNs is a very promising one for the description of DESs with alternative structural configurations for diverse purposes, such as structural analysis, performance evaluation, or optimisation, for example, for the development of decision support systems.

As future lines of research, we can envisage the application of this formalism to a wider range of sectors and DESs.

References

Bruzzone, A.G. and Longo, F. (2010) 'An advanced system for supporting the decision process within large-scale retail stores', *Simulation*, Vol. 86, No. 12, pp.742–762.

David, R. and Alla, H. (2005) *Discrete, Continuous and Hybrid Petri Nets*, Springer, Berlin.

Jensen, K. and Kristensen, L.M. (2009) *Colored Petri nets. Modelling and Validation of Concurrent Systems*, Springer, Berlin.

Jimenez, E. and Perez, M. (2004) 'Simulation and optimization of logistic and production systems using discrete and continuous Petri nets', *Simulation: Transactions of the Society for Modeling and Simulation International*, Vol. 80, No. 3, pp.142–152.

Jimenez, E., Martinez, E., Blanco, J., Perez, M. and Graciano, Ch. (2014) 'Methodological approach towards sustainability by integration of environmental impact in production system models through life cycle analysis: application to the Rioja wine sector', *Simulation: Transactions of the Society for Modeling and Simulation International*, Vol. 90, No. 2, pp.143–161.

Latorre, J.I. and Jiménez, E. (2012) 'Colored Petri nets as a formalism to represent alternative models for a discrete event system', *24th European Modelling and Simulation Symposium (EMSS 12)*, Vienna.

Latorre, J.I. and Jiménez, E. (2013a) 'Petri nets with exclusive entities for decision making', *International Journal of Simulation and Process Modelling*, Vol. 8, No. 1, pp.66–73.

Latorre, J.I. and Jiménez, E. (2013b) 'Simulation-based optimization of discrete event systems with alternative structural configurations using distributed computation and the Petri net paradigm', *Simulation: Transactions of the Society for Modeling and Simulation International*, Vol. 89, No. 11, pp.1310–1334.

Latorre, J.I., Jiménez, E. and Pérez, M. (2010a) 'Colored Petri nets as a formalism to represent alternative models for a discrete event system', *22nd European Modelling and Simulation Symposium (EMSS 10)*, Fez, Morocco, pp.247–252.

Latorre, J.I., Jiménez, E., Pérez, M., Blanco, J. and Martínez, E. (2010b) 'The alternatives aggregation Petri nets as a formalism to design discrete event systems', *International Journal of Simulation and Process Modelling*, Vol. 6, No. 2, pp.152–164.

Latorre, J.I., Jiménez, E. and Pérez, M. (2013a) 'The optimization problem based on alternatives aggregation Petri nets as models for industrial discrete event systems', *Simulation: Transactions of the Society for Modeling and Simulation International*, Vol. 89, No. 3, pp.346–361.

Latorre, J.I., Jiménez, E., Blanco, J. and Sáenz, J.C. (2013b) 'Decision support in the Rioja wine production sector', *International Journal of Food Engineering*, Vol. 9, No. 3, pp.267–278.

Latorre, J.I., Jiménez, E. and Pérez, M. (2014a) 'Sequence of decisions on discrete event systems modeled by Petri nets with structural alternative configurations', *Journal of Computational Science*, Vol. 5, No. 3, pp.387–394.

Latorre, J.I., Jiménez, E., Blanco, J. and Sáenz-Díez, J.C. (2014b) 'Optimal design of an olive oil mill by means of the simulation of a Petri net model', *International Journal of Food Engineering*, Vol. 10, No. 4, pp.573–582

Latorre-Biel, J.I., Jiménez-Macías, E., de la Parte, M.P., Blanco-Fernández, J. and Martínez-Cámara, E. (2014c) 'Control of discrete event systems by means of discrete optimization and disjunctive colored PNs: application to manufacturing facilities', *Abstract and Applied Analysis*, Article ID 821707, 16pp, doi:10.1155/2014/821707.

Latorre-Biel, J.I. and Jiménez-Macías, E. (2012) 'Simulation for education in business decision-making', *SCS M&S Magazine*, Vol. 2, No. 3, pp.59–65.

Longo, F. and Nicoletti, L. (2015) 'An advanced training environment for vessels' last mile navigation', *International Journal of Simulation and Process Modelling*, Vol. 10, No. 2, pp.117–131.

Longo, F., Nicoletti, L., Chiurco, A., Solis, A.O., Massei, M. and Diaz, R. (2013) 'Investigating the behavior of a shop order manufacturing system by using simulation', *SpringSim (EAIA)*, Vol. 7, pp.47–54.

Mota, M.M. and Piera, M.A. (2011) 'A compact timed state space approach for the analysis of manufacturing systems: key algorithmic improvements', *International Journal of Computer Integrated Manufacturing*, Vol. 24, No. 2, pp.135–153.

Mújica, M.A., Piera, M.A. and Narciso, M. (2010) 'Revisiting state space exploration of timed coloured Petri net models to optimize manufacturing system's performance', *Simulation Modelling Practice Theory*, Vol. 18, No. 9, pp.1225–1241.

Music, G. (2009) 'Petri net based scheduling approach combining dispatching rules and local search', in *Proceedings of the 21st European Modeling and Simulation Symposium (EMSS '09)*, Puerto de la Cruz, Spain, Vol. 2, pp.27–32.

Music, G., Loscher, T. and Breitenecker, F. (2008) 'Simulation based scheduling applying Petri nets with sequences and priorities', in *Proceedings of the UKSim 10th International Conference on Computer Modelling and Simulation (EUROSIM/UKSim '08)*, pp.455–460.

Narciso, M., Piera, M.A. and Guasch, A. (2010) 'A methodology for solving logistic optimization problems through simulation', *Simulation: Transactions of the Society for Modeling and Simulation International*, Vol. 86, No. 5, pp.369–389.

Nicoletti, L., Chiurco, A. and Spadafora, F. (2015) 'Advanced interoperable simulators for training in car terminals', *International Journal of Simulation and Process Modelling*, Vol. 10, No. 2, pp.132–143.

Piera, M.A. and Music, G. (2011) 'Coloured Petri net scheduling models: timed state space exploration shortages', *Mathematics and Computers in Simulation*, Vol. 82, No. 3, pp.428–441.

Piera, M.A., Narciso, M., Guasch, A. and Riera, D. (2004) 'Optimization of logistic and manufacturing system through simulation: a colored Petri net-based methodology', *Simulation: Transactions of the Society for Modeling and Simulation International*, Vol. 80, No. 3, pp.121–129.

Silva, M. (1993) 'Introducing Petri nets', in Di Cesare, F. (Ed.): *Practice of Petri Nets in Manufacturing*, pp.1–62, Chapman & Hall.

Wainer, G.A. (2016) 'Real-time simulation of DEVS models in CD+', *International Journal of Simulation and Process Modelling*, Vol. 11, No. 2, pp.138–153.

Xiao, Z. and Ming, Z. (2011) 'A method of workflow scheduling based on colored Petri nets', *Data & Knowledge Engineering*, Vol. 70, No. 2, pp.230–247.

Zaitsev, D.A. and Shmeleva, T.R. (2011) 'A parametric colored petri net model of a switched network', *International Journal of Communications, Network and System Sciences*, Vol. 4: No. 1, pp.65–76, Scientific Research Publishing Inc.