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COMPOSITION OF COMPOSABLE CELLULAR AUTOMATA WITH RESPECT TO THEIR DIMENSIONAL ATTRIBUTES

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ABSTRACT

The Composable Cellular Automata (CCA) specification formally defines a system for building cellular automata models that can be composed with other models. The intent is to build hybrid simulation models with uniquely modeled subsystems. It is feasible that two or more subsystems of a large, complex system are modeled using CCA. For the purposes of refining domain abstraction, reducing composition complexity, or improving model execution, the need to compose two or more CCA to create a single CCA may exist. It is then important for a modeler to understand the implications that specific disparities between the CCA have on their composition. To that end, this paper describes formal composition of CCA with respect to their dimensional attributes—cell indices and time.

1 INTRODUCTION

This paper defines closure properties of the dimensional attributes of Composable Cellular Automata (CCA). CCA are formally specified cellular automata that are developed with the intent of composing the model with other models to develop more complex hybrid system models (see Sarjoughian 2006, and Sarjoughian and Mayer 2011). CCA are defined in Mayer and Sarjoughian (2009) and summarized here. There are two major components of CCA, a network and the cells contained within the network. The network encapsulates the cells such that all external input and output (I/O) must be handled by the network itself. The cells represent the individual automaton that maintain state, produce output based upon state, and undergo state transition based upon current state and input.

Mayer and Sarjoughian (2009) discusses an approach to *mapping* I/O between two CCA networks. This is feasible and remains a viable solution, especially if the two CCA are distributed or extremely disparate across their elements. However, it does not address a direct composition approach to managing CCA that are alike. In other words, how CCA can be composed with other CCA and treated as a single CCA if they differ in specific ways. The utility of this is that it may simplify the composition of the CCA models with an additional, non-CCA model; thus, making the hybrid model more simplistic (see Mayer 2009; and Barton, Ullah, and Bergin 2010 as examples). Another purpose to composing two or more CCA is to reduce execution overhead; removing the need for multiple simulators, as an example. This paper discusses how CCA may be composed with respect to their dimensional properties—cell representation and time. The overall goal is to describe approaches to formally compose CCA, and provide an understanding of the implications of doing such when faced with specific disparities between the CCA systems.

The section that immediately follows provides a summary introduction of the composable cellular automata specification. Section 2 introduces properties that apply to all CCA. Next, in Section 3, CCA composition is discussed from the perspective of cell indices and time. Finally, Section 4 provides a summary of what is presented in the paper.

1.1 Review of CCA Specification

A Composable Cellular Automata network, N, is defined as

$$N = \langle X_N, Y_N, D, \{M_{ijk}\}, T, F \rangle, \tag{1}$$

and each cell component, M_{ijk}, within the network is specified by

$$M_{ijk} = \langle X_{ijk}, Y_{ijk}, Q_{ijk}, I_{ijk}, \delta_{ijk}, \lambda_{ijk}, T \rangle. \tag{2}$$

As discussed in (Mayer and Sarjoughian 2009), the first two elements of N, X_N and Y_N, are the input from an external system to the network and output from the network to an external system, respectively. D is a set of indices that uniquely identify each cell within the set of homogeneous cells, $\{M_{ijk}\}$, that belongs to the network (subscript ijk is the index for a unique element of a three-dimensional network). T is a finite set of time-ordered, time intervals that structures the discrete-time dynamics of the network. The last set, F, contains the mapping functions between the CCA cells and the network as a whole.

The cells, M_{ijk}, are defined by, X_{ijk} and Y_{ijk}, which are the input to and output from each cell. Each is a union of data internal and external to the network. $X_{ijk} = \dot{X}_{ijk} \cup \overline{X}_{ijk}$, which represents input from the cell's influencers and external input mapped to this cell, respectively. $Y_{ijk} = \dot{Y}_{ijk} \cup \overline{Y}_{ijk}$, which represents output to the cells that this cell influences and output that acts as part of the external output from this network, respectively. Q_{ijk} is the set of possible states for the component. I_{ijk} is the set of indices that identify this cell's influencers (i.e., its neighborhood). The component's state transition function is δ_{ijk}, and the output function is λ_{ijk}. $\delta_{ijk} : Q_{ijk} \times X_{ijk} \rightarrow Q_{ijk}$ and $\lambda_{ijk} : Q_{ijk} \rightarrow Y_{ijk}$. T is the same set of time-ordered, time intervals that exists in the network tuple in (1). The significance of this is that it ensures that every cell in the network is using the same set of time-ordered, time intervals and, therefore, every cell undergoes state transition at the same discrete time. It should also be noted that a specific cell has no a priori knowledge of the network, including characteristics such as total number of cells in the network and connectivity (e.g., Moore versus von Neumann networks).

2 BASIC PROPERTIES OF CCA

Let Φ and Ψ represent two distinct composable cellular automaton. The network specifications for Φ and Ψ are $N^{\Phi} = \langle X_N^{\Phi}, Y_N^{\Phi}, D^{\Phi}, \{M_{ijk}^{\Phi}\}, T^{\Phi}, F^{\Phi} \rangle$ and $N^{\Psi} = \langle X_N^{\Psi}, Y_N^{\Psi}, D^{\Psi}, \{M_{ijk}^{\Psi}\}, T^{\Psi}, F^{\Psi} \rangle$, respectively. Similarly, the specifications for the cell components of Φ and Ψ are $M_{ijk}^{\Phi} = \langle X_{ijk}^{\Phi}, Y_{ijk}^{\Phi}, Q_{ijk}^{\Phi}, I_{ijk}^{\Phi}, \delta_{ijk}^{\Phi}, \lambda_{ijk}^{\Phi}, T^{\Phi} \rangle$ and $M_{ijk}^{\Psi} = \langle X_{ijk}^{\Psi}, Y_{ijk}^{\Psi}, Q_{ijk}^{\Psi}, I_{ijk}^{\Psi}, \delta_{ijk}^{\Psi}, \lambda_{ijk}^{\Psi}, T^{\Psi} \rangle$, respectively.

Definition 1 $\Phi = \Psi$ is defined as all subset elements of tuples Φ and Ψ being equal. Formally,

$$\Phi = \Psi \Leftrightarrow (X_N^{\Phi} = X_N^{\Psi}) \wedge (Y_N^{\Phi} = Y_N^{\Psi}) \wedge (D^{\Phi} = D^{\Psi}) \wedge (\{M_{ijk}^{\Phi}\} = \{M_{ijk}^{\Psi}\}) \wedge (T^{\Phi} = T^{\Psi}) \wedge (F^{\Phi} = F^{\Psi}), \tag{3}$$

where $\{M_{ijk}^{\Phi}\} = \{M_{ijk}^{\Psi}\} \Leftrightarrow \forall ijk : (X_{ijk}^{\Phi} = X_{ijk}^{\Psi}) \wedge (Y_{ijk}^{\Phi} = Y_{ijk}^{\Psi}) \wedge (Q_{ijk}^{\Phi} = Q_{ijk}^{\Psi}) \wedge (I_{ijk}^{\Phi} = I_{ijk}^{\Psi}) \wedge (\delta_{ijk}^{\Phi} = \delta_{ijk}^{\Psi}) \wedge (\lambda_{ijk}^{\Phi} = \lambda_{ijk}^{\Psi}) \wedge (T^{\Phi} = T^{\Psi})$.

Definition 2 $\Phi \sim \Psi$ is defined as Φ and Ψ being *similar*, differing only in the time intervals subset, T. Formally,

$$\Phi \sim \Psi \equiv \{\Phi - \{T^{\Phi}\}\} = \{\Psi - \{T^{\Psi}\}\}, \tag{4}$$

where $T^{\Phi} \neq T^{\Psi}$. Note that $\Phi \sim \Psi \models T \in \{M_{ijk}^{\Phi}\} \neq T \in \{M_{ijk}^{\Psi}\} \because T \in N = T \in \{M_{ijk}\}$.

Definition 3 *Composition* of two CCA is a disjoint union of the CCA **plus** a composition tuple containing any new external I/O mappings to the resultant set of cells and any new influencers to specific cells. In other words, it is the union of each of the subset elements of the CCA tuples and each subset element is pairwise disjoint, the union of the network mapping function set with an *mapping composition set*, and the union of the influencer set of each cell with an *influencer composition set*. Formally, let $\Xi = \{F', \{I'_{ijk}\}\}$ be the composition tuple, where F' is the mapping composition set and $\{I'_{ijk}\}$ is the influencer composition set. Then, composition $\equiv \Phi \odot_{\Xi} \Psi = \langle \{X_N^\Phi \cup X_N^\Psi\}, \{Y_N^\Phi \cup Y_N^\Psi\}, \{D^\Phi \cup D^\Psi\}, \{\{M_{ijk}^\Phi\} \uplus \{M_{ijk}^\Psi\}\}, \{T^\Phi \cup T^\Psi\}, \{F^\Phi \cup F^\Psi \cup F'\}\rangle$, and $\{\{M_{ijk}^\Phi\} \uplus \{M_{ijk}^\Psi\}\} = \{\{M_{ijk}^\Phi\} \sqcup \{M_{ijk}^\Psi\}\} = \forall ijk : \langle \{X_{ijk}^\Phi \cup X_{ijk}^\Psi\}, \{Y_{ijk}^\Phi \cup Y_{ijk}^\Psi\}, \{Q_{ijk}^\Phi \cup Q_{ijk}^\Psi\}, \{I_{ijk}^\Phi \cup I_{ijk}^\Psi \cup I'_{ijk}\}, \{\delta_{ijk}^\Phi \cup \delta_{ijk}^\Psi\}, \{\lambda_{ijk}^\Phi \cup \lambda_{ijk}^\Psi\}, \{T^\Phi \cup T^\Psi\}\rangle$. Note that $\Xi = \emptyset \Rightarrow \Phi \odot_{\Xi} \Psi = \Phi \odot_{\emptyset} \Psi = \Phi \sqcup \Psi$.

Theorem 1 If $\Phi = \Psi$, then $\Phi \odot_{\emptyset} \Psi = \Phi$.

Proof. Let $\{I'_{ijk}\} = \emptyset$. Using Definition 1, substitute Ψ for Φ into the equation for $\Phi \odot_{I'_{ijk}} \Psi$ given in Definition 3. Then, $\Phi \odot_{\emptyset} \Psi = \langle \{X_N^\Phi \cup X_N^\Phi\}, \{Y_N^\Phi \cup Y_N^\Phi\}, \{D^\Phi \cup D^\Phi\}, \{\{M_{ijk}^\Phi\} \uplus \{M_{ijk}^\Phi\}\}, \{T^\Phi \cup T^\Phi\}, \{F^\Phi \cup F^\Phi\}\rangle$, and $\{\{M_{ijk}^\Phi\} \uplus \{M_{ijk}^\Phi\}\} = \forall ijk : \langle \{X_{ijk}^\Phi \cup X_{ijk}^\Phi\}, \{Y_{ijk}^\Phi \cup Y_{ijk}^\Phi\}, \{Q_{ijk}^\Phi \cup Q_{ijk}^\Phi\}, \{I_{ijk}^\Phi \cup I_{ijk}^\Phi\}, \{\delta_{ijk}^\Phi \cup \delta_{ijk}^\Phi\}, \{\lambda_{ijk}^\Phi \cup \lambda_{ijk}^\Phi\}, \{T^\Phi \cup T^\Phi\}\rangle$. By definition of a union of sets, $\{\{M_{ijk}^\Phi\} \uplus \{M_{ijk}^\Phi\}\} = \{M_{ijk}^\Phi\}$, and the first tuple simplifies to $\langle \{X_N^\Phi\}, \{Y_N^\Phi\}, \{D^\Phi\}, \{\{M_{ijk}^\Phi\}\}, \{T^\Phi\}, \{F^\Phi\}\rangle$, which is the specification for Φ. $\therefore$ if $\Phi = \Psi$, then $\Phi \odot_{\emptyset} \Psi = \Phi$. □

3 PROPERTIES OF DIMENSIONAL ATTRIBUTES

3.1 Cell Dimensionality

Definition 4 If two CCA have the same set of cell identifiers, D, and the same set of cell influencers, $\{I_{ijk}\}$, then the two CCA possess the same domain representation, $\mathfrak{D}$. Formally,

$$(D^\Phi = D^\Psi) \wedge (\forall ijk : I_{ijk}^\Phi = I_{ijk}^\Psi) \Leftrightarrow \mathfrak{D}^\Phi = \mathfrak{D}^\Psi. \tag{5}$$

D models the tessellation of the domain space while $\{I_{ijk}\}$ captures the abstraction of domain element interactions (specified by the network). Arbitrary values can be assigned to i, j, and k as labels in D. However, for the purposes of evaluation of a CCA there are two approaches. First, from a domain-neutral perspective, all indices must be assumed to use the same coordinate system, start at $(0,0,0)$, and then be numbered sequentially based upon movement in a respective dimension. Alternatively, the semantics of the values with respect to the domain must be considered. Thus, stating $D^\Phi = D^\Psi$ entails that all of the same discrete-elements of the domain are being represented by D. As examples of differences in D, consider a grid-shaped tessellation versus a hexagonal one. For differences in $\{I_{ijk}\}$ consider a Moore network versus a von Neumann network.

Definition 5 *Regions*, $\mathfrak{R}$, of CCA network, N, are a set of references to distinct sets of cells in the network, to which a specific external input value in X_N is mapped. Formally, $f : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$, where $f \in F$, $\mathfrak{r} \in \mathfrak{R}$, $x_N \in X_N$, $\bar{x}_{ijk} \in \overline{X}_{ijk}$, $ijk \in D$, and $0 \leq |\{\overline{X}_{ijk}\}| \leq |D|$. Regions are a domain-dependent implementation concept (similar to ports).

3.2 Composition with Disparate Indices

Confining a discussion of CCA differences to the cells themselves, $\{\Phi - \{D^\Phi, F^\Phi\}\} = \{\Psi - \{D^\Psi, F^\Psi\}\}$. Note that F is dependent on the indices in D, and so must be considered.

Theorem 2 Two single-celled CCA, Φ and Ψ, differing by set D (and, potentially, F and $\{I_{ijk}\}$ as well) can be composed into a third CCA, Ω.

Proof. Let $D^\Phi = \{(a,b,c)\}$ and $D^\Psi = \{(e,f,g)\}$; let $F^\Phi = \{f^\Phi\}$ and $F^\Psi = \{f^\Psi\}$, where $f^\Phi : (\mathfrak{r}^\Phi, x_N^\Phi) \mapsto \bar{x}_{abc}$ and $f^\Psi : (\mathfrak{r}^\Psi, x_N^\Psi) \mapsto \bar{x}_{efg}$; let $I_{abc}^\Phi = \{(a,b,c)\}$ and $I_{efg}^\Psi = \{(e,f,g)\}$, where $c \neq g$ and the remaining indice variables may be arbitrary values; and let $F' = \emptyset$ and $\{I'_{ijk}\} = \emptyset$.

Using Definition 3: $\Omega = \Phi \odot_\Xi \Psi \Rightarrow D^\Omega = D^\Phi \cup D^\Psi = \{(a,b,c),(e,f,g)\}$, $F^\Omega = F^\Phi \cup F^\Psi \cup F' = \{f^\Phi, f^\Psi\}$; and $\forall ijk : I_{ijk}^\Omega = I_{ijk}^\Phi \cup I_{ijk}^\Psi \cup I'_{ijk} = \{\{I_{abc}^\Omega\},\{I_{efg}^\Omega\}\}$, where $\{I_{abc}^\Omega\} = \{I_{abc}^\Phi\}$ and $\{I_{efg}^\Omega\} = \{I_{efg}^\Psi\}$. The unions of the remaining, non-disparate tuple elements sets are elementary where $\{\}^\Phi = \{\}^\Psi = \{\}^\Omega$. $\therefore N^\Omega = \langle X_N^\Omega, Y_N^\Omega, D^\Omega, \{M_{ijk}^\Omega\}, T^\Omega, F^\Omega \rangle$ and $\{M_{ijk}^\Omega\} = \langle X_{ijk}^\Omega, Y_{ijk}^\Omega, Q_{ijk}^\Omega, I_{ijk}^\Omega, \delta_{ijk}^\Omega, \lambda_{ijk}^\Omega, T^\Omega \rangle$ define Ω. Note that F now contains two mapping functions, one that maps to each cell. □

Corollary 3 Two single-celled CCA, Φ and Ψ, differing by set D (and, potentially, F and $\{I_{ijk}\}$) can be composed into a third CCA, Ω, if F' contains a mapping from the network to the collection of all cells within the network after composition (i.e., $F' = \{f\}$, where $\forall ijk, f : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$), and $I'_{ijk} = \emptyset$.

Proof. Let $F^\Phi = \{f_\phi\}$, where $\forall ijk \in D^\Phi, f_\phi : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$; $F^\Psi = \{f_\psi\}$, where $\forall ijk \in D^\Psi, f_\psi : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$; and $F' = \{f_\omega\}$, where $\forall ijk \in D^\Omega, f_\omega : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$, and let $I'_{ijk} = \emptyset$.

Then, $\Omega = \Phi \odot_\Xi \Psi \Rightarrow F^\Omega = \{F^\Phi \cup F^\Psi \cup F'\} = \{f_\phi, f_\psi, f_\omega\}$. As before, the unions of the remaining tuple element sets are elementary and Ω is properly defined. Note that three external I/O mappings now exist—one to each cell and a third to both cells. □

Corollary 4 Two single-celled CCA, Φ and Ψ, differing by set D (and, potentially, F and $\{I_{ijk}\}$) can be composed into a third CCA, Ω, if I'_{ijk} contains the index of the cell with which ijk is composed (i.e., $I'_{abc} = \{(e,f,g)\}$ and $I'_{efg} = \{(a,b,c)\}$), and $F' = \emptyset$.

Proof. Let $\forall ijk : \{I'_{ijk}\} = \{\{I_{abc}^\Phi\},\{I_{efg}^\Psi\}\}$, where $\{I_{abc}^\Phi\} = \{(e,f,g)\}$ and $\{I_{efg}^\Psi\} = \{(a,b,c)\}$, and let $F' = \emptyset$.

Then, $\Omega = \Phi \odot_\Xi \Psi \Rightarrow \forall ijk : I_{ijk}^\Omega = \{I_{ijk}^\Phi\} \cup \{I_{ijk}^\Psi\} \cup \{I'_{ijk}\} = \{\{I_{abc}^\Omega\},\{I_{efg}^\Omega\}\}$, where $\{I_{abc}^\Omega\} = \{(a,b,c), (e,f,g)\}$ and $\{I_{efg}^\Omega\} = \{(e,f,g),(a,b,c)\}$. As before, the unions of the remaining tuple element sets are elementary and Ω is properly defined. Note that the two cells now influence each other as well as themselves. □

Corollary 5 Given the associative property of a binary union, multiple CCA; differing only by sets D, F, and $\{I_{ijk}\}$; can be composed in any order. $(\Phi \odot_\Xi \Psi) \odot_\Xi \Upsilon = \Phi \odot_\Xi (\Psi \odot_\Xi \Upsilon)$. Furthermore, as the unions are disjoint, each mapping composition set, F', and influencer composition set, I'_{ijk}, may be given with respect to each composition or all at once with the last composition as a union of all respective composition sets.

Note that when it stated that the composition set is "with respect to" each composition, this means that the mappings and influencer indices are valid for the two CCA being composed (i.e., $ijk \in \{D^\Phi \cup D^\Psi\}$).

Whether or not the composed CCA have additional mappings and/or influence over a subset of the other to which it is composed, a network mapping to a set of dimensionally-disparate cells is a viable system. Thus, multiple CCA, differing only by sets D, F, and $\{I_{ijk}\}$ can be said to be *closed under composition*. This implies that a CCA may be built from a composition of one (dimensionally-unique) cell at a time or from a composition of cell sets containing at least one dimensionally-unique cell member between them.

Note that if two composed CCA contain a common "edge" cell then, even if the influencer composition set is $\emptyset$, the composed CCA will integrate the two CCA based upon the union of the influencers of the edge cell. For example, let Φ and Ψ be two two-celled CCA where $D^\Phi = \{(a,b,c),(e,f,g)\}$ and $D^\Psi = \{(e,f,g),(p,q,r)\}$; and let $I_{ijk}^\Phi = \{\{I_{abc}^\Phi\},\{I_{efg}^\Phi\}\}$, where $\{I_{abc}^\Phi\} = \{(a,b,c),(e,f,g)\}$ and $\{I_{efg}^\Phi\} = \{(e,f,g),(a,b,c)\}$. Similarly, let $I_{ijk}^\Psi = \{\{I_{efg}^\Psi\},\{I_{pqr}^\Psi\}\}$, where $\{I_{efg}^\Psi\} = \{(e,f,g),(p,q,r)\}$ and $\{I_{pqr}^\Psi\} = \{(p,q,r),(e,f,g)\}$; and let $F' = \emptyset$ and $I'_{ijk} = \emptyset$. Then, $\Omega = \Phi \odot_\emptyset \Psi \Rightarrow I_{efg}^\Phi \cup I_{efg}^\Psi \cup \emptyset = I_{efg}^\Omega = \{(e,f,g),(a,b,c),(p,q,r)\}$.

Note also that directional influence and automata networks such as torus can be implemented by excluding or including specific indices in I'_{ijk}. Furthermore, if in the preceding example the restriction for $F' = \emptyset$ is removed and instead $F' = \{f_\omega\}$, where $\forall ijk \in D^\Omega, f_\omega : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$; and $F^\Phi = \{f_\phi\}$, where $\forall ijk \in D^\Phi, f_\phi : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$; and $F^\Psi = \{f_\psi\}$, where $\forall ijk \in D^\Psi, f_\psi : (\mathfrak{r}, x_N) \mapsto \{\bar{x}_{ijk}\}$; then all three external I/O mapping functions, $\{f_\phi, f_\psi, and f_\omega\}$, will have at least one common cell to which they map.

3.3 Composition of Disparate Discrete-Time Segments

Attention is now turned to the composition of two CCA that differ with respect to their discrete-time segments, T. In this regard, $\Phi \sim \Psi$. It may not make much sense to compose two CCA with the same dimensional representation if they only vary in time. However, there is no dependency of D, I_{ijk}, or F on T; therefore, whether they are equal or not does not matter to what follows.

Theorem 6 If two similar single-celled CCA are composed, the resultant CCA is a system with the network and all cells possessing discrete-time segments from both.

Proof. Let $\Phi \sim \Psi$; and $T^\Phi = \{t_1^\Phi, t_2^\Phi, \ldots, t_m^\Phi\}$ and $T^\Psi = \{t_1^\Psi, t_2^\Psi, \ldots, t_n^\Psi\}$, where $T^\Phi \neq T^\Psi$ and $0 < m \leq n \leq \mathbb{N}^*$ and $\mathbb{N}^* \equiv \mathbb{N} - \{0, \infty\}$.

$\Omega = \Phi \odot_\Xi \Psi \Rightarrow T^\Omega = T^\Phi \cup T^\Psi$ at the network-level and $\forall ijk : T^\Omega = T^\Phi \cup T^\Psi$ at the cellular level. $T^\Omega = T^\Phi \cup T^\Psi = \{t_1^\Omega, t_2^\Omega, \ldots, t_{m+n}^\Omega\}$. Given that T is independent of index ijk, even at the cell-level, T^Ω at the cellular level is also $\{t_1^\Omega, t_2^\Omega, \ldots, t_{m+n}^\Omega\}$, retaining equality of discrete-time segments at the network and cellular levels. $\square$

4 CONCLUSION

With adherence to specification and an understanding of the dependencies between tuple elements, composable cellular automata (CCA) can be directly composed with each other. CCA are defined as a network sextuple, which contain cellular automata (cells) defined by a septuple of elements. Composition of CCA is defined as a disjoint union of the tuple elements, with the addition of a mapping composition set and a influencer composition set that can redefine the external input/output and stitch two CCA together, respectively. The properties of set unions are therefore used as a foundation for composition approaches.

The paper discusses CCA composition from the perspective of dimensional properties—cell indices and time. It discusses how two CCA that differ in their index sets can be composed to create a new CCA. This is valid for compositions containing mapping composition sets, influencer compositions sets, or containing or lacking both. The paper also described why the composition exhibits an associate property such that multiple CCA can be composed in any order. Thus, the CCA specification exhibits a closure under composition. Lastly, two CCA that differ only in their discrete-time segments may also be composed, yielding a CCA containing all discrete-timne segments from both original CCA.

The reader is cautioned that proper composition ensures that the system results can be verified to conform to specification. It does not assure validation with respect to the domain. Even if the original subsystems are valid, the composed system cannot be assumed to be valid. For example, changing the tessellation that represents the domain space or adding additional cell influencers may impact the overall simulation results. Similarly, while valid results may be obtained with CCA using two different sets of discrete-time segments, merging the two may not yield valid results. Just as a software developer should test the integration of two correct software components, a software modeler should validate the resultant composed model.

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