Distance-based Inter-cell Interference Coordination in Small Cell Networks: Stochastic Geometry Modeling and Analysis

Jonghun Yoon and Ganguk Hwang, *Member, IEEE*

***Abstract*—We propose a distance-based Inter-Cell Interference Coordination (ICIC) scheme in Small Cell Networks (SCNs). While most of the previous works focus on a randomly selected user called a typical user, we focus on an edge user because the main purpose of ICIC is to improve the performance of an edge user. Since there are many inactive Small cell Base Stations (SBSs) in SCNs, a simple criterion for an edge user such as being located near a cell boundary is not appropriate for SCNs. To accurately detect edge users experiencing severe performance degradation, we newly define an edge user in SCNs based on the nearest active neighbor SBS. We then apply our scheme only to edge users where SBSs within so-called the cooperation radius from each edge user cooperate. With the help of the stochastic geometry we obtain a semi-closed expression for the coverage probability of an edge user with our scheme. We investigate two trade-offs on the resource efficiency of a network and the coverage probability of an interior user. We then determine the optimal cooperation radius that maximizes the coverage probability of an edge user considering the two trade-offs. Our analytical results are validated through simulations.**

***Index Terms*—Stochastic geometry, Poisson point process, Small cell networks, Interference, Edge user, Interference coordination.**

I. Introduction

In Small Cell Networks (SCNs), Inter-cell Interference Coordination (ICIC) using coordinated radio resource management is a promising technique to improve the performance of a so-called edge user who experiences strong intra-tier interference [1], [2] and hence do not satisfy Quality of Service (QoS) such as coverage probability and capacity.

The performance of ICIC depends on the irregular network topology due to the small cell deployment and the spatial distribution of edge users. However, despite the importance of the spatial distribution of edge users in ICIC for performance optimization, several works analyzing ICIC have not considered the spatial distribution of edge users [3], [4].

An edge user usually refers to a user located near a cell boundary in a traditional cellular network. This has been a good criterion on user performance degradation. However, in SCNs, this is no longer a good criterion since there are many inactive Small cell Base Stations (SBSs) and the activation of an SBS changes dynamically due to network densification of SCNs [5], [6].

An earlier version of this paper was presented at the 1st International Workshop on Small Cell Networking for 5G (IEEE SCN2017), Macau, China, Jun. 12, 2017. J. Yoon and G. Hwang are with the Department of Mathematical Sciences, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Republic of Korea. (E-mail: guhwang@kaist.edu)

In this paper, an edge user in SCNs is newly defined based on the nearest active neighbor SBS to accurately detect user performance degradation, and a user-centric ICIC scheme suitable for SCN environment, called the distance-based ICIC scheme, is proposed. We provide an analytical framework based on stochastic geometry to deal with the spatial distribution of edge users in SCNs. With the help of the analytical framework we evaluate the performance of the proposed ICIC scheme and determine the optimal cooperation radius of the distance-based ICIC scheme considering the two trade-offs on the resource efficiency of a network and the coverage probability of an interior user.

A. SCNs and ICIC

Network densification is an inevitable response of fifth generation (5G) mobile networks to explosively increasing demand for data traffic. Recent reports predict that thousands of times mobile traffic is expected in the next ten years [7]. One of the solutions for solving this explosive data traffic increase is the network densification using small cells [5]. By deploying small cells at high density, operators can serve users closer, which improves spectral efficiency and capacity [8]. However, co-channel interference among base stations due to the full frequency reuse still limits the attainable performance of the network. Cross-tier interference between small cells and macro cells can be avoided with split-spectrum assignment [3], but a more technical interference management scheme is required to avoid intra-tier interference among SBSs.

The inter-cell interference has been recognized as the main bottleneck since 3GPP release 8 and there have been several approaches to manage inter-cell interference. ICIC is one of them. The concept of ICIC is described in short as follows: The performance degradation caused by inter-cell interference is particularly noticeable as a user approaches a cell edge. To provide satisfactory QoS to users close to cell edges, usually called edge users, ICIC manages the radio resources of nearby SBSs and hence improves the performance of edge users [1], [2].

ICIC can be divided into two types. One is cell-centric ICIC and the other is user-centric ICIC. Under cell-centric ICIC, the resource management is done by a pre-designed fixed frequency reuse pattern. These schemes have the advantage of simplicity and less signaling overhead. However, it is not suitable for SCN environment because it is difficult in cell-centric ICIC to cope with the dynamics of user locations and

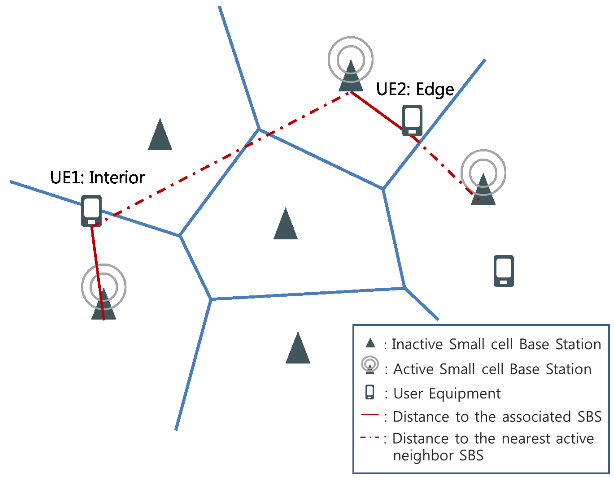


Fig. 1. UE1 and UE2 are located near small cell boundaries. However, while UE2 experiences severe performance degradation due to the nearest active neighbor SBS, UE1 does not experience severe performance degradation because the nearest active neighbor SBS is far away from it. From this viewpoint, it is reasonable to classify UE1 as an interior user and UE2 as an edge user.

channel variations. On the other hand, in user-centric ICIC the resource management is coordinated based on the actual locations of users through multi-cell cooperation. This is more complicated and needs more signaling overhead than cell-centric ICIC, but it is suitable for SCN environment because it can cope with the dynamics of user locations well and there are various efforts to overcome signaling overhead in 5G networks [4], [9].

B. Edge User in Traditional Cellular Network

The terminology *edge user* usually refers to a user located near a cell boundary in a traditional cellular network. This has been a good criterion on user performance degradation since all base stations are active in a traditional cellular network. However, this is no longer a good criterion in SCNs because there are many inactive SBSs in SCNs and the activation of an SBS changes dynamically depending on whether it has a user to server or not [5], [6]. Due to the existence of inactive SBSs in SCNs the fact that a user is located near a small cell boundary does not always imply that the user experiences severe performance degradation. From this viewpoint a better criterion on user performance degradation is necessary in SCNs. In this paper, the terminology *edge user* is defined carefully and differently from that in traditional cellular networks based on a better criterion on user performance degradation.

C. Related Works and Contributions

Conventionally, the grid model and network-level simulation were commonly used to evaluate the performance of a cellular network and they are also used to evaluate the performance of ICIC [10]. However, the grid model becomes intractable unless we have a limited network size, and does not capture the characteristics of practical networks since it is ideal. Practical networks have irregular network topologies, especially in SCNs due to small cell deployment. Recently, since a cellular network model based on stochastic geometry has been proposed [11], more and more cellular network analyses have been based on stochastic geometry that successfully captures the locations of randomly distributed BSs and users [12]. In such analyses the Homogeneous Poisson Point Process (HPPP) has been widely used because it plausibly captures the irregular network topology of practical networks, especially SCNs, and gives mathematically tractable results even when the network size is not limited.

ICIC has been also analyzed using stochastic geometry. In [13], the authors analyzed fractional frequency reuse schemes using independent thinning which is a type of cell-centric ICIC. In [4], the authors proposed and analyzed a user-centric ICIC that uses the signal strength order from the viewpoint of an ICIC-scheduled user. In [3], the authors proposed and analyzed a user-centric ICIC in SCNs where cross-tier interference between small cells and a macro cell is avoided with split-spectrum assignment and SBSs that send interfering signals with strength over a certain threshold to an ICIC-scheduled user stop using the allocated resource.

In addition to ICIC, different kinds of interference management schemes have been studied. In [14], an inter-tier interference coordination scheme in two tier cellular networks is analyzed where small cell transmitters around macro cell receivers do not use frequency channels used by macro cell receivers. In [15], an inter-tier interference avoidance scheme in two tier cellular networks is analyzed where SBSs access macro cell spectrum using the cognitive radio technique. In [16], the interference cancellation, a kind of interference suppression scheme using receiver processing, in spectrum-sharing networks is analyzed.

The main purpose of ICIC is to improve an edge user who experiences performance degradation due to inter-cell interference. Therefore, the performance of an ICIC scheme should be evaluated by the performance improvement of an edge user by ICIC. However, most of the existing works on user-centric ICIC consider a randomly selected user called a typical user, not an edge user, and analyze the performance improvement of the typical user by ICIC. Attempts focusing on an edge user rather than a typical user have been in other areas such as cell-centric ICIC and Coordinated Scheduling [13], [17]. Also meta distribution is proposed in a more general way to study the per-user performance rather than a typical user [18].

In this paper, an edge user in SCNs is newly defined based on the nearest active neighbor SBS and a user-centric ICIC scheme, called the distance-based ICIC scheme, is proposed to provide guaranteed QoS to edge users in SCNs. We use the coverage probability as a performance metric. For performance optimization, we first use stochastic geometry in our analysis to capture the spatial distribution of edge users in SCNs in detail which significantly affects the coverage probability. We then derive the coverage probability of an edge user and investigate two trade-offs on the resource efficiency of a network and the coverage probability of an interior user. Finally, we formulate an optimization problem and find the optimal cooperation radius in the distance-based ICIC scheme that improves the coverage probability of an edge user. The

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2018.2819998, IEEE Transactions on Wireless Communications
This is the author's version of an article that has been published in this journal. Changes were made to this version by the publisher prior to publication.
The final version of record is available at http://dx.doi.org/10.1109/TWC.2018.2819998

main contributions of this paper are summarized as follows.

1) We propose a user-centric ICIC scheme in SCNs and develop an analytical framework to optimize the coverage probability of an edge user. Our analytical framework is based on stochastic geometry and with the help of the analytical framework we derive the coverage probability of an edge user with ICIC.
2) To accurately detect users experiencing severe performance degradation caused by inter-cell interference among SBSs, we newly define an edge user in SCNs by considering whether neighbor SBSs are actually active or not. Through the asymptotic analysis of the coverage probabilities for our edge user and a traditional edge user, i.e., a user located near a cell boundary, we shows that as SBSs get more dense, our definition of an edge user becomes a better criterion than the traditional definition.
3) We investigate two trade-offs on the resource efficiency of a network and the coverage probability of an interior user from our analysis. Based on our investigation we derive the optimal cooperation radius that optimally improves the coverage probability taking into account the two trade-offs.

The rest of the paper is organized as follows: We explain our ICIC scheme in Section II. Section III presents the network model. Section IV derives the expression of several coverage probabilities and investigates the two trade-offs of our ICIC scheme. In Section V, we validate our model and investigate the performance improvement through numerical and simulation study.

II. The Interference Coordination Scheme

We propose a user-centric ICIC scheme using distance-based cooperation to enhance the edge user performance in a network consisting of a number of small cells. Each small cell has one SBS and SBSs in the network are connected via a backhaul to exchange the information for interference coordination.

A. Edge User in the Proposed Scheme

From the perspective of Section I-B, we define an edge user by considering whether neighbor SBSs are actually active or not as follows. We consider a user in an SCN and its associated SBS that is the nearest SBS to the user. We now define the degree of edge for the user as the ratio of the distance from the user to its associated SBS to the distance from the user to the nearest *active* neighbor SBS to the user. The degree of edge has a value between 0 and 1 because the associated SBS of the user is the nearest SBS and the nearest active neighbor SBS is much more far away than its associated SBS. When the value is close to 1, the user experiences high interference from its nearest active neighbor SBS and hence the performance of the user is degraded significantly.

We next consider a threshold to classify the users into edge users and interior users based on the degree of edge. That is, a threshold η is given such that a user is called an edge user if the user has the degree of edge greater than or equal to η. Similarly, a user is called an interior user if the user has the degree of edge less than η. So η is called the edge threshold from now on. Since ICIC is basically a technique to improve the performance of edge users, the edge threshold η must be set based on performance metric. As mentioned before we use the coverage probability as a performance metric. We will later explain how to determine the edge threshold η based on a given coverage probability criterion.

B. Description of the Interference Coordination Scheme

In the proposed distance-based ICIC scheme we assume the following.

1) Cross-tier interference between SBSs and their macro base stations is avoided with split-spectrum assignment. Therefore, our concern is intra-tier interference coordination among SBSs.
2) Each user is associated with its nearest SBS, called the associated SBS, and thus small cells are constructed as the voronoi diagram generated by SBSs.
3) If there is no user in a small cell, the SBS of the small cell is called an inactive SBS. Otherwise, the SBS is called an active SBS. Note that each inactive SBS does not transmit any signal because there is no user to serve.
4) The proposed ICIC scheme is employed in a certain sub-frame. We tag an arbitrary time slot in the sub-frame for analysis. The analysis of the appropriate sub-frame length or ratio is out of the scope of this paper.
5) Intra-cell Time Division Multiple Access (TDMA) is adopted. Each active SBS randomly selects one user in its cell to allocate a Resource Block (RB) at each time slot. We focus on an RB allocated at the tagged time slot, called the tagged RB. A user selected by its associated SBS to allocate the tagged RB, is called a served user.
6) Each SBS has location information about its neighbor SBSs and can receive activation information from its neighbor SBSs through the backhaul. In addition, each active SBS can estimate the location information of its served user.
7) The network information necessary for ICIC is exchanged prior to the tagged time slot. The information exchange procedure are out of the scope of this paper.
8) The edge threshold η is assumed to be given. The detailed explanation on how to determine η will be explained later.
9) The cooperation radius d for ICIC is assumed to be given. The optimal cooperation radius d will be analyzed and determined later.

Now the proposed distance-based ICIC scheme is performed as follows.

1) Each SBS exchanges its activation information with the neighbor SBSs through the backhaul prior to the tagged time slot.
2) Each active SBS uses its own location information and activation information to determine if its served user is an edge user.
3) Any active SBS with its served user being an edge user sends an ICIC cooperation request to its neighbor SBSs within the cooperation radius d from its served user via the backhaul.

4) If an active SBS with its served user being an interior user receives the ICIC cooperation request, the active SBS does not transmit any signal to its served user at the tagged time slot.[1]

III. SYSTEM MODEL

A. Spatial Model

The locations of the SBSs are modeled by a HPPP Φ^b in $\mathbb{R}^2$ with intensity λ^b and small cells are constructed by the Voronoi diagram of the SBSs. The locations of the users are modeled by an independent HPPP Φ^u in $\mathbb{R}^2$ with intensity λ^u. Without loss of generality, we consider a typical user denoted by $\mathbf{u}_o \in \Phi^u$ located at the Cartesian origin o by Slivnyak's theorem [20]. We assume that user $\mathbf{u}_o$ is a served user.

Let $\mathbf{x}_1$ denote the location of the nearest SBS and hence the associated SBS of $\mathbf{u}_o$ and $\mathbf{r}_1$ be the distance from user $\mathbf{u}_o$ to $\mathbf{x}_1$. Let $\mathbf{x}_2$ denote the location of the second nearest SBS of $\mathbf{u}_o$ and $\mathbf{r}_2$ be the distance from user $\mathbf{u}_o$ to SBS $\mathbf{x}_2$. Let $\mathbf{x}_{na}$ denote the location of the nearest active neighbor SBS of $\mathbf{u}_o$ and $\mathbf{r}_{na}$ be the distance from user $\mathbf{u}_o$ to SBS $\mathbf{x}_{na}$.

B. Channel Model

SBSs use transmit power P. The path loss exponent is given by $\alpha > 2$. For each $\mathbf{x} \in \Phi^b$, we add independent marks $\mathbf{h}_\mathbf{x} \in \mathbb{R}$ to denote the small-scale fading on the link from $\mathbf{x} \in \Phi^b$ to $\mathbf{u}_o$ at the tagged RB. $\mathbf{h}_\mathbf{x}$ is assumed to be exponentially distributed with unit mean (which corresponds to Rayleigh fading).[2] It is assumed that the network is interference limited and hence the thermal noise is ignored in our analysis. Because of this assumption, we assume $P = 1$ without loss of generality.

C. Signal to Interference Ratio

Due to the independent locations of SBSs and users, the number of users in each small cell is random and SBSs are classified into active SBSs and inactive SBSs. Since an inactive SBS does not transmit any signal, all users experience inter-cell interference only from active SBSs. So the interference without ICIC at user $\mathbf{u}_o$ is derived as follows.

For $\mathbf{x} \in \Phi^b$, define $\mathbf{A}_\mathbf{x}$ as

$$\mathbf{A}_\mathbf{x} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ is an active SBS,} \\ 0 & \text{if } \mathbf{x} \text{ is an inactive SBS.} \end{cases} \tag{1}$$

Then the interference without ICIC at user $\mathbf{u}_o$, denoted by $\mathbf{I}_{wo}$, is given by

$$\mathbf{I}_{wo} := \sum_{\mathbf{x} \in \Phi^b \setminus \{\mathbf{x}_1\}} P\mathbf{h}_\mathbf{x} \|\mathbf{x}\|^{-\alpha} \mathbf{A}_\mathbf{x}, \tag{2}$$

and the Signal to Interference Ratio (SIR) at user $\mathbf{u}_o$ without ICIC is given by

$$\mathbf{SIR}_{wo} = \frac{P\mathbf{h}_{\mathbf{x}_1}\mathbf{r}_1^{-\alpha}}{\mathbf{I}_{wo}}. \tag{3}$$

When the distance-based ICIC scheme is applied, any active SBS receiving an ICIC cooperation request does not transmit any signal to its served user if its served user is not an edge user. So the interference with ICIC at user $\mathbf{u}_o$, denoted by $\mathbf{I}_w$, is derived as follows.

For $\mathbf{x} \in \Phi^b$, define $\mathbf{E}_\mathbf{x}$ as

$$\mathbf{E}_\mathbf{x} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ serves an edge user,} \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

and define $\mathbf{N}_\mathbf{x}$ as

$$\mathbf{N}_\mathbf{x} = \begin{cases} 1 & \text{if } \mathbf{x} \text{ does not receive an ICIC cooperation request,} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Then the interference with ICIC at user $\mathbf{u}_o$ is

$$\begin{aligned} \mathbf{I}_w = & \sum_{\mathbf{x} \in \Phi^b \setminus \{\mathbf{x}_1\}} P\mathbf{h}_\mathbf{x} \|\mathbf{x}\|^{-\alpha} \mathbf{A}_\mathbf{x} (1 - \mathbf{E}_\mathbf{x}) \mathbf{N}_\mathbf{x} \\ & + \sum_{\mathbf{x} \in \Phi^b \setminus \{\mathbf{x}_1\}} P\mathbf{h}_\mathbf{x} \|\mathbf{x}\|^{-\alpha} \mathbf{A}_\mathbf{x} \mathbf{E}_\mathbf{x}. \end{aligned} \tag{6}$$

Note that when $\mathbf{u}_o$ is an edge user, the interference with ICIC at user $\mathbf{u}_o$ is equal to

$$\begin{aligned} \mathbf{I}_{w,edg} := & \sum_{\mathbf{x} \in \Phi^b \cap B_d^c \setminus \{\mathbf{x}_1\}} P\mathbf{h}_\mathbf{x} \|\mathbf{x}\|^{-\alpha} \mathbf{A}_\mathbf{x} (1 - \mathbf{E}_\mathbf{x}) \mathbf{N}_\mathbf{x} \\ & + \sum_{\mathbf{x} \in \Phi^b \setminus \{\mathbf{x}_1\}} P\mathbf{h}_\mathbf{x} \|\mathbf{x}\|^{-\alpha} \mathbf{A}_\mathbf{x} \mathbf{E}_\mathbf{x} \end{aligned} \tag{7}$$

where $B_d := \{x \in \mathbb{R}^2 : \|x\| \le d\}$.

Then the SIR at user $\mathbf{u}_o$ with ICIC is given by

$$\mathbf{SIR}_w = \frac{(\mathbf{E}_{\mathbf{x}_1} + (1 - \mathbf{E}_{\mathbf{x}_1})\mathbf{N}_{\mathbf{x}_1})\, P\mathbf{h}_{\mathbf{x}_1}\mathbf{r}_1^{-\alpha}}{\mathbf{I}_w}. \tag{8}$$

Note that if user $\mathbf{u}_o$ is an edge user, then

$$\mathbf{SIR}_w = \frac{P\mathbf{h}_{\mathbf{x}_1}\mathbf{r}_1^{-\alpha}}{\mathbf{I}_{w,edg}}. \tag{9}$$

D. Coverage Probability

For a given coverage threshold θ,

$$\begin{aligned} cp_{wo}(\theta \mid \cdot) &:= \mathbb{P}\left[\mathbf{SIR}_{wo} > \theta \mid \cdot\right] \text{ and} \\ cp_w(\theta, d \mid \cdot) &:= \mathbb{P}\left[\mathbf{SIR}_w > \theta \mid \cdot\right] \end{aligned}$$

are the conditional coverage probabilities of user $\mathbf{u}_o$ without and with ICIC given some condition $\cdot$, respectively.

For simplicity we drop *conditional* and use the term *the coverage probability* from now on.

[1]The ICIC scheme improves edge users performance by the concession from high-performance interior users at the sub-frame. This concept is similar to that of enhanced Inter-Cell Interference Coordination (eICIC) proposed in 3GPP LTE release [19].

[2]Although the rayleigh fading channel assumption simplifies the analysis, the presented analysis can be extended to a more general channel model including log-normal shadowing as in [11].

IV. PERFORMANCE ANALYSIS

In this section, we first analyze the coverage probabilities of edge user $\mathbf{u}_o$ without ICIC and the coverage probability of user $\mathbf{u}_o$ located near a small cell boundary for a comparison purpose. Then the coverage probabilities of edge user $\mathbf{u}_o$ with ICIC is analyzed. In addition, the resource efficiency of a network and the lower bound of the coverage probability of interior user $\mathbf{u}_o$ under ICIC are analyzed. For convenience, the terminologies and notations on user $\mathbf{u}_o$ used in the paper are listed in Table I.

Terminology/Notation	Definition
$\mathbf{r}_1$	Distance to the nearest SBS $\mathbf{x}_1$
$\mathbf{r}_2$	Distance to the second nearest SBS $\mathbf{x}_2$
$\mathbf{r}_{na}$	Distance to the nearest active neighbor SBS $\mathbf{x}_{na}$
Typical user	$\mathbf{u}_o$ without condition, i.e., a randomly selected user
Edge user	$\mathbf{u}_o$ with condition $\frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta$ for some $\eta \in [0,1]$
Interior user	$\mathbf{u}_o$ with condition $\frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta$ for some $\eta \in [0,1]$
Boundary user	$\mathbf{u}_o$ with condition $\frac{\mathbf{r}_1}{\mathbf{r}_2} \geq \zeta$ for some $\zeta \in [0,1]$

TABLE I: Terminology and Notation for user $\mathbf{u}_o$

A. Coverage Probability of an Edge User without ICIC

We first analyze the probability that user $\mathbf{u}_o$ is an edge user, denoted by p^u_{edg}, then the coverage probability of edge user $\mathbf{u}_o$ without ICIC, and finally the coverage probability of user $\mathbf{u}_o$ located near a small cell boundary for a comparison purpose.

Note that for a given edge threshold η, user $\mathbf{u}_o$ is an edge user when its degree of edge is greater than or equal to η, i.e., $\frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta$.

For a tractable analysis which takes into account the random positioning of active SBSs, suppose that $\mathbf{A}_\mathbf{x}$ in (1) for $\mathbf{x} \in \Phi^b$ is an independent Bernoulli random variable with success probability $p^{b,a}$ where $p^{b,a}$ is the probability that a typical SBS is active. It has been shown that $p^{b,a}$ is a function of the SBS-user intensity ratio $\frac{\lambda^b}{\lambda^u}$, given by [21]

$$p^{b,a} = 1 - \left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}. \tag{10}$$

Let $\Phi^{b,a} := \left\{\mathbf{x} \in \Phi^b : \mathbf{A}_\mathbf{x} = 1\right\}$, which denotes the locations of active SBSs and is approximated by an independently thinning of Φ^b with thinning probability $p^{b,a}$. Then the intensity of active SBSs $\Phi^{b,a}$, denoted by $\lambda^{b,a}$, satisfies $\lambda^{b,a} = p^{b,a}\lambda^b$.

To analyze the probability p^u_{edg} that user $\mathbf{u}_o$ is an edge user, we need the following lemma.

LEMMA 1: The joint probability density function (p.d.f.) of $\mathbf{r}_1$ and $\mathbf{r}_{na}$ is given by, for $r_1 < r_{na}$

$$\begin{aligned} & f_{\mathbf{r}_1,\mathbf{r}_{na}}(r_1, r_{na}) \\ & = p^{b,a}(2\pi\lambda^b)^2 r_1 r_{na} e^{-p^{b,a}\lambda^b\pi r_{na}^2} e^{-(1-p^{b,a})\lambda^b\pi r_1^2}. \end{aligned} \tag{11}$$

PROOF. From $\mathbb{P}\left[\mathbf{r}_{na} > r_{na} | \mathbf{r}_1 = r_1, \mathbf{r}_{na} > \mathbf{r}_1\right] = e^{-\lambda^{b,a}\pi(r_{na}^2 - r_1^2)}$, we obtain the following conditional p.d.f. of $\mathbf{r}_{na}$, given $\mathbf{r}_1$ as

$$f_{\mathbf{r}_{na}|\mathbf{r}_1}(r_{na}|r_1) = 2\lambda^{b,a}\pi r_{na} e^{-\lambda^{b,a}\pi(r_{na}^2 - r_1^2)}. \tag{12}$$

Multiplying the p.d.f. of $\mathbf{r}_1$ to the above equation, which is given in [22], we obtain the joint p.d.f. of $\mathbf{r}_1$ and $\mathbf{r}_{na}$ as follows:

$$\begin{aligned} & f_{\mathbf{r}_1,\mathbf{r}_{na}}(r_1, r_{na}) \\ & = 2\lambda^{b,a}\pi r_{na} e^{-\lambda^{b,a}\pi(r_{na}^2 - r_1^2)} 2\lambda^b \pi r_1 e^{-\lambda^b\pi r_1^2} \\ & = p^{b,a}(2\pi\lambda^b)^2 r_1 r_{na} e^{-p^{b,a}\lambda^b\pi r_{na}^2} e^{-(1-p^{b,a})\lambda^b\pi r_1^2}. \end{aligned}$$

■

Using Lemma 1, we derive the probability p^u_{int} that user $\mathbf{u}_o$ is an interior user and the probability p^u_{edg} that user $\mathbf{u}_o$ is an edge user.

LEMMA 2: The probability p^u_{int} that user $\mathbf{u}_o$ is an interior user and the probability p^u_{edg} that user $\mathbf{u}_o$ is an edge user, are explicitly given by

$$p^u_{int} = \frac{\eta^2}{1 - (1-\eta^2)\left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}} \tag{13}$$

$$p^u_{edg} = \frac{(1-\eta^2)\left(1 - \left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}\right)}{1 - (1-\eta^2)\left(1 + \frac{\lambda^u}{3.5\lambda^b}\right)^{-3.5}}, \tag{14}$$

respectively.

PROOF. Using (11) in Lemma 1, we have

$$\begin{aligned} \mathbb{P}\left[\frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right] &= \int_0^\infty \int_{\frac{r_1}{\eta}}^\infty f_{\mathbf{r}_1,\mathbf{r}_{na}}(r_1, r_{na})\, dr_{na}\, dr_1 \\ &= \frac{1}{1 - p^{b,a} + p^{b,a}/\eta^2}. \end{aligned}$$

Since the probability that user $\mathbf{u}_o$ is an edge user satisfies $p^u_{edg} = 1 - p^u_{int}$, from (10) we obtain the lemma. ■

REMARK 1: Note that (13) and (14) are functions of the SBS-user intensity ratio λ^b/λ^u and the edge threshold η. The larger the SBS-user intensity ratio λ^b/λ^u, the smaller p^u_{edg} since the nearest neighbor active SBS is relatively farther away than the nearest SBS. Also, the larger the edge threshold η, the smaller p^u_{edg}.

To analyze the coverage probability of edge user $\mathbf{u}_o$ without ICIC, we next derive the coverage probability of edge user $\mathbf{u}_o$ without ICIC for fixed r_1 and r_{na}. For later use we define a function $H(x, y)$ by

$$\begin{aligned} H(x,y) &= 2\int_{\frac{x}{y}}^\infty \frac{\theta v}{v^\alpha + \theta}\, dv \\ &= \frac{\theta^{\frac{2}{\alpha}}}{\operatorname{sinc}\left(\frac{2}{\alpha}\right)} - \left(\frac{x}{y}\right)^2 {}_2F_1\left(1, \frac{2}{\alpha}; 1 + \frac{2}{\alpha}; -\frac{1}{\theta}\left(\frac{x}{y}\right)^\alpha\right) \end{aligned} \tag{15}$$

where ${}_2F_1(a, b; c; z)$ is a hypergeometric function and $\operatorname{sinc}(x)$ is a sinc function.

LEMMA 3: For fixed r_1 and r_{na},

$$cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \geq \eta\right) = \frac{e^{-\pi p^{b,a}\lambda^b r_1^2 H(r_{na}, r_1)}}{1 + \theta\left(\frac{r_1}{r_{na}}\right)^\alpha} \tag{16}$$

where $p^{b,a}$ is given in (10) and $H(x, y)$ are given in (15).

The proof of Lemma 3 is provided in Appendix A.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2018.2819998, IEEE Transactions on Wireless Communications
The final version of record is available at http://dx.doi.org/10.1109/TWC.2018.2819998

The next proposition provides the coverage probability of edge user $\mathbf{u}_o$ without ICIC.

PROPOSITION 1:

$$cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) = \frac{p^{b,a}}{p^u_{edg}} \int_0^\infty \int_{r_1}^{\frac{r_1}{\eta}} \frac{r_1 r_{na}}{1+\theta\left(\frac{r_1}{r_{na}}\right)^\alpha} \times e^{-\frac{1}{2}\left(\left(p^{b,a}(H(r_{na},r_1)-1)+1\right)r_1^2 + p^{b,a} r_{na}^2\right)}\, dr_{na}\, dr_1 \quad (17)$$

where $p^{b,a}$, p^u_{edg} and $H(x,y)$ are given in (10), (14) and (15), respectively.

PROOF. From

$$cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) = \frac{1}{p^u_{edg}} \iint_{r_1 \geq \eta r_{na}} cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \geq \eta\right) f_{\mathbf{r}_1,\mathbf{r}_{na}}(r_1, r_{na})\, dr_{na}\, dr_1,$$

by substituting $cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \geq \eta\right)$ into (16) and employing the change of variables $r_1' = \sqrt{2\pi\lambda^b} r_1$ and $r_{na}' = \sqrt{2\pi\lambda^b} r_{na}$, our proposition immediately follows. ■

REMARK 2: Note that (17) is a function of the path loss exponent α, the SBS-user intensity ratio λ^b/λ^u, the edge threshold η, and the coverage threshold θ.

In the following corollary of which proof is provided in Appendix B, we prove the decreasing property of (17) and derive the limiting value of (17) as λ^b goes to ∞.

COROLLARY 1:

(a) $cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right)$ is a decreasing function in η.

(b) $\lim_{\lambda^b \to \infty} cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) = \frac{2\eta^2}{1-\eta^2} \int_\eta^1 \frac{1}{(1+\theta y^\alpha) y^3}\, dy.$

REMARK 3: From Proposition 1 and Corollary 1 we can determine the edge threshold η as follows. We assume that a coverage probability criterion ϕ is given *a priori*. The coverage probability criterion implies an allowable coverage probability for a user without ICIC, i.e., if the coverage probability of a user is smaller than ϕ, then the user experiences a severe difficulty in communication. Recall that our ICIC scheme is applied to improve the coverage probability of an edge user. So it is reasonable to define an edge user as a user whose coverage probability is severely small and needs to be improved. From this viewpoint and the decreasing property of the coverage probability given in Corollary 1, for a given coverage probability criterion ϕ we can determine η that satisfies $\phi = cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right)$. Note that the edge threshold η is a function of the path loss exponent α, the SBS-user intensity ratio λ^b/λ^u, the coverage threshold θ, and the coverage probability criterion ϕ.

For a comparison purpose, in what follows we analyze the coverage probability of a user located near a cell boundary and show that the fact that a user is located near a cell boundary does not always implies that the user experiences severe performance degradation in SCNs.

If $\frac{\mathbf{r}_1}{\mathbf{r}_2} \geq \zeta$ for some ζ, we could say that user $\mathbf{u}_o$ is located near a cell boundary. Recall that $\mathbf{r}_1$ and $\mathbf{r}_2$ are the distances from the user to the nearest and the second nearest SBSs, respectively. This user is called a boundary user to distinguish it from our edge user.

With the same argument used to derive Proposition 1, we derive the next proposition that provides the coverage probability of boundary user $\mathbf{u}_o$.

PROPOSITION 2:

$$cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_2} \geq \zeta\right) = \frac{1}{1-\zeta^2} \int_0^\infty \int_{r_1}^{\frac{r_1}{\zeta}} \left((1-p^{b,a}) + p^{b,a} \frac{1}{1+\theta\left(\frac{r_1}{r_2}\right)^\alpha}\right) r_1 r_2 \times e^{-\frac{1}{2}\left(p^{b,a} H(r_2,r_1) r_1^2 + r_2^2\right)}\, dr_2\, dr_1 \quad (18)$$

where $p^{b,a}$ and $H(x,y)$ are given in (10) and (15), respectively.

The proof of Proposition 2 is provided in Appendix C.

REMARK 4: Note that (18) is a function of the path loss exponent α, the SBS-user intensity ratio λ^b/λ^u, the value ζ and the coverage threshold θ.

Similarly to Corollary 1, the following corollary provides the decreasing property of (18) and the limiting value of (18) as λ^b goes to ∞.

COROLLARY 2:

(a) $cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_2} \geq \zeta\right)$ is a decreasing function in ζ.

(b) $\lim_{\lambda^b \to \infty} cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_2} \geq \zeta\right) = 1.$

PROOF. (a) is obtained by the same argument as in Corollary 1-(a). (b) immediately follows from $\frac{r_1}{r_2} \leq 1$ and $\lim_{\lambda^b \to \infty} p^{b,a} = 0$. ■

REMARK 5: Corollary 2 shows that as the network gets more dense, the fact that a user is located near a cell boundary cannot be a good criterion on performance degradation caused by inter-cell interference because neighbor SBSs are likely to be inactive even if they are close to the user. On the other hand, as we can see from Corollary 1, our definition for the edge user can detect users experiencing performance degradation caused by inter-cell interference regardless of the network intensity because it considers only active neighbor SBSs. This is also verified numerically in Section V-A.

B. ICIC Cooperation Request

Next, for later use we derive the probability that a typical SBS does not receive any ICIC cooperation request, denoted by $p_{on}(d)$. In this section, we consider a typical SBS located at the Cartesian origin o. Since a typical SBS receives an ICIC cooperation request if there exist edge users served by their associated SBSs within a distance d from itself, the probability $p_{on}(d)$ is given by

$$p_{on}(d) = \mathbb{P}_o\left[\Phi^{u,s}_{edg}(B_d \setminus V) = 0\right] \quad (19)$$

where $\mathbb{P}_o[\cdot]$ denotes the Palm probability [20] for a typical SBS, $\Phi^{u,s}_{edg}$ denotes the locations of served users who are edge users and V is the voronoi cell of the typical SBS. The reason why the voronoi cell V of the typical SBS is removed in the probability is that the typical SBS obviously receives ICIC

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2018.2819998, IEEE Transactions on Wireless Communications
The final version of record is available at http://dx.doi.org/10.1109/TWC.2018.2819998

cooperation request from an edge user served by another SBS in another cell, not its own cell.

To calculate $p_{on}(d)$ in a tractable way, we use an independent thinning approximation[3] for $\Phi^{u,s}_{edg}$. Recall that each active SBS randomly selects one user in its cell to allocate the tagged RB. To approximate the locations of served users, we need to know the probability that a user is selected by its associated SBS, denoted by $p^{u,s}$, and hence actually served at the tagged RB. It has been shown that $p^{u,s}$ is a function of the SBS-user intensity ratio $\frac{\lambda^b}{\lambda^u}$, given by [21]

$$p^{u,s} = p^{b,a}\frac{\lambda^b}{\lambda^u} \tag{20}$$

where $p^{b,a}$ is given in (10). Using (14) and (20), $\Phi^{u,s}_{edg}$ are approximated by an independent thinning of Φ^u with thinning probability $p^u_{edg}p^{u,s}$. It then follows that the intensity of $\Phi^{u,s}_{edg}$, denoted by $\lambda^{u,s}_{edg}$, is given by $\lambda^{u,s}_{edg} = p^u_{edg}p^{u,s}\lambda^u$. Moreover, by (20), we have

$$\lambda^{u,s}_{edg} = p^u_{edg}p^{b,a}\lambda^b. \tag{21}$$

V is also hard to handle because of its random cell shape. For this reason, we approximate the probability $p_{on}(d)$ using the largest disk included in V. Let $\mathbf{r}_m$ be the radius of the largest disk included in V. The p.d.f. of $\mathbf{r}_m$ denoted by $f_{\mathbf{r}_m}(r)$ is given in [23] by

$$f_{\mathbf{r}_m}(r) = 8\pi\lambda^b r e^{-4\pi\lambda^b r^2}. \tag{22}$$

Using the independent thinning approximation for $\Phi^{u,s}_{edg}$ and the p.d.f. of $\mathbf{r}_m$, we finally obtain the following result.

LEMMA 4: A lower bound of $p_{on}(d)$, denoted by $\underline{p_{on}}(d)$, is given by

$$\begin{aligned}&\underline{p_{on}}(d) = \\ &e^{-p^u_{edg}p^{b,a}\pi\lambda^b d^2}\frac{4}{4-p^u_{edg}p^{b,a}}\left(1-e^{-\left(4-p^u_{edg}p^{b,a}\right)\pi\lambda^b d^2}\right) \\ &+ e^{-4\pi\lambda^b d^2}\end{aligned} \tag{23}$$

where $p^{b,a}$ and p^u_{edg} are given in (10) and (14), respectively.

PROOF. Let $\mathbf{r}_m$ be the radius of the largest disk included in V. It then follows from (19), (22) and Slivnyak's theorem that

$$\begin{aligned}p_{on}(d) &= \mathbb{P}_o\left[\Phi^{u,s}_{edg}(B_d\setminus V)=0\right] \\ &\geq \mathbb{P}_o\left[\Phi^{u,s}_{edg}(B_d\setminus B_{\mathbf{r}_m})=0\right] \\ &= \int_0^d e^{-\lambda^{u,s}_{edg}\pi(d^2-r^2)}8\pi\lambda^b r e^{-4\pi\lambda^b r^2}\,dr + e^{-4\pi\lambda^b d^2} \\ &= e^{-\lambda^{u,s}_{edg}\pi d^2}\frac{4\lambda^b}{4\lambda^b-\lambda^{u,s}_{edg}}\left(1-e^{-\left(4\lambda^b-\lambda^{u,s}_{edg}\right)\pi d^2}\right) \\ &\quad + e^{-4\lambda^b\pi d^2}\end{aligned}$$

where $\lambda^{u,s}_{edg} = p^u_{edg}p^{b,a}\lambda^b$ given in (21). ∎

In Section V, we will verify that the simulation results and the analytical results using $\underline{p_{on}}(d)$ instead of $p_{on}(d)$ are well matched.

REMARK 6: Note that (23) is a function of the value the SBS-user intensity ratio λ^b/λ^u ,$\sqrt{\lambda^b}d$ and the edge threshold η. The larger the SBS-user intensity ratio λ^b/λ^u is, the smaller the intensity of served users who are edge users, which increases the value of $\underline{p_{on}}(d)$. Also, the larger the cooperation radius d, the smaller the value of $\underline{p_{on}}(d)$ since a typical SBS is more likely to receive a cooperation request. Finally, the larger the edge threshold η, the smaller the number of edge users and hence the larger the value of $\underline{p_{on}}(d)$.

C. Coverage Probability of an Edge User with ICIC

From now on we assume that user $\mathbf{u}_o$ is an edge user. In this case we analyze the coverage probability of edge user $\mathbf{u}_o$ with ICIC.

In order to consider the interference with ICIC at edge user $\mathbf{u}_o$ given in (7) as a tractable way, we use independent thinning approximations for the locations of SBSs as follows. Recall $\mathbf{A_x}$, $\mathbf{E_x}$, and $\mathbf{N_x}$ given in (1), (4), and (5), respectively. Let $\Phi^{b,a}_{edg} := \left\{\mathbf{x}\in\Phi^b : \mathbf{A_x}\mathbf{E_x}=1\right\}$ which denotes the locations of active SBSs that serve their edge users and $\Phi^{b,a}_{int,on} := \left\{\mathbf{x}\in\Phi^b : \mathbf{A_x}(1-\mathbf{E_x})\mathbf{N_x}=1\right\}$ which denotes the locations of active SBSs that serve their interior users but do not receive any ICIC cooperation request. By assuming that $\mathbf{A_x}$, $\mathbf{E_x}$ and $\mathbf{N_x}$ are independent Bernoulli random variables with success probabilities $p^{b,a}$, p^u_{edg} and $p_{on}(d)$, respectively,[4] $\Phi^{b,a}_{edg}$ is approximated by an independent thinning of Φ^b with thinning probability $p^u_{edg}p^{b,a}$ and $\Phi^{b,a}_{int,on}$ is approximated by an independent thinning of Φ^b with thinning probability $p_{on}(d)p^u_{int}p^{b,a}$. It then follows that the intensity of $\Phi^{b,a}_{edg}$, denoted by $\lambda^{b,a}_{edg}$, satisfies that $\lambda^{b,a}_{edg} = p^u_{edg}p^{b,a}\lambda^b$. Similarly, the intensity of $\Phi^{b,a}_{int,on}$, denoted by $\lambda^{b,a}_{int,on}$, satisfies that $\lambda^{b,a}_{int,on} = p_{on}(d)p^u_{int}p^{b,a}\lambda^b$.

With the help of above independent thinning approximations, the coverage probability of edge user $\mathbf{u}_o$ with ICIC is derived in the next proposition of which proof are provided in Appendix D.

PROPOSITION 3: $cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}}\geq\eta\right)$ is equal to (26).

REMARK 7: Note that (26) is a function of the path loss exponent α, the SBS-user intensity ratio λ^b/λ^u, the edge threshold η, the coverage threshold θ and the value $\sqrt{\lambda^b}d$. Moreover, (26) is an increasing function in d.

D. Optimal Cooperation Radius

In this section we investigate two fundamental trade-offs in our ICIC scheme. We then determine the optimal cooperation radius that optimally improve the coverage probability of edge users taking into account the two trade-offs.

[3]When our ICIC scheme is employed, the locations of users and BSs become correlated. However, if the correlation does not affect the coverage probability significantly, we might use a thinned PPP as an approximation and this will be verified through simulation results. The use of thinned PPP as an approximation is widely used in many previous works, e.g. [4], [13] even when there exists some correlation between locations of users and base stations.

[4]Since each active SBS uniform-randomly selects one user in their cell to serve, the probability that an active SBS serves an edge user is equal to p^u_{edg}.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2018.2819998, IEEE Transactions on Wireless Communications
This is the author's version of an article that has been published in this journal. Changes were made to this version by the publisher prior to publication.
The final version of record is available at http://dx.doi.org/10.1109/TWC.2018.2819998

1) Resource efficiency of a network: The first trade-off is the resource efficiency of a network which is defined by

$$R_{eff}(d) := \lim_{W \uparrow \mathbb{R}^2} \mathbb{E}\left[\frac{\Phi^{b,a}_{edg}(W) + \Phi^{b,a}_{int,on}(W)}{\Phi^{b,a}(W)}\right] \tag{27}$$

where $\Phi^{b,a}$ is the locations of active SBSs, $\Phi^{b,a}_{edg}$ is the locations of active SBSs that serve their edge users, $\Phi^{b,a}_{int,on}$ is the locations of active SBSs that serve their interior users but do not receive any ICIC cooperation request, and W denotes a square of area w^2 centered at the origin. When the cooperation radius d is large, many active SBSs with its served user being an interior user are likely to receive an ICIC cooperation request and hence do not serve any users in their cells, which obviously causes a waste in network resources. So our ICIC scheme should be designed to ensure a certain level of resource efficiency.

In what follows we derive a closed form expression of the resource efficiency of a network.

PROPOSITION 4:

$$R_{eff}(d) = 1 - p^u_{int}\left(1 - p_{on}(d)\right). \tag{28}$$

where p^u_{int} and $p_{on}(d)$ are given in (13) and (23), respectively.

The proof of Proposition 4 is provided in Appendix E.

REMARK 8: Note that (28) is a function of the value $\sqrt{\lambda^b}d$, the SBS-user intensity ratio λ^b/λ^u and the edge threshold η. Moreover, (28) is a decreasing function in d.

COROLLARY 3:

$$\lim_{d \to \infty} R_{eff}(d) = p^u_{edg}. \tag{29}$$

PROOF. From $\lim_{d \to \infty} p_{on}(d) = 0$, our corollary immediately follows. ■

2) Coverage probability of an interior user: The second trade-off is the coverage probability of an interior user with ICIC which is given by (8)

$$cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right) = \mathbb{P}\left[\frac{\mathbf{N}_{\mathbf{x}_1} P \mathbf{h}_{\mathbf{x}_1} \mathbf{r}_1^{-\alpha}}{\mathbf{I}_w} > \theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right] \tag{30}$$

where $\mathbf{N}_{\mathbf{x}_1}$ and $\mathbf{I}_w$ are given in (5) and (6), respectively. For the same reason as in the resource efficiency of a network, as the cooperation radius increases, the coverage probability of an interior user decreases. Although our ICIC scheme aims to increase the coverage probability of edge users by the concession from interior users, it is important to ensure that the decrease in the coverage probability of an interior user due to ICIC is maintained within an allowable range.

The coverage probability of an interior user with ICIC is more difficult to handle than that of an edge user since the network topology and spatial correlation seen from an interior user are more complex than those seen from an edge user. Thus, in what follows we drive a lower bound of the coverage probability of an interior user with ICIC assuming that $\mathbf{N}_{\mathbf{x}_1}$ is an independent Bernoulli random variable with success probability $p_{on}(d)$.

PROPOSITION 5:

$$\begin{aligned} & cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right) \geq \\ & \frac{p_{on}(d)}{p^u_{int}}\left(\frac{1}{1 + p^{b,a}\frac{2\theta}{\alpha-2}{}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\theta\right)}\right. \\ & \left. - p^u_{edg}\, cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right)\right) \end{aligned} \tag{31}$$

where $p^{b,a}$, p^u_{int}, p^u_{edg}, $p_{on}(d)$, and $cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right)$ are given in (10), (13), (14), (23), and (17), respectively.

PROOF. Our proposition immediately follows from

$$\begin{aligned} & cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right) \\ & = p_{on}(d)\mathbb{P}\left[\frac{P\mathbf{h}_{\mathbf{x}_1}\mathbf{r}_1^{-\alpha}}{\mathbf{I}_w} > \theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right] \\ & \geq p_{on}(d)\mathbb{P}\left[\frac{P\mathbf{h}_{\mathbf{x}_1}\mathbf{r}_1^{-\alpha}}{\mathbf{I}_{wo}} > \theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right] \text{ since } \mathbf{I}_{wo} \geq \mathbf{I}_w \\ & = p_{on}(d)\frac{\mathbb{P}\left[\mathbf{SIR}_{wo} > \theta\right] - p^u_{edg}\mathbb{P}\left[\mathbf{SIR}_{wo} > \theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right]}{p^u_{int}} \end{aligned}$$

where $\mathbf{I}_{wo}, \mathbf{SIR}_{wo}, \mathbf{I}_w$ are given in (2), (3), (6) and $\mathbb{P}\left[\mathbf{SIR}_{wo} > \theta\right] = \frac{1}{1 + p^{b,a}\frac{2\theta}{\alpha-2}{}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\theta\right)}$ by the same argument as in Theorem 2 in [11]. ■

REMARK 9: Note that RHS of (31) is a function of the path loss exponent α, the SBS-user intensity ratio λ^b/λ^u, the edge threshold η, the coverage threshold θ and the value $\sqrt{\lambda^b}d$. Note further that the RHS of (31) is equal to $p_{on}(d)cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right)$, which is a decreasing function in d.

From the perspective described above, to determine the optimal cooperation radius we have to consider two constraints on the resource efficiency of a network and the coverage

$$\begin{aligned} cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) = & \frac{p^{b,a}}{p^u_{edg}}\left(\int_{\sqrt{2\pi\lambda^b}d\eta}^{\sqrt{2\pi\lambda^b}d}\int_{\sqrt{2\pi\lambda^b}d}^{\frac{r_1}{\eta}} C_1(r_1, r_{na})\, dr_{na}\, dr_1 + \int_{\sqrt{2\pi\lambda^b}d}^{\infty}\int_{r_1}^{\frac{r_1}{\eta}} C_1(r_1, r_{na})\, dr_{na}\, dr_1\right. \\ & \left. + \int_0^{\sqrt{2\pi\lambda^b}d\eta}\int_{r_1}^{\frac{r_1}{\eta}} C_2(r_1, r_{na})\, dr_{na}\, dr_1 + \int_{\sqrt{2\pi\lambda^b}d\eta}^{\sqrt{2\pi\lambda^b}d}\int_{r_1}^{\sqrt{2\pi\lambda^b}d} C_2(r_1, r_{na})\, dr_{na}\, dr_1\right). \end{aligned} \tag{26}$$

Here, $p^{b,a}$, p^u_{int}, p^u_{edg}, $p_{on}(d)$, and $H(x, y)$ are given in (10), (13), (14), (23), and (15), respectively, and

$C_1(r_1, r_{na}) := \left(1 + (1 - p_{on}(d))p^u_{int}\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\right)\frac{r_1 r_{na}}{1+\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}} e^{-\frac{1}{2}\left(\left(p^{b,a}\left((p^u_{edg}+p^u_{int}p_{on}(d))H(r_{na}, r_1)-1\right)+1\right)r_1^2 + p^{b,a}r_{na}^2\right)}$,

$C_2(r_1, r_{na}) := \left(1 + p^u_{int}\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\right)\frac{r_1 r_{na}}{1+\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}} e^{-\frac{1}{2}\left(\left(p^{b,a}\left(p^u_{edg}H(r_{na}, r_1)+p^u_{int}p_{on}(d)H(\sqrt{2\pi\lambda^b}d, r_1)-1\right)+1\right)r_1^2 + p^{b,a}r_{na}^2\right)}$.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2018.2819998, IEEE Transactions on Wireless Communications
The final version of record is available at http://dx.doi.org/10.1109/TWC.2018.2819998

probability of an interior user, which results in the following optimization problem on the cooperation radius d.

Let a_1 be the minimum allowable resource efficiency of a network and a_2 be the value between 0 and 1 so that $a_2 \cdot cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right)$ is the minimum allowable coverage probability of an interior user. For given $a_1,\, a_2 \in (0,1)$,

$$\begin{aligned} &\text{maximize: } cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) \\ &\text{subject to:} \\ &\quad R_{eff}(d) \geq a_1 \text{ and} \\ &\quad cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right) \geq a_2 \cdot cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right). \end{aligned} \tag{32}$$

REMARK 10: We focus on improving the performance of edge users at the cost of the performance degradation of interior users. In our optimization problem (32), we consider a constraint on the coverage probability of an interior user, so that we can prevent the performance degradation of an interior user. From this viewpoint, the impact of our ICIC scheme on the network throughput is not significant.

Since we do not have an exact expression for $cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right)$, we cannot find the optimal cooperation radius d^*, but we can determine a suboptimal cooperation radius $\underline{d}^*$ easily from Proposition 4 and our lower bound of $cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} < \eta\right)$ in Proposition 5, which satisfies

$$p_{on}(\underline{d}^*) = \max\left(\frac{a_1 - p^u_{edg}}{p^u_{int}}, a_2\right). \tag{33}$$

In Section V-E, we will verify that the ICIC performance using the suboptimal cooperation radius $\underline{d}^*$ is close to the ICIC performance using the optimal cooperation radius d^*.

V. NUMERICAL EVALUATION AND SIMULATIONS

In this section we provide numerical results based on our analysis and simulation results for the proposed ICIC scheme using MATLAB. Let $W \subset \mathbb{R}^2$ be a finite square observation window whose edge length is 4000 m. For each realization, the SBSs are distributed on W as an HPPP with intensity λ^b and users are distributed as an HPPP with intensity λ^u. 10^5 realizations are averaged to obtain the simulation results. The approximations and numerical evaluation of the integrals in our analysis are validated by simulations.

A. Two Performance Degradation Criterions

In Section IV-A, from Corollary 1 and Corollary 2, we see that as the network gets more dense, the fact that a user is located near a cell boundary is not a good criterion for performance degradation caused by inter-cell interference, but our definition for the edge user is. In this section, we compare the two performance degradation criteria numerically.

The results are plotted in Fig. 2. The figure shows that regardless of the value λ^b, the coverage probability of an edge user is away from that of a typical user, which implies that the edge user obviously experiences performance degradation. On the other hand, the coverage probability of a boundary user becomes closer to that of a typical user and goes to 1 as λ^b increases. This implies that a boundary user might not experience severe performance degradation when λ^b is relatively large and hence the consideration of a boundary user in ICIC might not make sense. From this observation, we see that our definition of an edge user provides a better criterion on performance degradation caused by inter-cell interference in SCNs.

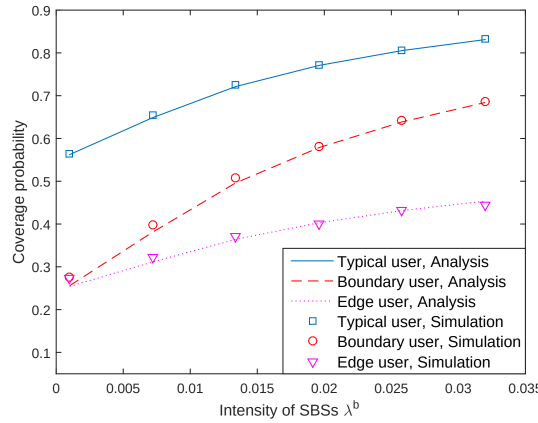


Fig. 2. The comparison of two performance degradation criteria ($\theta = 0$ dB, $\lambda^u = 10^{-2}$ m^{-2}, $\eta = \zeta = 0.8$).

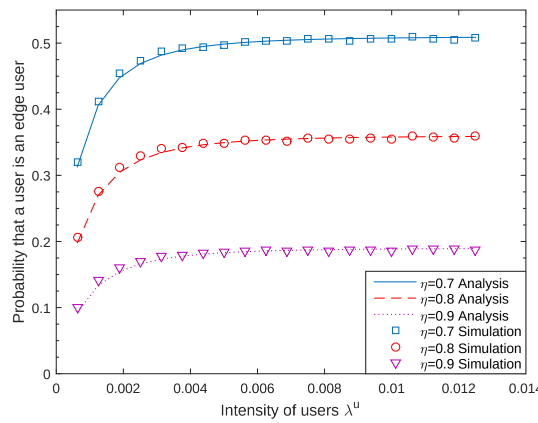


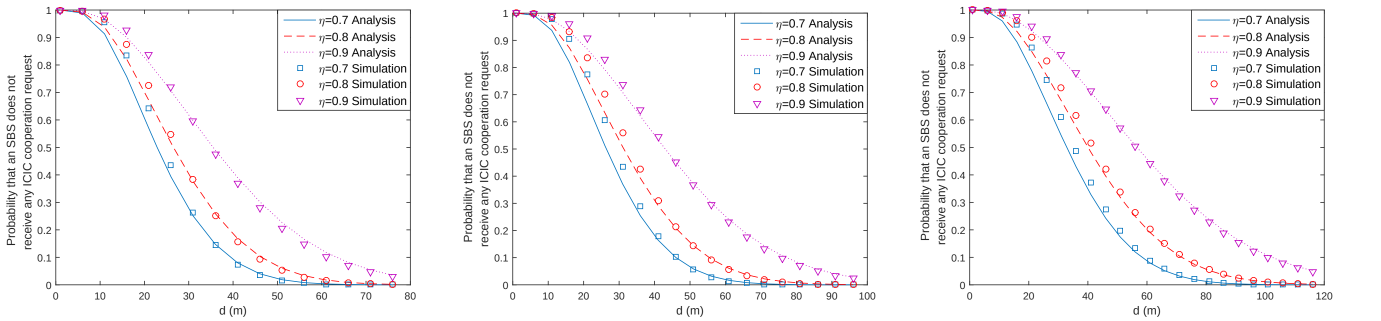
Fig. 3. The validation of p^u_{edg} for different η ($\lambda^b = 10^{-3}$ m^{-2}).

B. Approximation of Thinning Probabilities

Thinning probabilities p^u_{edg} and $p_{on}(d)$ are approximated in our analysis. So we first verify the approximations by simulation. Since $p^{b,a}$ and $p^{u,s}$ are already verified in [21], we omit them.

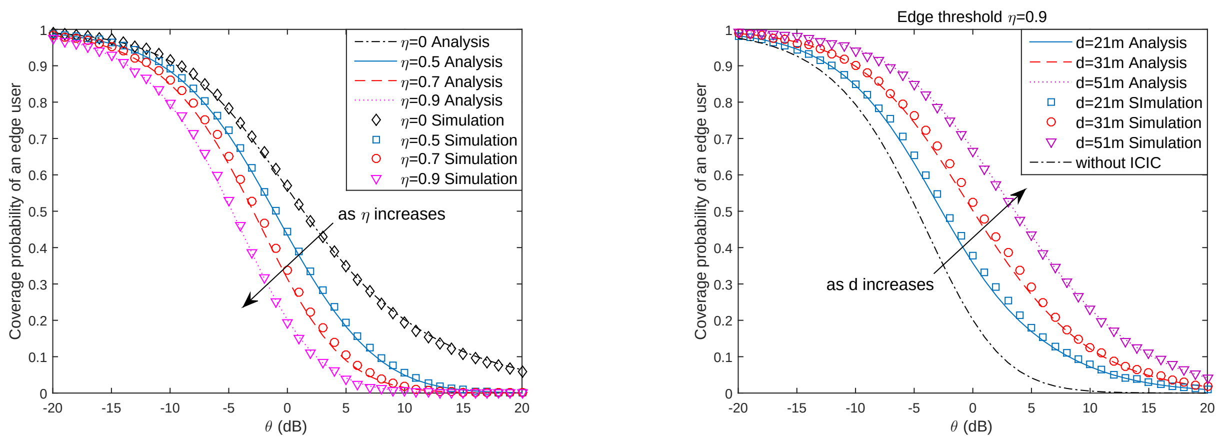
First, we consider the probability p^u_{edg} that a user is an edge user. The analytical and simulation results are plotted in Fig. 3. The figure shows that our analytical results are well matched with the simulation results, which verifies our analysis on p^u_{edg}. As can be seen from the figure, the smaller λ^u, i.e., the larger the SBS-user intensity ratio λ^b/λ^u, the smaller p^u_{edg} since the nearest neighbor active SBS of a user is relatively farther away than its nearest SBS. We also see that the larger the edge threshold, the smaller p^u_{edg}.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2018.2819998, IEEE Transactions on Wireless Communications
The final version of record is available at http://dx.doi.org/10.1109/TWC.2018.2819998



(a) $\lambda^u = 10 \cdot 10^{-3}$ m^{-2} (b) $\lambda^u = 2 \cdot 10^{-3}$ m^{-2} (c) $\lambda^u = 1 \cdot 10^{-3}$ m^{-2}

Fig. 4. The validation of $p_{on}(d)$ by $\underline{p_{on}}(d)$ for different λ^u and η ($\lambda^b = 10^{-3}$ m^{-2}).



(a) without ICIC (b) with ICIC

Fig. 5. The coverage probability of an edge user: (a) the coverage probability of an edge user without ICIC for different η ($\alpha = 4$, $\lambda^b = 10^{-3}$ m^{-2}, $\lambda^u = 10^{-2}$ m^{-2}). (b) the coverage probability of an edge user with ICIC for different d ($\alpha = 4$, $\lambda^b = 10^{-3}$ m^{-2}, $\lambda^u = 10^{-2}$ m^{-2}, $\eta = 0.9$).

Next, we consider the probability $p_{on}(d)$ that a typical SBS dose not receive an ICIC cooperation request. Since it is difficult to compute $p_{on}(d)$ analytically, we derive a lower bound $\underline{p_{on}}(d)$ instead and use it in our analysis. To verify whether the lower bound $\underline{p_{on}}(d)$ is tight we compare the simulation results of $p_{on}(d)$ and the lower bound $\underline{p_{on}}(d)$. The results are plotted in Fig. 4. As seen in the figure, two results are very close to each other,[5] from which we expect that the use of the lower bound $\underline{p_{on}}(d)$ provides a good estimation on the coverage probability in our analysis.

C. Coverage Probability of an Edge User without and with ICIC

We consider the coverage probability without ICIC of an edge user and plot our analytical and simulation results in Fig. 5(a). The figure shows that the simulation results coincide with the analytical results. From the figure we also see that, as η increases, the coverage probability decreases and the decrease becomes more significant for high values of the coverage threshold θ. From this observation, we see that it is important to use the ICIC scheme especially when the coverage threshold θ is not too small in order to improve the coverage probability of an edge user.

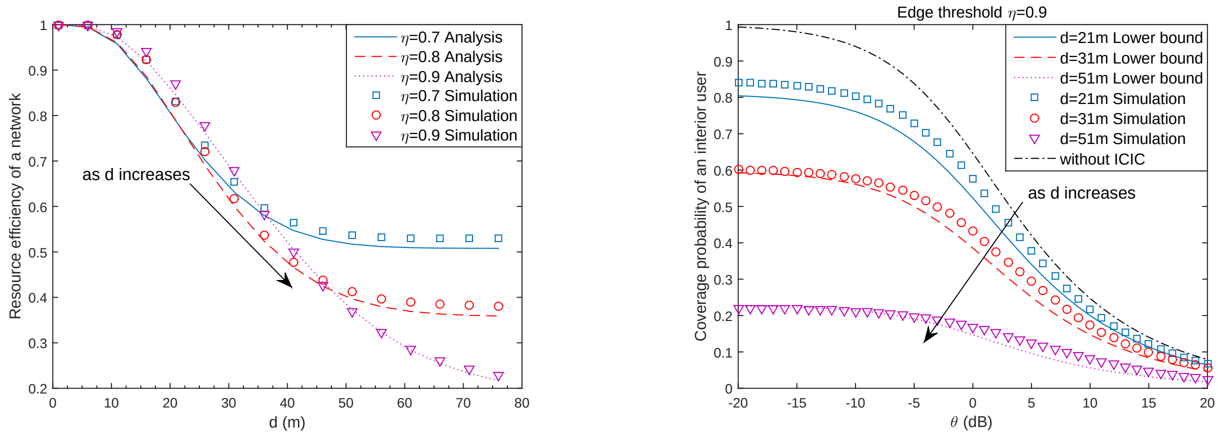
We next consider the coverage probability of an edge user when our ICIC scheme is applied. The analytical and simulation results in this case are plotted in Fig. 5(b). From the figure we see that both results coincide. Moreover, we see that, as the cooperation radius d increases, the coverage probability of an edge user increases as we can easily expect.

D. Two Trade-offs of ICIC

In this section, we validate the two trade-offs of our ICIC scheme analyzed in section IV-D.

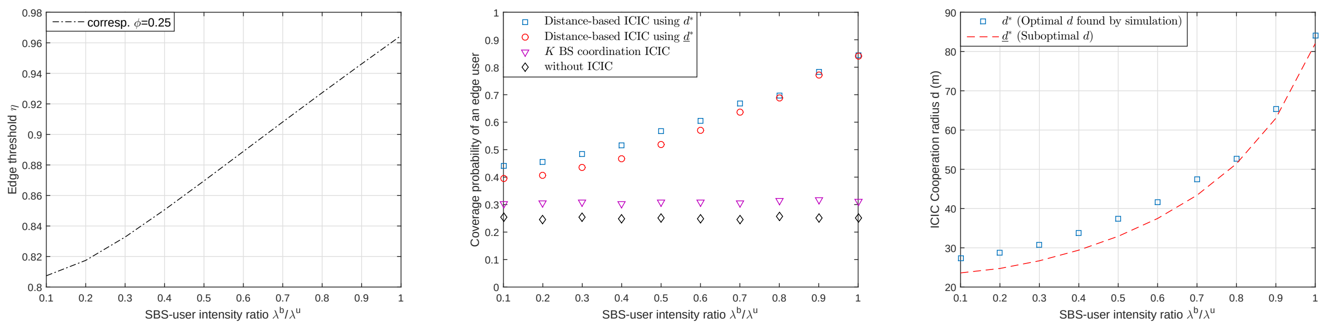
First, we consider the resource efficiency of a network. The analytical and simulation results are plotted in Fig. 6(a). The figure shows that the resource efficiency decreases as the cooperation radius d increases. This shows a trade-off between the cooperation radius and the resource efficiency. Moreover, Fig. 6(a) shows that, as the the cooperation radius d increases, the resource efficiency converges. This is because only edge

[5] In fact, $\underline{p_{on}}(d)$ is a lower bound when the locations of edge users who are served are well approximated by an independent homogeneous poisson point process. However, this approximation works well when the cooperation radius d is not too large. That is why $\underline{p_{on}}(d)$ is not a lower bound when d is large in Fig. 4.



(a) Resource efficiency of a network (b) Coverage probability of an interior user with ICIC

Fig. 6. The two trade-offs of ICIC: (a) the resource efficiency for different η ($\lambda^b = 10^{-3}$ m^{-2}, $\lambda^u = 10^{-2}$ m^{-2}). (b) the coverage probability of an interior user for different d ($\alpha = 4$, $\lambda^b = 10^{-3}$ m^{-2} $\lambda^u = 10^{-2}$ m^{-2}, $\eta = 0.9$).



(a) The value of η for $\phi = 0.25$ (b) Coverage probability (c) ICIC cooperation radius

Fig. 7. The performance comparison with the other ICIC scheme for different SBS-user intensity ratios λ^b/λ^u ($\alpha = 4$, $\lambda^b = 10^{-3}$m^{-2}, $\theta = 0$ dB, $a_1 = a_2 = 0.6$): For each ratio λ^b/λ^u, edge users are identified by the edge threshold η corresponding to the coverage probability criterion $\phi = 0.25$ in Fig. 7(a). In Fig. 7(b), the circles are obtained through simulation using the suboptimal value $\underline{d}^*$ from (33), while the squares are obtained by using the optimal value d^* through simulation. The values of d^* and $\underline{d}^*$ are plotted in Fig. 7(c).

users are likely to be served as the cooperation radius becomes large.

We next consider the coverage probability of an interior user when our ICIC scheme is applied. The analytical and simulation results are plotted in Fig. 6(b), which shows that our lower bound in (31) is quite close to the simulation results and the coverage probability of an interior user decreases as the cooperation radius d increases. This shows a trade-off between the cooperation radius and the coverage probability of an interior user.

E. Optimal Cooperation Radius and Performance Comparison

In this section, based on our observation on the two trade-offs we determine the optimal cooperation radius given in section IV-D and compare the proposed ICIC scheme with the other user-centric ICIC scheme called the K BS-coordination ICIC in [4].[6] In the K BS-coordination ICIC, ICIC-scheduled users are randomly selected from among served users. Each ICIC-scheduled user sends an ICIC cooperation request to its $K-1$ nearest active neighbor SBSs. Then, an active SBS receiving the ICIC cooperation request does not transmit any signal to its served user.[7] Note that the K BS-coordination ICIC focus on not an edge user, but a typical user.

Fig. 7 shows the comparison result for different SBS-user intensity ratios λ^b/λ^u. For each ratio λ^b/λ^u, edge users are identified by the edge threshold η corresponding to the coverage probability criterion $\phi = 0.25$ in Fig. 7(a). For the proposed scheme, we use the optimal cooperation radius d satisfying (32) with $a_1 = a_2 = 0.6$, and for the K BS-coordination ICIC, we use $K = 2$ which optimally improves their ICIC-scheduled users while keeping the resource efficiency greater than 0.6. In Fig. 7(b), we can observe that: 1) Our distance-based ICIC scheme significantly improves the coverage probability of an edge user than the K BS-coordination ICIC. When the K BS-coordination ICIC randomly select ICIC-scheduled users from among served users,

[6]To the best of the authors' knowledge, [4] is the only paper that analyze a user-centric ICIC scheme in one tier cellular network to improve multiple ICIC-scheduled users, where ICIC-scheduled users are randomly selected users, not edge users.

[7][4] focus on the case $p^{b,a} \approx 1$. For the comparison we modified their ICIC scheme to be suitable for the case $p^{b,a} \leq 1$.

edge users might not be selected for ICIC. This is the main reason why our distance-based ICIC scheme performs better than the K BS-coordination ICIC. 2) The use of the suboptimal cooperation radius $\underline{d}^*$ provides near optimal performance, so it can be used in practice. In addition, Fig. 7(c) shows the difference between optimal and suboptimal cooperation radii, which is not significant from the viewpoint of the coverage probability as seen in Fig. 7(b).

VI. Conclusions

To improve the coverage probability of edge users, we proposed a new user-centric ICIC scheme in a small cell network and developed an analytical framework based on the stochastic geometry to analyze the performance of the proposed ICIC scheme. In our analytical framework we carefully defined an edge user based on performance degradation and considered the spatial distribution of edge users that was not considered in most of the previous works. With the help of the analytical framework we derived a semi-closed expression for the coverage probability of an edge user with the proposed ICIC scheme. Considering the two trade-offs on the resource efficiency of a network and the coverage probability of an interior user, we formulated an optimization problem and obtained the optimized ICIC cooperation radius. Our analytical results were validated through numerical and simulation results.

Acknowledgment

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (NRF-2017R1A2B4008581).

Appendix

A. Proof of Lemma 3

Let $\mathbf{I}_{wo\setminus B_{r_{na}}} := \sum_{\mathbf{x}\in\Phi^b\setminus B_{r_{na}}} \mathbf{h}_{\mathbf{x}}\|\mathbf{x}\|^{-\alpha}\mathbf{A}_{\mathbf{x}}$ where $B_{r_{na}} := \left\{x\in\mathbb{R}^2 : \|x\|\le r_{na}\right\}$. Then,

$$\begin{aligned}
&cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \ge \eta\right)\\
&= \mathbb{P}\left[\mathbf{SIR}_{wo} > \theta \mid \frac{r_1}{r_{na}} \ge \eta\right], \text{ where } \mathbf{SIR}_{wo} \text{ is given in (3)}\\
&= \mathbb{P}\left[\mathbf{h}_{\mathbf{x}_1} > \theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\mathbf{h}_{\mathbf{x}_{na}} + \theta r_1^{\alpha}\mathbf{I}_{wo\setminus B_{r_{na}}}\right]\\
&= \mathcal{L}_{\mathbf{h}_{\mathbf{x}_{na}}}\left(\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\right)\mathcal{L}_{\mathbf{I}_{wo\setminus B_{r_{na}}}}\left(\theta r_1^{\alpha}\right) \qquad (34)
\end{aligned}$$

where $\mathcal{L}_{\mathbf{h}_{\mathbf{x}_{na}}}(s) := \mathbb{E}\left[e^{-s\mathbf{h}_{\mathbf{x}_{na}}}\right]$ and $\mathcal{L}_{\mathbf{I}_{wo\setminus B_{r_{na}}}}(s) := \mathbb{E}\left[e^{-s\mathbf{I}_{wo\setminus B_{r_{na}}}}\right]$.

Note that

$$\mathcal{L}_{\mathbf{h}_{\mathbf{x}_{na}}}\left(\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\right) = \frac{1}{1+\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}} \text{ and} \qquad (35)$$

$$\begin{aligned}
\mathcal{L}_{\mathbf{I}_{wo\setminus B_{r_{na}}}}\left(\theta r_1^{\alpha}\right) &= \exp\left\{-2\pi\lambda^{b,a}r_1^2\int_{\frac{r_{na}}{r_1}}^{\infty}\frac{\theta v}{v^{\alpha}+\theta}\,dv\right\}\\
&= \exp\left\{-\pi p^{b,a}\lambda^b r_1^2 H(r_{na},r_1)\right\}. \qquad (36)
\end{aligned}$$

From (34), (35) and (36), our lemma immediately follows. ■

B. Proof of Corollary 1

(a) Let μ_η be a finite measure such that $\mu_\eta(x,\infty) := \int_x^{\infty}\frac{f_{\mathbf{r}_{na}|\mathbf{r}_1}(r_{na}|r_1)}{p^u_{edg}}\mathbb{1}_{\left[r_1,\frac{r_1}{\eta}\right]}(r_{na})\,dr_{na}$ where $f_{\mathbf{r}_{na}|\mathbf{r}_1}(r_{na}|r_1)$ is given in (12).
Consider $\eta_1 < \eta_2$. Note that μ_{η_1} stochastically dominates μ_{η_2}, i.e., $\mu_{\eta_1}(x,\infty) > \mu_{\eta_2}(x,\infty)$. Recall $cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \ge \eta\right)$ given in (16) and note that $cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \ge \eta\right)$ is a nonnegative increasing function for r_{na}. From this, we obtain

$$\begin{aligned}
&\int_{-\infty}^{\infty} cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \ge \eta\right)\mu_{\eta_1}(dr_{na})\\
&> \int_{-\infty}^{\infty} cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \ge \eta\right)\mu_{\eta_2}(dr_{na}).
\end{aligned} \qquad (37)$$

Recall $cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta\right)$ given in (17) and note that

$$\begin{aligned}
&cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta\right)\\
&= \mathbb{E}\left[\int_{-\infty}^{\infty} cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{r_{na}} \ge \eta\right)\mu_\eta(dr_{na})\right].
\end{aligned} \qquad (38)$$

From (37) and (38), Corollary 1-(a) immediately follows.

(b) Let Y be a random variable with its probability density function $f_Y(y) = \frac{2\eta^2}{(1-\eta^2)y^3}$ for $0 < \eta \le y < 1$. Using Lemma 1, it can be proved that conditional random variables $\mathbf{r}_1$ and $\frac{\mathbf{r}_1}{\mathbf{r}_{na}}$ given $\frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta$ converge 0 and Y in distribution as λ^b goes to ∞, respectively. By Slutsky's theorem and Continuous mapping theorem, this implies that a conditional random vector $\left(\lambda^{b,a}\mathbf{r}_1^2, \frac{\mathbf{r}_1}{\mathbf{r}_{na}}\right)$ given $\frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta$ converges $(0,Y)$ in distribution as λ^b goes to ∞.
Let $g(x,y) := \frac{1}{1+\theta y^{\alpha}}\exp\left\{-2\pi x\int_{1/y}^{\infty}\frac{\theta v}{v^{\alpha}+\theta}\,dv\right\}$ and note that $g(x,y)$ is a bounded continuous function. Recall $cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta\right)$ given in (17) and note that

$$\mathbb{E}\left[g\left(\lambda^{b,a}\mathbf{r}_1^2, \frac{\mathbf{r}_1}{\mathbf{r}_{na}}\right) \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta\right] = cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta\right). \qquad (39)$$

By (39) and Portmanteau theorem, we obtain

$$\lim_{\lambda^b\to\infty}\mathbb{E}\left[g\left(\lambda^{b,a}\mathbf{r}_1^2, \frac{\mathbf{r}_1}{\mathbf{r}_{na}}\right) \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \ge \eta\right] = \mathbb{E}\left[g(0,Y)\right]. \qquad (40)$$

From (40),

$$\lim_{\lambda^b \to \infty} cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) = \frac{2\eta^2}{1-\eta^2}\int_{\eta}^{1} \frac{1}{(1+\theta y^{\alpha})y^3}\,dy$$

immediately follows. ■

C. Proof of Proposition 2

To analyze the coverage probability of a boundary user, we first obtain the followings by using the same arguments used to derive Proposition 1.

The joint probability density function (p.d.f.) of $\mathbf{r}_1$ and $\mathbf{r}_2$ is given by, for $r_1 < r_2$

$$f_{\mathbf{r}_1,\mathbf{r}_2}(r_1, r_2) = (2\pi\lambda^b)^2 r_1 r_2 e^{-\lambda^b \pi r_2^2}, \tag{41}$$

the probability that a user is a boundary user is

$$\mathbb{P}\left[\frac{\mathbf{r}_1}{\mathbf{r}_2} \geq \zeta\right] = 1 - \zeta^2 \tag{42}$$

and the coverage probability of boundary user $\mathbf{u}_o$ for fixed r_1 and r_2 is

$$\begin{aligned} &cp_{wo}\left(\theta \mid \frac{r_1}{r_2} \geq \zeta\right) \\ &= \left(1 - p^{b,a} + p^{b,a}\frac{1}{1+\theta\left(\frac{r_1}{r_2}\right)^{\alpha}}\right) e^{-\pi p^{b,a}\lambda^b r_1^2 H(r_2, r_1)} \end{aligned} \tag{43}$$

where $p^{b,a}$ and $H(x, y)$ are given in (10) and (15), respectively.

From (41), (42) and (43), the coverage probability of boundary user $\mathbf{u}_o$, $cp_{wo}\left(\theta \mid \frac{\mathbf{r}_1}{\mathbf{r}_2} \geq \zeta\right)$ is evaluated. And by employing the change of variables $r_1' = \sqrt{2\pi\lambda^b} r_1$ and $r_{na}' = \sqrt{2\pi\lambda^b} r_{na}$, our proposition immediately follows. ■

D. Proof of Proposition 3

To analyze the coverage probability of edge user $\mathbf{u}_o$ with ICIC, we first derive the coverage probability of edge user $\mathbf{u}_o$ with ICIC for fixed r_1 and r_{na}, i.e., $cp_w\left(\theta, d \mid \frac{r_1}{r_{na}} \geq \eta\right)$.

Suppose that $r_{na} > d$. Since r_{na} is the distance from edge user $\mathbf{u}_o$ to the nearest active neighbor SBS, it is obvious that there is no active SBS in $B_{r_{na}}$, so we disregard the region $B_{r_{na}}$ in the derivation. Given r_1 and r_{na}, it then follows that the interference $\mathbf{I}_{w,edg}$ given in (7) is decomposed into three terms; the interference from the nearest active neighbor SBS, the interference from active SBSs in $\Phi_{edg}^{b,a} \setminus B_{r_{na}}$ that serve edge users, and the interference from active SBSs in $\Phi_{int,on}^{b,a} \setminus B_{r_{na}}$ that serve interior users and do not receive an ICIC cooperation request. That is, $\mathbf{I}_{w,edg}$ is rewritten as

$$\begin{aligned} \mathbf{I}_{w,edg} = &\left(\frac{1}{r_{na}}\right)^{\alpha}(\mathbf{E}_{\mathbf{x}_{na}} + (1-\mathbf{E}_{\mathbf{x}_{na}})\mathbf{N}_{\mathbf{x}_{na}})\mathbf{h}_{\mathbf{x}_{na}} \\ &+ \mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,edg} + \mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,int,on} \end{aligned} \tag{44}$$

where $\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,edg} := \sum_{\mathbf{x}\in\Phi_{edg}^{b,a}\setminus B_{r_{na}}} \mathbf{h}_{\mathbf{x}}\|\mathbf{x}\|^{-\alpha}$ and $\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,int,on} := \sum_{\mathbf{x}\in\Phi_{int,on}^{b,a}\setminus B_{r_{na}}} \mathbf{h}_{\mathbf{x}}\|\mathbf{x}\|^{-\alpha}$.

It then follows that

$$\begin{aligned} &cp_w\left(\theta, d \mid \frac{r_1}{r_{na}} \geq \eta\right) \\ &= \mathbb{P}\left[\mathbf{SIR}_{w,edg} > \theta \mid \frac{r_1}{r_{na}} \geq \eta\right] \\ &\quad \text{, where } \mathbf{SIR}_{w,edg} \text{ is given in (9)} \\ &= \mathbb{P}\Bigg[\mathbf{h}_{\mathbf{x}_1} > \theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}(\mathbf{E}_{\mathbf{x}_{na}} + (1-\mathbf{E}_{\mathbf{x}_{na}})\mathbf{N}_{\mathbf{x}_{na}})\mathbf{h}_{\mathbf{x}_{na}} \\ &\qquad + \theta r_1^{\alpha}\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,edg} + \theta r_1^{\alpha}\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,int,on}\Bigg] \\ &= \mathbb{E}\left[\exp\left\{-\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}(\mathbf{E}_{\mathbf{x}_{na}} + (1-\mathbf{E}_{\mathbf{x}_{na}})\mathbf{N}_{\mathbf{x}_{na}})\mathbf{h}_{\mathbf{x}_{na}}\right\}\right] \\ &\quad \times \mathcal{L}_{\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,int,on}}(\theta r_1^{\alpha})\,\mathcal{L}_{\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,edg}}(\theta r_1^{\alpha}) \end{aligned} \tag{45}$$

where $\mathcal{L}_{\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,int,on}}(s) := \mathbb{E}\left[e^{-s\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,int,on}}\right]$ and $\mathcal{L}_{\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,edg}}(s) := \mathbb{E}\left[e^{-s\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,edg}}\right]$.

Note that

$$\begin{aligned} &\mathbb{E}\left[\exp\left\{-\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}(\mathbf{E}_{\mathbf{x}_{na}} + (1-\mathbf{E}_{\mathbf{x}_{na}})\mathbf{N}_{\mathbf{x}_{na}})\mathbf{h}_{\mathbf{x}_{na}}\right\}\right] = \\ &1 - (p_{edg}^u + p_{int}^u p_{on}(d)) + (p_{edg}^u + p_{int}^u p_{on}(d))\frac{1}{1+\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}}, \end{aligned} \tag{46}$$

$$\mathcal{L}_{\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,int,on}}(\theta r_1^{\alpha}) = \exp\left\{-2\pi\lambda_{int,on}^{b,a} r_1^2 \int_{\frac{r_{na}}{r_1}}^{\infty} \frac{\theta v}{v^{\alpha}+\theta}\,dv\right\} \tag{47}$$

and

$$\mathcal{L}_{\mathbf{I}_{w,edg\setminus B_{r_{na}}}^{b,a,edg}}(\theta r_1^{\alpha}) = \exp\left\{-2\pi\lambda_{edg}^{b,a} r_1^2 \int_{\frac{r_{na}}{r_1}}^{\infty} \frac{\theta v}{v^{\alpha}+\theta}\,dv\right\}. \tag{48}$$

From (45), (46), (47) and (50), we obtain, for $r_{na} > d$,

$$\begin{aligned} &cp_w\left(\theta, d \mid \frac{r_1}{r_{na}} \geq \eta\right) \\ &= \left((1-p_{on}(d))p_{int}^u + (p_{edg}^u + p_{int}^u p_{on}(d))\frac{1}{1+\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}}\right) \\ &\quad \times e^{-(p_{edg}^u + p_{int}^u p_{on}(d))p^{b,a}\lambda^b \pi r_1^2 H(r_{na}, r_1)} \end{aligned} \tag{49}$$

where $p^{b,a}$, p_{int}^u, p_{edg}^u, $p_{on}(d)$, and $H(x, y)$ are given in (10), (13), (14), (23) and (15), respectively.

Next, suppose that $r_{na} \leq d$. In this case, when we compute the interference from active SBSs that serve interior users and do not receive any ICIC cooperation requests, we disregard the region B_d because d is the cooperation radius and $r_{na} \leq d$. Note that the interference from active SBSs that serve edge users is the same as the case $r_{na} > d$.

Let $\mathbf{I}^{b,a,int,on}_{w,edg\setminus B_d} := \sum_{\mathbf{x}\in\Phi^{b,a}_{int,on}\setminus B_d} \mathbf{h}_{\mathbf{x}}\|\mathbf{x}\|^{-\alpha}$. It then follows that

$$\begin{aligned}
&cp_w\left(\theta, d \mid \frac{r_1}{r_{na}} \geq \eta\right) \\
&= \mathbb{P}\left[\mathbf{SIR}_{w,edg} > \theta \mid \frac{r_1}{r_{na}} \geq \eta\right] \\
&\quad \text{, where } \mathbf{SIR}_{w,edg} \text{ is given in (9)} \\
&= \mathbb{P}\left[\mathbf{h}_{\mathbf{x}_1} > \theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\mathbf{E}_{\mathbf{x}_{na}}\mathbf{h}_{na} + \theta r_1^{\alpha}\mathbf{I}^{b,a,edg}_{w,edg\setminus B_{r_{na}}} \right. \\
&\qquad \left. + \theta r_1^{\alpha}\mathbf{I}^{b,a,int,on}_{w,edg\setminus B_d}\right] \\
&= \mathbb{E}\left[\exp\left(-\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\mathbf{E}_{\mathbf{x}}\mathbf{h}_{na}\right)\right]\mathcal{L}_{\mathbf{I}^{b,a,edg}_{w,edg\setminus B_{r_{na}}}}\left(\theta r_1^{\alpha}\right) \quad (50) \\
&\quad \times \mathcal{L}_{\mathbf{I}^{b,a,int,on}_{w,edg\setminus B_{r_d}}}\left(\theta r_1^{\alpha}\right)
\end{aligned}$$

where $\mathcal{L}_{\mathbf{I}^{b,a,edg}_{w,edg\setminus B_{r_{na}}}}(s) := \mathbb{E}\left[e^{-s\mathbf{I}^{b,a,edg}_{w,edg\setminus B_{r_{na}}}}\right]$ and $\mathcal{L}_{\mathbf{I}^{b,a,int,on}_{w,edg\setminus B_d}}(s) := \mathbb{E}\left[e^{-s\mathbf{I}^{b,a,int,on}_{w\setminus B_d}}\right]$.

Note that

$$\begin{aligned}
&\mathbb{E}\left[\exp\left(-\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}\mathbf{E}_{\mathbf{x}_{na}}\mathbf{h}_{na}\right)\right] \\
&= 1 - p^u_{edg} + p^u_{edg}\frac{1}{1+\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}} \text{ and} \qquad (51)
\end{aligned}$$

$$\mathcal{L}_{\mathbf{I}^{b,a,int,on}_{w,edg\setminus B_d}}\left(\theta r_1^{\alpha}\right) = \exp\left\{-2\pi\lambda^{b,a}_{int,on} r_1^2 \int_{\frac{d}{r_1}}^{\infty} \frac{\theta v}{v^{\alpha}+\theta}\,dv\right\}. \qquad (52)$$

From (50), (51) and(52), we obtain, for $r_{na} \leq d$,

$$\begin{aligned}
&cp_w\left(\theta, d \mid \frac{r_1}{r_{na}} \geq \eta\right) \\
&= \left(p^u_{int} + p^u_{edg}\frac{1}{1+\theta\left(\frac{r_1}{r_{na}}\right)^{\alpha}}\right) \qquad (53) \\
&\quad \times e^{-p^{b,a}\lambda^b \pi r_1^2\left(p^u_{edg}H(r_{na},r_1) + p^u_{int}p_{on}(d)H(d,r_1)\right)}
\end{aligned}$$

where $p^{b,a}$, p^u_{int}, p^u_{edg}, $p_{on}(d)$, and $H(x,y)$ are given in (10), (13), (14), (23) and (15), respectively.

From (49) and (53), we finally evaluate the coverage probability of edge user $\mathbf{u}_o$ with ICIC.

$$\begin{aligned}
&cp_w\left(\theta, d \mid \frac{\mathbf{r}_1}{\mathbf{r}_{na}} \geq \eta\right) = \\
&\frac{1}{p^u_{edg}}\iint_{r_1 \geq \eta r_{na}} cp_{wo}\left(\theta \mid \frac{r_1}{r_{na}} \geq \eta\right) f_{\mathbf{r}_1,\mathbf{r}_{na}}(r_1, r_{na})\,dr_{na}\,dr_1, \qquad (54)
\end{aligned}$$

and the integral domain is divided into four regions as follows.

$$\begin{aligned}
R_1 &:= \left\{(r_1, r_{na}) : \eta d < r_1 < d, d < r_{na} < \frac{r_1}{\eta}\right\}, \\
R_2 &:= \left\{(r_1, r_{na}) : d < r_1 < \infty, r_1 < r_{na} < \frac{r_1}{\eta}\right\}, \\
R_3 &:= \left\{(r_1, r_{na}) : 0 < r_1 < \eta d, r_1 < r_{na} < \frac{r_1}{\eta}\right\}, \text{ and} \\
R_4 &:= \left\{(r_1, r_{na}) : \eta d < r_1 < d, r_1 < r_{na} < d\right\}.
\end{aligned}$$

Using (49) over R_1 and R_2 and (53) over R_3 and R_4, (54) is evaluated. By employing the change of variables $r_1' = \sqrt{2\pi\lambda^b}r_1$ and $r_{na}' = \sqrt{2\pi\lambda^b}r_{na}$, our proposition immediately follows. ■

E. Proof of Proposition 4

Let $\Phi^{b,a}_{int,off}$ be the locations of active SBSs with its served user being an interior user which receive ICIC cooperation requests.

$$\begin{aligned}
R_{eff}(d) &= 1 - \lim_{W\uparrow\mathbb{R}^2}\mathbb{E}\left[\frac{\Phi^{b,a}_{int,off}(W)}{\Phi^{b,a}(W)}\right] \\
&= 1 - \lim_{W\uparrow\mathbb{R}^2}\mathbb{E}\left[\frac{\Phi^{b,a}_{int,off}(W)}{\Phi^{b,a}(W)}\mathbb{1}_{\{\Phi^{b,a}(W)>0\}}\right].
\end{aligned} \qquad (55)$$

Since $\Phi^{b,a}_{int,off}(W) \sim \text{Poisson}(p^u_{int}\left(1-p_{on}(d)\right)\lambda^{b,a}w^2)$, $(\Phi^{b,a}(W) - \Phi^{b,a}_{int,off}(W)) \sim \text{Poisson}((p^u_{edg} + p^u_{int}p_{on}(d))\lambda^{b,a}w^2)$ and they are independent by the assumption on independent thinning,

$$\begin{aligned}
&\lim_{W\uparrow\mathbb{R}^2}\mathbb{E}\left[\frac{\Phi^{b,a}_{int,off}(W)}{\Phi^{b,a}(W)}\mathbb{1}_{\{\Phi^{b,a}(W)>0\}}\right] \\
&= \lim_{w\to\infty}\frac{p^u_{int}\left(1-p_{on}(d)\right)\lambda^{b,a}w^2}{\lambda^{b,a}w^2}\left(1-e^{-\lambda^{b,a}w^2}\right) \\
&= p^u_{int}\left(1-p_{on}(d)\right).
\end{aligned} \qquad (56)$$

From (55) and (56), our proposition immediately follows. ■

REFERENCES

[1] A. S. Hamza, S. S. Khalifa, H. S. Hamza and K. Elsayed, “A Survey on Inter-Cell Interference Coordination Techniques in OFDMA-Based Cellular Networks,” *IEEE Communications Surveys & Tutorials,* vol. 15, no. 4, pp. 1642-1670, Fourth Quarter 2013.

[2] M. K. Hasan et al., “Inter-cell interference coordination in LTE-A HetNets: A survey on self organizing approaches,” *2013 IEEE International Conference on Computing, Electronic and Engineering (ICCEEE),* Khartoum, 2013, pp. 196-201.

[3] S. A. R. Zaidi, D. C. McLernon, M. Ghogho and M. A. Imran, “Cloud empowered Cognitive Inter-cell Interference Coordination for small cellular networks,” *2015 IEEE International Conference on Communication Workshop (ICCW),* London, 2015, pp. 2218-2224.

[4] X. Zhang and M. Haenggi, “A Stochastic Geometry Analysis of Inter-Cell Interference Coordination and Intra-Cell Diversity,” *IEEE Transactions on Wireless Communications,* vol. 13, no. 12, pp. 6655-6669, Dec. 2014.

[5] M. Kamel, W. Hamouda and A. Youssef, “Ultra-Dense Networks: A Survey,” *IEEE Communications Surveys & Tutorials,* vol. 18, no. 4, pp. 2522-2545, Fourthquarter 2016

[6] J. Park, S. L. Kim and J. Zander, ”Tractable Resource Management With Uplink Decoupled Millimeter-Wave Overlay in Ultra-Dense Cellular Networks,” *IEEE Transactions on Wireless Communications,* vol. 15, no. 6, pp. 4362-4379, June 2016.

[7] X. Chen, J. Wu, Y. Cai, H. Zhang and T. Chen, "Energy-Efficiency Oriented Traffic Offloading in Wireless Networks: A Brief Survey and a Learning Approach for Heterogeneous Cellular Networks," *IEEE Journal on Selected Areas in Communications,* vol. 33, no. 4, pp. 627-640, April 2015.
[8] S. Lee and K. Huang, "Coverage and Economy of Cellular Networks with Many Base Stations," *IEEE Communications Letters,* vol. 16, no. 7, pp. 1038-1040, July 2012.
[9] B. U. Kazi, M. Etemad, G. Wainer and G. Boudreau, "Signaling overhead and feedback delay reduction in heterogeneous multicell cooperative networks," *2016 International Symposium on Performance Evaluation of Computer and Telecommunication Systems (SPECTS),* Montreal, QC, 2016, pp. 1-8.
[10] A. D. Wyner, "Shannon-theoretic approach to a Gaussian cellular multiple-access channel," *IEEE Transactions on Information Theory,* vol. 40, no. 6, pp. 1713-1727, Nov 1994.
[11] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on Communications,* vol. 59, no. 11, pp. 3122.3134, Nov. 2011.
[12] H. ElSawy, E. Hossain and M. Haenggi, "Stochastic Geometry for Modeling, Analysis, and Design of Multi-Tier and Cognitive Cellular Wireless Networks: A Survey," *IEEE Communications Surveys & Tutorials,* vol. 15, no. 3, pp. 996-1019, Third Quarter 2013.
[13] T. D. Novlan, R. K. Ganti, A. Ghosh and J. G. Andrews, "Analytical Evaluation of Fractional Frequency Reuse for OFDMA Cellular Networks," *IEEE Transactions on Wireless Communications,* vol. 10, no. 12, pp. 4294-4305, December 2011.
[14] M. Afshang, Z. Yazdanshenasan, S. Mukherjee and P. H. J. Chong, "Hybrid division duplex for HetNets: Coordinated interference management with uplink power control," *IEEE International Conference on Communication Workshop (ICCW),* London, 2015, pp. 106-112.
[15] H. ElSawy and E. Hossain, "Two-Tier HetNets with Cognitive Femtocells: Downlink Performance Modeling and Analysis in a Multichannel Environment," *IEEE Transactions on Mobile Computing,* vol. 13, no. 3, pp. 649-663, March 2014.
[16] J. Lee, J. G. Andrews and D. Hong, "Spectrum-Sharing Transmission Capacity with Interference Cancellation," *IEEE Transactions on Communications,* vol. 61, no. 1, pp. 76-86, January 2013.
[17] S. Y. Jung, H. k. Lee and S. L. Kim, "Worst-Case User Analysis in Poisson Voronoi Cells," *IEEE Communications Letters,* vol. 17, no. 8, pp. 1580-1583, August 2013.
[18] M. Haenggi, "The Meta Distribution of the SIR in Poisson Bipolar and Cellular Networks," *IEEE Transactions on Wireless Communications,* vol. 15, no. 4, pp. 2577-2589, April 2016.
[19] M. I. Kamel and K. M. F. Elsayed, "Performance evaluation of a coordinated time-domain eICIC framework based on ABSF in heterogeneous LTE-Advanced networks," *IEEE Global Communications Conference (GLOBECOM),* Anaheim, CA, 2012, pp. 5326-5331.
[20] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks,* Volume I.Theory. Delft, The Netherlands: NOW, 2009.
[21] S. M. Yu and S. L. Kim, "Downlink capacity and base station density in cellular networks," *2013 11th International Symposium and Workshops on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt),* Tsukuba Science City, 2013, pp. 119-124.
[22] Martin Haenggi, "Stochastic geometry for wireless networks," Cambridge University Press, 2012.
[23] Calka, Pierre. "The distributions of the smallest disks containing the Poisson-Voronoi typical cell and the Crofton cell in the plane," *Advances in Applied Probability,* 34.04 (2002): 702-717.

Jonghun Yoon received his Ph.D. degree in in mathematical sciences from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in Feb. 2018. Since Mar. 2018, he has been working as a Senior Engineer for Samsung Research, Seoul, South Korea. His dissertation focused on inter-cell interference coordination techniques for 5G small cell networks and cellular network modeling using stochastic geometry. He received the B.S. degree in mathematics from Korea University, Seoul, South Korea, in 2012 and the M.S. degree in mathematical sciences from KAIST in 2014.

His research interests include performance evaluation of communication systems, modeling wireless communication networks and optimization in communication networks.

Ganguk Hwang (M'03) received his B.Sc., M.Sc., and Ph.D. degrees in Mathematics (Applied Probability) from Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Republic of Korea, in 1991, 1993, and 1997, respectively. From February 1997 to March 2000, he was with Electronics and Telecommunications Research Institute (ETRI), Daejeon, Republic of Korea. From March 2000 to February 2002, he was a visiting scholar at the School of Interdisciplinary Computing and Engineering in University of Missouri - Kansas City. Since March 2002, he has been with the Department of Mathematical Sciences and Telecommunication Engineering Program at KAIST, where he is currently a Professor. From August 2010 to July 2011, he was a visiting scholar at the Department of Electrical Engineering, University of Washington, Seattle. His research interests include teletraffic theory, performance analysis of communication systems, quality of service provisioning for wired/wireless networks, and cross-layer design and optimization for wireless networks.