

Five things to know about modeling and simulation

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Abstract

Modeling is as old as humanity. It is one of the ways in which we experience the world, teach our children, and entertain ourselves. The digital computer, on the other hand, is approximately 60 years old but as computing power increases and access to technology becomes easier, more disciplines are using statistical and computational simulations. From the humanities to social sciences, scholars are advocating for a computational branch of their field of study. This is very exciting, and we want to make sure that all disciplines stay connected and share their insights as they grow in their respective areas. Religion is a complex system that consists of humans, society, culture and social constructs that have evolved over millennia. The study of religion relies on empirical approaches to collect and analyze data to generate or validate existing theories. Modeling and simulation allows us to venture beyond statistical observation and into an exploration of the causal relationships between the different aspects of religion. It provides us with (1) the ability to understand the system as a whole, (2) the possibility of projecting how religion will evolve in the future and (3) the capability to compare, contrast and merge seemingly conflicting theories of religion. In this article, we present five critical things that scholars in psychology of religion should know about the discipline of Modeling and Simulation. The goal of this short primer is to highlight the universal aspects of Modeling and Simulation and to provide a unifying view that transcends disciplines.

Keywords

Analytics, computational model, experimentation, modeling and simulation, statistical modeling

Model, referent, abstraction, and problem situation

Modeling is ingrained in most of us at an early stage. Children model their environment and simulate the behavior of adults around them. Adults learn from mentors and act as role models to one another. As the scientific level, a systematic approach to modeling is expected to lead to an explainable insight into a phenomenon. This is why modeling is central to almost every discipline. In Mathematics, Physics and Biology, models are expressed formally or semi-formally. In the social sciences and the humanities, models are expressed as theories, typically presented narratively. Today, computers are used predominantly to explore models but modeling itself predates the advent of computers. This is an important distinction because it means that scholars in the

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psychology of religion do not have to invent new theories or radically change their discipline to use Modeling and Simulation (M&S). They simply have to use computers to explore the models (i.e. theories) they are interested in exploring.

The story of every model begins with a referent. A *model* is a purposeful abstraction of an observation of a referent. A *referent* is the thing to which the model refers to and is often thought of as the perfect version of the model. The referent can be some aspect of the world we live in or an imaginary object yet to be developed. In any case, we assume that the referent is not completely accessible because of its size, complexity, the ethics involved in accessing it, or simply because of the unreasonable amount of time it would take to fully access it. Be that as it may, we wish to understand, explain or change some aspect of the referent. To do so, we make many assumptions, some of which we are aware of and some not. Then, we formulate theories, collect data and perform analyses, which we then use to gain insight into the referent. In M&S, the process of reducing a referent to a simplified form using a set of assumptions is the beginning of the modeling process (Tolk, Diallo, Padilla, & Herencia-Zapana, 2013; Robinson, 2008) and ends with a *reference model*. For example, we might consider a religious ritual—how it starts, its various stages, its empirically verified cognitive and emotional effects on the participants, and its downstream consequences for the individuals and community involved. All of the part-theories relevant to understanding the religious ritual would be incorporated and rendered as insight-generating components of a larger synthesized theory, which helps to express the target phenomenon in a reference model.

It is important to note that due to the nature of the referent, we are bound by what we can observe and the modeling question we wish to pose. Given a modeling question, different observers can arrive at widely different models even if they have only slight differences in their assumptions. Consequently, there is almost an infinite number of models for a single referent because a model is a purposeful simplification of an observation of a referent (Robinson, 2008). A *conceptual model* is the subset of the reference model that answers the modeling question (Robinson, 2008). For example, we might be particularly interested in behavioral changes in the wake of a ritual, or—more specifically—in the conditions under which people are more forgiving after a ritual than they were before, and how this effect might accumulate as the ritual is repeated over the years. Alternatively, we might be interested in the way religious authority is perceived and its ability to reinforce or disrupting existing authority or belief structures within the community (see Upal, 2005a, 2005b, or Bainbridge, 2006 for examples), and the long-term cumulative effects on authority of repeated participation in the ritual (see McCorkle & Lane, 2012). These two research interests would lead to two different conceptual models, specifying the same reference model; indeed, the previous approaches to these subjects that have utilized M&S have resulted in vastly different models. Moreover, the very same research question could be pursued in a variety of ways, so a referent can yield a very large number of conceptual models even for the same modeling question (e.g. understanding the dynamics of motivation and leadership in doctrinal religions in the theory of Divergent Modes of Religiosity has at least three published computer models discussing it: Whitehouse, Kahn, Hochberg, and Bryson (2012); McCorkle and Lane (2012); and Lane (2018)).

Depending on the discipline, reference models can be captured as theories, mathematical equations, logical specifications or a combination of the three. In Natural Sciences where there are common observations of the referent through standard instruments and even standard modeling questions, most conceptual models are thought of as universal because they have been used over centuries and have the consensus of their community. In those fields, it is common to reuse models once there is a match between the referent and the modeling question. This is not the case for other disciplines where such consensus is hard to achieve. This is due at least in part to the fact

that these researchers deal with a constantly changing and evolving referent that is often culturally located, relationally constituted, and hermeneutically multi-faceted, making it hard to observe and measure through standard instrumentation. In those disciplines, we are dealing with problem situations (Checkland, 1999, pp. 45–56) where defining a referent is a serious problem; naturally, this issue is familiar to scholars of religion and culture (e.g. McCauley & Lawson, 1996; Smith, 2004; Spiro, 1966). In those cases, the modeling process takes on an even more critical role and must be conducted rigorously because the resulting reference model is a reflection of the decisions made by model developers. *Modelers should be aware that the reference model is only a version of the referent that reflects the assumptions that they made and the modeling question they intend to answer.* For example, the ritual referent will be interpreted by the researcher against background cultural knowledge and the ritual theories helping to define the reference model and the conceptual model may well be incomplete, leaving researchers guessing about some critical aspects of the ritual dynamics. These assumptions might be simply woven into the conceptual model used to guide research, but with experience it is possible to identify such assumptions. Indeed, the modeling process can draw our attention even to hidden assumptions we might normally miss using other methods.

For most referents, it is often very difficult to directly derive a conceptual model that satisfactorily answers the modeling question. One trick commonly used in M&S is to simplify the referent into a system. A *system* is a collection of connected parts that transform a set of inputs into a set of outputs (Checkland, 1999). This “systems view” of modeling forces us to comprehend the world as inputs, outputs and connected parts but makes it easier to derive a model of the referent (its relation to a strict cognitive understanding of psychology is worth considering). However, it is important to note that a system is a simplification of a referent. Therefore, the reference model becomes a second order representation of the referent (the referent is formulated as a system, which is then expressed as a reference model). For example, we could use Lawson and McCauley’s (1990) model of ritual to generate a systems perspective on our referent ritual (see Braxton, 2008; Lane, Shults, & McCauley, 2019). Or we could generate a system using the Whitehouse (2004; also see Whitehouse et al., 2012 and Lane, 2018, for examples), Bell (1997), Turner, Abrahams, and Harris (2017), or Rappaport (1999) models of ritual—or others, or a synthesis of several of these. We could then ask a modeling question about the emotional effect of ritual on individuals (e.g. Lane, 2018) or investigate the impact of the personality of participants on the effectiveness of a ritual. Using a systems conceptuality also means that it is possible to model inputs, parts, relationships and outputs separately, which greatly simplifies model design and evaluation. For some referents, collecting data is a big challenge due to practical and ethical concerns, such as dealing with religious extremism or new religious movements (as discussed in Lane, 2013, 2017a in relation to M&S as a tool to study religion and cognition). The ability to model data that serves as input to a simulation is called *input modeling* (Zouaoui & Wilson, 2003). It is an important aspect of M&S because it allows the development of simulations where data is scarce or absent. It also allows us to model the uncertainty and variability that is present in some referents and gives us the power to design repeatable experiments with a model of the referent. For example, we might have data from a variety of studies on frequency of ritual participation, social-network maps of participants (e.g. Lane, 2018), and even data from monitoring devices on physiological state through the time-course of the ritual activity (e.g. Konvalinka et al., 2011; Xygalatas, 2012). If such data do not exist, we might be inspired to collaborate with researchers in a position to collect that data in order to strengthen the model.

In summary, we engage in modeling because we want to study referents that cannot be accessed completely. Models are purposeful simplifications of an observation of a referent. Each discipline uses the language that best suits its field of study. It is possible to translate between languages and

have multiple equivalent formulations of a model. For instance, it is thinkable that cognitive science and social theory could yield equivalent models of a ritual process, each registering everything the other does in its characteristic way. In cases where there is consensus on observations of the referent and there are standardized instruments, existing models can be reused. However, in problem situations, the problem, once captured, becomes the referent. The hermeneutical sophistication implied in this description is second-nature to humanities scholars and social scientists, in one way, but the modeling process can actually deepen and render more precise expert awareness of hermeneutical nuances.

Systems, simulators, humans and computers

In M&S, the purpose of developing a model is to simulate it. A simulation is the execution of a model over time (Law, Kelton, & Kelton, 1991) and there are at least three simulation modes. A *live simulation* is the execution of a model by humans. A *virtual simulation* is the execution of a model using a combination of humans and computers. A *constructive simulation* is the execution of a model using only computers. In the abstract, one can see each of these modes being employed already in the psychology and cognitive science of religion: live simulations are used to model phenomena for experimental conditions in the lab; virtual simulations are often used in lab settings employing economic games, where participants play against computer programs to study behaviors such as cooperation or altruism, and constructive simulations are being employed more recently to use computers to study entire religious systems (see Braxton, Upal, & Nielbo, 2012; Lane, 2017b for reviews of their use in the study of religion and the history of cognitive science and religion respectively). The selection of a simulation mode depends on the modeling question and the ability of a computer to execute some aspect of the model. In some cases, all three modes are used simultaneously in what is called a live-virtual-constructive simulation. Constructive simulations require that a model be expressed in a rigid formal language that is “computable.” The underlying computational models are usually faster and less expressive than their live or virtual counterparts. Picture a computer representation of people with their artificial emotions and social networks participating in a ritual—the constructive simulation mode—versus actual human beings with human emotions and social relationships participating in a ritual on a theater stage as part of a dramatic play—the live mode. The differences matter and each mode of simulation has its characteristic advantages.

In all three cases, we need a human or computer simulator to execute the simulation. A *simulator* generates a simulation from a model (Wainer, 2009). Different simulators may produce different simulations for the same model. In the case of computable models, the simulator is also a computable model meaning the simulation is now a fourth-order representation of the referent: referent to system (first order) to model (second order) to simulator (third order) to simulation (fourth order). A *simulation model* is the artifact that is executed by the simulator. The simulation model takes the form of computer code for a computational model in the constructive mode and that of a scenario or script in the live mode. Since the constructive mode is the rising form among scholars, we will focus on computational models in this section. However, it is important to keep in mind that this is only one mode and not the whole of M&S. To fully take advantage of the power of simulation, it is essential to carefully consider which mode best suits a study and not automatically assume that the computational approach is the only option.

Let us return to the nature of computational models and their executable derivative which we call simulation models. As one might expect, modelers can choose from several types of simulation models. Usually, that decision is driven by how the system is defined. A system is called

deterministic if its outputs are only determined by its inputs and initial conditions. A system is stochastic if one or more variables has uncertainty or variability. A system is static if its outputs only depend on its inputs, and it is called dynamic if its outputs depend on its inputs and its state. For instance, a model that depicts the effect of natural hazards and mortality salience on religiosity as a series of events and reactions (Shults et al., 2017) or a church's recruitment process as a process where potential members are vetted and carefully filtered through a well-defined series of steps is said to be dynamic whereas a model that depicts the process as a random selection is called static. Statistical models are the most common static models, and they are supported by several commercially available simulators (Excel, SPSS, SAS, R, etc.). Dynamic models range from time-driven mathematical models to event-driven discrete systems (Fishwick, 2007). The market for dynamic simulators is also vibrant with powerful open-source and commercial packages. They usually consist of an engine to generate random numbers (Park & Miller, 1988), an analytics package and an engine to execute simulations (Gould, Tobochnik, & Christian, 1988).

Once a system is defined, modelers use a paradigm to specify the simulation model. A paradigm in M&S is a method for describing a system (Sokolowski & Banks, 2011). Since M&S is applied in almost every discipline, there are many paradigms (Mustafee, Katsaliaki, Fishwick, & Williams, 2012) that have been developed to account for the referent of interest, questions of interest as well as what can be observed and measured. The selection of a paradigm depends on the modeling question and the amount of information we have about the system. Once a paradigm is selected, the simulation model is formulated and implemented in a simulation environment that usually consists of a simulator as described above, an experimentation package and a visualization component. Assuming we have a reference model, a modeling question, a simulation model and data, we can build a computable model that is a fourth-order representation of the referent.

At this point, the all-important question is how we ensure that the simulation is useful. We explore this question in the next section.

Verification, validation and correspondence

Verification and Validation (V&V) are the most important activities in M&S. To better explain V&V, let's think of models (reference, conceptual and simulation) as theories that consist of a collections of connected statements. Verification is the process of determining that the simulation is correct. By correct, we mean that the simulation model is "free of error," or, more formally, that every statement of the simulation model is also a statement of the conceptual model. We also mean that the pairing of the simulation model with the simulator is correct, or, more formally, that every statement of the simulation generated by the simulator is a statement of the reference model. Validation is the process of evaluating whether the (second-order) conceptual model is correct. In this case, being correct means that (1) every statement of the conceptual model is a statement of the reference model and (2) every statement of the reference model is a statement of the referent. For example, if the simulation allows people to leave in the middle of the ritual even though this is not a possibility within the conceptual model, the simulation would not be valid. The relevant fixes would be to rein in the simulation model so that it matches the conceptual model or to expand the conceptual model to render the simulation model consistent with it. If the (first-order) reference model doesn't allow for such departures in the middle of a ritual but the (second-order) conceptual model does, then we have a validation problem, which can be fixed at either the reference-model level or the conceptual-model level.

Picturing this multi-level feedback helps us see how working on a (fourth-order) simulation model can draw attention to problematic assumptions in the (second-order) conceptual model and even in the (first-order) reference model. This kind of conceptual checking is invaluable for evaluating theories of complex phenomena of the sort studied in the field of psychology of religion. Examples of new insights or specifications of a theory being generated about theories relevant to the psychology of religion have already been found in relation to Ritual Competency Theory (see Lane et al., 2019), the Divergent Modes of Religiosity Theory (Lane & McCorkle, 2012; Lane, 2015, 2018), the Supernatural Punishment hypothesis (Lane, 2017c), cultural epidemiology (Kaše, Hampejs, & Pospíšil, 2018), social identity theory (Upal, 2015), and costly signaling theory Wildman and Sosis (2011). We don't just have to figure out somehow what we are silently assuming; the simulation process can draw our attention to some of those hidden assumptions. In M&S settings, it is common to differentiate verification and validation by stating that verification is building the simulation right and validation is building the right thing. Although presented as two distinct and independent processes, in practical terms, verification and validation are so intertwined that they can be conceived as a single activity. Depending on the size and complexity of the simulation, verification can be a semi-automated process that uses techniques from software engineering, mathematical logic, systems engineering, and statistics (Sargent, 2013).

Validation, on the other hand, is a series of processes that are human-centric and collaborative. Since the reference model is a result of simplifying assumptions, those assumptions must be validated as being "reasonable" for the purpose of the study. For instance, is it reasonable to assume in the (first-order) reference model that people can't leave the ritual in question in the middle of it? Given the large number of assumptions we are likely to make in a reference model, it is important to become aware of and render explicit as many assumptions as possible in order to achieve high levels of confidence in the validation process.

Non-trivial models have a high potential for contradictions to emerge. Almost all models in psychology of religion would be non-trivial in the relevant way, such are the complexities of human minds and behavior, particularly religious behavior. Contradictions occur when two or more assumptions cannot be true simultaneously. This is usually the case when multiple theories, perhaps from multiple disciplines or worldviews, are involved in the modeling process. When that happens, it is important to resolve contradictions in the conceptual model and not in the reference model. The modeling question usually guides which assumptions to keep and which to remove but the reference model should reflect the modeling process as it occurred. Emergence, as it is used within the bounds of M&S, happens when a logical combination of assumptions leads to the appearance of new assumptions that are not explicitly stated (for a discussion on the uses and application of "emergence" to the psychology of religion in an M&S context, see Lane (2018) and Wildman and Shults (2018)). On the surface, we don't see the lurking contradiction, but the emergent capabilities of a simulation model may expose it. For example, we may not notice that two of the theories contributing to the (first-order) reference model make different assumptions about whether people can leave the ritual in the middle of it. We weren't thinking about people leaving so we never made that assumption explicit. However, the process of building the simulation model makes us aware of the contrasting assumptions and forces us to explicitly state our assumptions.

Emergence can be positive or negative. In an exploratory study, where the goal is to let the simulation walk through all the possible paths a system can take, emergence is positive and encouraged. In confirmatory studies where the goal is to test a hypothesis in a repeatable way, emergence is not desirable because it can affect outcomes in unpredictable ways. For instance, when exploring

our ritual model, we'd be delighted to see emergent features of the artificial ritual system that weren't programmed into the low-level system components as this is fertile ground for novel insights. But if we want to test the hypothesis that social networks are strengthened through the ritual process using a specific measure that presupposes the same group of people are present at the beginning and the end of the ritual, but our model sometimes shows people leaving in the middle, we have a problem in the form of an incompatibility between emergent features of the model and our hypothesis-testing activities.

The task of determining if a (fourth-order) simulation model is an accurate representation of its referent (zeroth-order) is also called validation. This can be confusing, and we wish different words were in use for these two kinds of validation questions. It would be less confusing to refer to this checking-accurate-representation process as *correspondence*, which we will do here. The way to establish correspondence depends on the type and purpose of the simulation model. For static models, the degree of correspondence might be established by comparing input/output pairs against relevant data (e.g. correlations between ritual role and post-ritual emotional state). For dynamic models, correspondence might be established by comparing the trajectory of a model (input, state, output) with historical records (e.g. the way a person's behavior changes over time in the wake of repeated ritual participation). When data or trajectory information is not available, correspondence can be established by comparison with other models.

In a *confirmatory study*, establishing correspondence is critical since the simulation model acts as a proxy for the referent. In that case, it is important to identify data sets early in the model-development process to ensure that we will have the relevant means to evaluate correspondence when the time comes. In an *exploratory study*, correspondence is hard or sometimes impossible to establish. In that case, the correspondence process increases our confidence in the simulation model. In both types of study, the degree to which a model should correspond to the referent depends on the intended use of model or its "goodness of fit." For training purposes where the simulation model is used to stimulate trainees, a high degree of correspondence is not usually required so long as the right insights are generated and learned by trainees. For a scientific study, whether exploratory or confirmatory, we usually require a higher degree of correspondence. It is important to establish a desired level of correspondence very early in the study to ensure that expectations are set and understood and that there is enough information to complete the process. For many studies in psychology of religion, a relatively high degree of correspondence is important if we are to have confidence in the insights generated through running the simulation model and analyzing its results. Thus, it makes sense to look to build computational models in subject domains that have an abundance of data. Sometimes, though, psychologists of religion might build a model to explore the effects of changing conditions in ways that haven't been experimentally evaluated in the existing literature (for a further discussion on correspondence in relation to simulation of theories in the psychology of religion, see Lane, 2018, pp. 338–347). For instance, what would happen if the traditional food critical to the ritual were changed to another kind of food? This could be explored in a simulation even if we had never tried it in real life, and simulation predictions could then be tested in real-world experiments.

Data, analytics and visualization

In M&S, data can itself be treated as a model; in this conceptuality, each data value is a model of a simulation-related concept from the perspective of the modeler. Data can be static, a permanent record not expected to change (e.g. date of birth). Data can also be dynamic, meaning

it is associated with a timestamp and is susceptible to change (e.g. age). Input data tend to be static for a given simulation run while output data tend to be dynamic. Simulation models need data in the form of inputs, but they also produce data in the form of outputs. A deterministic simulation requires only a single simulation run to obtain outputs. For example, a deterministic model of personality-relative changes in emotion during a ritual would only need to be run once for each combination of personality type and ritual conditions. By contrast, when a dynamic model is needed to capture uncertainty and variability in a referent phenomenon, multiple simulation runs are necessary to achieve stable knowledge of the input–output patterns and data are modeled using probability distribution functions (PDFs) on which we can run statistical tests for comparison against real-world data when it is available. Simulators represent this variability through random number generators, which can themselves be biased in various ways depending on the underlying algorithms. Executing simulations several times and presenting results as interval estimates with a range and a confidence level helps to account for any bias. For example, when we lack a deterministic account of the way personality impacts ritual emotions, we may need to build stochasticity into the simulation model by randomly selecting for a set of plausible choices how a ritual participant is reacting emotionally to proceedings. We'd need to execute such a simulation a number of times with the same starting parameters in order to discover what the typical emotional reactions are.

The goal of M&S is to generate insight into the referent by answering the modeling question. Yet data produced by a simulation tells us about the simulation and not necessarily about the referent. Even if we assume the simulation is valid and has a high degree of correspondence, it is still a simplification of the referent—and those simplifications can be very dramatic when it is human minds and relationships we are studying, as against airplane wings or manufacturing processes. In most research topics within the psychology of religion, we will need additional interpretation to help relate what we know from the simulation to what we are entitled to infer about the referent. The role of analytics and visualization is to help human beings gain insight into the referent from data generated by the simulation.

Analytics and visualization are becoming even more critically important as computing power and the size of datasets generated by simulations continue to increase. The type of analysis performed depends on the purpose of the simulation and the intended audience. However, for the most part, the goal is to reduce complexity and identify the leading source of change within the simulation model. For example, a typical question might be whether ritual efficacy depends more on the social network of the participants or on the authenticity and charisma of the ritual leaders. To identify the leading sources of change, we rely on descriptive statistics and advanced analytics methods. Principle component analysis (PCA) allows us to explore multi-dimensional datasets with interdependent variables to identify the leading sources of variance and identify the structure of the data (Jolliffe, 2011). Multivariate linear regression allows us to use a set of independent variables and compute the value of one or more dependent variables called responses (Johnson & Wichern, 2004). We also rely on partial least squares (PLS) to identify between two or more sets of data (Chin, 1998).

Once we identify the key variables of the model, we can conduct virtual experiments to explore the behavior space of the simulation. A *Monte Carlo experiment* involves randomly varying input variables in order to observe the behavior of output variables. A *calibration experiment* allows us to measure how closely a simulation matches a set of predetermined data or pattern of data. An *optimization experiment* tells us how well a simulation performs relative to an objective. *Parameter variation* allows us to systematically vary input variables and record data on both the input and the output variables. Each type of experiment yields datasets that can be further analyzed to generate insight into the simulation and ultimately into the referent. For example, we

could ask the simulation to identify the conditions under which participants' emotions related to a willingness to forgive are maximized by participation in the ritual process. This would be an optimization experiment, and we could analyze the data produced by the simulation to understand the identified optimal conditions in more detail.

Summary and conclusion

In summary, there are cases when studying religion where it is too dangerous, difficult or unethical to obtain data. Even in cases where data are available, we are often interested in asking "why" and "how" questions of religion. We wish to understand the underlying mechanisms behind the emergence of religion and even predict religious change in the 50 years. The discipline of M&S offers hundreds of methods to generate insight into a complex phenomenon generation of insight through analysis. The M&S process consists of identifying a referent and using a modeling question to create an executable simulation model that we verify and validate. We execute the simulation model to collect data that we analyze to gain insight in the referent. We can also use the simulation model to (1) make projections into the future, (2) compare and contrast alternative scenarios and (3) generate new theories. Appendix 1 provides a use case example of the process we describe in this article and Appendix 2 gives the reader additional material and tools to start their journey into M&S.

Religion is an important aspect of human life and will likely continue to be an important driver in key events. We hope that scholars use M&S not only to investigate the major role religion plays in peace and conflict in a technologically divided world. M&S can also help scholars in the psychology of religion understand how religion (or lack thereof) help people with the uncertainty of a connected but sometimes alienating society. We hope that in the near future, M&S will be a staple in the toolbox of most scholars in the psychology of religion.

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Appendix I

Use case example

We provide a use case example on how to apply the M&S process to a hypothetical study.

Appendix I. (Continued)

Step	Description	Example	Outcome
Identify a referent	A situation that we wish to explain or understand.	Several surveys report that people who participate in religious rituals regularly are said to be happier and more trustworthy. There are several possible explanations for this observation, but there is no consensus on the nature of the relationship between happiness and regular practice of religious rituals.	A document describing the referent
Develop a reference model	The reference model captures the description of the problem situation as it is understood for the purpose of the study.	The referent is the relationship between ritual participation and happiness. We focus on rituals in the Catholic church as they are performed weekly. We assume that members attend church service three times a week, and there are no holidays or prolonged periods of absence. We assume that members are employed male and female adults between the ages of 18 and 65 of with at least a high school degree. We assume that the service leader is a duly appointed church leader who knows the ritual perfectly and does not make mistakes. We assume that members are married and have between 0 and 5 children. The total number of members is fixed but the number of people at a service What is the role of rituals on the happiness of Catholics?	Reference model with a list of assumption
Identify a modeling question	The purpose of the study on the central question that the model needs to answer		Modeling question
Develop a conceptual model	The conceptual model is the part of referent we need to capture in order to answer the modeling question.	We conceptualize the reference model as a system. The system is defined by rituals led by a priest and performed daily by affiliated members during a church service. The inputs are personality types of the members and the outputs are the degree of happiness and the ratio of people who are happy over people who are unhappy. Members attend a service led by a church leader. There are several types of services with varying degree of significance and impact. People leading the service has varying levels of charisma and ability to inspire and engage members.	A text or diagram showing the components and relationships of the model
Develop a simulation model	A simulation model is the executable version of the conceptual model. It is the conceptual model with the additional logic explicitly spelled out.	We implement the conceptual model using a constructive simulation platform. Members are represented as computer agents with big five personality attributes and a happiness attribute. Church leaders are modeled as computer agents with big five attributes and an additional charisma variable. Services are modeled as grid cells in a 10 by 10 grid. Each cell has an intensity value denoting the intensity of the service. In the model, members seek a charismatic leader and fellow members to congregate around for a service. If they find a suitable group and leader they hold a service and update their happiness attribute based on the happiness of other participating members, the charisma of the leader and the intensity of the service. Each step in the model represents a day.	A software simulation with inputs and outputs

Appendix I. (Continued)

Step	Description	Example	Outcome
Model the inputs	Develop a model that captures how inputs to the simulation model will be represented	Personality traits for catholic church members mirror those if the population at large and therefore will be modeled using normal distributions. We will ensure that personality traits that are correlated in the real-world remain correlated in the model. The type of service is modeled as a log-normal distribution with most services being of medium intensity and very few services being of very high intensity.	Distributions or datasets for each input of the simulation model
Verify the simulation model	Ensure that there are no errors in the software model	We test the simulation by running it multiple times and using statistical methods to ensure that correlations of personality traits in the real world hold in the model, members are not participating in more than three rituals a week and happiness values are updating in accordance with the theories we are using to increase or decrease the values.	A document that shows the test that were performed and the analysis that shows that we are confident that the simulation model is free of errors
Validate the simulation model	Ensure that the simulation model is a faithful representation of the reference model	We run the simulation several times and compare our results with survey data. We validate our algorithms against existing theory. We use our model to predict future happiness levels and compare our results with projections by other scholars.	A document describing the validation test and results proving that the model is a plausible representation of the referent.
Conduct experiments and analyze results	Design an experiment to gain insight into the modeling question. Decide if an exploratory or confirmatory study is needed.	We design an experiment to test the effects of the size of a church, the intensity and frequency of a service and the charisma of leaders on the happiness of members. We statistical analyses to calculate the correlation between the inputs and the outputs. Our analysis shows that with a 95 percent significance level, the level of happiness is strongly correlated with the charisma of the leader (64%). We run a second experiment to investigate conditions under which happiness is maximized. Results show that churches with a medium size (less than 5000 members) and a balanced mix of charismatic leaders performing mostly medium intensity services is the best solution. Finally we run the model several thousand times and analyze the results. The model shows that there is a threshold effect for the size of the church and the frequency of high intensity services meaning there is an upper and lower bound above and below which happiness ceases to increase and in fact decreases.	A document that explains the design of experiment and frames the answer to the modeling question in the form of insights.

Appendix 2. Recommended readings and resources.

Type	Reference	Description
Tool	Methapr: Upcoming	A tool that allows users to build models without computer programming
Tool	Netlogo: https://ccl.northwestern.edu/netlogo/index.shtml	A free open source tool with great examples on how to build models
Document	Balci, O. (1998, December)	A description of verification and validation techniques and challenges in M&S
Document	Banks, J., & Chwif, L. (2011)	An overview of issues and challenges inherent to computer modeling and simulation
Document	Robinson, S. (2008)	An introduction to conceptual modeling including why it is a critical step of M&S
Document	Law, A. M. (2003, December).	A best practice guide from one the key scientists in the field of M&S
Document	Diallo, S. Y., Gore, R. J., Padilla, J. J., and Lynch, C. J. (2015).	An overview of the field of M&S including the major players and a historical perspective on how M&S emerged from the shadows of Systems Science and Computer Science
Document	Lane, J. E. (2017a), Shults, F. L., Lane, J. E., Wildman, W. J., Diallo, S., Lynch, C. J., and Gore, R. (2017).	Examples of modeling and simulation efforts in the study of religion