PRIMARY HEALTHCARE DELIVERY NETWORK SIMULATION
USING STOCHASTIC METAMODELS

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ABSTRACT

A discrete-event simulation (DES) of the network of primary health centers (PHCs) in a region can be used to evaluate the effect of changes in patient flow on operational outcomes across the network, and can also form the base simulation to which simulations of secondary and tertiary care facilities can be added. We present a DES of a network of PHCs using stochastic metamodels developed from more detailed DES models of PHCs ('parent' simulations), which were developed separately, for comprehensively analyzing individual PHC operations. The stochastic metamodels are DESs in their own right. They are simplified versions of the parent simulation with full-featured representations of only those components relevant to the analysis at hand. We show that the outputs of interest from the metamodels and the parent simulations (including the network simulations) are statistically similar and that our metamodel-based network simulation yields reductions of up to 80% in runtimes.

1   INTRODUCTION

In India, primary health centres (PHCs) are the fundamental component of public healthcare delivery. They are the first point of contact for the public with a formally trained medical doctor, and they provide outpatient care, conduct community health and outreach programmes, limited emergency and inpatient care, and also attend to childbirth and antenatal care patients (IPHS Guidelines 2012). While the number of PHCs in India has increased substantially from 9,115 during 1981-85 to 25,650 in 2017 (Government of India M of H& FW 2018), their operational performance and impact on enhancing access to public healthcare have not adequately been assessed. Therefore, a simulation-based assessment of the operational capability of these facilities to respond to changes in demand would inform investment decisions, for example, the number of new PHCs to be funded or existing facilities extended and upgraded. This would require the development of several models which would simulate the operations of the individual PHCs as well as that of the network of PHCs operating in a given region, such as a district. The model of an individual PHC can be used to analyze and improve the operational efficiency of the centre and determine how key outcomes such as patient waiting time and resource utilization respond to changes in demand. A model of a district-level PHC network, on the other hand, helps us (a) evaluate the effect of changes in patient flow on the public primary healthcare delivery network across a region (e.g., a sudden influx of population in a particular area within a district), (b) evaluate the effect of locating new PHCs on operational outcomes at other PHCs in the same network (location-allocation), (c) evaluate the operational outcomes of referral mechanisms that operate within the PHCs (e.g., divert patients from one PHC to another depending on demand/congestion) and, potentially, between PHCs and higher-level care (e.g., secondary or tertiary level of care).

In this paper, we describe the development of a discrete-event simulation (DES) of the PHC network within a district in India. Such network simulations, if developed using full-featured simulations of the

individual facilities that are part of the network, often incur considerable computational expense in their operation. For example, Mustafee et al. (2009) investigated the blood ordering policies of a UK National Blood Service (NBS) supply chain that consisted of one NBS Process, Testing and Issuing (PTI) facility and several hospitals (this is conceptually similar to a PHC network), and their experiments showed an exponential rise in execution time when individual DES models of hospitals were being added to the network simulation. Distributed simulation was used in this study (ibid.) to reduce execution time. In this work, we were confronted with a similar problem (large execution time), but rather than using distributed simulation approach, we employ a *stochastic metamodeling approach*. The stochastic metamodels we develop are DES models in their own right and are developed from the full-featured parent models.

How long were the run times in our study and which motivated us to use the metamodeling approach? The version of the PHC network simulation that we developed using full-featured individual PHC DESs required nearly 42 hours for 200 replications on a computer with 8 GB RAM with 2 cores. PHC network simulation using stochastic metamodels were therefore developed from the full-featured individual PHC simulations. We hereafter refer to these detailed simulation models of PHC operations as the 'parent' PHC simulations, and these are developed for the separate purpose of analyzing and identifying methods of improving individual PHC operations in a comprehensive manner. The stochastic metamodels that we develop for constructing the PHC network simulation are abstractions of the parent PHC simulations – they are simplified simulations constructed using information from the parent PHC simulation, but contain only those subsystems of the PHC, and therefore of the parent PHC simulation, that are relevant to the analysis at hand. Other subsystems of the PHC/parent-simulation are abstracted and represented in the parameterization of the metamodel using data generated from the parent model.

The PHC network simulation developed using the stochastic metamodels yield substantial savings in computational runtime while retaining the flexibility in conducting what-if analyses that a simulation offers. We show numerically that the distributions of outcomes of interest such as average outpatient waiting time, doctor, nurse and bed utilizations estimated using both the parent model and the metamodel are statistically identical while achieving reductions of up to 76% in model runtime. We also implement a simulation of the network of PHCs using both the metamodel and the parent model and demonstrate that the key outcomes are statistically similar while achieving approximately 80% reductions in runtimes.

Previous work related to healthcare facility network simulations have focused on reuse and adaptation of models developed for one facility for other facilities (Penn et al. 2019), on managing patient flow out of hospitals into long-term care facilities (Patrick et al. 2015), integration of secondary and tertiary care facilities (Ortíz-barrios et al. 2017), distributed simulation for blood supply chain network simulation (Mustafee et al. 2009, Katsaliaki et al. 2009, Blake & Hardy 2014), etc. Overall, our work demonstrates how simpler simulations conceptualized from more complicated facility operation simulations can be used in the development of facility network simulations that incur a substantially shorter computational runtimes, especially in the context of healthcare delivery.

The remainder of the paper is organized as follows. Section 2 discusses existing work. Section 3 provides a brief background on PHCs and the development of the parent PHC models. Section 4 is on the implementation of the PHC metamodels and the PHC network simulation.

2 LITERATURE REVIEW

We focus our literature review on healthcare facility network simulations, generic modelling framework development and model reuse within DES, and also briefly discuss how our use of stochastic metamodels fits within the metamodeling literature.

A majority of DES studies in healthcare facility modelling typically focus on single facility or single unit (within a larger facility) operations (Zhang 2018, Günal & Pidd 2010). (Swisher et al. 2001) develop a DES model of an outpatient clinic within a physician network, and propose the use of the clinic simulation to simulate the entire network with minor modifications relevant to each individual clinic. However, they do not implement the network simulation themselves. Katsaliaki et al. (2009) and Mustafee et al. (2009), in related articles, compare a standard DES approach for simulating a large blood distribution supply chain network (from donor to recipient) to a distributed simulation approach. The authors found that the distributed approach reduces computational expense when the number of facilities in the network exceeded

a certain threshold intrinsic to the network being simulated. Patrick et al. (2015) developed a DES model to aid in capacity planning for long-term care in community health facilities of patients leaving hospitals, and hence modelled patient flow in a small network with hospitals and long-term care facilities. A similar problem was also tackled by Bae et al. (2019), who developed a simulation of patient flow across facilities associated with long-term care to forecast the demand for long-term care. Blake and Hardy (2014) develop a generic simulation modelling framework in conjunction with response surface metamodel optimization to develop optimal inventory management policies for a donated blood supply chain network. Ortíz-barrios et al. (2017) developed a DES model to evaluate the effect of integrating patient flow between secondary and a tertiary care health facilities on patient waiting times for outpatient appointments in the internal medicine department. In this context, our study demonstrates the development of a simulation of the primary healthcare delivery network in an Indian district. To the best of our knowledge, this is the first study which models primary healthcare delivery network simulation using stochastic metamodels of individual facility simulations.

During our visits to multiple PHCs, we found that while similar operational patterns are followed in each PHC, there were differences in staffing levels, number of beds, presence/absence of childbirth care facilities, etc. Thus, we developed multiple configurations (3 configurations) of an archetype PHC simulation framework to represent operations of each PHC (9 PHCs) in the district. Hence, in this context, we also briefly discuss the generic simulation modelling and simulation model adaptation and reuse literature. Multiple studies discuss reconfigurable and reuse simulations in a general setting (Mackulak et al. 1998, Steele et al. 2002, Robinson et al. 2004, Pidd & Carvalho 2006, Kaylani et al. 2008, Fletcher & Worthington 2009, Penn et al. 2019), and/or in the context of applications focused on healthcare (Swisher et al. 2001, Fletcher & Worthington 2009, Günal & Pidd 2010, Mustafee et al. 2011, Di Mascolo & Gouin 2013, Weerawat et al. 2013, Penn et al. 2019). While these studies describe the development of reusable DES frameworks for physician clinics (Swisher et al. 2001, Mustafee et al. 2011), generic hospital simulation frameworks (Günal & Pidd 2010), and that of their subunits (Weerawat et al. 2013), (Penn et al. 2019), our literature survey did not identify a demonstration of model reuse, especially in conjunction with metamodel development, for network simulations. In our model, we develop an archetype PHC operations model that we then modify depending upon the specific configuration in operation at each node in the PHC network in the district under consideration. Our model demonstrates how the archetype model can be modified and used in a facility network simulation, either as-is or using metamodels developed from each version of the archetype.

Given the substantial body of literature on metamodeling work in the simulation literature (Kleijnen & Sargent 2000, Barton 2009), we focus on stochastic metamodels that can be used in what-if analyses and provide probability distributions for their outputs (without requiring a separate metamodel for variance estimation). A key metamodel class capable of such analyses includes dynamic Bayesian network (DBN) metamodels (Poropudas & Virtanen 2007, Poropudas & Virtanen 2010, Poropudas & Virtanen 2011, Kelleher et al. 2018). DBN metamodels describe the time-evolution of the state of the system probabilistically by deriving a network of conditional probability distributions for the states of the system at various points in time (including the case of continuous time, achieved via interpolation). Constructing DBN metamodels involves selecting an optimal set of time instants at which simulation data must be collected to estimate the probability distributions for the simulation state, and conducting what-if analyses involves fixing the simulation state at a given point in time and updating the simulation state conditional probability distributions using the observed value of the simulation state. Another related technique involves metasimulation, as demonstrated by Bauer et al. (2004). They develop a simulation of internet and telephone networking protocols and propose the use of a random search approach to the design of experiments for finding key parameters and interaction patterns that affect key network performance measures. Thus they utilized their parent simulation to construct a simpler simulation that captures the impact of key parameters (including their interactions) on their key performance measure.

With respect to these approaches, our stochastic metamodel is a DES model in its own right, constructed using outputs from the parent simulation. In contrast to DBN metamodels and the metasimulation approach, our approach is likely to require fewer replications of the parent model to build the metamodel, because the parent model subsystems of interest are explicitly simulated in the metamodel. Thus, the multiple input/output replication sets required in an experimental design framework to fit the curve/function

replacing the simulated subsystem are likely to not be required with our approach. For example, in our case, we only require one set of replications from the parent model – the same set generated as part of routine analysis – to build our metamodels. Further, given that our metamodel is a DES in its own right, it is capable of generating multiple outputs associated with the analysis from a single set of replications, and retains the flexibility that the parent model would offer in terms of conducting what-if analyses. The trade-off, however, lies in reduction of computational expense (in terms of runtimes) – other metamodels may incur considerable computational expense in their construction, whereas their execution times are negligible. In contrast, while constructing our metamodels may incur lesser computational expense, its runtimes are not negligible compared to deterministic or DBN metamodels. However, as will be demonstrated in the following sections, our approach still yields reductions in runtimes up to 80% compared to the PHC network simulations constructed using the parent models, which can represent significant savings.

3   PHC PARENT MODEL DEVELOPMENT

A PHC aims to deliver integrated curative, promotive, and preventive healthcare at the primary level in the Indian context. Depending upon the region, PHCs were designed to serve populations of 30,000 in the plains and 20,000 persons in hilly or tribal areas. They primarily provide outpatient services (8 hours/day for 6 days a week); however, they also have a small inpatient department to handle childbirth cases and patients requiring inpatient care and monitoring for brief periods (less than 24 hours) - for instance, management of injuries and accidents, first aid, and other emergency conditions. PHC staff also provide counseling and medical care to persons requiring antenatal (ANC) care and conduct public health outreach programmes regarding disease prevention and immunization (IPHS Guidelines 2012).

3.1   PHC Simulation: Parent Model

The parent DES model of the PHC was developed as part of another study conducted to comprehensively analyze the operations of individual PHCs (Shoaib & Ramamohan 2020). The parent model is full-featured in that it contains all operationally relevant systems associated with providing medical care of the PHC. The parent model simulates PHC operations with one or two doctors (depending upon the staffing level at the PHC type under consideration), two nurses (an outpatient nurse for conducting checks related to non-communicable diseases, referred to hereafter as the NCD nurse, and a nurse attending to inpatient and childbirth cases, referred to hereafter as the staff nurse), a pharmacist, a clinical laboratory technician and multiple patient types (outpatients, inpatients, childbirth patients, and patients requiring antenatal care). We also include resources such as inpatient and childbirth beds in the parent model.

Over the course of nearly 1.5 years from May 2018 onwards, we visited nine PHCs in a nearby district with a mix of urban, semi-urban, and rural populations to understand PHC operations and collect data regarding specific aspects of their operations. We observed that while most PHCs follow a certain standard pattern of operations, there were significant variations between PHCs in terms of staffing level, patient load, and whether certain classes of medical services were offered (e.g., childbirth care was not offered in all PHCs visited). Therefore, while a single simulation model would not be able to capture this operational diversity, modified versions of a single archetype model that would represent each specific PHC would be able to do so. Thus, out of the nine PHCs visited, four were identified as the PHC type that conforms most closely to the government guidelines (IPHS Guidelines 2012), and hence the archetype model was developed based on the operations of these PHCs. This model is referred to below as configuration 1, and configurations 2 and 3 (note that configuration 3 PHCs do not offer childbirth care services) are modifications of this archetype PHC operations model. Patient load and staffing levels of each configuration are described in Table 1. A more detailed description of the government guidelines for PHC operation, our visits to the PHCs, input data collected and the grouping of PHCs into the archetype (configuration 1) and other configurations is provided in (Shoaib & Ramamohan 2020). We now describe the patient flow in a PHC as captured by the archetype model.

Table 1: Operational characteristics of PHCs by configuration.

Configuration (number of PHCs)	Average patient load per day				Number of doctors, staff nurses/shift
	Outpatient	Inpatient	Childbirth	Antenatal	
Configuration 1 (4)	120-140	0.5	1	1	2, 1
Configuration 2 (2)	60-80	0.5	0.5	0.5	1, 1
Configuration 3 (3)	60-80	0.5	--	--	1, 1

3.2 Patient Flow in the Parent PHC Model

Patients visiting PHCs can be categorized as, (a) outpatients, (b) inpatients, (c) childbirth patients, (d) ANC patients. Outpatients first go to the outpatient room, where patients aged less than 30 years directly consult the doctor. Other outpatients consult first with a nurse, who checks the blood pressure, body temperature, blood glucose level, weight, etc. before doctor consultation. After consulting the doctor, patients are either sent to the in-house laboratory for tests or they exit after visiting the pharmacy. Two types of tests are conducted at laboratories: (a) tests for which reports are provided within 5–10 minutes, and (b) tests that require several hours for reports. Note that all patients must exit after visiting the pharmacy (even if they do not require medicines), as it also serves as a registration counter for the PHC.

Inpatient and childbirth patients are assumed to have priority for care when compared to outpatients (we assume non-preemptive priority). If a doctor is available, inpatients are attended first and are then sent to the IPD ward where they are kept under the supervision of the staff nurse. The childbirth case is similar to that of inpatients; however, instead of occupying an inpatient bed upon arrival, they occupy the labour room (bed). After childbirth, the patient is shifted to the inpatient bed. ANC patients are attended to primarily by the staff nurse. If it is a first visit, after counseling from the staff nurse, routine laboratory tests are performed at the laboratory, after which they exit the system via the pharmacy after collecting medicines and supplements. As per the Indian Public Health Standards guidelines (IPHS Guidelines 2012), ANC patients are advised to make four visits to the healthcare facility for routine examinations, medications and counselling. The first visit is within 12 weeks of pregnancy, followed by the second visit between 14 and 26 weeks, third visit is scheduled between 28 and 34 weeks and the last visit is between 36 weeks and the term.

3.3 Key Model Inputs and Parent Model Simulation Outcomes

The input parameters used in the parent model are shown in Tables 2 and 3. Table 2 lists service time estimates common to all configurations. The distributions of the doctor's consultation time, and the service times at the pharmacy and laboratory were determined by finding the best-fit distribution to data collected (service/consultation times recorded using stopwatches) for each parameter during our visits across the PHCs. For inpatient and childbirth parameters, we were unable to access data for lengths of stay – it is unclear if they are recorded. Therefore, for these parameters, we determined distributions and their parameter estimates based on interviews with nurses and doctors. Note that all arrivals were determined to be Poisson, and their parameters (average arrival rates) were estimated using the average arrival rates from Table 1 (also the header of Table 3 in Section 3.4). More details regarding the input parameter estimation process are provided in (Shoaib and Ramamohan 2020).

The results show that all configurations are substantially underutilized. For example, for configurations 2 and 3, the doctor's utilization increases from 27% to 37% despite the decrement in outpatient arrivals (from 130 to 60), since only one doctor is present during the OPD hours in config 2 (compared to two doctors in configuration 1). Further, inpatient and labour bed occupancies are also low for all configurations. Regardless of the low labour bed occupancies, a substantial proportion of childbirth patients do not receive care at the PHC they visit first. This is because these patients may arrive when the labour bed is occupied by another childbirth patient and therefore leave the system without being admitted. The underutilization of PHCs is explained by the fact that only approximately 30% of patients seek care at public healthcare facilities in India, and also by the relatively very low consultation times for outpatients with doctors (a mean consultation time < 1 minute) , which are a fraction of those in developed settings.

Simulation experiments conducted in (Shoaib and Ramamohan 2020) show that if average consultation times are increased to more reasonable values (2.5-5 minutes) and outpatient loads are increased (i.e., if more patients seek care at public healthcare facilities), then resource utilization increase substantially.

Table 2 : Service/consultation time parameters common to all configurations.

Service Time Parameters (minutes)	Distribution	Source
Doctor consultation time: outpatients	Normal (0.87; 0.21^2)	Observation
Pharmacy service time	Normal (2.08; 0.72^2)	Observation
Laboratory service time	Normal (3.45; 0.82^2)	Observation
NCD nurse service time	Uniform (2, 5)	Interviews with nurses; Observation
Doctor consultation time: inpatient	Uniform (10, 30)	Interviews with doctors
Staff nurse service time: inpatient	Uniform (30, 60)	Interviews with the nurses
Staff nurse service time: childbirth cases	Uniform (120, 240)	Interviews with the nurses

Table 3: Operational outcomes for each PHC configuration from the parent simulation.

	Configuration-1 (2/130/0.5/1/1/6/1)*	Configuration-2 (1/60/0.5/0.5/0.5/6/1)*	Configuration-3 (1/60/0.5/0/0/6/0)*
	Mean (95% CI)	Mean (95% CI)	Mean (95% CI)
Doctor utilization	26.8% (26.7; 26.8)	37.2% (37.1; 37.3)	35.4% (35.3; 35.4)
NCD Nurse utilization	86.5% (86.3; 86.8)	46.9% (46.8; 47)	46.8% (46.7; 46.9)
Staff nurse utilization	32.3% (32.1; 32.4)	24.3% (24.1; 24.4)	16 % (16; 16)
Pharmacist utilization	64.3%(64.2; 64.4)	28.8%(28.8; 28.9)	28.9% (28.8; 28.9)
Lab Utilization	55.9% (55.7; 56)	25.4% (25.3; 25.4)	23.9% (23.9; 24)
Inpatient bed utilization	9.3% (9.2; 9.4)	5.5 % (5.4; 5.6)	1.1% (1.1; 1.1)
Labour bed utilization	28.3% (28.1; 28.5)	15.3% (15.1; 15.4)	Not applicable
Mean length of OPD queue (number of patients)	0 (0; 0)	0.007 (0.006; 0.007)	0.001 (0.001; 0.001)
OPD queue waiting time (min)	0.009 (0.008; 0.010)	0.171 (0.165; 0.178)	0.034 (0.033; 0.034)
Pharmacy queue waiting time (min)	1.025 (1.021;1.029)	0.244 (0.243; 0.246)	0.232 (0.231; 0.233)
Lab queue waiting time (min)	2.084 (2.073;2.094)	0.606 (0.601; 0.610)	0.571 (0.567; 0.575)
Fraction of childbirth cases referred	15.6% (15.2; 16)	8.8% (8.3; 9.2)	Not applicable

Number of doctors per shift / outpatients per day / inpatients per day / childbirth cases per day / antenatal care patients per day / inpatient beds available / labour beds available

4   IMPLEMENTATION OF THE INDIVIDUAL METAMODELS

The metamodels are constructed as abstractions of the original model, including only those resources relevant for the analysis. In contrast to the full-featured original model, only resources directly linked to provision of medical care and that are relevant to the analysis question are included in the metamodel as comprehensively as in the parent simulation. For example, in the version of the metamodel we develop, ANC patients were excluded, as they affect only the staff nurse's utilization, and given their small arrival rate and service time, they do not have a significant effect on the staff nurse's utilization. This is consistent with typical metamodeling practice – for example, in the case of deterministic metamodel development, functions that relate only outcomes of interest (e.g., average waiting time) and model inputs of interest are developed (Barton 1998). Thus, only outpatients, inpatients and childbirth patients were included in the version of the metamodel we present here. Further, instead of 4-6 beds, the number of beds in each PHC configuration simulation was reduced to 2, with one bed representing the set of beds for inpatients and the second representing the childbirth beds. The distribution of the length of stay in the inpatient beds was also

scaled down appropriately. Note that childbirth patients are shifted to inpatient beds after a certain period to free up the childbirth bed for other cases, so the distribution of the duration of time spent in the inpatient bed by the childbirth patient is also scaled down accordingly. Finally, the clinical laboratory and the pharmacy were also excluded in the metamodel. However, the time spent by patients at the clinical laboratory for their tests (denoted by the random variable f) and the time spent by patients at the pharmacy (denoted by the random variable g) for getting their medications and for registration were included in the metamodel by holding patients in the PHC system for the corresponding durations before they exited the system. The distributional parameters of f and g were estimated using data from the parent model.

Note that we combine the inpatient beds into a single bed because in this version of the metamodel of the parent PHC model, the focus of our analysis is the outpatient care and childbirth care process. This is because of the following reasons: (a) the majority of the patient load at a PHC is due to outpatients, and (b) the proportion of childbirth patients referred elsewhere because of unavailability of the childbirth bed is also a key outcome of interest for our analysis. Hence the key outcomes we are interested in are the waiting time before seeing the doctor, the doctor utilization and NCD nurse utilization, the total time spent in the system for outpatients, and the fraction of childbirth cases referred elsewhere. However, if inpatient care outcomes (e.g., time spent by inpatients in the system, inpatient bed utilization) was also a key area of focus for this analysis, then the inpatient beds could also be included separately in the metamodel. More generally, the specific set of resources and processes included in a comprehensive manner in the metamodel depends on the focus of the analysis, especially when a metamodel is being constructed for a system with multiple types of resources (medical personnel, equipment/facilities) and job (patients). We summarize the differences between the parent models and all configurations of the metamodels in the Tables 4 and 5. Patient flow through the archetype version of the metamodel (configuration 1) is depicted in Figure 1.

Table 4: Parent model and metamodel staffing and resource characteristics for all configurations.

	Parent Model	Metamodel
Patient Types	**(a)** Outpatients; **(b)** Inpatients; **(c)** Childbirth patients[a]; **(d)**ANC patients	**(a)** Outpatients; **(b)** Inpatients; **(c)** Childbirth patients[a]
Resources	**(a)** Doctor(s)[b]; **(b)** NCD nurse; **(c)** Staff nurse; **(d)** Inpatient beds (4-6); **(e)** Childbirth bed[a]; **(f)** Laboratory; **(g)** Pharmacy	**(a)** Doctor(s)[b]; **(b)** NCD nurse; **(c)** Staff nurse; **(d)** One inpatient bed; **(e)** One childbirth bed[a]

[a]For configuration 3, childbirth patients (and beds) are not applicable. [b]The number of doctors remains the same between the parent model and the metamodel.

Table 5: Comparison between parent model and metamodel input parameter estimates.

Parameter (minutes)	Parent Model	Metamodel
Doctor service time		
(a) Outpatient	$N(0.87, 0.21^2)$	$N(0.87, 0.21^2)$
(b) Inpatient	$U(10, 30)$	$U(10, 30)$
(c) Childbirth case	$U(30, 60)$	$U(30, 60)$
NCD Nurse service time	$U(2, 5)$	$U(2, 5)$
Staff Nurse service times		
(a) IPD	$U(30, 60)$	$U(30, 60)$
(b) Childbirth	$U(120, 240)$	$U(120, 240)$
Length of stay in beds		
(a) Inpatient	$T(60, 360, 180)$	$T(10, 60, 30)$
(b) Childbirth	$U(240, 1440)$	$U(40, 240)$
Laboratory	Service time: $N(3.456, 0.823^2)$	f: total time spent at the laboratory Configuration 1: $N(5.685, 0.824^2)$ Configuration 2: $N(4.09, 0.823^2)$ Configuration 3: $N(4.064, 0.823^2)$

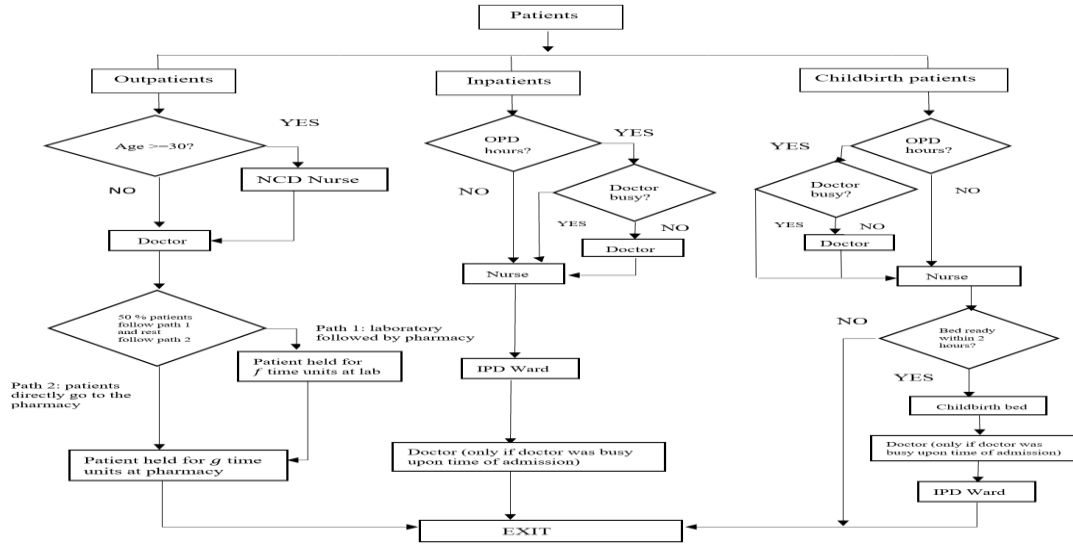| Pharmacy	Service time: N(2.083, 0.72^2)	g: total time spent at the pharmacy Configuration 1: N(3.212,0.723^2) Configuration 2: N(2.348,0.723^2) Configuration 3: N(2.341,0.723^2)

Figure 1: Patient flow in the metamodel.

4.1   ANALYSIS OF THE RESULTS OF THE METAMODEL AND THE PARENT MODEL

Based on Tables 4 and 5, metamodels of the parent simulations of each configuration were developed. Outcomes from the metamodel and the parent model were estimated from 200 replications generated on an Intel m5 laptop with 8 GB RAM (we were unable to access workstations due to COVID-19 lockdowns). These results were generated by running both simulations, for each configuration, for a period of 365 simulated days, with a 180 day warm up period. Two-sample t-tests were conducted to check whether the averages of the estimate of each outcome from the parent model and the metamodel are statistically identical at a 5% level of significance. This is consistent with methods used to assess the accuracy of stochastic metamodels where the distributions of metamodel output estimators are available – for example, with DBNs (Poropudas & Virtanen 2011). For deterministic metamodels, curve-fitting measures of accuracy such as the root mean-squared error and the mean absolute percentage error are typically used (Kleijnen & Sargent 2000). Recently, a misspecification test for such metamodels has also been proposed (Wang et al. 2017).

The null and alternate hypotheses for each t-test that was conducted are:

H$_0$: Mean value of the parent model outcome is equal to that of the metamodel ($\mu_{parent} = \mu_{metamodel}$)

H$_a$: Mean value of the parent model outcome is unequal to that of the metamodel ($\mu_{parent} \neq \mu_{metamodel}$)

The results for each configuration are summarized in Table 6. Note that p-values obtained were larger than 0.05 for all outcomes except for the staff nurse utilization, inpatient length of stay and childbirth patient length of stay (Table 6). Thus, for these outcomes, we fail to reject the null hypothesis, and hence it is reasonable to assume the means of the distributions of these outcomes are identical. With regard to the staff nurse utilization, the associated p-value < 0.05 because ANC patients were excluded from the metamodels. However, when they are included in the metamodel, as seen in the last row of the results for configuration 1, we see that the p-value for staff nurse utilization also is > 0.05. With regard to the inpatient and childbirth patient lengths of stay, because the focus of our analysis is outpatient outcomes, resource utilizations and the proportion of childbirth cases referred elsewhere (due to lack of availability of a childbirth bed for more than two hours), we scale down the length of stay variables for these patient types and hence the utilization values of their associated resources are statistically similar, but the length of stay values are not.

Note that the above formulation of the hypothesis test assumes that the accepted truth (the null hypothesis) is that $\mu_{parent} = \mu_{metamodel}$. While there is precedent for this in the literature (Poropudas & Virtanen 2011), it can be argued that a hypothesis test where the reverse is assumed for the null hypothesis might be more appropriate (i.e., $\mu_{parent} \neq \mu_{metamodel}$). Under such a formulation, we could reasonably conclude that the metamodel is accurate at $\alpha = 0.05$ if the p-value for the test is < 0.05. We applied such hypothesis tests to the outcomes listed in Table 6, and for all outcomes and configurations, the results of the tests were the same.

We see that reductions in runtimes per replication range from 30%-76% across three configurations. The relatively lower reduction in runtime per replication for configuration 3 is likely because it is a less complex simulation when compared to configurations 1 and 2, with a single doctor and no childbirth services. However, when the entire simulation is considered, the reduction in time is nearly 50% for configuration 3 as well, likely because of across replication data storage and computation of summary statistics. For configurations 1 and 2, reductions in runtime remain approximately the same. In general, as the number of replications increase, it is likely that further reductions in runtime will be achieved.

Table 6 : Parent model and metamodel simulation outcomes.

Outcome[a]	Configuration 1		Configuration 2		Configuration 3[b]	
	Parent	Metamodel	Parent	Metamodel	Parent	Metamodel
OPD queue waiting time	0.009 (0.002)	0.009 (0.002)	0.183 (0.022)	0.186 (0.0219)	0.0998 (0.008)	0.100 (0.007)
Doctor's utilization	31.5% (0.2)	31.5% (0.2)	41.8% (0.3)	41.8% (0.4)	41.1% (0.26)	41.1% (0.3)
NCD Nurse utilization	85.6% (0.7)	85.5% (0.8)	51.2% (0.5)	51.2% (0.5)	51.4% (0.5)	51.3% (0.6)
Staff Nurse utilization	32.2% (0.8)	30.3%* (0.7)	24.4% (0.5)	23.3%* (0.6)	15.7% (0.1)	15.68% (0.1)
Bed utilization	9.3% (0.4)	9.3% (0.4)	5.6% (0.3)	5.6% (0.4)	0.89% (0.06)	0.9% (0.07)
NCD Nurse waiting time	2.593 (0.073)	2.605 (0.077)	0.835 (0.04)	0.838 (0.041)	0.760 (0.029)	0.756 (0.025)
Outpatient length of stay in the PHC	9.694 (0.060)	9.707 (0.056)	7.401 (0.056)	7.411 (0.053)	7.287 (0.038)	7.294 (0.041)
Inpatient length of stay in the PHC	255.33 (5.034)	92.074* (2.208)	254.93 (5.00)	90.75* (1.96)	264.77 (5.53)	99.55 (1.54)
Childbirth patient length of stay in the PHC	1556.34 (26.248)	862.17* (10.68)	1538.41 (29.56)	841.32* (11.21)	Not applicable	
Childbirth bed utilization	28.1% (1.2)	28.1% (1.2)	15.3%* (3.91)	15.4%* (3.92)	Not applicable	
Proportion of childbirth patients turned away	0.150 (0.016)	0.153 (0.018)	0.0846 (0.016)	0.086 (0.0211)	Not applicable	
Runtime per replication[c]	288.618	69.237*	110.434	32.022*	45.506	30.103*
Net simulation runtime[c]	958.58	230.73	372.26	120.31	132.80	70.31
% differences in runtimes – per replication, net	76.01%, 75.93%		71.00%, 67.68%		33.85%, 47.05%	
Staff nurse utilization (when ANC patients are included)[d]	32.2% (0.8)	32.1% (0.9)	Not applicable		Not applicable	

Notes: Standard errors of the means of the outcomes are given in the parentheses. [a]All lengths of stay and waiting time outcomes are in minutes. [b]Configuration 3 does not offer childbirth care services, hence associated outcomes are

not applicable for this configuration. ^cRuntimes per replication are in seconds, and runtimes for the entire simulation are in minutes. ^dSimulation experiment conducted only for the archetype PHC (configuration 1) for illustrative purposes.
*P- value for this outcome comparison < 0.05.

5   IMPLEMENTATION OF THE METAMODEL-BASED NETWORK SIMULATION

After we validated the metamodels, we developed a simulation of the network of 9 PHCs in the district using the metamodels of the parent models of each PHC configuration. The number of PHCs of each configuration in the network is provided in Table 1, and the average patient arrival rates for each configuration are provided in the header of Table 3.

In order to verify that the PHC network simulation constructed using the individual PHC metamodels is an accurate representation of a PHC network simulation constructed using the parent PHC models, we constructed both versions, and tested whether the means of outcomes measured across the network were statistically identical for both versions. These results were also generated from 200 replications over a period of 365 simulation days, with a 180 day warm up period. The accuracy of the metamodel was also assessed in the same manner as for the individual facility metamodels, with two-sample t-tests conducted to check for statistically significant differences between the means of simulation outcomes (null and alternate hypotheses same as for individual facility parent models and metamodels).

The results from both versions of the PHC network simulations are summarized in Table 7. Similar trends are evident with regard to the outcomes of the t-tests. We see similar runtime reduction outcomes as well. Once again, we anticipate greater reductions with an increase in the number of replications, and this will form an avenue for our future research.

Table 7: Outcomes from parent model and metamodel based network simulations.

Simulation Outcome^a	Parent Model	Metamodel	p-value
Outpatient waiting time	0.057 (0.079)	0.058 (0.081)	0.209
Doctor's utilization	33.4% (0.069)	33.4% (0.070)	0.858
NCD nurse utilization	68.4% (0.183)	68.4% (0.183)	0.544
Staff nurse utilization	24.9% (0.076)	23.7% (0.066)	0.0062
Inpatient bed utilization	5.7% (0.038)	5.7% (0.039)	0.534
NCD nurse waiting time	1.800 (0.962)	1.803 (0.960)	0.471
Length of stay (outpatient)	8.629 (1.248)	8.627 (1.249)	0.682
Length of stay (inpatient)	257.56 (4.605)	93.22 (4.09)	7.89E-16
Length of stay (childbirth cases)	1549.18 (13.37)	857.54 (9.81)	4.59E-12
Childbirth bed utilization	23.6% (6.6)	23.7% (6.6)	0.611
Proportion of childbirth patients turned away	0.131(0.035)	0.130 (0.037)	0.758
Runtime per replication (seconds)	668.076	132.977	--
Net simulation runtime (minutes)	2568.38	615.56	--
% differences in runtimes – per replication, net	80.10%, 76.05%		--

Notes: Standard errors of the means of the outcomes are given in the parentheses. ^aAll lengths of stay and waiting time outcomes are in minutes.

6   CONCLUSION

In this work, we present preliminary work from the development of a DES of a network of primary health centers in India, developed using stochastic metamodels – DESs in their own right – of more comprehensive simulations of individual PHCs. We demonstrate that the facility network simulation developed using the stochastic metamodels yields statistically similar simulation output estimates for outcomes of interest while achieving reductions in runtimes up to 80%.

In comparison to metamodels developed by fitting curves to describe the relationship between simulation inputs and outputs, the metamodels we develop here offer the flexibility of simulations in terms

of conducting what-if analyses without necessarily requiring a specific experimental design framework for constructing the metamodel. However, it also has some limitations. For example, as the parent simulation model becomes more complex, developing its metamodel may require generation of simulation outputs not collected while generating the standard set of outputs from the parent simulation. Further, our approach, while offering more flexibility, sacrifices the non-trivial execution time reductions achieved by a metamodel developed via curve-fitting. However, a metamodel developed using our approach is likely to generate multiple outputs for a given set of inputs, whereas a curve-fit metamodel would likely be required for every input-output relationship.

There exists several avenues for future research, for example, extending the facility network simulation to include secondary and tertiary levels of care using their stochastic metamodel counterparts; demonstrating the use of such facility network simulations in optimal facility location problems using simulation optimization methods, especially when the number of feasible solutions is discrete and small; accurately characterizing the reductions in runtime with increase in complexity of the parent simulation, number of replications; metamodel development only of subsystems of complex parent models; and finally, a general framework for the development of such metamodels from parent simulations, including a characterization of the additional simulation replications required to develop the metamodel (if necessary).

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