Modular construction of compact Petri net models

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Abstract: The use of modelling formalisms for the design of discrete event systems presents many advantages, such as the possibility of structural analysis of the model or performance evaluation. However, the difficulty of the process to obtain an appropriate model of the system requires the use of methodologies to ease the work of the designers. In this paper, two main subjects are discussed. On the one hand, the modular construction of Petri nets alleviates the design process by the use of blocks that can be assembled to build up a complete Petri net model. On the other hand, the development of decision support systems may require the assessment of the performance and properties of complete models obtained from different combinations of modular blocks. The formalism of the alternatives aggregation Petri net may help in the development of compact and efficient models that may reduce the use of scarce computer resources.

Keywords: modular Petri nets; alternatives aggregation Petri nets; decision support systems; performance evaluation.

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1 Introduction

The modelling and simulation methodologies, as a base for the development of decision support systems, can be applied to many different problems that range from the design to the optimal operation of a discrete event system (Narciso et al., 2010; Nicoletti et al., 2015). In some of these cases, especially in the design of a discrete event system (Jimenez et al., 2014; Jimenez and Perez 2004), it is usual that the structure of the real system is not completely defined, but it should be clarified after making the subsequent decisions (Mota and Piera, 2011; Music, 2009; Piera and Music, 2011).

The modular construction of models allows the designer to use encapsulated blocks to construct the model of the system, easing the process of modelling. Furthermore, this situation is very common in design projects, where a discrete event system can be constructed from subsystems, supplied by diverse manufacturers, which can be combined in many different ways. Moreover, it is common that the designer does not know which is the best combination of blocks for the goals of the system in process of being designed. For this reason, an automatic testing of the different possible combinations of the blocks for build up of complete models would alleviate the modelling process (Music et al., 2008; Xiao and Ming, 2011).

Furthermore, as this problem is intensive in the use of computer resources, the development of adequate methodologies for obtaining compact and efficient models is a crucial issue in the development of decision support systems based on modelling and simulation (Wainer, 2016; Zaitsev and Shmeleva, 2011).

The development of decision support systems based on modelling and simulation has been discussed by Bruzzone and Longo (2010) and Longo et al. (2013). The range of application of these decision support systems is broad, including the food industry (Latorre et al., 2013b, 2014b).

The use of the Petri nets as a versatile paradigm for modelling discrete event systems is considered in Silva et al. (1993), David and Alla (2005) and Jensen and Kristensen (2009). In particular Piera et al. (2004) and Latorre et al. (2013a) describe Petri net models for simulation. Moreover, Mújica et al. (2010) is oriented to the application of simulation for quantifying a performance evaluation in the context of an optimisation process.

A particularly difficult problem consists of designing a discrete event system whose model should be chosen among a set of alternatives. In this case, it is convenient that the model of the system includes a set of exclusive entities (Latorre et al., 2010b).

Moreover, an optimisation process may be based on the simulation of a set of selected feasible decisions, chosen from a solution pool. In this case, the choice of the most promising decisions may be performed, for instance, by means of a search methodology guided by a metaheuristic (Latorre and Jimenez, 2013a, 2013b).

In the following section, the topic of the modular Petri nets is discussed. Moreover, in Section 3 brief comments on the concept of alternatives aggregation Petri net are provided, while in Section 4, a discussion of the application of this formalism to represent a modular Petri net constructed as a sequence from combinations of four Petri subnets is given.

The next section is focused on the conclusions and the future research work, while the last one is devoted to the bibliography.

2 Modular Petri nets

The construction process of models of discrete event systems that approximate complex real systems may be considered more an art than a precise and algorithmic procedure.

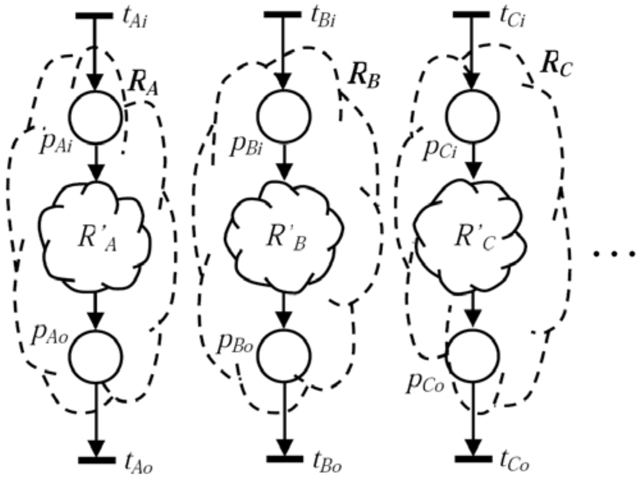
One of the most common methodologies for coping with the modelling process of a complex discrete event system is the bottom-up approach. According to this idea a model is developed for every one of the subsystems in which the complete system can be divided. The level of detail required for every model depends on its purpose and application.

This methodology derives naturally to the concept of modular construction of the Petri net model of a discrete event system. This idea implies the definition of a set of Petri net modules, ready to be assembled for the construction of complex models of real systems.

As an example, let us consider a set of four Petri subnets called R_A, R_B, R_C, and R_D. These Petri nets should present a compatible interface to be connected to other subnets. In the subnets considered in this paper, a single input link transition and a single output transition appear. Moreover, the input link transition presents a single output place, belonging to the considered subnet. Similarly, the only output link transition presents a single input place, also belonging to the considered subnet.

See Figure 1 for a simplified representation of three of the mentioned Petri subnets, as well as some of their constituent elements, such as the input and output transitions and their output and input places.

Figure 1 Three Petri subnets defined for the construction of a modular Petri net

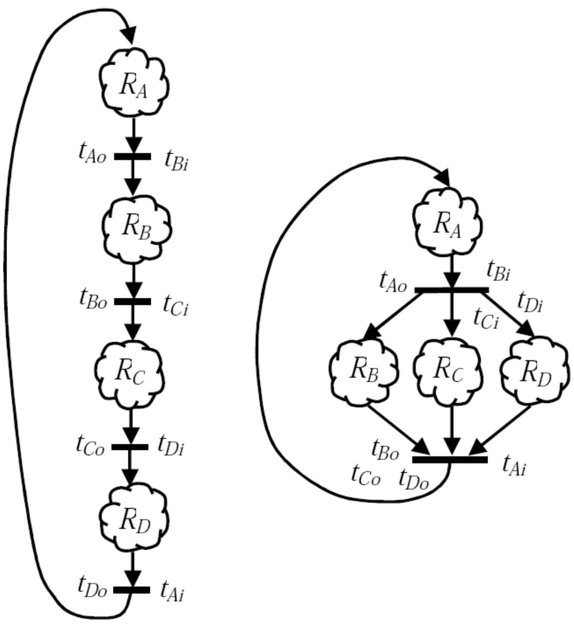


In Figure 1, the input link transitions of the Petri subnets R_A, R_B, and R_C are, respectively, t_{Ai}, t_{Bi}, and t_{Ci}. On the other hand, the output link transitions of these same Petri subnets, in the same order, are t_{Aoh}, t_{Bo}, and t_{Co}.

These subnets might be combined in different ways to build up the complete model of a real system. Prior to a detailed analysis of every resulting model, it may be difficult to foresee the performance of any of them.

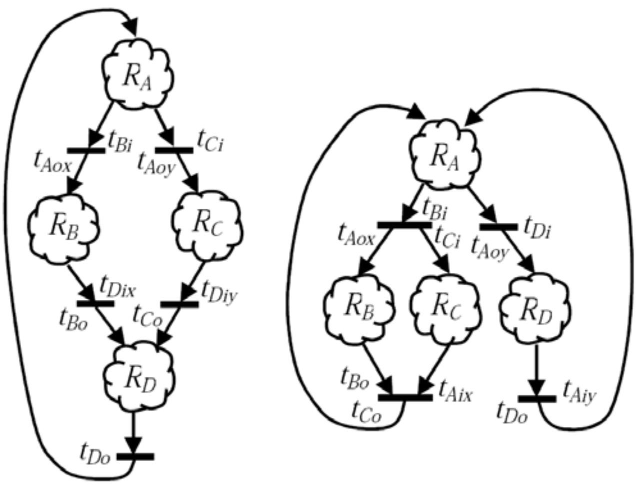
For this reason, a procedure can be defined in order to construct a set of feasible solutions for the complete model of the system by combining the subnets in appropriate ways. A second step in this procedure would be to develop a performance analysis of every complete model and, eventually, to compare the desired performance parameters calculated for every complete model in order to decide the best combination of subnets. Figures 2 and 3 show examples of combinations of the Petri subnets R_A, R_B, R_C, and R_D leading to complete models for a real system.

Figure 2 Different feasible combinations of four subnets



(a) (b)

Figure 3 More feasible combinations of four subnets



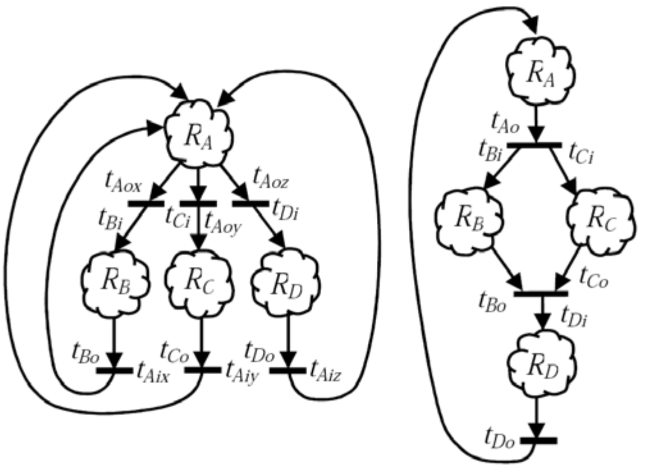
(a) (b)

A Petri net model of a discrete event system can be developed for different purposes, such as performing structural analysis, calculating a certain subset of the set of reachable states, or for performance evaluation.

Especially in this last case, it is crucial for the success of the operation to use an efficient algorithm able to cope with the, sometimes, very costly process in terms of computer resources and time. One methodology, broadly used, that can virtually cope with every model, no matter how complex it is, is simulation.

Regarding the previous considerations, an important goal in the process of modelling a Petri net for performance evaluation is to obtain a formal description of the original system, as simple and reduced as possible. Hence, the costly process of simulation might be developed in affordable time and computer resources.

Figure 4 More feasible combinations of four subnets



(a) (b)

It may be noticed the possibility of having different input or output link transitions for a given Petri subnet in a certain complete model. Also, consider that these multiple input or output link transitions present, respectively, a single output or input place. This possibility is illustrated in both Petri net models depicted in Figure 3.

One important application for performance evaluation of Petri net models using simulation consists of decision-making support with the purpose of designing a real system. The feasible models of the system in process of being designed can be compared by means of the quantitative result of a performance evaluation of every candidate model.

In the modular construction of a Petri net model, it may be interesting to test different or even all the feasible combinations of subnets that can be obtained. Every feasible solution is a candidate for being selected as the final model of the system in a design process; hence, every solution is an alternative model for the system. For this reason, a Petri net formalism able to represent alternative Petri nets, such as one containing a set of exclusive entities, should be considered (Latorre et al., 2014c).

3 Alternatives aggregation Petri nets

The existence of alternative models for the development of a given discrete event system, requires the use of specific formalisms, able to cope with the particularities of this kind of design problems.

A family of formalisms, based on the Petri net paradigm, specially developed for this purpose are the ones based on the concept of exclusive entities, leading to formalisms such the set of alternative Petri nets or the alternatives aggregation Petri nets (Latorre et al., 2014a; Latorre and Jimenez, 2012a, 2012b).

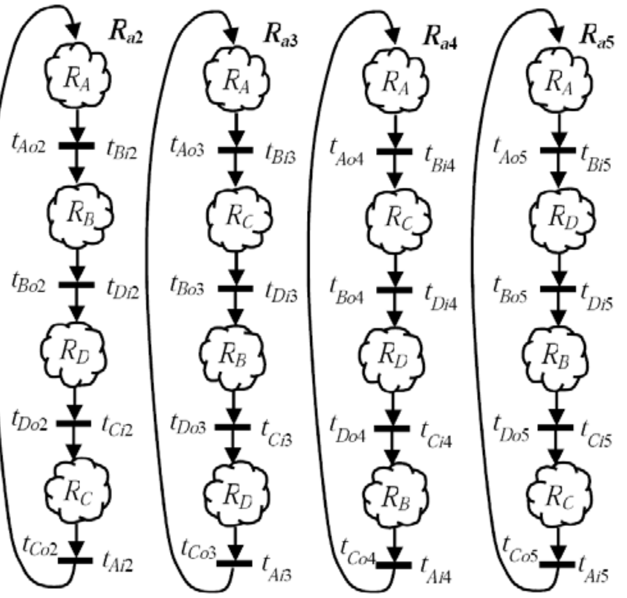
Both formalisms will be extensively used in this paper.

4 Sequences of four subnets

In this section and in the following one, diverse modular Petri net models constructed from different combinations of the same four subnets will be considered. This section in particular deals with a strict sequence of the four Petri subnets called R_A, R_B, R_C, and R_D as they were mentioned in Section 2.

All the possible complete Petri net models, built up from different combinations of the four Petri subnets in a sequence will be considered, as feasible solutions for the design process of a real system. One of the feasible solutions is presented in Figure 2(a), called R_{a1}, while most of the rest of them, R_{a2} to R_{a5} are presented in Figure 5. The only remaining modular Petri net, not represented in a figure and called R_{a6}, presents the sequence of Petri subnets R_A, R_D, R_C, and R_B and its representation is less interesting than the remaining five feasible modular Petri nets, as it will be shown when constructing the alternatives aggregation Petri net.

Figure 5 Different combinations of four Petri subnets for constructing modular Petri nets



The six modular Petri nets may be feasible models of a real system in process of being designed. In this design process, a decision should be made regarding the modular Petri net that best complies with the objectives of the real system, usually measured or quantified by means of performance parameters.

In fact, the modular Petri nets are alternative Petri nets; hence, the model of the real system in process of being designed can be represented by a formalism containing a set of exclusive entities. Furthermore, using the appropriate formalism, it is possible to reduce considerably the computational resources required to solve the associated decision-making problem.

Regarding previous results in other case-studies, the formalism of the alternatives aggregation Petri nets is chosen for modelling the real system in process of being designed, that is to say, to represent in a single model the six alternative Petri nets, removing from the model the redundant information.

One of the algorithms for obtaining an alternatives aggregation Petri nets from a set of alternative Petri nets (Latorre et al., 2010a) states that any of the alternative Petri nets may be chosen as the seed for the resulting the alternatives aggregation Petri net model. See Figure 2(a), where R_{a1} has been selected for this purpose. In this seed, every link transition should be associated to a choice variable a_1 as a guard of the transition itself.

The next steps of the algorithm for the construction of an alternatives aggregation Petri net from a set of alternative Petri nets belong to an iterative procedure, where every new alternative Petri net is added to the seed of the alternatives aggregation Petri net by including the new subnets (in this case-study there is not any of them) and all the link transitions associated to a guard function, given by the choice variable a_i that corresponds to the alternative Petri net R_{ai} containing the link transitions.

Following this algorithm, its second step consists of adding to the seed of the alternatives aggregation Petri net the alternative Petri net called R_{a2} (see Figure 5). Due to the fact that this alternative Petri net does not present any subnet that is not already included in R_{a1}, then the only modification of the seed of the alternatives aggregation Petri net introduced by the alternative Petri net R_{a2} is the addition of the link transitions. In the case of R_{a2}, the link transitions are (see Figure 5) t_{Ao2}, t_{Bo2}, t_{Do2}, and t_{Co2}, or, what is the same, the transitions called t_{Bi2}, t_{Di2}, t_{Ci2}, and t_{Ai2}. These transitions are added to the seed of the alternatives aggregation Petri net associated to the choice variable a_2.

Once the new link transitions have been included in the seed of the alternatives aggregation Petri net, it is possible to apply a reduction rule, which groups together the quasi-identical transitions, modifying the associated function of choice variables. If it is possible to apply this reduction rule to a given operation, then the resulting model will be simpler, since it contains a lower number of transitions. In fact, a couple of quasi-identical transitions verify, having the same set of input and output places, as well as the same weight in the input and output arcs. Moreover, the transitions should be associated to different functions of choice variables, otherwise the transitions would be identical instead of quasi-identical ones.

In the example of this second step of the algorithm, a link transition of R_{a2} has been merged with t_1, just by constructing an associated function of choice variables with the logic operator OR applied to both choice variables a_1 and a_2 (see Figure 6). The link transitions that have been added in this second step of the algorithm appear in Figure 6 and are named t_5, t_6, and t_7.

Figure 6 Second step in the construction of the alternatives aggregation Petri net

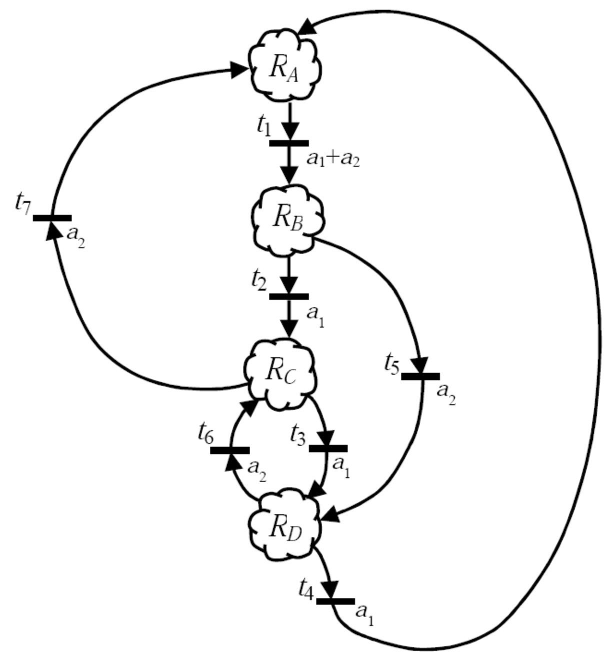
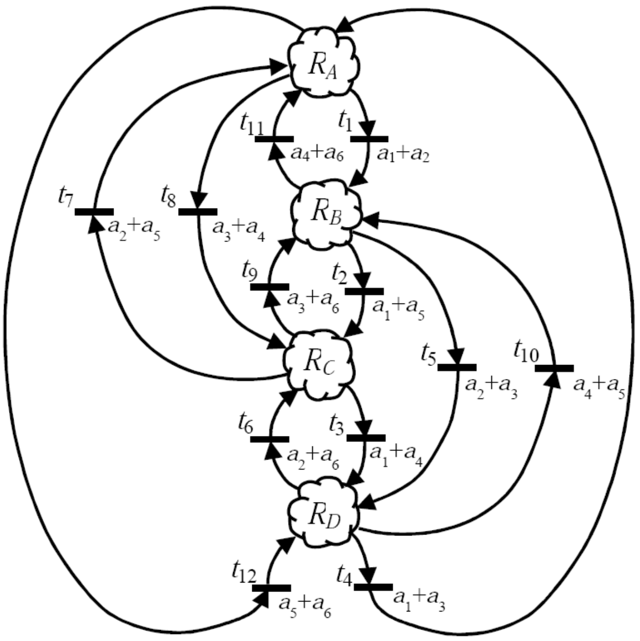


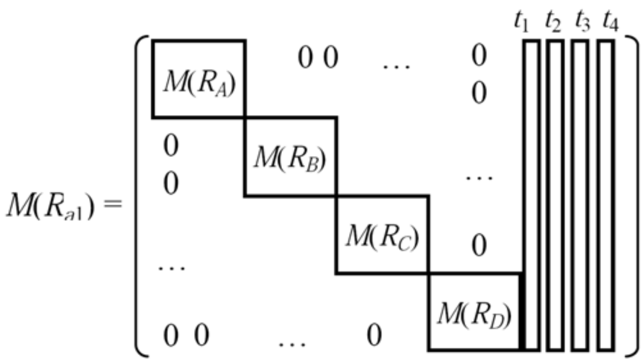
Figure 7 Complete alternatives aggregation Petri net



In Figure 6, the second step of the algorithm can be seen, while the complete alternatives aggregation Petri net is shown in Figure 7.

In order to complete the alternatives aggregation Petri net depicted in Figure 8, it has been necessary to include five more link transitions, in addition to the four ones introduced by R_{a1} and to the three new transitions delivered by R_{a2}.

Figure 8 Incidence matrix of an alternative Petri net



In the following paragraphs, the compacity in the matricial representation of both models of a system in process of being designed by means of a set of alternative Petri nets will be discussed (see Figure 5). The compacity of the alternatives aggregation Petri net is based in the fact that a large amount of redundant information, present in the set of alternative Petri nets, has been removed from the model: the Petri subnets, which appear in every alternative Petri net but the first.

The incidence matrix of any of the six alternative Petri nets has a dimension that can be calculated as follows:

Let us consider that the dimension of a Petri net is given by the multiplication of the number of rows and the number of columns of the associated incidence matrix.

According to this idea, the dimensions of the four Petri subnets of this case-study are:

$$M(R_A) \in M_{Ar'Ac},$$

where A_r is the number of rows of the incidence matrix of R_A and A_c is its number of columns. Analogously:

$$M(R_B) \in M_{Br'Bc};\ M(R_C) \in M_{Cr'Cc};$$
$$M(R_D) \in M_{Dr'Dc}$$

On the other hand, any of the alternative Petri nets R_{a1}, R_{a2}, R_{a3}, R_{a4}, R_{a5}, and R_{a6} present the same dimension, which can be calculated as it is described below.

$$M(R_{a1}), M(R_{a2}), M(R_{a3}),$$
$$M(R_{a4}), M(R_{a5}), M(R_{a6}) \in M_{r'c}$$

where

$$r = A_r + B_r + C_r + D_r \tag{1}$$

$$c = A_c + B_c + C_c + D_c + 4 \tag{2}$$

It should be considered that r is the number of rows of the incidence matrix of R_{ai}, with $i = 1,\ldots, 6$. On the other hand, c is the number of columns of the incidence matrix of R_{ai}, with $i = 1,\ldots, 6$.

The number 4 that appears in the expression (2) is originated by the four link transitions included in every R_{ai}, with $i = 1,\ldots, 6$.

Figure 8 shows a representation of the incidence matrix of any of the alternative Petri nets

The calculation of the resulting alternatives aggregation Petri net, R_{AA}, can be developed in a similar way.

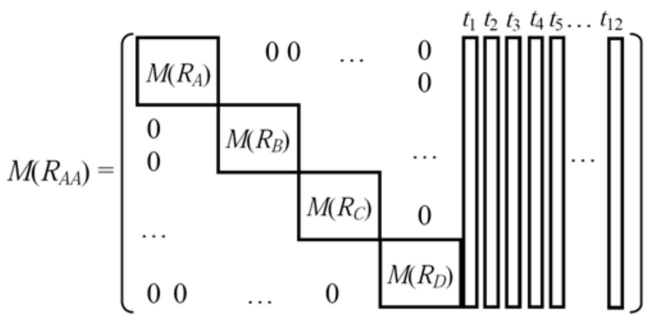
$$M(R_{AA}) \in M_{r'\times c'},$$

where

$$r' = A_r + B_r + C_r + D_r \tag{3}$$

$$c' = A_c + B_c + C_c + D_c + 4 + 8 \tag{4}$$

Figure 9 Incidence matrix of the alternatives aggregation Petri net



The comments on the previous expressions (3) and (4) are the same as done before. However, the number 8 in (4) is a consequence of the fact that the aggregation of alternative Petri nets to the seed of the alternatives aggregation Petri net introduces 8 new link transitions.

Figure 9 shows the incidence matrix of the alternatives aggregation Petri net.

In order to compare the convenience of using one of both models of a modular Petri net for developing a decision support system, the computer resources required to execute an optimisation algorithm using the simulation of one of both models can be compared.

In fact, some important computer requirements depend on the size of the model itself. Hence, the comparison between the set of alternative Petri nets and the alternatives aggregation Petri nets can be performed calculating a size ratio, defined in the following way:

$$\text{size ratio} = \frac{\text{size of the alternatives aggregation Petri net}}{\text{size of the set of six alternative Petri nets}}$$

$$\text{size ratio} = \frac{r' \times c'}{6 \times r \times c} \qquad (5)$$

$$\text{size ratio} = \frac{\left(A_r + B_r + C_r + D_r\right) \times \left(A_c + B_c + C_c + D_c + 12\right)}{6 \times \left(A_r + B_r + C_r + D_r\right) \times \left(A_c + B_c + C_c + D_c + 4\right)}$$

Owing to the fact that the number of rows is the same in the alternatives aggregation Petri net and in the alternatives aggregation Petri net, it is possible to cancel this number in the numerator and the denominator of the expression. As a result, it is possible to see that the size ratio does not depend on the number of places of the Petri nets.

$$\text{size ratio} = \frac{\left(A_c + B_c + C_c + D_c + 12\right)}{6 \times \left(A_c + B_c + C_c + D_c + 4\right)}$$

As an example, if every subnet presents 5 internal transitions, the size ratio has the value:

$$\text{size ratio} = \frac{(5+5+5+5+12)}{6 \times (5+5+5+5+4)} = \frac{1}{4.5}$$

In other words, for a small size of the Petri subnets, 5 internal transitions, the alternatives aggregation Petri net is 4.5 times smaller than the equivalent set of alternative Petri nets.

The calculation of the amount of redundant information removed from the set of alternative Petri nets is:

$$100 \times (1 - \text{size ratio}) = 100 \times \left(1 - \frac{1}{4.5}\right) \cong 77.8\%$$

Finally, it is possible to calculate the upper bound in both parameters: the size ratio and the amount of removed redundant information in the model of the system:

Let us call x_A, x_B, x_C, and x_D the number of columns of the incidence matrices of the Petri subnets R_A, R_B, R_C, and R_D respectively. Let us call $x = x_A + x_B + x_C + x_D$.

$$\lim_{x \to \infty} (\text{size ratio}) = \lim_{x \to \infty} \left(\frac{(x+12)}{6 \times (x+4)} \right) = \frac{1}{6}$$

On the other hand, the upper bound of the percentage of redundant information removed from the set of alternative Petri nets is:

$$100 \times (1 - \text{size ratio}) = 100 \times \left(1 - \frac{1}{6}\right) \cong 83.3\%$$

As it can be seen, the alternatives aggregation Petri net outperforms the complete set of alternative Petri nets, while this last formalism is more intuitive and easy to apply for the modelling of discrete event systems.

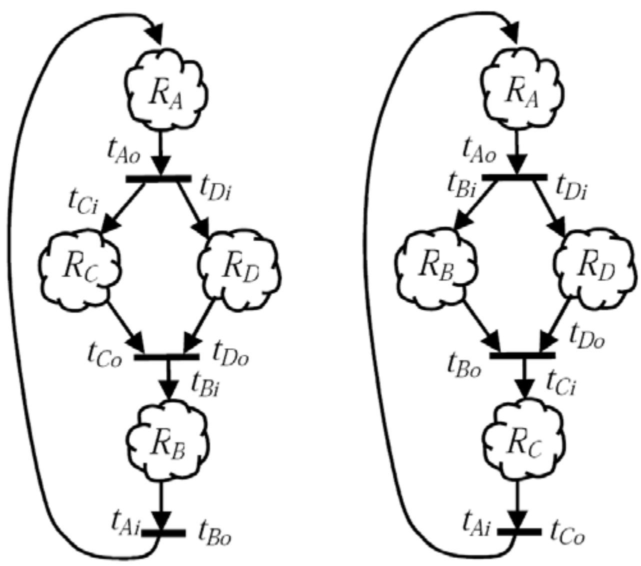
It is also interesting to point out that the model based on the alternatives aggregation Petri net should add some additional information to the model itself: the functions of choice variables associated to every link transition.

5 Sequential and parallel combination of subnets

This section will detail the analysis of the modular construction of a Petri net, from a set of four subnets, in the configuration represented in Figure 4(b). The feasible combinations of the four different subnets in the layout that corresponds to this structure reaches a number of 12, since the order in the sequential arrangement is important, while it is not the order of the subnets in the parallel configuration. This fact is a consequence of the symmetry of the structure. Hence, there is as a result a set of 12 alternative Petri nets: $\{R_{b1}, R_{b2}, \ldots, R_{b12}\}$, and therefore, 12 representations of feasible alternative structural configurations for a discrete event systems.

Figure 10 represents, in addition to Figure 4(b), some of these combinations. In Table 1, it is possible to find all the feasible combinations of the subnets $\{R_A, R_B, R_C, R_D\}$, which, for improving the clarity in the exposition, have been shown as $\{A, B, C, D\}$.

Figure 10 Representation of some of the feasible combinations



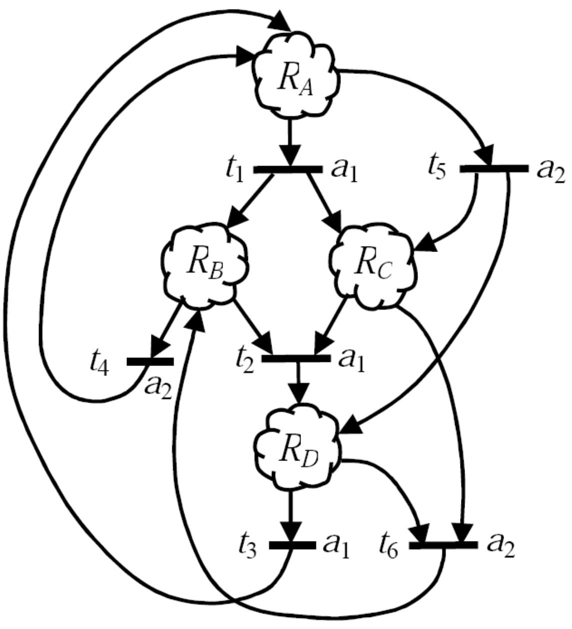
Among these Petri net models the optimal configuration for the discrete event system might be found. In order to alleviate the computational requirements for developing a performance evaluation of the system under every one of the structural configurations, a compact Petri net representation of the model may be constructed. This compact model can be obtained by the use of the alternatives aggregation Petri nets, able to remove redundant information, present in the different alternative Petri nets, describing each particular configuration.

Table 1 Feasible combinations of subnets

R_{b1}	R_{b2}	R_{b3}	R_{b4}	R_{b5}	R_{b6}
A	A	A	B	B	B
B C	C D	B D	A C	A D	C D
D	B	C	D	C	A
R_{b7}	R_{b8}	R_{b9}	R_{b10}	R_{b11}	R_{b12}
C	C	C	D	D	D
A B	A D	B D	A B	A C	B C
D	B	A	C	B	A

One algorithm, appropriate for constructing an alternatives aggregation Petri net from a set of alternative Petri nets, can be applied as explained in the following [Latorre and Jimenez (2013) and Latorre et al. (2010a)]. This algorithm is the same as the one put into practice in the previous section for a sequence of four subnets. In this application, the seed of the alternatives aggregation Petri net is *Rb*1, which can be found in Figure 4(b). The second step in the application of the algorithm consists of adding to the seed the second alternative Petri net, R_{b2}. This addition implies to include in the alternatives aggregation Petri net the new subnets that are present in the added Petri net (R_{b2}), as well as all the link transitions between the subnets of the newly added Petri net (R_{b2}). These link transitions should be associated to the choice variable a_i corresponding to the alternative Petri net R_{bi}, in this case a_2. The result of this step can be seen in Figure 11.

Figure 11 First step in the application of the algorithm for constructing an alternatives aggregation Petri net



In this first step it is not possible to apply any reduction rule for merging several quasi-identical transitions and simplifying the resulting alternatives aggregation Petri net.

The following steps in the application of the construction algorithm to the creation of *al* alternatives aggregation Petri net from the 12 alternative Petri nets lead to a net with the same subnets than any of the alternatives and with three additional link transitions originated in each alternative Petri net. The characteristics of the link transitions do not allow to apply any reduction rule to decrease its number.

As a consequence, the dimensions of the incidence matrix can be calculated as indicated in the following:

$$M\left(R_{AA2}\right) \in M_{r2^{\cdot}c2},$$

where

$$r_2 = A_r + B_r + C_r + D_r \quad (6)$$

$$c_2 = A_c + B_c + C_c + D_c + 36 \quad (7)$$

Analogously to the previous section, it is possible to calculate a size ratio, which quantifies the reduction in the size of the model of a discrete event system when a set of alternative Petri nets is substituted by an equivalent alternatives aggregation Petri net.

In this arrangement, the number of alternative Petri nets is 12

$$\text{size ratio} = \frac{r_2 \times c_2}{12 \times r \times c} \quad (8)$$

$$\text{size ratio} = \frac{\left(A_r + B_r + C_r + D_r\right) \times \left(A_c + B_c + C_c + D_c + 36\right)}{12 \times \left(A_r + B_r + C_r + D_r\right) \times \left(A_c + B_c + C_c + D_c + 4\right)}$$

Simplifying the previous expression, it is possible to obtain

$$\text{size ratio} = \frac{\left(A_c + B_c + C_c + D_c + 36\right)}{12 \times \left(A_c + B_c + C_c + D_c + 4\right)}$$

As an example, let us consider that every subnet presents five internal transitions; hence, the size ratio has the value:

$$\text{size ratio} = \frac{(5+5+5+5+36)}{12 \times (5+5+5+5+4)} = \frac{7}{36} = \frac{1}{5.143} = 0.19\hat{4}$$

In other words, for a small size of the Petri subnets, five internal transitions, the alternatives aggregation Petri net is 5.143 times smaller than the equivalent set of alternative Petri nets.

It is also possible to calculate the amount of redundant information removed from the set of alternative Petri nets in the alternatives aggregation Petri nets:

$$100 \times (1 - \text{size ratio}) = 100 \times \left(1 - \frac{7}{36}\right) \cong 80.\hat{5}\%$$

Finally, it is possible to calculate the upper bound in both parameters: the size ratio and the amount of removed redundant information in the model of the system.

Let us call x_A, x_B, x_C, and x_D the number of columns of the incidence matrices of the Petri subnets R_A, R_B, R_C, and R_D respectively. Let us call $x = x_A + x_B + x_C + x_D$.

$$\lim_{x\to\infty} (\text{size ratio}) = \lim_{x\to\infty}\left(\frac{(x+36)}{12\times(x+4)}\right) = \frac{1}{12}$$

On the other hand, the upper bound of the percentage of redundant information removed from the set of alternative Petri nets is:

$$100\times(1-\text{size ratio}) = 100\times\left(1-\frac{1}{12}\right) \cong 91.\hat{6}\%$$

It is possible to see that the upper bounds in the size ratio and in the amount of removed data in the model are more favourable for this arrangement of the subnets than the strict sequence of subnets. It can be seen that the size rate is in inverse proportion to the number of alternative Petri nets, substituted by a single alternatives aggregation Petri net.

6 Petri net equivalent to two subnets arrangements

It is also possible to calculate the size ratio of an alternatives aggregation Petri net equivalent to both arrangements presented in the present and the previous sections.

The equivalent alternatives aggregation Petri net presents the same number of rows and number of columns, the amount corresponding to the subnets and the addition of the number of link transitions coming from both arrangements: 12 and 36. Moreover, in this case the number of alternative Petri nets is 18, where six of these nets arise from the strict sequence of subnets and the other 12 nets come from the sequential and parallel arrangement, described in the previous section. In effect:

$$\text{size ratio} = \frac{(A_r+B_r+C_r+D_r)\times(A_c+B_c+C_c+D_c+48)}{18\times(A_r+B_r+C_r+D_r)\times(A_c+B_c+C_c+D_c+4)}$$

Simplifying the previous expression, it is possible to obtain

$$\text{size ratio} = \frac{(A_c+B_c+C_c+D_c+48)}{18\times(A_c+B_c+C_c+D_c+4)}$$

As an example, let us consider that every subnet presents five internal transitions; hence, the size ratio has the value:

$$\text{size ratio} = \frac{(5+5+5+5+48)}{18\times(5+5+5+5+4)} = \frac{17}{108} = \frac{1}{6.35} = 0.1574$$

In other words, for a small size of the Petri subnets, five internal transitions, the alternatives aggregation Petri net is 6.35 times smaller than the equivalent set of alternative Petri nets.

The amount of redundant information removed from the set of alternative Petri nets in the alternatives aggregation Petri nets is:

$$100\times(1-\text{size ratio}) = 100\times\left(1-\frac{17}{108}\right) \cong 84.26\%$$

Finally, it is possible to calculate the upper bound in both parameters: the size ratio and the amount of removed redundant information in the model of the system.

Let us call x_A, x_B, x_C, and x_D the number of columns of the incidence matrices of the Petri subnets R_A, R_B, R_C, and R_D respectively. Let us call $x = x_A + x_B + x_C + x_D$.

$$\lim_{x\to\infty} (\text{size ratio}) = \lim_{x\to\infty}\left(\frac{(x+36)}{18\times(x+4)}\right) = \frac{1}{18}$$

On the other hand, the upper bound of the percentage of redundant information removed from the set of alternative Petri nets is:

$$100\times(1-\text{size ratio}) = 100\times\left(1-\frac{1}{18}\right) \cong 94.\hat{4}\%$$

As it can be seen, all the parameters and upper bounds are more favourable as the number of alternative Petri nets that are substituted by a single alternatives aggregation Petri net grows.

7 Conclusions and future research lines

In this paper, some considerations on the modular construction of Petri net models have been introduced. Furthermore, the application of these models to decision-support systems based on simulation requires the development of exigent algorithms in terms of computer resources.

In order to overcome or at list palliate this problem, a transformation of a non-efficient model based on a set of alternative Petri nets into an alternatives aggregation Petri net is discussed. Two parameters have been defined and calculated: the size ratio, to quantify the relative size between both models, and the percentage of redundant information that has been removed in the alternatives aggregation Petri net but not in the original set of alternative Petri nets.

As conclusions, it can be stated that the modular construction of Petri net models is a promising research line to develop decision support systems to construct models of discrete event systems. On the other hand, there are formalisms, such as the alternatives aggregation Petri nets, able to reduce significantly the size of a model in the case of models composed of sequences of four Petri subnets.

As future research actions, it can be considered to extend the discussion of these methodologies and results to other layouts in the modular Petri nets, as well as considering a larger number of subnets and different constitutions of the subnets.

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