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A MULTI-OBJECTIVE PERSPECTIVE ON ROBUST RANKING AND SELECTION

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ABSTRACT

In this study, we consider the robust Ranking and Selection problems with input uncertainty. Instead of adopting the minimax analysis in the classical robust optimization, we propose a novel method to approach this problem from the perspective of multi-objective optimization and Pareto optimality. More specifically, the performances of each design under various scenarios are reformulated as multiple objectives, and in this case, robust Ranking and Selection becomes a multi-objective Ranking and Selection. In order to determine the number of simulation replications to various scenarios of each design, a bi-level convex optimization is formulated by maximizing the surrogate of the large deviation rate function of the probability of false selection ($P(FS)$). Numerical results show the efficiency of the proposed procedure (PR-OCBA) compared with other methods.

1 INTRODUCTION

One successful and widely-used approach to dealing with the intractable complex optimization with randomness given finite candidate designs for decision-makers is the Ranking and Selection (R&S) which determines how to allocate the simulation budget efficiently to each design such that the "best" can be identified with high probability. A general formulation of R&S can be illustrated by $\arg\min_{i\in S} E[\boldsymbol{H}(i,\omega|\theta)]$, where the decision-maker wants to find the optimal design i from a finite set S with the smallest mean of the random variable $\boldsymbol{H}(i,\omega|\theta)$. Here, ω represents the intrinsic uncertainty within the simulation model, while θ and $\boldsymbol{H}(\cdot)$ are the input parameters and input models which are assumed to be given for most classical R&S work. However, it can be quite difficult to select the correct input model and the associated parameters in practice, which leads to the problem of R&S with input uncertainty or robust R&S.

To tackle the R&S with input uncertainty, one typical way borrowed from the classical robust optimization is the Wald's maximin or minimax models. This approach intends to find designs with the best worst-case performance, which can be mathematically formulated as $\arg\min_{i\in S}\max_{(\theta,\boldsymbol{H})\in\mathscr{U}} E[\boldsymbol{H}(i,\omega|\theta)]$ where Θ is the input parameter set, $\mathscr{H}$ is the input model set, and $\mathscr{U}=\Theta\times\mathscr{H}$ is the input uncertainty set. For example, Gao, Xiao, Zhou, and Chen (2016) developed an asymptotically optimal budget allocation strategy under Gaussian assumption to select the best designs on the worst-case performance.

Despite the fact that for a realized scenario $(\theta,\boldsymbol{H})$, the performance $E[\boldsymbol{H}(i^*,\omega|\theta)]$ of the optimal designs i^* given by the minimax model cannot be worse than their worst-case performance, such minimax models are criticized for their conservative decisions especially when the worst case is unlikely to happen in practice. Furthermore, this approach is from the perspective of the risk-averse decision-makers, while the risk-neutral and risk-seeking decision-makers might want to find the designs with optimal average-case performance and best-case performance, respectively. Finally, as pointed by Iancu and Trichakis (2013), such paradigm can produce suboptimal designs without the property of Pareto optimality in practice, and thus leads to inefficiency.

To address these issues and provide more flexibility, we propose a novel approach from the perspective of multi-objective optimization and Pareto optimality to handle the robust R&S. More specifically, as presented in (1), the performances of each design under various scenarios are reformulated as multi-objectives where each scenario is treated as an objective measure, and we want to identify all the Pareto robust designs which are non-dominated by the others. It is noteworthy that the number of input uncertain scenarios can be positive infinite, but we can construct a finite uncertainty set by selecting some representative scenarios. Such scenarios might be selected as the quantiles of the performance of each design, or the status of the decision environment and the decisions by the adversarial player (e.g. sunny or rainy, stock prices go up or down, silent or betray).

$$\arg\min_{i\in S} \quad \{E[\boldsymbol{H}(i,\omega|\theta)], \forall(\theta,\boldsymbol{H}) \in \mathscr{U}\}. \tag{1}$$

2 ASYMPTOTIC OPTIMAL ALLOCATION STRATEGY

In this study, we consider the robust R&S problem of identifying the best designs from a finite set S of r designs. For all the r designs, their input uncertainty set $\mathscr{U}$ is assumed to be identical and contains s scenarios. The robust R&S problem is reformulated as a multi-objective R&S problem as discussed earlier. Let S_p and S_p^c denote the Pareto set and its complementary set. Then $P(FS) = P(\bigcup_{i\in S_p} \bigcup_{l\in S, l\neq i} l \hat{\prec} i \cup \bigcup_{j\in S_p^c} \bigcap_{l\in S, l\neq j} l \hat{\not\prec} j)$ where $l \hat{\prec} i$ represents that design i is dominated by l empirically, and $l \hat{\not\prec} j$ represents that design j is not dominated by l empirically. Relying on the Bonferroni inequality, the bounds of $P(FS)$ are derived, and the surrogate of the large deviation rate function of $P(FS)$ as a concave function for the optimal allocation can be further proposed by the Gärtner-Ellis theorem. Finally, the optimal allocation problem is defined by maximizing the surrogate of the rate function which turns into a bi-level convex optimization problem.

Please refer to the contributed paper by Liu, Gao, and Lee (2017) for more detailed illustration of the methodology and numerical results in this study.

3 CONCLUSIONS AND FUTURE RESEARCH

This study proposes a novel procedure which tackles the robust R&S from the perspective of multi-objective optimization and Pareto optimality. In addition, it is observed that the optimal allocation problem is a bi-level convex optimization which can be solved easily even for large scale problems. The numerical results show the promising performance of the proposed approach (PR-OCBA) than the compared methods.

Future research work can consider how to utilize the correlation information between performances of each design under different scenarios to propose more efficient budget allocation strategies. In addition, a closed-form budget allocation strategy should be developed instead of solving the optimal allocation problem based on convex optimization solver. Furthermore, stochastic dominance and risk measure can be considered to compare the performance of each design when the probability distribution of scenarios in the input uncertainty set is known.

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