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The evaluation of pedestrians' behavior using *M/G/C/C* analytical, weighted distance and real distance simulation models

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Abstract *M/G/C/C* analytical and simulation models have long been used to evaluate the performance of a large and complex topological network. However, such evaluation is only founded on a network's total arrival rate and its weighted distance. Thus, this paper discusses some concepts and issues in building an *M/G/C/C* simulation model of a complex geometric system where all its arrival sources and their exact distances to the end of their networks (i.e., corridors) have been taken into consideration in measuring the impacts of various evacuation rates to its throughput, blocking probability, expected service time and expected number of pedestrians. For this purpose, the Dewan Tuanku Syed Putra hall, Universiti Sains Malaysia, Malaysia has been selected as a case study for various evaluations of complex pedestrian flows. These results were analyzed and compared with the results of our analytical and weighted distance simulation models. We then documented the ranges of arrival rates for each of the model where their results exhibited significant discrepancies and suggest ideal rates to best evacuate occupants from the hall. Our model can be utilized by policy makers to recommend suitable actions especially in emergency cases and be modified to build and measure the performance of other real-life complex systems.

Keywords *M/G/C/C* state dependent · Discrete-event simulation · Queuing system · Finite capacity · Topological network

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1 Introduction

A Discrete Event Dynamic System (DEDS) is a system whose behavior changes only at specified discrete time points. Typical examples of DEDS include service (e.g., bank and healthcare), telecommunication (e.g., call center and computer network), transportation (e.g., airport and seaport) and manufacturing (e.g., production line) systems. To analyze their behavior, various modeling and analysis techniques have been proposed and used; e.g., Petri nets (Holloway et al. 1997; Piedrafita and Villarroel 2011), perturbation analysis (Yao and Cassandras 2011), queuing theory (Cassandras and Lafortune 1999; Xia 2014) and computer simulation (Banks et al. 2010).

Queuing theory explores the effects of a capacity constrained resources on common performance measures of a system; such as its queue length and throughput. In most queuing systems, their resources' service times are assumed to have exponentially distributed regardless of the number of competing entities. However, in some systems, the current number of competing entities dynamically adjusts their resources' service times. These scenarios are typical for entities flowing through a limited space layout; e.g., pedestrians walking through a corridor or vehicles travelling on a road. The entities' behavior can be modeled using an *M/G/C/C* state dependent queuing network (Cheah and Smith 1994; Cruz and Smith 2007; Cruz et al. 2005; Smith and Cruz 2014).

M/G/C/C state dependent queuing networks have long been used to model and evaluate real-life systems where their service rates are decayed with the number of residing entities. Jain and Smith (1997) used *M/G/C/C* approaches to analyze traffic flow on roadways. The analysis of results showed that in almost all the cases, an analytical model slightly underestimated the performance of the flow compared to its counterpart Discrete Event Simulation (DES) model. Cruz et al. (2005) and Cruz and Smith (2007) implemented *M/G/C/C* approaches to analyze the performance of a ten-story high-rise building which has the same length and width of staircases. By assuming arrival rates were equally assigned to each floor, they found that light traffic increased the blocking probabilities in the lower levels while heavy traffic increased the blocking probabilities in the upper levels. Bedell and Smith (2012) utilized *M/G/C/C* approaches to study the flow of multi-products in a complex manufacturing system. They discovered that Poisson inter-arrivals would create Poisson inter-departures and the same results were also applied for other types of distributions, even for heavy blocking probabilities. *M/G/C/C* approaches have also been utilized to design the optimal evacuation routes in transportation networks (Stepanov and Smith 2009), find optimal routings in a closed queuing network (Smith 2011) and optimize building evacuation networks (Weiss et al. 2012).

1.1 Background of the study

Kawsar et al. (2012) used the *M/G/C/C* analytical approach to analyze the impacts of various evacuation (arrival or entrance) rates to source corridors and their subsequent (i.e., intermediate and exit) corridors in the DTSP (Dewan Tuanku Syed Putra, Universiti Sains Malaysia, Malaysia) hall. The performance of each corridor in terms of its throughput, blocking probability, expected service time and expected number of pedestrians was measured based on two premises. First, all arrival sources along a source corridor were assumed to be located at the beginning of the corridor. For this, its performance was approximated based on the total arrival rate of its arrival sources and the weighted travelling distance of the arrival sources to the end of the corridor. The weighted travelling distance was calculated using the formula developed by Yuhaski and Smith (1989). Second, the arrival rate to an intermediate or exit corridor was based on the throughputs of its previous corridors. For example, if the corridor was previously connected by two corridors then its arrival rate was the total throughput of these two corridors. Its throughput then became part of the arrival rate for its next corridor, and this process continued until the exit corridors. Based on these premises, the result analysis showed that it was crucial to control the entrance rate to each source corridor to increase the DTSP's overall throughput, and the best entrance rates which only maximize the throughput of source corridors did not guarantee the best overall throughput of the hall since these arrival rates might create congestions along their subsequent corridor links.

Khalid et al. (2013) compared these analytical results with the mean performance measures generated by their Arena Discrete Event Simulation (Altiok and Melamed 2007; Kelton 2009) model. How Arena could be programmed to handle the instantaneous service rates in *M/G/C/C* networks using its available modules has been discussed in detail. As considered by the analytical model, the simulation model was used to iteratively measure the performance of each available corridor based on its arrival rate which was the total arrival rate of its arrival sources or the total throughput of its previous corridors. In brief, they found that for each corridor the best arrival rate for flowing pedestrians generated by their simulation model was a slightly smaller compared to the best arrival rate reported by the analytical model, and this eventually caused the analytical and simulation results exhibited a little discrepancy. Both models however generated almost the same performance measures for other smaller or higher arrival rates. The simulation model also showed that there was a tiny range of arrival rates for each corridor where its blocking probabilities fluctuated significantly across replications and these values occurred right before the blocking started.

1.2 Motivation

Previous works and methodologies related to pedestrian traffic flows (e.g., Cheah and Smith 1994; Cruz and Smith 2007; Cruz et al. 2005; Fruin 1971; Mitchell and Smith 2001; Smith 2001; Smith and Cruz 2014; Weiss et al. 2012) measures the performance of a corridor based on three input parameters; i.e., pedestrian arrival rates streamed from a single arrival source located at the beginning of the corridor, its length through which pedestrians have to travel and its capacity. However, multiple arrival sources scattered along a corridor are typical for facilities with complex corridor geometry; e.g., storey buildings and stadiums. The conversion of the corridor as another single corridor of an arrival rate λ' (i.e., the total arrival rate of its available arrival sources and/or the throughput of its previous corridors) and length L' (i.e., the distance averaged from the arrival sources to the end of the corridor which is used as pedestrian travel distance) is thus necessary for its performance approximation using an *M/G/C/C* analytical model. As mentioned earlier, Kawsar et al. (2012) and Khalid et al. (2013) used this approach to simplify their DTSP analytical and simulation models.

It is significant to see if such a premise really replicates pedestrians' behavior while travelling from various arrival sources throughout a network. As a test bed, we chose the DTSP hall for two main reasons. First, it has complex topological structures with various source (with multiple arrival sources), intermediate and exit corridors which later form series, splitting and merging network topologies, and these make our pedestrian traffic modeling quite complicated. The actual behavior of pedestrian flow throughout the structures can then be compared with the previous results (Kawsar et al. 2012; Khalid et al. 2013). Second, it is a place where all important events of the university are held (e.g., convocation ceremony and grand staff meeting), and thus our findings can be utilized by its policy makers to understand pedestrian movement throughout the hall and eventually optimize the evacuation process during emergency situations; e.g., fire and earthquake.

1.3 Objectives of the paper

The main objective of this paper is to design and develop simulation models for simulating pedestrian traffic flows throughout the DTSP hall. This is a challenging task since its actual topological structure formed by various source, intermediate and exit corridors has to be replicated to accurately measure its performance. Additionally, the source corridors which have multiple arrival sources from which pedestrians arrive stochastically and walk to their nearest corridors with relevant speed depending on the number of residing pedestrians throughout the network must properly be modeled to represent their arrival and walking behavior. The dynamic speed is modeled based on an *M/G/C/C* state dependant queuing network and embedded in our Arena simulation models in order to evaluate the performance along the corridor links especially their throughputs and congestions. Although the models have been developed using Arena, their development processes are however adaptable to any high-level simulation software; e.g., SIMUL8 (Concannon et al. 2007) and ExtendSim (Strickland 2011).

There are two versions of simulation models for the evaluation purposes. The weighted distance model replicates the behavior movement of pedestrians from corridor to corridor in the hall. However, their arrival rates from various arrival sources along a source corridor are summed up and their travelling distance are modeled as the weighted distance of these arrival sources to the end of the corridor. The real distance model meanwhile considers all actual locations of these arrival sources and their distance to the end of the corridor. Both models are then used to evaluate the impacts of various evacuation rates to each of the corridor links and find the ideal evacuation rates to best evacuate the occupants from the hall. The results are then analyzed and compared with the results of its analytical model.

The main purpose of the comparison is to observe any significant differences in pedestrian behavior generated by the three models. The *M/G/C/C* analytical model abstracts general pedestrian behavior based on previous empirical studies (e.g., Hankin and Wright 1958; O'Flaherty and Parkinson 1972) and relevant assumptions to mathematically express the effect of pedestrian density to the performance of a corridor. For a complex structure of corridors as in the DTSP hall, its global performance based on an analytical method is thus approximated by measuring the performance of each corridor separately (and its impacts to other corridors in the topological network) and then analyzing the performance of all available exit corridors. Simulation models are otherwise used to entirely capture and model relevant characteristics of pedestrian movement from corridor to corridor in the network as in the real situation based on the logic of the *M/G/C/C* density-flow approach and calculate its performance based on the relevant number of pedestrians and their total spending time at a certain point of time; e.g., the throughput which is defined as the number of departing pedestrians divided by simulation length, and expected service time which is the time spent by the pedestrians divided by simulation length. Since the current number and time spent are determined by pedestrian arrival rates, we further investigate and compare how modeling a source corridor with multiple arrival sources as another equivalent corridor (as in the weighted distance model) and replicating the source corridor as its real structure (as in the real distance model) affect its performance and the global performance of the hall. The simulation models are then used to validate the analytical results.

In general, the three models should also be compared with real data to justify the validity of the pedestrian flows out of the hall especially during the evacuation process. However, since this paper is to validate the analytical results performed by Kawsar et al. (2012) and propose the evacuation rates to best evacuate occupants from the hall, we ignore this aspect. In the near future, it is our hope to compare our results with the real data to see how much their difference from reality. However, collecting real behavioral data based on crowd and egress under the emergency situation is limited and difficult since this task requires large amounts of observable data based on a proper methodology especially when dealing with unpredictable human behavior . It is also not possible to predict the emergency situation on the basis of the data collected in a normal situation. Thus, at this moment we only simulate the emergency situation based on some assumptions; e.g., pedestrian may walk at their fastest speed.

1.4 Contributions of the paper

The main contribution of this paper is to present the concepts and methodologies for building a simulation model of a complex *M/G/C/C* state dependent queuing system. Considering assumptions of an analytical model, various analyses of its behavior through the computer simulation correspond to that through its analytical model. However, the ability of the simulation model to analyze and optimize pedestrian behavior as occurred in reality based on relevant evacuation rates makes it a valuable tool for managing the system and planning its emergency evacuation.

1.5 Outline of the paper

We organized this paper as follows. In Section 2, we first compare the simulation results of our basic weighted distance *M/G/C/C* simulation model with the results of Cruz, Smith and Medeiros's simulation model (Cruz et al. 2005) to ensure that this model replicated and simulated whatever logic considered by the analytical model. Some discrepancies between both models are reported and discussed. Section 3 briefly discusses the modeling issues and challenges which we faced while developing and building the real distance simulation model using the Arena software. Approaches on how such the model can be built by extending our previous weighted simulation model are also discussed. Section 4 validates the model to ensure that it really replicates pedestrian behavior and then compares its results with the analytical and weighted distance simulation models. Reports on how both simulation results correspond to the analytical results in abstract and graphical forms and some discussions on these are documented and discussed. Based on various analyses, we then propose some recommendations on the best practical approach to evacuate occupants from the hall. Finally, Section 5 summarizes the findings and presents some conclusions and future work.

2 Comparing our basic weighted distance model with the available *M/G/C/C* simulation model

The *M/G/C/C* analytical model considers the capacity of a circulation space (e.g., corridor) as its number of servers whose service times in terms of the average walking speed decrease with the increase of traffic density. Starting with the initial speed of V_1=1.5 m/s, the speed is then dynamically adjusted accordingly based on the current number of residing pedestrians and becomes 0 when the number is equal to the capacity of the corridor; i.e., 5 × length × width. At this state, the forward movement is halted and arriving pedestrians are blocked from entering the space. How frequent the pedestrians can enter the space at a particular time can thus be controlled by setting their arrival rates.

The current arrival rates and walking speed are used to calculate the probability of the number of pedestrians in the space which is then expended to measure its performance such as the throughput, blocking probability, expected number of pedestrians and expected service time. The effect of these parameters to the performance of the space has mathematically been represented in the analytical *M/G/C/C* model and is discussed in detail in previous studies (e.g., Cheah and Smith 1994; Cruz and Smith 2007; Cruz et al. 2005). Based on this mechanism, two *M/G/C/C* simulation models have been developed. The first model was by Cruz, Smith and Medeiros (Cruz et al. 2005) which used the C++ language while the other model was by Khalid et al. (Khalid et al. 2013) which used Arena as an implementation tool and serves as our basic weighted distance simulation model. Thus, the purpose of this section is to show that our basic weighted distance simulation model exhibits the same behavior as their simulation model. For this, we consider a single corridor with relevant length and width, feed it with various arrival rates and then compare its performance measures with the results generated by their model.

As theirs, all results of our simulation models were run for 20,000 s and 30 replications. The replication length of 20,000 s is sufficiently enough since through our visual inspection of the output data, its steady state reported by our simulation models has long been reached. In this stationary phase, the random behavior of all performance measures does not depend on the value of simulation time anymore. The results for a single node's performance measures are presented in Tables 1 and 2. Table 1 displays the results for a corridor of 8 m long and 2.5 m wide under various arrival rates. Table 2 displays the results for a corridor of 8 m long with various width and arrival rates. The width and length are set as the properties of the corridor (see Khalid et al. 2013 for more explanation) to primarily calculate its capacity; i.e., the maximum number of pedestrians allowed to reside at a time. Additionally, the length determines the time spent by a pedestrian to cross the corridor (based on the current speed) which is then used to calculate its expected service time and expected number of entities. Note that Simulation[a] denotes Cruz, Smith and Medeiros's simulation model while Simulation[b] denotes our simulation model. λ, θ, *p(c)*, *L* and *W* respectively represent the arrival rate, the throughput, the blocking probability, the expected service time and the expected number of entities of the corridor. In the tables, simulation results are shown in the format of $a[b, c]$. a is the mean for a particular performance measure based on the 30 replications. b and c are the lower and upper values of the performance measure; i.e., the 95% confidence interval on the expectations of the performance measure which is respectively given by $a - t_{n-1,1-\alpha/2}\frac{s}{\sqrt{n}}$ and $a + t_{n-1,1-\alpha/2}\frac{s}{\sqrt{n}}$ where s is the sample standard deviation, n is the number of replications (i.e., 30 replications), α=0.05 and $t_{n-1,1-\alpha/2}$ is the upper $1 - \alpha/2$ critical point from the t distribution with $n - 1°$ of freedom.

From Table 1, we can clearly observe that the increase in arrival rates will increase the blocking probabilities, and eventually decrease the throughput. Both models displayed almost the same results except for λ=2.7 pedestrians/second (ped/s) where our simulation model reported a slightly higher blocking probability compared to the analytical result and the result of Cruz, Smith and Medeiros's simulation model. Table 2 also reports almost similar simulation results except for a corridor of width 4.5 m with λ=5 ped/s. In this case, our model reported a lower blocking probability, i.e., 0.04 compared to their simulation model reporting

Table 1 Single node performance measures versus arrival rate for the 8 × 2.5 m corridor

λ	Model	$p(c)$	θ	L	W
1.0	Analytical	0.00	1.00	6.02	6.02
	Simulation[a]	0.00 [0.00, 0.00]	1.00 [1.00, 1.00]	6.02 [5.99, 6.02]	6.02 [6.02, 6.02]
	Simulation[b]	0.00 [0.00, 0.00]	1.00 [1.00, 1.00]	6.02 [6.02, 6.04]	6.02 [6.02, 6.02]
2.0	Analytical	0.00	2.00	14.49	7.24
	Simulation[a]	0.00 [0.00, 0.00]	2.00 [1.99, 2.00]	14.46 [14.42, 14.49]	7.24 [7.23, 7.25]
	Simulation[b]	0.00 [0.00, 0.00]	2.00 [1.99, 2.00]	14.46 [14.43, 14.49]	7.24 [7.23, 7.25]
2.7	Analytical	0.01	2.66	29.90	10.98
	Simulation[a]	0.01 [0.00, 0.02]	2.67 [2.65, 2.68]	27.91 [26.28, 29.55]	10.49 [9.81, 11.18]
	Simulation[b]	0.06 [0.03, 0.10]	2.53 [2.43, 2.62]	41.59 [32.29, 50.89]	17.76 [12.76, 22.75]
3.0	Analytical	0.33	2.01	96.96	48.31
	Simulation[a]	0.32 [0.32, 0.33]	2.03 [2.02, 2.04]	95.21 [94.52, 95.90]	46.95 [46.37, 47.53]
	Simulation[b]	0.34 [0.34, 0.35]	1.97 [1.96, 1.98]	97.44 [96.75, 98.14]	49.57 [48.96, 50.19]
4.0	Analytical	0.51	1.96	99.01	50.53
	Simulation[a]	0.51 [0.51, 0.51]	1.96 [1.96, 1.96]	98.77 [98.75, 98.79]	50.43 [50.41, 50.45]
	Simulation[b]	0.52 [0.52, 0.52]	1.93 [1.93, 1.93]	99.76 [99.75, 99.77]	51.66 [51.65, 51.67]

[a] Denotes Cruz, Smith and Medeiros's simulation model

[b] Denotes our simulation model

the blocking probability of 0.11. In this case, the blocking probability reported by our model was close to that generated by the analytical model which was 0.00.

Tables 3, 4 and 5 respectively report the results for series, splitting and merging network topologies. For both of the series and splitting topologies, our simulation model measured slightly higher blocking probabilities for the first node compared to analytical and Cruz, Smith and Medeiros's results. Thus, our model reported slightly lower throughputs for other consequence nodes. For the merging corridor, our model reported a completely different blocking probability for node 3, i.e., 0.01 compared to 0.51 and 0.50 for analytical and simulation results reported in Cruz et al. (2005). We believe that our model reported the correct blocking probability if we follow the mathematical equation for measuring the throughput; i.e., $\theta = \lambda(1-p(c))$. In general, we are confident that our weighted simulation model has replicated the logic of the *M/G/C/C* model.

3 Modeling issues and approach of the real distance simulation model

The DTSP hall has large and complex structures. Its real map and its simplified structures for our modeling purposes can be obtained in Kawsar et al.'s paper (Kawsar et al. 2012). However, we reattach the structures (see Fig. 1) to avoid any cross references. The numbers represent the indexes of the corridors, the alphabets *S, T, U, V, W, X, Y* and *Z* represent the different seating arrangements and *A′, B′, C′* and *D′* are the exits to other corridors. Briefly, DTSP has a total of 15 *M/G/C/C* networks (i.e., 6 source corridors, 3 intermediate corridors and 6 exit corridors) with different lengths and widths which later form a number of series, splitting and merging topologies.

There are 48 arrival sources (which symbolize entry/exit points to/from a row of chairs) located throughout the source corridors (corridor 6, corridor 7, corridor 8, corridor 9, corridor

Table 2 Single node performance measures versus arrival rate for the 8 m and various width corridor

λ	Width	Model	*p(c)*	*θ*	*L*	*W*
2.5	1.0	Analytical	0.68	0.79	39.53	50.02
		Simulation[a]	0.68 [0.68, 0.68]	0.79 [0.79, 0.79]	39.46 [39.45, 39.46]	49.98 [49.96, 49.99]
		Simulation[b]	0.69 [0.69, 0.69]	0.78 [0.78, 0.78]	39.93 [39.93, 39.93]	51.42 [51.42, 51.43]
	1.5	Analytical	0.52	1.19	59.05	49.69
		Simulation[a]	0.52 [0.52, 0.52]	1.19 [1.19, 1.19]	58.91 [58.90,58.93]	49.59 [49.57, 49.61]
		Simulation[b]	0.53 [0.53, 0.54]	1.16 [1.16, 1.16]	59.87 [59.06, 59.87]	51.55 [51.54, 51.56]
	2.0	Analytical	0.36	1.61	77.71	48.32
		Simulation[a]	0.35 [0.35, 0.35]	1.62 [1.61, 1.62]	76.87 [76.63, 77.11]	47.50 [47.23, 47.76]
		Simulation[b]	0.38 [0.37, 0.38]	1.55 [1.55, 1.56]	79.28 [79.10, 79.45]	50.99 [50.79, 51.18]
	2.5	Analytical	0.00	2.50	21.07	8.43
		Simulation[a]	0.00 [0.00, 0.00]	2.50 [2.49, 2.50]	21.00 [20.94, 21.06]	8.41 [8.40, 8.43]
		Simulation[b]	0.00 [0.00, 0.00]	2.49 [2.47, 2.51]	22.71 [19.41, 26.01]	9.23 [7.60, 10.85]
	3.0	Analytical	0.00	2.50	18.39	7.36
		Simulation[a]	0.00 [0.00, 0.00]	2.50 [2.49, 2.50]	18.35 [18.31, 18.39]	7.35 [7.34, 7.36]
		Simulation[b]	0.00 [0.00, 0.00]	2.50 [2.49, 2.50]	18.38 []18.33, 18.44]	7.36 [7.35, 7.36]
5.0	2.0	Analytical	0.69	1.56	79.54	51.00
		Simulation[a]	0.69 [0.69, 0.69]	1.56 [1.56, 1.56]	79.40 [79.39, 79.40]	50.95 [50.95, 50.96]
		Simulation[b]	0.69 [0.69, 0.69]	1.55 [1.55, 1.55]	79.86 [79.85, 79.86]	51.67 [51.67, 51.67]
	3.0	Analytical	0.53	2.34	119.10	50.86
		Simulation[a]	0.53 [0.53, 0.53]	2.34 [2.34, 2.34]	118.83 [118.81, 118.84]	50.76 [50.75, 50.77]
		Simulation[b]	0.54 [0.54, 0.54]	2.32 [2.32, 2.32]	119.72 [119.72, 119.73]	51.70 [51.69, 51.71]
	4.0	Analytical	0.37	3.14	158.24	50.46
		Simulation[a]	0.37 [0.37, 0.37]	3.17 [3.16, 3.17]	156.04 [155.55, 156.53]	49.29 [49.01, 49.57]
		Simulation[b]	0.38 [0.37, 0.38]	3.12 [3.11, 3.13]	157.91 [157.35, 158.47]	50.69 [50.36, 51.03]
	4.5	Analytical	0.11	4.45	95.66	21.49
		Simulation[a]	0.00 [0.00, 0.00]	4.99 [4.97, 5.00]	46.80 [45.54, 48.05]	9.39 [9.10, 9.68]
		Simulation[b]	0.04 [0.00, 0.07]	4.99 [4.99, 5.00]	61.67 [48.36, 74.98]	13.66 [9.70, 17.62]
	5.0	Analytical	0.00	5.00	40.94	8.19
		Simulation[a]	0.00 [0.00, 0.00]	5.00 [4.99, 5.00]	40.88 [40.80, 40.96]	8.18 [8.18, 8.19]
		Simulation[b]	0.00 [0.00, 0.00]	5.00 [4.99, 5.00]	40.88 [40.80, 40.96]	8.18 [8.18, 8.19]

[a] Denotes Cruz, Smith and Medeiros's simulation model

[b] Denotes our simulation model

10 and corridor 11) with different distances to the end of the corridors. Table 6 shows various arrival sources for each source corridor and their distances to the end of the corridor; i.e., before occupants start travelling to their next relevant corridors (intermediate and/or exit corridors).

The flow of occupants throughout the hall can be summarized as follow. Occupants from seating arrangement *S* and *T* will first come to source corridor 11, and they will then choose their nearest corridor to exit. Similarly for seating arrangement *U* and *V*, occupants will first come to source corridor 10, and they will then choose their nearest corridor to exit. For seating arrangements *W, X, Y* and *Z*, occupants will first come to source corridors 6, 7, 8 and 9 respectively. Every seating arrangement has a row of chairs with its own distance to the end of its corridor.

Table 3 Results for 3-node series topology (λ=3.0)

Measure	Node 1		Node 2		Node 3	
	Analytical	Simulation[a], Simulation[b]	Analytical	Simulation[a], Simulation[b]	Analytical	Simulation[a], Simulation[b]
p(c)	0.33	0.33 [0.32, 0.33]	0.00	0.01 [0.00, 0.01]	0.00	0.01 [0.00, 0.03]
		0.35 [0.34, 0.35]		0.00 [0.00, 0.00]		0.00 [0.00, 0.00]
θ	2.01	2.02 [2.01, 2.03]	2.01	2.02 [2.01, 2.02]	2.01	2.02 [2.01, 2.02]
		1.96 [1.95, 1.96]		1.96 [1.95, 1.96]		1.96 [1.95, 1.96]
L	96.96	95.87 [95.35, 96.40]	14.56	16.44 [15.00, 17.88]	16.51	16.51 [15.23, 17.79]
		98.10 [97.61, 98.59]		[14.16, 14.52]		14.33 [14.15, 14.52]
W	48.31	47.53 [47.10, 47.96]	7.26	8.15 [7.44, 8.86]	8.19	8.19 [7.56, 8.81]
		50.16 [49.73, 50.59]		7.33 [7.26, 7.41]		7.33 [7.26, 7.11]

[a] Denotes Cruz, Smith and Medeiros's simulation model

[b] Denotes our simulation model

From these various distances of a set of arrival sources, we calculated its average travelling distance to the end of its corridor. The average distance, capacity (i.e., 5 × corridor length × corridor width) and the summation of arrival rates for the arrival sources were then used as input parameters for our previous analytical (Kawsar et al. 2012) and basic weighted distance simulation models (Khalid et al. 2013) to evaluate its performances measures. Thus, in the basic weighted distance simulation model, we only needed one *Create* module to generate pedestrians' arrivals and one *Assign* module to store and assign their average travelling distance for every source corridor. The *Create* module is the birth node for arrival of pedestrians to our model's boundary. It creates a sample of pedestrians arriving to the corridor according to the exponential distribution of an arrival rate (λ) as considered in the *M/G/C/C* mathematical model. However, the λ has to be converted to $1/\lambda$ since the creation of entities in Arena is based on time between arrivals; i.e., the time separating consecutive arrivals of pedestrians which originate in this module.

Table 4 Results for 3-node split topology (λ=3.0)

Measure	Node 1		Node 2		Node 3	
	Analytical	Simulation[a], Simulation[b]	Analytical	Simulation[a], Simulation[b]	Analytical	Simulation[a], Simulation[b]
p(c)	0.33	0.32 [0.32, 0.33]	0.00	0.00 [0.00, 0.00]	0.00	0.00 [0.00, 0.00]
		0.35 [0.34, 0.35]		0.00 [0.00, 0.00]		0.00 [0.00, 0.00]
θ	2.01	2.03 [2.02, 2.04]	1.20	1.22 [1.21, 1.23]	0.80	0.81 [0.81, 0.82]
		1.96 [1.95, 1.97]		1.18 [1.17, 1.18]		0.78 [0.78, 0.79]
L	96.96	95.04 [94.23, 95.84]	7.48	7.61 [7.55, 7.67]	4.70	4.76 [4.73, 4.80]
		98.04 [97.38,98.71]		7.30 [7.24, 7.36]		4.58 [4.54, 4.61]
W	48.31	46.82 [46.16, 47.49]	6.21	6.24 [6.23, 6.26]	5.86	5.87 [5.86, 5.88]
		50.12 [49.54, 50.70]		6.21 [6.19, 6.23]		5.85 [5.84, 5.85]

[a] Denotes Cruz, Smith and Medeiros's simulation model

[b] Denotes our simulation model

Table 5 Results for 3-node merge topology ($\lambda_1 = \lambda_2 = 3.0$)

Measure	Node 1		Node 2		Node 3	
	Analytical	Simulation[a], Simulation[b]	Analytical	Simulation[a], Simulation[b]	Analytical	Simulation[a], Simulation[b]
p(c)	0.67	0.68 [0.68, 0.68]	0.67	0.68 [0.68, 0.68]	0.51	0.50 [0.50, 0.50]
		0.67 [0.67, 0.68]		0.68 [0.67, 0.69]		0.01 [0.01, 0.01]
θ	0.98	0.97 [0.97, 0.97]	0.98	0.97 [0.97, 0.97]	1.96	1.93 [1.93, 1.93]
		0.98 [0.96, 1.00]		0.96 [0.93, 0.98]		1.93 [1.93, 1.93]
L	99.51	98.63 [98.60, 98.67]	99.51	98.61 [98.56, 98.67]	99.02	99.76 [99.75, 99.76]
		99.54 [99.48, 99.59]		99.55 [99.19, 99.92]		99.77 [99.76, 99.77]
W	101.6	102.0 [101.8, 102.2]	101.6	101.9 [101.8, 102.1]	50.54	51.70 [51.70, 51.71]
		101.96 [99.76, 104.16]		104.56 [102.25, 106.88]		51.71 [51.70, 51.71]

[a] Denotes Cruz, Smith and Medeiros's simulation model

[b] Denotes our simulation model

In the real simulation model, all these arrival sources and their distances must explicitly be modeled. Therefore, we must have a set of *Create* and *Assign* modules to respectively represent each arrival source with its own independent arrival rate and to set the pedestrians' travelling distance to the end of its corridor. For example, corridor 6 has three arrival sources. The first arrival source is 0.73125 m to corridor 1, the second arrival source is 5.00625 m to either corridor 1 or corridor 2 and the third arrival source is 0.73125 m to corridor 2. Thus, we must have three *Create* modules and three *Assign* modules to specify their arrival rates and to assign these distances to the end of the corridor. The logic of pedestrians' flow in corridor 6 can visually be presented using the activity diagram as in Fig. 2. It describes the pedestrians' states in corridor 6 over a time interval. *TBA* denotes time between arrivals (i.e., the inverse of arrival rates) while *delay for the walking activity* is a function of the corridor's capacity, the current number of pedestrians and their travelling distances. Other source corridors simply follow the same logic. Figure 3 shows how the logic is structured in Arena simulation software.

Notice that each *Create* module (*Create W1*, *Create W2* and *Create W3*) is later followed by a *Decide* module (*Proceed Ped W1*, *Proceed Ped W2* and *Proceed Ped W3*). Its main purpose is to either queue generated pedestrians or to destroy them (*Destroy Ped Corr6*) when the number of blocked pedestrians waiting to enter the corridor has been reached. In our case, we intentionally set the number to 300 pedestrians. This approach is so useful especially for arrival rates which cause high blocking probabilities since storing insignificant pedestrians in a queue will consume computer memory. Destroying these insignificant pedestrians will not influence the calculation of the expected number of occupants and the expected service time of the corridor since both performance measures are calculated for pedestrians that have entered the corridors. Their arrivals must however be recorded (through *Accumulate Corr6 Arrival*) in order to calculate the corridor's blocking probabilities.

The DTSP hall consists of many series, splitting and merging network topologies. To clearly represent the locations for arrival sources, and the start and the end of corridors, we use *Station* modules (e.g., *Station W1*, *Station W2*, *Station W3* and *Station_EndCorridor6*). These modules ease us to channel pedestrian flows from corridor to corridor. Such movement

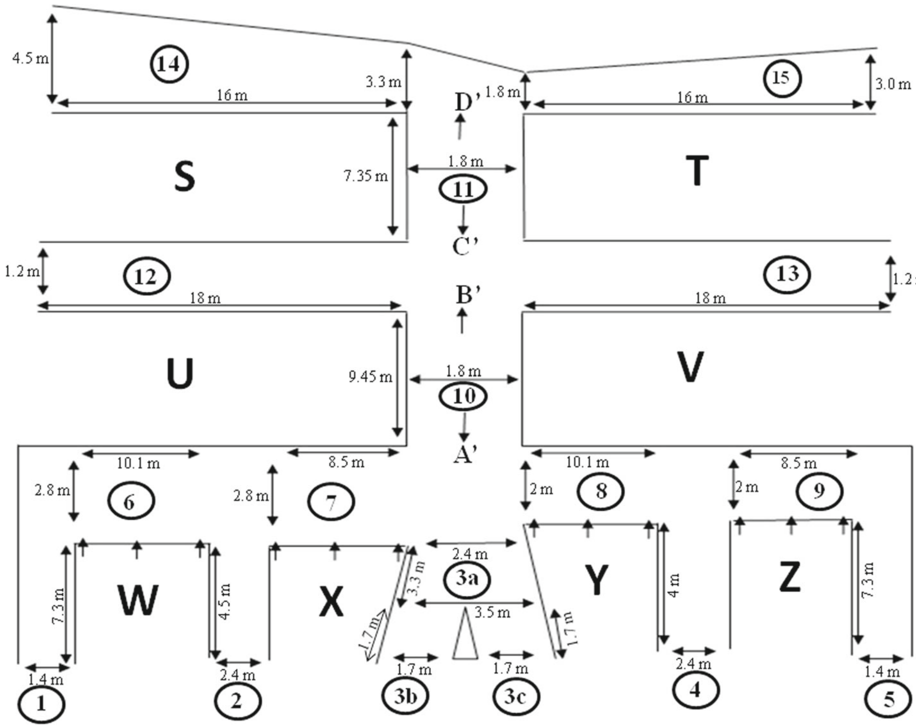

Fig. 1 A graphical representation of DTSP's hall

employs *Route* modules (e.g., *Route Corr6 to InitStation, Route to Corr1* and *Route to Corr2*) with their route time is set to zero since our purpose is to only route pedestrians to their next corridor after they have been delayed for a certain amount of time.

Table 6 Detail information on source corridor

Source corridor	No. of source	Length × width (meter)	Distances of arrival source (meter)	Average length (meter)	Relevant exit / intermediate corridor
6	3	10.1 × 2.8	L_1=0.73125, L_2=5.00625, L_3=0.73125	2.156	1 and 2
7	3	8.5 × 2.8	L_1=0.73125, L_2=3.88125, L_3=0.73125	1.780	2 and 3
8	3	10.1 × 2.0	L_1=0.73125, L_2=5.00625, L_3=0.73125	2.156	3 and 4
9	3	8.5 × 2.0	L_1=0.73125, L_2=3.88125, L_3=0.73125	1.780	4 and 5
10	20	9.45 × 1.8	L_1=L_2=L_3=L_4=0.9 L_5=L_6=L_7=L_8=1.8 L_9=L_{10}=L_{11}=L_{12}=2.7 L_{13}=L_{14}=L_{15}=L_{16}=3.6 L_{17}=L_{18}=L_{19}=L_{20}=4.5	2.700	3, 12, and 13
11	16	7.35 × 1.8	L_1=L_2=L_3=L_4=0.91 L_5=L_6=L_7=L_8=1.82 L_9=L_{10}=L_{11}=L_{12}=2.73 L_{13}=L_{14}=L_{15}=L_{16}=3.64	2.275	12, 13, 14 and 15

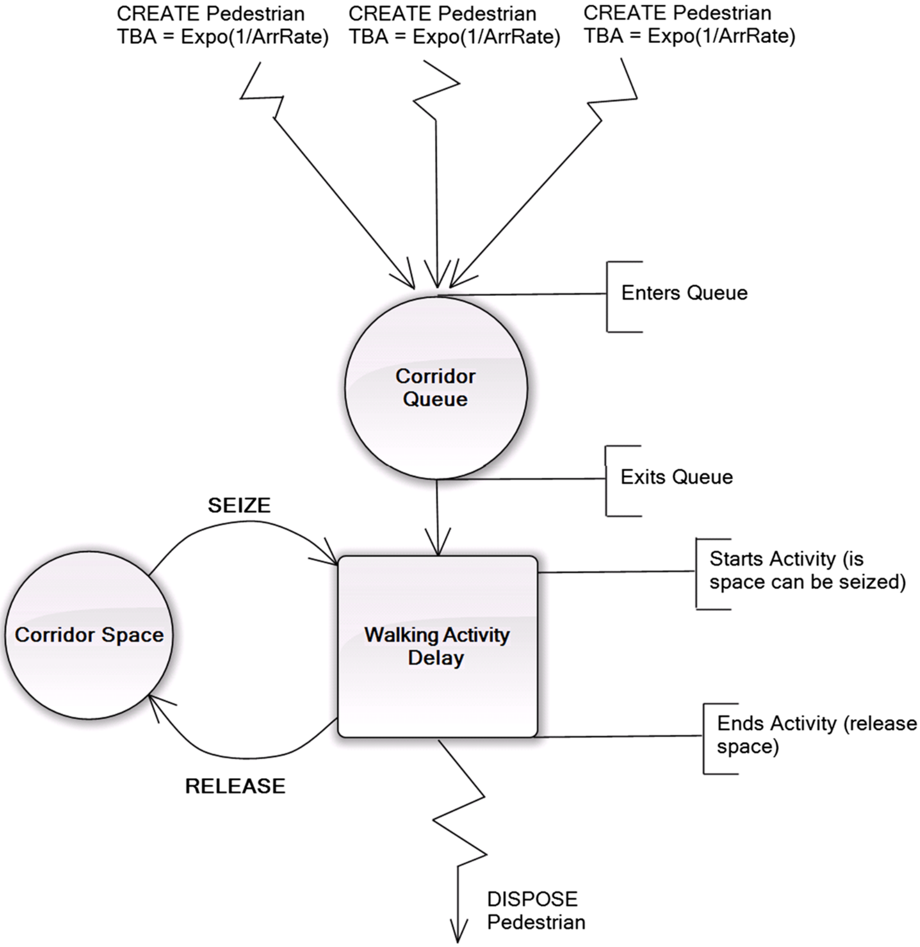

Fig. 2 Activity diagram of corridor 6

The appearance of many *Create* and other following relevant modules on a single computer screen complicates our vision to effectively trace and understand the logic of pedestrian flows in the hall. To tackle this problem, the hall has been partitioned into *submodels* (smaller models) so that we can easily model and trace pedestrians' behavior in every single corridor. These smaller models can optionally contain deeper *submodels*. Each of these hierarchical levels can have their own full workspaces which can contain their own simulation blocks and statistical reports; e.g., graphs, variables, etc. Clicking a submodel brings us to its lower level while closing the submodel brings us to its upper level. This feature helps us control the crowd of modules and eventually assists us to trace the logic and behavior of the corridor. Figure 4 shows how our real simulation model has been partitioned into a number of smaller models; e.g., submodels for *Corridor 6*, *Corridor 7*, etc. We also have an *M/G/C/C* engine submodel (*Simulation Engine*) which locates all modules for simulating pedestrians' walking through corridors as discussed in our previous paper (Khalid et al. 2013). Briefly, each corridor is represented using a queue and pedestrians' walking speeds are simulated using *Delay* modules. Although this demands the use of many *Create* and *Delay* modules, their use is important in order to accurately represent the pedestrians' behavior.

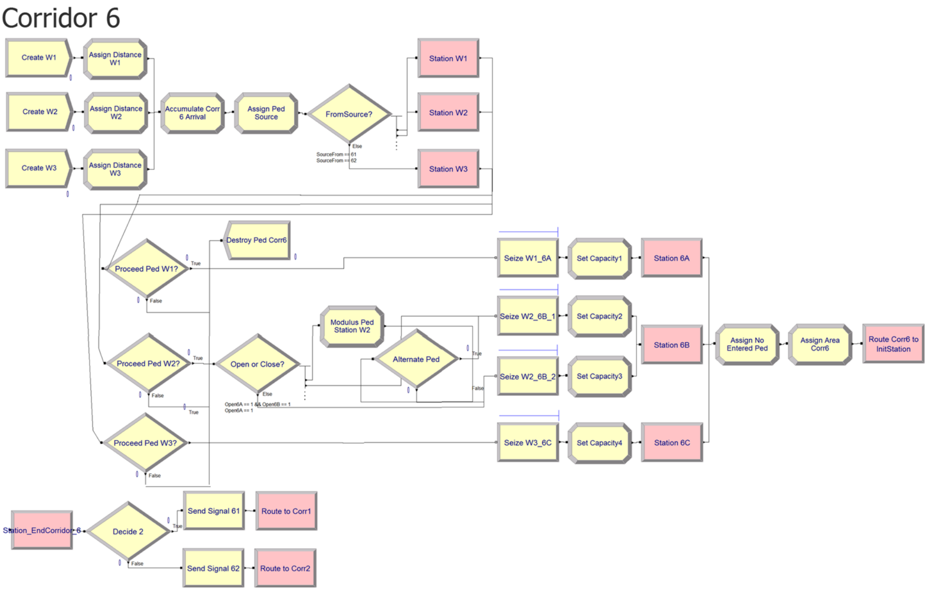

Fig. 3 Multiple arrival sources and distances for corridor 6

In our analytical and weighted distance simulation models, we assume that pedestrians will travel to their nearest corridor. Thus, for each source corridor, half of generated pedestrians travel to one side of the corridor and another half travel to another side. This logic can

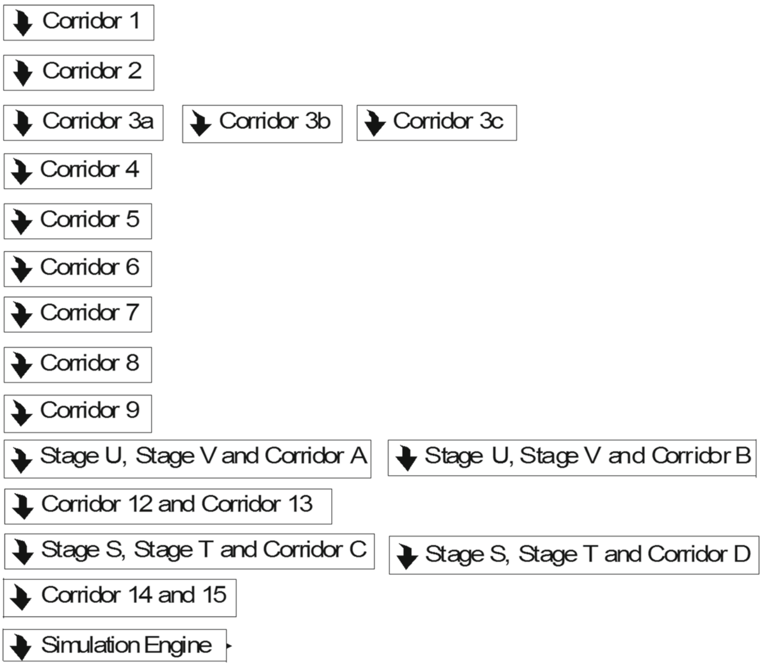

Fig. 4 Partitioning the model

straightly be modeled if the corridor has the even number of arrival sources. For example, there are 16 arrival sources along corridor 11. Thus, 8 arrival sources (i.e., 4 arrival sources from the upper left and 4 arrival sources from the upper right) will generate and channel pedestrians to corridor 14 or corridor 15. Other 8 arrival sources (4 arrival sources from the lower left and 4 arrival sources from the lower right) will generate and channel pedestrians to corridor 12 or corridor 13.

This logic must however be handled carefully for a source corridor which has odd number of arrival sources if we wish to compare its results with the results of its analytical and weighted distance simulation models. For example, corridor 6 has three arrival sources. Thus, the right and the left arrival sources will generate pedestrians who travel to corridor 1 and corridor 2 respectively. Pedestrians from the middle arrival source must be split, 50% of them travel to the corridor 1 and another 50% travel to corridor 2. This splitting can be done using two options. The first option is we split the pedestrians based on 50% chance using a *Decide* module. Alternatively, we modulus the number of accumulated pedestrians with 2 and test the returned result for odd or even. We then alternately channel the pedestrians to corridor 1 and corridor 2. This approach ensures that the pedestrians will be flowed equally. We chose the second option.

Our approach of representing walking time in a corridor through the use of storing, searching and removing pedestrians from a queue poses a major animation issue. This issue arises since the service time (walking time) in an *M/G/C/C* network depends on its current number of residing pedestrians. This variably service time hinders us from using a *Route* module to animate pedestrian movement from corridor to corridor. However, basic animation of pedestrians' movement in a flowchart, queues, variables and graphs are still supported.

Simulating pedestrians' flow in all corridors at a time to evaluate their performances will consume time and computer memory since there are many events (arrival, departure, etc.) which must be created, sorted and executed appropriately. Thus, we measured the performance of each corridor separately. For this, we provided two control points (through the use of variables) which control the creation of pedestrians to the left or/and to the right for each source corridor. For example, we can flow all pedestrians from corridor 6 to corridor 1 and corridor 2, or alternatively we can route 50% of the pedestrians in corridor 6 to only corridor 1 or corridor 2. However, the maximum number of pedestrians allowed to enter the corridor (whether full of half of the capacity) must correctly be modeled in order to accurately calculate the current speed in the corridor. Through these options of *open and close* relevant source arrivals, we can evaluate the performances of source corridors and their relevant intermediate and exit corridors separately. This approach accelerates the evaluation process and saves computer memory. However, the arrival rates for a middle arrival source for source corridors which have the odd number of arrival sources (i.e., corridor 6, corridor 7, corridor 8 and corridor 9) must be decreased to half. This ensures that no creations are made for other half pedestrians who will influence the corridor's performance measures. For example, if the arrival rate set for the corridor is 4 ped/s, the middle corridor should only generate 2 ped/s.

Other modeling issue relates to how we flow pedestrians from corridor to corridor. In the real physical hall, pedestrians can only travel to their next corridor if its remaining space is available; i.e., the maximum is 5 pedestrians in a meter square. In the simulation analogy, pedestrians can only release their current corridor if they can seize a space in the next corridor. Thus, only source corridors should have blocking probabilities while intermediate or exit corridors should have zero blocking probabilities. However, our simulation model just allows pedestrians to leave their current corridor once they have finished travelling the corridor, regardless of there is a space or not in the next corridor. If there is no space, the pedestrians will be queued, accumulated for blocking probability calculation and wait until there is a space in the next corridor. We have to follow this logic since it is used in the analytical model and this enables us to validate their outputs. However, our simulation model can be altered to suite each of the perspectives with simple modifications.

4 Experimental result

4.1 Running the model

We did not run the model on the Arena platform. We instead used its *Process Analyzer* to assess the performance of the DTSP hall. All scenarios' (in terms of arrival rates) mean performance measures were calculated based on 30 replications with each scenario was carried out for 20,000 s. While running the scenarios, there were typical occasions where the file crashed or did not generate any reports. Rerunning the scenario was very time consuming especially for evacuation rates which caused blocking.

4.2 Validating the model

After verifying all the logic of the real distance simulation model, we then validated it to ensure that pedestrians' behavior in each corridor had been replicated correctly. For this, we twisted the model to be the weighted distance simulation version. The trick was we set the *distance to travel* attribute (in the *Assign* module) for each arrival source in a source corridor to its weighted travelling distance. With this setting, pedestrians would travel to the end of the corridor with the same distance regardless of their arrival sources.

We then tested the model for a set of evacuation rates and compared its outputs with the outputs of the weighted distance simulation model. Note that evacuation rates used in the weighted distance simulation model for a particular source corridor must be divided with the number of its arrival sources before it could be input to each of the arrival source in the real distance simulation model. We purposely selected the evacuation rates which caused blocking since both simulation models would definitely generate almost the same outputs in case of there was no blocking probabilities. Samples of the outputs for both models are shown in Table 7. Sim^{w} denotes the weighted distance simulation model while Sim^{r} denotes the real distance simulation model. As before, λ, θ, $p(c)$, L and W respectively represent the arrival rate, the throughput, the blocking probability, the expected service time and the expected number of pedestrians in a corridor.

Table 7 shows that in most cases the real distance simulation model reported almost the same results with its weighted distance simulation model. The small differences of the outputs for certain evacuation rates especially in corridor 6 (λ=14.60), corridor 9 (λ=10.50) and corridor 10 (λ=7.00) were caused by the nature of random number generation and the internal computational issues. Simple tests with the same summation of arrival rates but with the different number of sources showed that there were different numbers of generated entities for each case which then caused small discrepancies in the final outputs. For

Table 7 Comparison between the weighted distance and the real distance simulation models

Source corridor	λ	Model	θ	$p(c)$	L	W
6	14.60	Sim^w	12.3386 [11.7000, 12.9700]	0.1279 [0.0822, 0.1736]	80.8386 [65.2000, 96.4700]	6.9468 [5.3020, 8.5920]
		Sim^r	12.3764 [11.7800, 12.9700]	0.1516 [0.1108, 0.1924]	89.0361 [75.0400, 103.0000]	7.6757 [6.2290, 9.1220]
	20.00	Sim^w	10.1657 [10.1700, 10.1700]	0.4917 [0.4914, 0.4914]	140.9074 [140.9000, 140.9000]	13.8611 [13.8600, 13.8600]
		Sim^r	10.1658 [10.1700, 10.1700]	0.4916 [0.4913, 0.4919]	140.9101 [140.9000, 140.9000]	13.8612 [13.8600, 13.8600]
7	15.00	Sim^w	10.9265 [10.6800, 11.1800]	0.2714 [0.2548, 0.2880]	107.7527 [103.1000, 112.4000]	9.9536 [9.3720,10.5400]
		Sim^r	11.1743 [10.9100, 11.4400]	0.2545 [0.2364, 0.2726]	103.0908 [98.0400, 108.1000]	9.3312 [8.6950, 9.9670]
	25.00	Sim^w	10.3252 [10.3300, 10.3300]	0.5867 [0.5865, 0.5869]	118.9515 [118.9000, 119.0000]	11.5205 [11.5200,11.5200]
		Sim^r	10.3253 [10.3300, 10.3300]	0.5867 [0.5865, 0.5870]	118.9497 [118.9000, 119.0000]	11.5203 [11.5200, 11.5200]
8	10.00	Sim^w	8.9933 [8.6530, 9.3330]	0.0998 [0.0657, 0.1339]	52.0894 [42.6500, 61.5300]	6.141 [4.8040, 7.4780]
		Sim^r	9.0823 [8.6790, 9.4860]	0.0908 [0.0505, 0.1312]	49.7074 [38.4600, 60.9500]	5.9680 [4.3570, 7.5780]
	15.00	Sim^w	7.2394 [7.2390, 7.2400]	0.5172 [0.5169, 0.5175]	100.9362 [100.9000, 100.9000]	13.9427 [13.9400, 13.9400]
		Sim^r	7.2395 [7.2390, 7.2400]	0.5170 [0.5166, 0.5174]	100.9360 [100.9000, 100.9000]	13.9424 [13.9400, 13.9400]
9	10.50	Sim^w	7.5631 [7.4870, 7.6400]	0.2794 [0.2724, 0.2865]	81.3844 [79.8500, 82.9200]	10.7816 [10.4800, 11.0800]
		Sim^r	7.6812 [7.5767, 7.7857]	0.2681 [0.2581, 0.2781]	78.9656 [76.8246, 81.1066]	10.3180 [9.9180, 10.7180]
	14.00	Sim^w	7.3863 [7.3860, 7.3870]	0.4719 [0.4716, 0.4723]	84.9402 [84.9300, 84.9500]	11.4997 [11.5000, 11.5000]
		Sim^r	7.3858 [7.3850, 7.3860]	0.4723 [0.4719, 0.4726]	84.9472 [84.9400, 84.9500]	11.5014 [11.5000, 11.5000]
10	7.00	Sim^w	4.9705 [4.9340, 5.0070]	0.2891 [0.2839, 0.2944]	82.1348 [81.0700, 83.2000]	16.5411 [16.2100, 16.8700]
		Sim^r	5.0715 [4.9950, 5.1480]	0.2747 [0.2636, 0.2859]	79.1329 [76.8600, 81.4000]	15.6716 [15.0200, 16.3200]
	20.00	Sim^w	4.8749 [4.8750, 4.8750]	0.7559 [0.7558, 0.7561]	84.9683 [84.9700, 84.9700]	17.4297 [17.4300, 17.4300]
		Sim^r	4.8751 [4.8750, 4.8750]	0.7559 [0.7558, 0.7561]	84.9734 [84.9700, 84.9700]	17.4299 [17.4300, 17.4300]
11	6.50	Sim^w	4.5673 [4.5500, 4.5850]	0.2969 [0.2940, 0.2998]	64.7160 [64.2800, 65.1500]	14.1735 [14.0300, 14.3200]
		Sim^r	4.5785 [4.5580, 4.5990]	0.2951 [0.2919, 0.2983]	64.4910 [64.0000, 64.9900]	14.0907 [13.9200, 14.2600]
	7.00	Sim^w	4.5274 [4.5240, 4.5310]	0.3524 [0.3515, 0.3534]	65.7551 [65.6800, 65.8300]	14.5241 [14.5000, 14.5500]
		Sim^r	4.5221 [4.5200, 4.5240]	0.3536 [0.3527, 0.3544]	65.8661 [65.8200, 65.9100]	14.5654 [14.5500, 14.5800]

^{w} Denotes the weighted distance simulation model

^{r} Denotes the real distance simulation model example, our simple test on finding the number of created entities with the summation of arrival rates of 15 ped/s but with 1 source and 3 sources, and each was run for 30 replications and 20,000 s reported that Arena has generated 299,740 and 299,480 entities respectively. The comparison of other outputs for evacuation rates which either caused no blocking or blocking also showed that both simulation models reported almost the same mean performance measures.

Since the real distance simulation model exhibits the same behavior as the weighted simulation model, and the weighted distance model is consistent with Cruz, Smith and Medeiros's simulation model, we are confident that our real distance simulation model replicated the pedestrians' behavior in the hall correctly.

4.3 Comparing the models

We then assigned the travelling distance for each arrival source with its exact distance value as in Table 6. The model's outputs were later compared with the outputs of its analytical and weighted distance simulation models. To graphically illustrate the discrepancies of these three models in evaluating the source corridors' performances, we plot their blocking probabilities and throughputs over evacuation rates. Figures 5 and 6 show how the three models differ in terms of reporting the throughputs and blocking probabilities for the source corridors. We can clearly notice how the blocking probability affects the throughput and eventually the expected service time and the expected number of pedestrians in each source corridor (see Appendix 1 and Appendix 2).

From these graphs, four patterns of model outputs can be observed. First, all of the three models (analytical, weighted distance and real distance) generated almost the same performance measures for an evacuation rate which did not cause blocking in each source corridor. Second, both simulation models started blocking probabilities at almost the same evacuation rate and this evacuation rate happened earlier compared to the analytical model. Third, the real distance simulation model generated slightly higher blocking probabilities for corridor 6, corridor 7, corridor 8 and corridor 9 than the other two models. As a result, the other performance measures for these corridor exhibited significant discrepancies in the throughput and expected numbers of pedestrians. Fourth, the three models reported almost the same blocking probabilities for corridors which contain many arrival sources (corridor 10 and corridor 11) and these made their other performance measures were almost the same after the evacuation rates which caused blocking.

To clearly see how the outputs of the three models differ for the corridor 6, corridor 7, corridor 8 and corridor 9, we table their outputs in details. Tables 8, 9, 10 and 11 show the performance measures generated by the three models for evacuation rates which induced blocking. Tables 12 and 13 meanwhile show the performances of corridor 10 and corridor 11 after the blocking.

From these tables, we can scrutinize the range of evacuation rates for each source corridor where the three models exhibited discrepancies. Table 14 reports the range of evacuation rates which the two simulation models differ from its analytical model. Compared to the analytical results, both simulation models underestimated the performance measures of every source corridor. We can also observe that in most cases, the real distance simulation model underestimated the analytical model compared to the weighted distance simulation model.

Note that we only tested both simulation models for the maximum evacuation rate of 25 ped/s. We did not test any evacuation rates which were greater than this value for two main reasons. First, our main purpose is to find the ideal evacuation rate which can best evacuate occupants from the hall, and this value will cause no or very small congestion. Second, such an evacuation rate will definitely create very high blocking probabilities, since many pedestrians

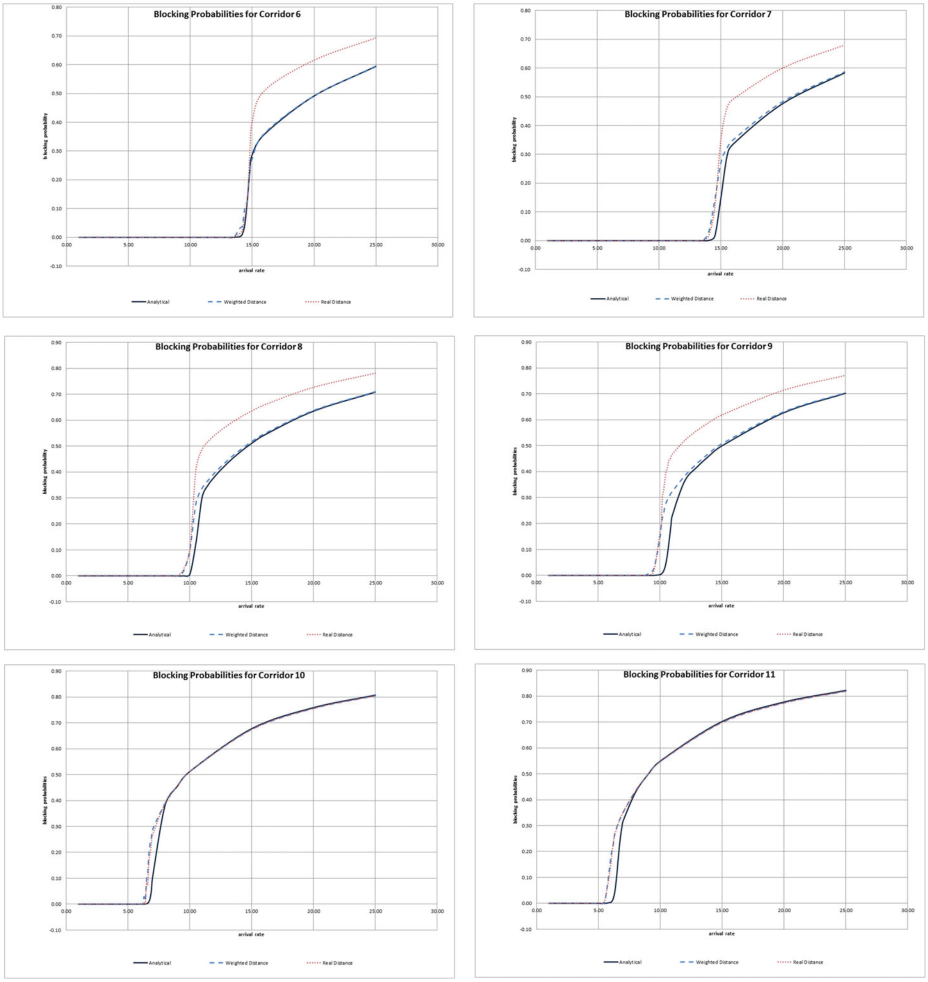

Fig. 5 Throughputs for source corridors

will be blocked from entering a corridor while other pedestrians are travelling very slowly in the corridor. Thus, running the scenario based on 30 replications, each of which would be run for 20,000 s is very time consuming. However, we may expect that any increment on arrival rates above this value will make the discrepancies of outputs between the real distance simulation model and its analytical model become smaller since the maximum value of blocking probabilities is 1.0.

4.4 Searching the best evacuation rates for source corridors

As in the analytical model, analyses of throughputs of both of the simulation models also showed that the maximum throughputs of source corridors occurred for evacuation rates which just start blocking. To find these best values, we used an optimization tool; i.e., *OptQuest* (Laguna and Marti 2003a). *OptQuest* utilizes *Tabu Search* (Glover and Laguna 1997), *Scatter*

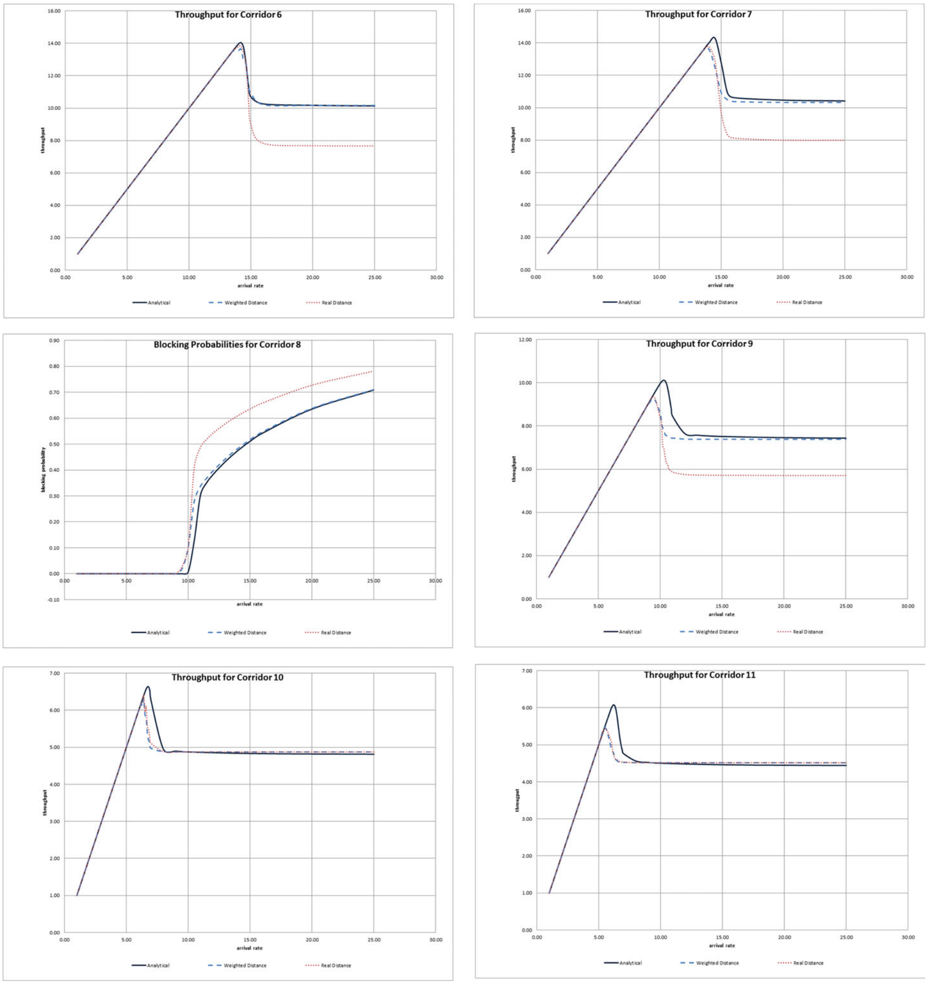

Fig. 6 Blocking probabilities for source corridors

Search (Laguna and Marti 2003b) and other heuristic and meta-heuristic methods to intelligently move and search the best value of an objective function in a specified control space. In our case, the objective is to maximize the throughput of each source corridor and its controls space is an identified range of evacuation rates which contain the optimal evacuation rate. The range of the evacuation rates for each source corridor is as in Table 14.

Table 15 compares the best evacuation rates for each source corridor measured by the three models. The best values reported by the two simulation models were based on 20 numbers of potential scenarios; each of which was run for 10 replications and 20,000 s. We had to run optimization using these settings since it was very time consuming for *OptQuest* to move, search and report the best evacuation rate for each source corridor based on the greater number of potential scenarios and replications. Even using both settings, our record showed that it was almost 1500 min for *OptQuest* to report the best evacuation rates for each source corridor. We may expect some discrepancies in the results if we change both settings since the quality of the best evacuation rate highly depends on these two factors. Running *OptQuest* with more replication and simulation numbers will certainly generate more accurate results. Note that we cannot report the low and high values of each performance measures for both simulation models since *OptQuest* does not provide such values.

From Table 15, we can make four conclusions. First, all the three models reported almost the same best evacuation rate for each source corridor. Second, in most cases, the analytical model reported slightly higher throughputs compared to both simulation models or in other words, the simulation models underestimated the performance measures of source corridors compared to the analytical model. Third, the best throughput of each source corridor reported by the analytical model happened at a slightly higher value of an evacuation rate compared to the simulation models. Fourth, there is no obvious pattern which can clearly differentiate both of the simulation models.

Based on the best evacuation rate for each source corridor, we then evaluated the performances of intermediate and exit corridors. Their performances based on the maximum throughputs of source corridors are shown in Table 16. All the three models reported that the maximum throughputs of source corridors created high congestions in subsequent corridors; i.e., corridor 1, corridor 2, corridor 3a, corridor 4, corridor 5, corridor 12 and corridor 13. At these levels of maximum throughputs which would later be arrival rates for the exit corridors, the expected numbers of occupants reached the capacities of the corridors and their expected service times became longer. Thus, occupants required much longer time to exit from the hall. This situation eventually caused small throughputs at the end. Notice that these exit corridors are actually serial or merging corridors of the source corridors.

For corridors 14 and corridor 15, the blocking probabilities were zero because of their high capacities and less arrival rates which had been divided to half from corridor 11. The expected service times were slightly greater than that for a single occupant (i.e., 16/1.5 = 10.6667 s). Meanwhile, there was a high blocking probability in corridor 3a since this corridor is a merging corridor of corridor 7, corridor 8 and corridor 10. Such a high congestion resulted in small throughput for corridor 3a. This small throughput was further divided into corridors 3b and 3c. In addition to their short lengths, this is the reason why we can observe that there were very small blocking probabilities in these two corridors. Summing the throughputs of all exit corridors (see Table 16), the total throughput of the hall was 13.0582 ped/s, 12.8241 ped/s and 12.8766 ped/s for the analytical, weighted distance and real distance models respectively. As a conclusion, if occupants rush to enter source corridors, then there will be high blockings of exit corridors and the overall throughput will decrease.

4.5 Controlling the evacuation rates to best evacuate occupants from the hall

We have observed that the maximum throughputs of source corridors would create high blockings in exit corridors and eventually decrease the throughput of the hall. Thus, restricting the rates of occupants to enter source corridors is crucial to maximize the throughput of the hall. We can also see that there are no blocking probabilities in corridor 14 and corridor 15. Therefore, we could channel all occupants from corridor 11 to these two corridors. This modification needs us to reassign the new distance values for relevant arrival source in corridor 11 (that has previously been channeled to corridor 12 and corridor 13) to the corridor 14 and corridor 15. Under these new restrictions, we reran *OptQuest* for the real distance simulation

Table 8 Comparison of outputs corridor 6

λ	Model	$p(c)$	θ	L	W
13.5000	Analytic	13.4994	0.0000	30.9914	2.2958
	Sim^w	13.4960[13.4900, 13.5100]	0.0000[0.0000, 0.0000]	30.9942[30.9300, 31.0600]	2.2936[2.2920, 2.2960]
	Sim^r	13.5003[13.4900, 13.5100]	0.0000[0.0000, 0.0000]	30.9803[30.9400, 31.0200]	2.2958[2.2920, 2.3000]
14.0000	Analytic	13.9652	0.0025	34.4873	2.4695
	Sim^w	13.5614[13.1400, 13.9900]	0.0313[0.0009, 0.0617]	36.6931[30.3400, 43.0500]	3.6370[2.4200, 4.8540]
	Sim^r	13.8129[13.4400, 14.1800]	0.0134[−0.0130, 0.0399]	34.9021[31.8600, 37.9500]	2.8138[1.9680, 3.6600]
14.2000	Analytic	14.0419	0.0111	38.9129	2.7712
	Sim^w	13.6415[13.2500, 14.0400]	0.0396[0.0118, 0.0674]	40.5631[34.3000, 46.8300]	3.8280[2.8270, 4.8290]
	Sim^r	13.8317[13.4500, 14.2100]	0.0254[−0.0015, 0.0524]	46.1549[35.1900, 57.1200]	3.0648[2.3710, 3.7590]
14.4000	Analytic	13.7712	0.0437	51.9059	3.7692
	Sim^w	12.9373[12.3500, 13.5200]	0.1013[0.0604, 0.1421]	51.4742[40.5600, 62.3800]	6.0293[4.5300, 7.5280]
	Sim^r	13.4010[12.7100, 14.0900]	0.0690[0.0210, 0.1171]	68.7230[54.1900, 83.2500]	4.3395[2.9320, 5.7470]
14.6000	Analytic	12.7616	0.1259	81.2885	6.3698
	Sim^w	12.7293[12.0600, 13.4000]	0.1279[0.0822, 0.1736]	65.6703[52.2500, 79.0900]	6.9468[5.3020, 8.5920]
	Sim^r	12.6985[11.8100, 13.5900]	0.1299[0.0690, 0.1908]	89.8071[74.8900, 104.7000]	6.0060[4.2690, 7.7430]
15.0000	Analytic	10.6869	0.2875	131.6891	12.3225
	Sim^w	10.9071[10.6700,11.1400]	0.2726[0.2570, 0.2882]	123.4910[116.5000, 130.5000]	11.5954[10.9200, 12.2700]
	Sim^r	8.9599[8.4580, 9.4620]	0.4022[0.3685, 0.4358]	123.0560[115.4000, 130.7000]	14.2655[13.0000, 15.5300]
16.0000	Analytic	10.2612	0.3587	140.0790	13.6514
	Sim^w	10.2095[10.1900,10.2300]	0.3615[0.3603, 0.3627]	139.3861[138.9000, 139.8000]	13.7301[13.6800, 13.7800]
	Sim^r	7.8277[7.7890, 7.8660]	0.5104[0.5079, 0.5129]	140.2472[140.0000, 140.5000]	17.8117[17.6700, 17.9500]
20.0000	Analytic	10.1727	0.4914	140.9409	13.8547
	Sim^w	10.1657[10.1700,10.1700]	0.4917[0.4914, 0.4914]	140.8251[140.8000, 140.8000]	13.8611[13.8600, 13.8600]
	Sim^r	7.6802[7.6780, 7.6830]	0.6157[0.6154, 0.6159]	140.9040[140.9000, 140.9000]	18.3361[18.3300, 18.3400]
25.0000	Analytic	10.1348	0.5946	141.3100	13.9430
	Sim^w	10.1639[10.1600,10.1600]	0.5931[0.5929, 0.5933]	140.8623[140.9000, 140.9000]	13.8651[13.8647, 13.8655]
	Sim^r	7.6637[7.6620, 7.6650]	0.6932[0.6930, 0.6933]	140.9280[140.9000, 140.9000]	18.3806[18.3800, 18.3900]

[w] Denotes the weighted distance simulation model

[r] Denotes the real distance simulation model

Table 9 Comparison of outputs corridor 7

λ	Model	$p(c)$	θ	L	W
13.50	Analytic	13.4997	0.0000	25.3009	1.8742
	Sim^w	13.5056[13.4900, 13.5200]	0.0000[0.0000, 0.0000]	25.3085[25.2700, 25.3500]	1.8739[1.8720, 1.8750]
	Sim^r	13.4925[13.4800, 13.5000]	0.0000[0.0000, 0.0000]	25.2822[25.2400, 25.3300]	1.8738[1.8710, 1.8760]
14.00	Analytic	13.9909	0.0007	27.5260	1.9674
	Sim^w	13.6133[13.3200, 13.9000]	0.0278[0.0070, 0.0487]	37.0234[29.7300, 44.3100]	2.8174[2.1570, 3.4780]
	Sim^r	13.7726[13.4200, 14.1200]	0.0159[−0.0091, 0.0410]	30.7274[25.3500, 36.1100]	2.3444[1.6980, 2.9910]
14.50	Analytic	14.2849	0.0148	34.5432	2.4182
	Sim^w	12.4592[11.8200, 13.1000]	0.1402[0.0959, 0.1845]	73.1561[59.3800, 86.9300]	6.3846[4.9430, 7.8260]
	Sim^r	12.8548[11.9700, 13.7400]	0.1132[0.0522, 0.1741]	52.2844[40.0700, 64.4900]	4.8254[3.2250, 6.4260]
15.00	Analytic	12.7589	0.1494	75.9530	5.9529
	Sim^w	10.9265[10.6800, 11.1800]	0.2714[0.2548, 0.2880]	107.7527[103.1000, 112.4000]	9.9536[9.3720, 10.5400]
	Sim^r	9.6827[9.2080, 10.1600]	0.3540[0.3222, 0.3858]	98.1814[92.2800, 104.1000]	10.4976[9.4360, 11.5600]
15.50	Analytic	10.9228	0.2953	111.7883	10.2344
	Sim^w	10.4943[10.4400, 10.5500]	0.3226[0.3191, 0.3261]	116.2016[115.3000, 117.1000]	11.0777[10.9400, 11.2100]
	Sim^r	8.3217[8.1960, 8.4470]	0.4627[0.4546, 0.4708]	115.3678[114.0000, 116.8000]	13.9005[13.5500, 14.2500]
16.00	Analytic	10.6248	0.3359	116.443	10.9596
	Sim^w	10.3778[10.3600, 10.3900]	0.3508[0.3497, 0.3519]	118.1701[117.9000, 118.4000]	11.3873[11.3500, 11.4300]
	Sim^r	8.1248[8.0890, 8.1610]	0.4919[0.4897, 0.4942]	117.6559[117.3000, 118.0000]	14.4845[14.3800, 14.5900]
20.00	Analytic	10.4674	0.4766	117.8681	11.2605
	Sim^w	10.3266[10.3300, 10.3300]	0.4835[0.4832, 0.4838]	118.9262[118.9000, 118.9000]	11.5165[11.5200, 11.5200]
	Sim^r	7.9935[7.9920, 7.9950]	0.6002[0.6000, 0.6005]	118.8714[118.9000, 118.9000]	14.8710[14.8700, 14.8700]
25.00	Analytic	10.4166	0.5833	118.2747	11.3544
	Sim^w	10.3252[10.3300, 10.3300]	0.5867[0.5865, 0.5869]	118.9515[118.9000, 119.0000]	11.5205[11.5200, 11.5200]
	Sim^r	7.9852[7.9840, 7.9860]	0.6803[0.6800, 0.6805]	118.9066[118.9000, 118.9000]	14.8909[14.8900, 14.8900]

^{w} Denotes the weighted distance simulation model

^{r} Denotes the real distance simulation model

Table 10 Comparison of outputs for corridor 8

λ	Model	$p(c)$	θ	L	W
9.00	Analytic	9.0000	0.0000	19.6917	2.1880
	Sim^w	9.0007[8.9920, 9.0100]	0.0000[0.0000, 0.0000]	19.6917[19.6500, 19.7300]	2.1878[2.1860, 2.1900]
	Sim^r	8.9936[8.9850, 9.0020]	0.0000[0.0000, 0.0000]	19.6502[19.6100, 19.7000]	2.1849[2.1810, 2.1890]
9.50	Analytic	9.4989	0.0001	21.9006	2.3056
	Sim^w	9.3204[9.1500, 9.4910]	0.0183[0.0003, 0.0363]	27.9327[21.9700, 33.9000]	3.0991[2.2920, 3.9060]
	Sim^r	9.2492[8.9470, 9.5510]	0.0263[−0.0054, 0.0581]	26.8571[20.8600, 32.8600]	3.1360[2.1070, 4.1640]
10.00	Analytic	9.9399	0.0060	26.3468	2.6506
	Sim^w	8.9933[8.6530, 9.3330]	0.0998[0.0657, 0.1339]	52.0894[42.6500, 61.5300]	6.141[4.8040, 7.4780]
	Sim^r	8.9958[8.4540, 9.5380]	0.1002[0.0461, 0.1544]	41.5717[32.3800, 50.7700]	5.3232[3.6260, 7.0200]
10.50	Analytic	9.1496	0.1286	60.0732	6.5657
	Sim^w	7.5503[7.4550, 7.6460]	0.2800[0.2707, 0.2893]	93.8997[91.7300, 96.0700]	12.4745[12.0400, 12.9100]
	Sim^r	6.1753[5.9610, 6.3890]	0.4115[0.3910, 0.4320]	90.8694[87.7500, 93.9900]	14.9490[13.9700, 15.9300]
11.00	Analytic	7.6604	0.3036	95.4085	12.4548
	Sim^w	7.2831[7.2690, 7.2970]	0.3374[0.3360, 0.3388]	100.1245[99.8500, 100.4000]	13.7483[13.6800, 13.8100]
	Sim^r	5.6276[5.5850, 5.6700]	0.4882[0.4844, 0.4921]	99.0375[98.5100, 99.5600]	17.6104[17.3900, 17.8400]
12.00	Analytic	7.4218	0.3815	99.2339	13.3706
	Sim^w	7.2448[7.2420, 7.2470]	0.3961[0.3957, 0.3965]	100.8539[100.8000, 100.9000]	13.9208[13.9100, 13.9300]
	Sim^r	5.4877[5.4810, 5.4940]	0.5420[0.5413, 0.5427]	100.7130[100.6000, 100.8000]	18.3527[18.3200, 18.3900]
15.00	Analytic	7.3414	0.5106	100.0133	13.6232
	Sim^w	7.2394[7.2390, 7.2400]	0.5172[0.5169, 0.5175]	100.9362[100.9000, 100.9000]	13.9427[13.9400, 13.9400]
	Sim^r	5.4638[5.4630, 5.4650]	0.6353[0.6351, 0.6356]	100.8701[100.9000, 100.9000]	18.4616[18.4600, 18.4700]
20.00	Analytic	7.2998	0.6350	100.4176	13.7562
	Sim^w	7.2386[7.2380, 7.2390]	0.6377[0.6376, 0.6379]	100.9526[101.0000, 101.0000]	13.9464[13.9500, 13.9500]
	Sim^r	5.4540[5.4530, 5.4550]	0.7271[0.7269, 0.7273]	100.9037[100.9000, 100.9000]	18.5010[18.5000, 18.5000]
25.00	Analytic	7.2826	0.7087	100.5855	13.8118
	Sim^w	7.2380[7.2380, 7.2380]	0.7103[0.7101, 0.7104]	100.9536[101.0000, 101.0000]	13.9477[13.9500, 13.9500]
	Sim^r	5.4502[5.4500, 5.4510]	0.7818[0.7817, 0.7819]	100.9089[100.9000, 100.9000]	18.5146[18.5100, 18.5200]

[w] Denotes the weighted distance simulation model

[r] Denotes the real distance simulation model

Table 11 Comparison of outputs for corridor 9

λ	Model	$p(c)$	θ	L	W
9.00	Analytic	9.0000	0.0000	16.1561	1.7951
	Sim^w	8.9578[8.8780, 9.0380]	0.0044[−0.0046, 0.0134]	17.8141[14.4100, 21.2200]	2.0166[1.5630, 2.4700]
	Sim^r	8.9938[8.9850, 9.0030]	0.0000[0.0000, 0.0000]	16.1494[16.1200, 16.1800]	1.7956[1.7930, 1.7980]
9.50	Analytic	9.4991	0.0001	17.9404	1.8886
	Sim^w	9.2524[9.0620, 9.4430]	0.0261[0.0061, 0.0462]	25.7982[19.7400, 31.8500]	2.903[2.0970, 3.7090]
	Sim^r	9.3603[9.1940, 9.5260]	0.0150[−0.0025, 0.0325]	20.5245[17.4900, 23.5600]	2.2443[1.8240, 2.6640]
10.00	Analytic	9.9723	0.0028	20.8172	2.0875
	Sim^w	8.5768[8.2580, 8.8960]	0.1425[0.1109, 0.1742]	55.4031[47.5400, 63.2700]	6.7508[5.6160, 7.8850]
	Sim^r	8.3644[7.7580, 8.9710]	0.1639[0.1035, 0.2243]	45.1722[35.9500, 54.4000]	6.2400[4.5550, 7.9250]
10.50	Analytic	9.9710	0.0504	34.3058	3.4406
	Sim^w	7.5631[7.4870, 7.6400]	0.2794[0.2724, 0.2865]	81.3844[79.8500, 82.9200]	10.7816[10.4800, 11.0800]
	Sim^r	6.2851[6.0820, 6.4890]	0.4010[0.3815, 0.4204]	77.7722[75.0700, 80.4700]	12.5458[11.7800, 13.3100]
11.00	Analytic	8.4552	0.2313	69.8442	8.2605
	Sim^w	7.4405[7.4250, 7.4560]	0.3234[0.3218, 0.3249]	84.0143[83.7500, 84.2800]	11.2923[11.2300, 11.3500]
	Sim^r	5.8572[5.8170, 5.8970]	0.4674[0.4637, 0.4710]	83.5404[83.1000, 83.9800]	14.2707[14.1000, 14.4400]
12.00	Analytic	7.6419	0.3632	82.9006	10.8481
	Sim^w	7.3943[7.3910, 7.3970]	0.3830[0.3824, 0.3835]	84.8321[84.7900, 84.8700]	11.4727[11.4600, 11.4800]
	Sim^r	5.7543[5.7480, 5.7610]	0.5203[0.5196, 0.5211]	84.7033[84.6500, 84.7600]	14.7202[14.6900, 14.7500]
15.00	Analytic	7.5151	0.4990	83.9574	11.1719
	Sim^w	7.3851[7.3850, 7.3850]	0.5072[0.5069, 0.5075]	84.9545[84.9500, 84.9600]	11.5035[11.5000, 11.5000]
	Sim^r	5.7202[5.7190, 5.7210]	0.6184[0.6182, 0.6186]	84.9182[84.9100, 84.9200]	14.8454[14.8400, 14.8500]
20.00	Analytic	7.4605	0.6270	84.3951	11.3123
	Sim^w	7.3840[7.3840, 7.3840]	0.6305[0.6303, 0.6307]	84.9653[84.9600, 84.9700]	11.5066[11.5100, 11.5100]
	Sim^r	5.7113[5.7110, 5.7120]	0.7143[0.7142, 0.7144]	84.9374[84.9300, 84.9400]	14.8717[14.8700, 14.8700]
25.00	Analytic	7.4385	0.7025	84.5720	11.3694764
	Sim^w	7.3837[7.3840, 7.3840]	0.7045[0.7043, 0.7046]	84.9665[84.9700, 84.9700]	11.5073[11.5100, 11.5100]
	Sim^r	5.7098[5.7090, 5.7100]	0.7715[0.7714, 0.7716]	84.9467 [84.9400,84.9500]	14.8772[14.8800, 14.8800]

[w] Denotes the weighted distance simulation model

[r] Denotes the real distance simulation model

Table 12 Comparison of outputs for corridor 10

λ	Model	$p(c)$	θ	L	W
6.00	Analytic	6.0000	0.0000	16.4811	2.7469
	Sim^{w}	5.9986[5.9920, 6.0050]	0.0000[0.0000, 0.0000]	16.4785[16.4400, 16.5100]	2.7470[2.7440, 2.7500]
	Sim^{r}	5.9949[5.9800, 6.0100]	0.0011[−0.0012, 0.0035]	16.8604[16.0200, 17.7000]	2.8149[2.6630, 2.9670]
6.50	Analytic	6.4908	0.0014	19.7816	3.0476
	Sim^{w}	5.8886[5.6460, 6.1310]	0.0939[0.0567, 0.1310]	44.0060[34.2400, 53.7700]	8.0682[5.9800, 10.1600]
	Sim^{r}	5.9568[5.7140, 6.2000]	0.0831[0.0456, 0.1206]	41.1269[31.2700, 50.9900]	7.4931[5.4080, 9.5790]
7.00	Analytic	6.2524	0.1068	46.2203	7.3924
	Sim^{w}	4.9705[4.9340, 5.0070]	0.2891[0.2839, 0.2944]	82.1348[81.0700, 83.2000]	16.5411[16.2100, 16.8700]
	Sim^{r}	5.1094[5.0340, 5.1850]	0.2692[0.2580, 0.2804]	77.9841[75.7200, 80.2500]	15.3294[14.6800, 15.9800]
8.00	Analytic	4.9440	0.3820	84.1638	17.0233
	Sim^{w}	4.8832[4.8810, 4.8850]	0.3892[0.3885, 0.3899]	84.7699[84.7300, 84.8100]	17.3595[17.3400, 17.3800]
	Sim^{r}	4.8838[4.8800, 4.8880]	0.3891[0.3882, 0.3899]	84.7700[84.7200, 84.8200]	17.3577[17.3300, 17.3800]
9.00	Analytic	4.8977	0.4558	84.7419	17.3024
	Sim^{w}	4.8762[4.8760, 4.8770]	0.4571[0.4568, 0.4575]	84.9167[84.9100, 84.9200]	17.4146[17.4100, 17.4200]
	Sim^{r}	4.8760[4.8730, 4.8790]	0.4576[0.4570, 0.4582]	84.9070[84.8900, 84.9200]	17.4131[17.4000, 17.4200]
10.00	Analytic	4.8753	0.5125	85.0150	17.4380
	Sim^{w}	4.8753[4.8750, 4.8760]	0.5121[0.5116, 0.5125]	84.9289[84.9300, 84.9300]	17.4203[17.4200, 17.4200]
	Sim^{r}	4.8756[4.8730, 4.8780]	0.5119[0.5114, 0.5124]	84.9299[84.9200, 84.9300]	17.4195[17.4100, 17.4300]
15.00	Analytic	4.8342	0.6777	85.5186	17.6904
	Sim^{w}	4.8744[4.8740, 4.8740]	0.6747[0.6745, 0.6749]	84.9521[84.9500, 84.9500]	17.4283[17.4300, 17.4300]
	Sim^{r}	4.8754[4.8730, 4.8780]	0.6747[0.6744, 0.6749]	84.9468[84.9500, 84.9500]	17.4235[17.4100, 17.4300]
20.00	Analytic	4.821067	0.758947	85.68009	17.77202
	Sim^{w}	4.8749[4.8750, 4.8750]	0.7559[0.7558, 0.7561]	84.9683[84.9700, 84.9700]	17.4297[17.4300, 17.4300]
	Sim^{r}	4.8740[4.8710, 4.8770]	0.7561[0.7559, 0.7563]	84.9499[84.9500, 84.9500]	17.4293[17.4200, 17.4400]
25.00	Analytic	4.814567	0.807417	85.76028	17.81267
	Sim^{w}	4.8751[4.8750, 4.8750]	0.8048[0.8047, 0.8049]	84.9748[84.9700, 84.9800]	17.4303[17.4300, 17.4300]
	Sim^{r}	4.8746[4.8720, 4.8770]	0.8049[0.8047, 0.8051]	84.9518[84.9500, 84.9500]	17.4276[17.4200, 17.4400]

^{w} Denotes the weighted distance simulation model

^{r} Denotes the real distance simulation model

Table 13 Comparison of outputs for corridor 11

λ	Model	$p(c)$	θ	L	W
5.00	Analytic	5.0000	0.0000	10.9050	2.1810
	Sim^w	5.0010[4.9940, 5.0080]	0.0000[0.0000, 0.0000]	10.9075[10.8800, 10.9300]	2.1810[2.1790, 2.1830]
	Sim^r	4.9993[4.9920, 5.0060]	0.0000[0.0000, 0.0000]	10.8985[10.8700, 10.9300]	2.1800[2.1770, 2.1830]
5.50	Analytic	5.4998	0.0000	12.8948	2.3446
	Sim^w	5.4697[5.4160, 5.5230]	0.0057[−0.0038, 0.0152]	14.5861[11.7300, 17.4400]	2.7084[2.0840, 3.3320]
	Sim^r	5.4426[5.3800, 5.5060]	0.0101[−0.0015, 0.0217]	15.8313[12.4100, 19.2600]	2.9668[2.2420, 3.6920]
6.00	Analytic	5.9766	0.0039	16.3810	2.7409
	Sim^w	4.9306[4.8170, 5.0440]	0.1777[0.1586, 0.1967]	51.7996[47.9200, 55.6800]	10.6587[9.6930, 11.6200]
	Sim^r	5.0984[4.9220, 5.2750]	0.1498[0.1202, 0.1793]	46.0769[40.0100, 52.1400]	9.3787[7.9520, 10.8100]
6.50	Analytic	5.7179	0.1203	38.7446	6.7760
	Sim^w	4.5673[4.5500, 4.5850]	0.2969[0.2940, 0.2998]	64.7160[64.2800, 65.1500]	14.1735[14.0300, 14.3200]
	Sim^r	4.5933[4.5700, 4.6170]	0.2938[0.2902, 0.2974]	64.1371[63.5700, 64.7100]	13.9700[13.7800, 14.1600]
7.00	Analytic	4.7647	0.3193	62.5775	13.1336
	Sim^w	4.5274[4.5240, 4.5310]	0.3524[0.3515, 0.3534]	65.7551[65.6800, 65.8300]	14.5241[14.5000, 14.5500]
	Sim^r	4.5298[4.5240, 4.5350]	0.3525[0.3516, 0.3533]	65.7113[65.6200, 65.8000]	14.5067[14.4700, 14.5400]
8.00	Analytic	4.5628	0.4297	65.5398	14.3640
	Sim^w	4.5180[4.5180, 4.5180]	0.4348[0.4343, 0.4353]	65.9382[65.9300, 65.9400]	14.5947[14.5900, 14.6000]
	Sim^r	4.5208[4.5180, 4.5230]	0.4342[0.4335, 0.4349]	65.9374[65.9300, 65.9400]	14.5853[14.5800, 14.5900]
10.00	Analytic	4.5035	0.5496	66.1507	14.6886
	Sim^w	4.5166[4.5160, 4.5170]	0.5479[0.5476, 0.5483]	65.9614[65.9600, 65.9600]	14.6041[14.6000, 14.6000]
	Sim^r	4.5179[4.5150, 4.5200]	0.5478[0.5473, 0.5483]	65.9608[65.9600, 65.9600]	14.6001[14.5900, 14.6100]
15.00	Analytic	4.4630	0.7025	66.5707	14.9161
	Sim^w	4.5158[4.5160, 4.5160]	0.6988[0.6986, 0.6990]	65.9661[65.9700, 65.9700]	14.6079[14.6100, 14.6100]
	Sim^r	4.5158[4.5130, 4.5180]	0.6989[0.6986, 0.6992]	65.9662[65.9700, 65.9700]	14.6078[14.6000, 14.6200]
20.00	Analytic	4.4495	0.7775	66.7115	14.9931

Table 13 (continued)

λ	Model	$p(c)$	θ	L	W
	Sim^w	4.5156[4.5160, 4.5160]	0.7740[0.7739, 0.7741]	65.9668[65.9700, 65.9700]	14.6087[14.6100, 14.6100]
	Sim^r	4.5156[4.5140, 4.5180]	0.7739[0.7738, 0.7741]	65.9672[65.9700, 65.9700]	14.6088[14.6000, 14.6200]
25.00	Analytic	4.4427	0.8223	66.7826	15.0321
	Sim^w	4.5155[4.5150, 4.5160]	0.8194[0.8193, 0.8195]	65.9671[65.9700, 65.9700]	14.6090[14.6100, 14.6100]
	Sim^r	4.5148[4.5120, 4.5180]	0.8193[0.8191, 0.8194]	65.9673[65.9700, 65.9700]	14.6113[14.6000, 14.6200]

[w] Denotes the weighted distance simulation model

[r] Denotes the real distance simulation model

model to find the ideal evacuation rates of the source corridors. *OptQuest* was run based on 150 considered simulations and 10 replications for each scenario. Table 17 shows the comparison between the ideal levels of evacuation rates for source corridors reported by the analytical and the real distance simulation modes.

From Table 17, the configuration of evacuation rates to source corridors to achieve the best throughput was quite different. The analytical model recommends that we control 4.600 ped/s to enter corridor 6, 0.6000 (corridor 7), 0.6000 (corridor 8), 4.600 (corridor 9), 5.200 (corridor 10) and 3.4500 (corridor 11) to optimize the throughput of the hall. The real distance simulation model meanwhile recommends that we restrict 4.600 ped/s to enter corridor 6, 0.2352 (corridor 7), 0.2287 (corridor 8), 4.7695 (corridor 9), 5.2000 (corridor 10) and 3.4500 (corridor 11) to achieve the best overall throughput. Thus, both models reported the different best throughput; i.e., 16.1102 ped/s (the analytical model) and 14.9932 ped/s (the real simulation model). Once again, the real simulation model underestimated the performances of the hall compared to the analytical model.

4.6 Recommending appropriate actions

Analyses of the outputs of the three models showed that it was crucial to control evacuation rates to each source corridor so that the throughput of the hall can be optimized. Any higher evacuation values than its best value will cause congestion and eventually decrease the throughput. Any lower values than the best value will otherwise let pedestrians walk faster. However, this situation does not improve the throughput at the end.

The optimal throughput reflects the minimum time to vacant the hall. In other words, any increment in the throughput will decrease the time to vacant occupants from the hall. Thus, in order to vacant the hall fast, the university's policy makers should consider these actions. First, they have to ensure that occupants from their seating arrangement enter their source corridors within the ideal level of entrance rates. We recommend them to implement the ideal evacuation rates suggested by the real distance simulation model; i.e., 4.600 ped/s (corridor 6), 0.2350 (corridor 7), 0.2287 (corridor 8), 4.7695 (corridor 9), 5.2000 (corridor 10) and 3.4500 (corridor 11) to achieve the best overall throughput of 14.9932 ped/s. These ideal evacuation rates are almost the same with that suggested by the analytical model except for corridor 7 and corridor 8 whose values are much lower compared to the values recommended by

Table 14 The range of arrival rates

Source corridor	Range of λ (Sim^w)	Range of λ (Sim^r)
6	$14.00 \le \lambda \le 15.00$	$\lambda \ge 14.00$
7	$14.00 \le \lambda \le 15.50$	$\lambda \ge 14.00$
8	$9.50 \le \lambda \le 11.00$	$\lambda \ge 9.50$
9	$9.50 \le \lambda \le 11.00$	$\lambda \ge 9.50$
10	$6.50 \le \lambda \le 7.00$	$6.50 \le \lambda \le 7.00$
11	$6.00 \le \lambda \le 7.00$	$6.00 \le \lambda \le 7.00$

[w] Denotes the weighted distance simulation model

[r] Denotes the real distance simulation model

Table 15 The best arrival rates for the three models

Source	Analytical model			Simulation model ($Sim^{w)}$)				Simulation model (Sim^{r})			
	λ	θ	$p(c)$	Range of λ	λ	θ	$p(c)$	Range of λ	λ	θ	$p(c)$
6	14.1800	14.0436	0.0096	[13.00, 14.50]	14.1800	14.0477	0.0087	[13.50, 14.50]	14.2216	14.2244	0.0000
7	14.4600	14.2904	0.1173	[13.00, 14.50]	13.9135	13.9134	0.0122	[13.50, 14.50]	14.0094	14.0155	0.0000
8	10.1100	9.9744	0.0134	[9.00, 10.50]	9.7385	9.6700	0.0787	[9.00, 10.50]	9.5757	9.57777	0.0000
9	10.2900	10.1213	0.0164	[9.00, 10.00]	9.4660	9.4763	0.0000	[9.00, 10.50]	9.7500	9.4610	0.0300
10	6.7500	6.6422	0.0160	[6.00, 6.50]	6.4289	6.3915	0.0062	[6.00, 7.00]	6.3838	6.3878	0.0000
11	6.2100	6.0807	0.0208	[5.00, 6.50]	5.6407	5.6231	0.0036	[5.00, 6.50]	5.7250	5.5397	0.0326

[w] Denotes the weighted distance simulation model

[r] Denotes the real distance simulation model

Table 16 Performance measures of the intermediate and exit corridors

Type	Corridor	Model	θ	$p(c)$	L	W
Intermediate	3a	Analytic	2.2793	0.8525	48.8260	21.4219
		Sim^w	2.2667	0.8486	48.9687	21.6040
		Sim^r	2.2666	0.8485	48.9670	21.6033
Exit	1	Analytic	1.0647	0.8484	51.8202	48.6714
		Sim^w	1.0877	0.8463	50.9282	46.8239
		Sim^r	1.0874	0.8468	50.9182	46.8243
	2	Analytic	1.8682	0.8681	53.8474	28.8231
		Sim^w	1.8605	0.8674	53.9818	29.0153
		Sim^r	1.8603	0.8680	53.9747	29.0144
	3b	Analytic	1.1391	0.0005	1.9680	1.7278
		Sim^w	1.1339	0.0000	1.9619	1.7301
		Sim^r	1.1333	0.0000	3.1789	2.8049
	3c	Analytic	1.1391	0.0005	1.9680	1.7278
		Sim^w	1.1339	0.0000	1.9619	1.7301
		Sim^r	1.1333	0.0000	3.1785	2.8047
	4	Analytic	1.8751	0.8134	47.7686	25.4753
		Sim^w	1.8614	0.8055	47.9615	25.7669
		Sim^r	1.8612	0.8072	47.9555	25.7662
	5	Analytic	1.0674	0.7891	51.7301	48.4651
		Sim^w	1.0876	0.7703	50.9204	46.8173
		Sim^r	1.0876	0.7763	50.9175	46.8167
	12	Analytic	0.9322	0.7069	107.5821	115.4055
		Sim^w	0.9240	0.8453	107.5666	116.4099
		Sim^r	0.9249	0.6924	107.5907	116.3230
	13	Analytic	0.9322	0.7069	107.5821	115.4055
		Sim^w	0.9240	0.8453	107.5666	116.4099
		Sim^r	0.9249	0.6924	107.5905	116.3222
	14	Analytic	1.5202	0.0000	18.1050	11.9100
		Sim^w	1.4049	0.0000	16.5675	11.7926
		Sim^r	1.4319	0.0000	16.8586	11.7738
	15	Analytic	1.5202	0.0000	19.9720	13.1382
		Sim^w	1.4062	0.0000	18.0962	12.8684
		Sim^r	1.4318	0.0000	18.3647	12.8265

^{w} Denotes the weighted distance simulation model

^{r} Denotes the real distance simulation model

the analytical model; i.e., 0.6000 respectively. In a worst case scenario, the ideal evacuation rates of the real simulation model will thus cause no or little congestion along the corridor paths to the exit doors. Second, all of our analyses assume that occupants are not blocked by any means of obstacles outside the hall. Any external blockages will definitely reduce the overall throughput. Thus, we can obtain almost the same throughput if we can ensure that other disturbance factors are not existed. Third, such evacuation rates must be acknowledged to occupants and be regularly practiced by them even though there are no emergency cases. Such a

Table 17 Performance measures of the intermediate and exit corridors

Type	Corridor	Model	λ	θ	$p(c)$	L	W
Source	6	Analytic	4.6000	4.6000	0.0000	7.3127	1.5897
		Sim^r	4.6000	4.6002	0.0000	7.3153	1.5902
	7	Analytic	0.6000	0.6000	0.0000	0.7198	1.1997
		Sim^r	0.2352	0.2354	0.0000	0.2811	1.1943
	8	Analytic	0.6000	0.6000	0.0000	0.8764	1.4607
		Sim^r	0.2287	0.2291	0.0000	0.3309	1.4447
	9	Analytic	4.6000	4.6000	0.0000	6.3714	1.3851
		Sim^r	4.7695	4.7707	0.0000	6.6621	1.3964
	10	Analytic	5.2000	5.2000	0.0000	12.9875	2.4976
		Sim^r	5.2000	5.2021	0.0000	12.9881	2.4967
	11	Analytic	3.4500	3.3781	0.0208	21.0002	6.2165
		Sim^r	3.4500	3.4140	0.0092	15.7968	4.7518
Intermediate	3a	Analytic	3.2000	3.0537	0.0457	19.7975	6.4831
		Sim^r	2.8333	2.6422	0.0665	23.1231	9.1543
Exit	1	Analytic	2.3000	1.0877	0.5271	51.0504	46.9341
		Sim^r	2.3000	1.0884	0.5254	50.8953	46.7608
	2	Analytic	2.6000	2.5051	0.0365	20.4604	8.1676
		Sim^r	2.4176	2.1848	0.0963	29.5179	14.2444
	3b	Analytic	1.5269	1.4990	0.0182	3.9610	2.6423
		Sim^r	1.3211	1.3205	0.0003	2.3395	1.7584
	3c	Analytic	1.5269	1.4990	0.0182	3.9610	2.6423
		Sim^r	1.3211	1.3205	0.0003	2.3346	1.7550
	4	Analytic	2.6000	2.4951	0.0403	18.8793	7.5665
		Sim^r	2.4999	1.9793	0.2072	41.0895	20.9475
	5	Analytic	2.3000	1.0877	0.5271	51.0504	46.9341
		Sim^r	2.3854	1.0883	0.5427	50.8974	46.7677
	12	Analytic	1.3000	1.2792	0.0160	31.9190	24.9523
		Sim^r	1.3005	1.2997	0.0000	25.8381	19.8795
	13	Analytic	1.3000	1.2792	0.0160	31.9190	24.9523
		Sim^r	1.3005	1.2997	0.0000	25.8375	19.8794
	14	Analytic	1.6891	1.6891	0.0000	20.4228	12.0912
		Sim^r	1.7070	1.7061	0.0000	20.5801	12.0608
	15	Analytic	1.6891	1.6891	0.0000	22.9379	13.5803
		Sim^r	1.7070	1.7059	0.0000	23.0566	13.5102

^{r} Denotes the real distance simulation model

systematic flow is better than allowing occupants to freely or randomly travel to whatever directions they want. This is important to ensure that they are not panic whenever emergency situations happen since panic tends to create havoc in the entire system and significantly decreases the throughputs.

In this paper, we only search the best evacuation rates of source corridors to vacant occupants from the DTSP hall under relevant conditions. We do not derive its optimal evacuation rates which maximize its throughput. However, the optimal evacuation rates based on the analytical model could be obtained by finding the optimal evacuation rate of each available corridor in the hall through the use of calculus and numerical analysis methods and inputting these values to its *network flow programming* model which considers pedestrian flows in the whole topological network to maximize its throughput. For the simulation model, we can only derive the near-optimal evacuation rates of the hall since the simulation model is only an input–output model; i.e., it produces outputs based on relevant input data which typically relate to certain types of distributions.

5 Conclusions and future research

We have implemented the *M/G/C/C* approaches for modeling a complex real-life system; i.e., the DTSP hall, Universiti Sains Malaysia, Malaysia. Its performance measures reported through the adjustment of evacuation rates can be utilized by the university's policy makers to best evacuate occupants from the hall and to recommend appropriate actions especially in emergency cases. Modeling and evaluating other much more complex *M/G/C/C* systems (e.g., Saie Area, Masjidil Haram, Mecca, Saudi Arabia during a Hajj period) is relatively easy through the implementation and extension of the concepts and methodologies discussed throughout this paper.

We could model *M/G/C/C* networks using a conveyer approach if its velocity can be altered during runtime. In this case, each conveyer represents a corridor and its velocity represents pedestrians' current walking speed in the corridor. Any flexibility which allows such a modification offers some advantages especially in providing good animation of pedestrian flows in the hall. We have also investigated the potential use of Arena's primitive modules in the *Blocks* panel to model the *M/G/C/C* mechanism. This closed to programming styles of modules such as *While*, *EndWhile*, *If*, *ElseIf*, etc. offer more flexibility in programming the logic of *M/G/C/C* networks and can help model users to understand its flow better.

All analytical results for the DTSP hall were measured when it is on a steady state (i.e., during its stable condition). In order to correctly validate these results, other statistical analysis strategies of outputs from steady state simulations should be employed. Note that a steady state simulation is not supposed to have initial conditions and is supposed to go forever. However, all simulation models have to be initialized and stopped somehow. Thus, in order to neutralize the bias of the initial conditions, a warm-up period is imposed where all performance measures are calculated after this time point and the run length should be long enough. If the warm-up is not too long (i.e., it is shorter relative to the run length), the *truncated replication* approach which replicates the simulation for a number of replications with each replication contains the warm-up period could be employed. If the warm-up period is fairly long, we can employ the *batch means* approach which only requires a really long run (to ensure uncorrelated observations) with the warm-up period at the start and then breaks the run into a few large batches. Details on these can be obtained in many DES text books (e.g., Altiok and Melamed 2007; Banks et al. 2010; Kelton 2009; Rossetti 2010; Wainer and Mosterman 2010).

Our future researches include finding the ideal routes which can best flow occupants from the hall in order to minimize its evacuation time. At this moment, we assume that occupants will travel to their nearest intermediate or exit corridors. However, we may improve the overall throughput if we channel them to appropriate intermediate or exit corridors. For example, we could channel most of pedestrians from corridor 10 to corridor 12 or corridor 13 than to corridor

3, since corridor 3 will be filled by some of pedestrians from corridor 7 and corridor 8. We could also channel most of pedestrians from corridor 6 to corridor 1 in order to decrease the number of pedestrians entering corridor 2, since corridor 2 will be filled by some pedestrians from corridor 7. How far our premises are true can be validated using *OptQuest*. We will also attempt to evaluate the performances of the hall using other distributions of evacuation rates. These include constant, normal, triangular, etc. By doing this, we can compare how the performance measures of the hall vary across different types of statistical distributions.

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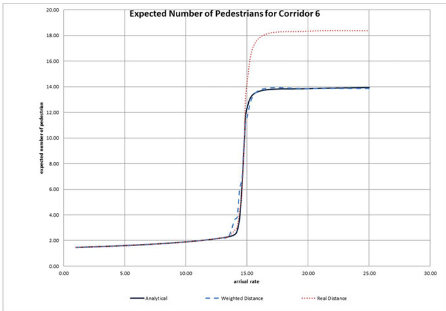
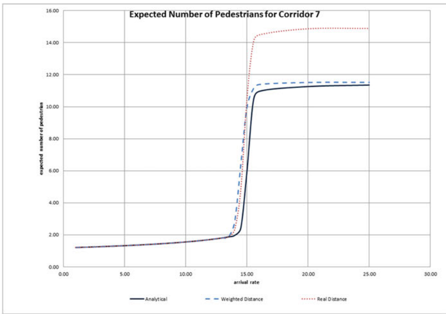
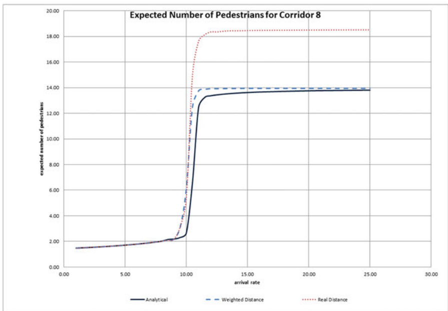
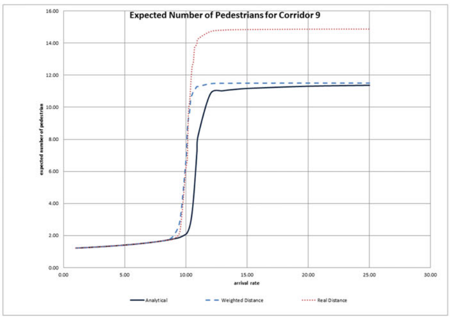
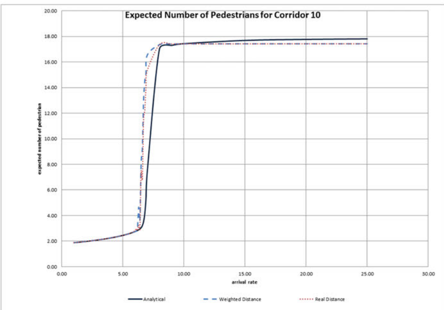
Appendix 1. The expected service time for source corridors

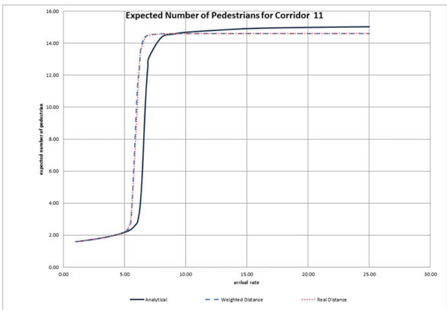




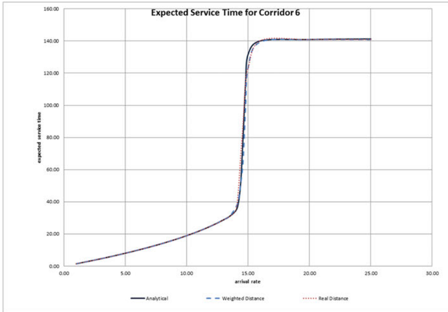


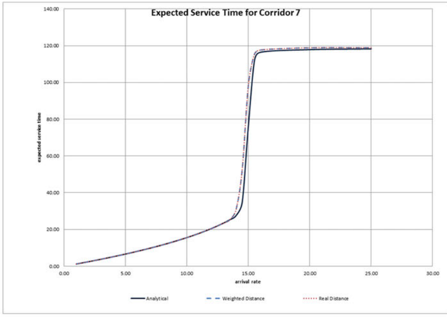


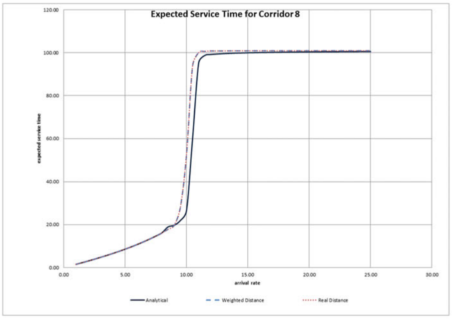


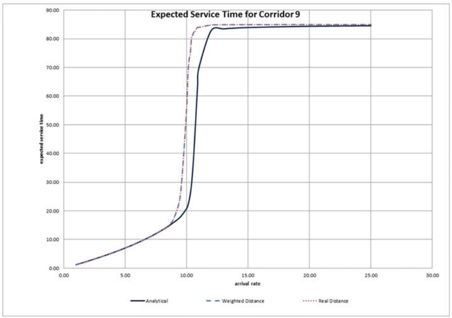


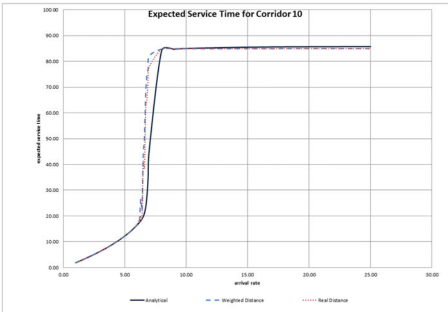
Appendix 2. The expected number of pedestrians for source corridors

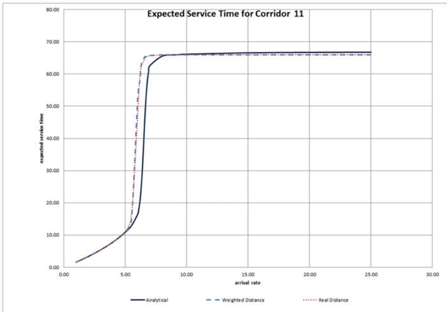












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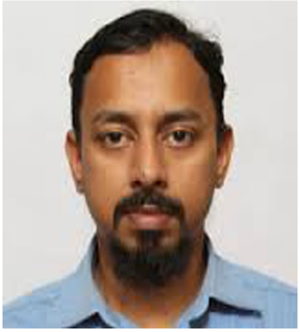

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