A holistic approach to assessment of value of information (VOI) with fuzzy data and decision criteria.

VILELA IBARRA, M.J.

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A holistic approach to assessment of value of information (VOI) with fuzzy data and decision criteria

Martin Jose Vilela Ibarra

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DECLARATION

This thesis, and the research underpinning it, is my own work. It has not been submitted in any previous application for a degree. The sources of information have been acknowledged in the text and included in the references of the thesis.

Signed:……………………………………………………………………………

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Whilst this thesis is the result of my own work, a few people have accompanied me during this doctoral study, and I would like to take this opportunity to recognize them.

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ABSTRACT

The research presented in this thesis integrates theories and techniques from statistical analysis and artificial intelligence to develop a more coherent, robust and complete methodology for assessing the value of acquiring new information in the context of the oil and gas industry.

The classical methodology for value of information assessment has been used in the oil and gas industry since the 1960s even though it is only recently that more applications have been published. It is commonly acknowledged that, due to the large number of data acquisition actions and the capital investment associated with it, the oil and gas industry is an ideal domain for developing and applying value of information assessments.

In this research, three main gaps in the classical methodology for value of information are identified and addressed by integrating three existing techniques from other domains. Firstly, the research identifies that the technique design of experiments can be used in value of information for providing a holistic assessment of the complete set of uncertain parameters, selecting the ones that have the most impact on the value of the project, and supporting the selection of the data acquisition actions for evaluation. Secondly, the fuzziness of the data is captured through membership functions, and the expected utility value of each financial parameter is estimated using the probability of the states conditioned to the membership functions (in the classical methodology, this is conditioned to crisp values of the data). Thirdly, a fuzzy inference system is developed for making the value of information assessment, capturing the decision-making human logic into the assessment process, and integrating several financial parameters into one.

The proposed methodology is applied to a case study describing a value of information assessment in an oil field where two alternatives for data acquisition are discussed. The case study shows how the three techniques can be integrated within the previous methodology, resulting in a more complete theory. It is observed that the technique of design of experiments provides a full identification of the input parameters affecting the value of the project and allows a proper selection of the data acquisition actions. In the case study, it is concluded that when the fuzziness of the data is included in the assessment, the value of the data decreases compared with the case where data are assumed to be crisp; this result means that the decision concerning the value of acquiring new data depends on whether the fuzzy nature of the data is included in the assessment and on the difference between the project value with and without data acquisition. The fuzzy inference system developed for this case study successfully follows the logic of the decision maker and results in a straightforward system to aggregate decision criteria. Sensitivity analysis of the parameters of two different membership functions is made, reaching consistent results in both cases.

Keywords: value of information, fuzzy logic, fuzzy inference system, uncertainty, oil and gas industry, design of experiments, risk attitude, reservoir.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

CAPEX	Capital Expenditure or investment
Cp	centipoise
d	Day
Datum	Reference depth
DOE	Design of Experiments
DPI	Discounted profit-to-investment ratio
E&P	Exploration and Production
EMV	Expected Monetary Value
EUT	Expected Utility theory
EUV	Expected Utility Value
EV	Expected value
EVT	Expected Value Theory
FIS	Fuzzy Inference System
IRR	Internal Rate of Return
Km	Kilometre
m	Meter
mD	Mili-Darcy
MSCF	Thousand standard cubic feet

NPV	Net Present Value
OPEX	Operational Expenditures
Psi	Pounds per square inch
SEUT	Subjective Expected Utility Theory
s_{orw}	Oil saturation relative to water
STB	Standard Barrel of Oil
s_{wirr}	Irreducible water saturation
TVDSS	True vertical depth sub sea
vNM	von Neumann and Morgenstern
VOI	Value of Information

Chapter One

Introduction and Justification of Research

1.1. INTRODUCTION

The aim of this chapter is to introduce this research project and to outline the subjects underpinning this research. The research discussed in this thesis is founded on the classical methodology of the value of information (VOI), and applications in the oil and gas industry. It contributes to the development of a more robust methodology for valuing data acquisition in projects related to the exploitation of hydrocarbons and integrates theories that have not yet been used in classical methodologies, such as the design of experiments (DOE), fuzzy logic and fuzzy inference systems (FIS).

1.2. BACKGROUND TO THE THESIS

VOI is a normative theory (i.e. related to how people should decide in a rational manner) for making decisions concerning data acquisition. It was developed in the early 1960s in research by Schlaifer (1959), Grayson (1960), Raiffa and Schlaifer (1961) and Newendorp (1967). The purpose of VOI is to assess the benefits of gathering information prior to making a decision.

Most decision-based problems in the real world are characterised by uncertainty, and the oil and gas industry is not an exception. Several elements need to be estimated in order to assess the benefits of exploring and producing oil and gas fields: (i) the series of future rewards (hydrocarbon production during the project period); (ii) the series of current and future investments (the costs of wells, flowlines, surface facilities); (iii) future operating costs; (iv) future hydrocarbon prices; and (v) the financial terms included in the contract (taxes, royalties, etc.). Of these elements, at least the first four carry uncertainty.

One of the key aspects of VOI is the possibility of assessing whether it is worthwhile to acquire new information, and this depends on the possibility of using new data to change decisions that would be made differently without that information. This is an important requirement for meaningful data acquisition and is based on the belief that value is associated with a reduction in uncertainty or an increase in confidence (Bratvold, 2007).

For many of the uncertain input parameters that are responsible for the determination of the value of a project, there is nothing that can be done to change the perception of their uncertainties; however, there are cases in which data can be collected that can change our understanding of the uncertainties related with one or more input parameters, and these changes may impact on the project assessment.

A review of the published VOI applications in the oil and gas industry (Moras et al., 1987; Gerhardt and Haldorsen, 1989; Stibolt and Lehman, 1993) shows that most VOI applications are defined based on specific actions. This means that the classical approach is an activity based VOI that assesses the impact (positive or negative) of the proposed data acquisition on the project's value. Using this approach, the decision maker loses the chance to take a holistic view of the full set of uncertainties of the project and to assess all the (data acquisition) actions that can increase its value, including the possible interactions between variables that can influence the best alternative for gathering new data, which is a project-based VOI. The authors of this research found only one reference (Coopersmith and Cunningham, 2002) that included an attempt to capture a complete view of the potential increase in the project's value via data acquisition; in addition, this reference lacks a methodological approach for reaching that goal and seems to be more a declaration of intention than a thorough analysis of this approach for carrying out VOI assessments. It is proposed to overcome this existing gap in the classical methodology by using DOE techniques.

As discussed later in Chapter 2 (Literature Review), uncertainty plays a key role in using decision analysis techniques to assess decision problems. In the VOI references reviewed in this chapter, uncertainty is ascribed to randomness, which is managed using probabilistic tools and techniques. Bellman and Zadeh (1970) show, however, that uncertainties arise not only due to randomness (i.e. inaccuracies in measurement or a lack of information) but also due to fuzziness in the data. For these authors, this factor is a major source of uncertainty in many decision problems. The classical methodology for VOI does not include a consideration of the fuzziness of the data, and this represents a gap in the classical VOI methodology. Filling this gap is one of the main contributions of the current work.

There is also another kind of fuzziness present in the VOI methodology, and this is related to the outcomes of the VOI assessment. In classical VOI, the outcomes of a project's assessments are crisp numbers; however, in practice, a decision maker's reasoning follows fuzzy logic more closely than crisp logic. Human logic does not have crisp boundaries in the same way as Boolean logic (zero or one, black or white), and involves degrees rather than extreme values. This form of reasoning can be implemented, and conclusions can be derived using fuzzy logic and FIS (FIS is explained in Chapter 2). In this research work, an FIS is developed to assess the VOI associated with an oil field exploitation project.

In the 18th century (see Chapter 2, Literature Review), researchers acknowledged that each decision maker has a particular attitude to risk that could be described using mathematical expressions. It was also understood that a theory for assessing decision problems should include this attitude to risk. Although Grayson (1960) and Howard (1966) have shown how risk attitude can be integrated into the classical VOI assessment, a review of the published literature on VOI applied to the oil and gas industry shows that the risk attitude of the decision maker is rarely included in these assessments; typically, the criterion used to assess whether additional data should be acquired is based on the expected monetary value. The use of this criterion is equivalent to assuming that the decision maker is risk-neutral, which is generally not the case. In the current research, which establishes a complete methodology for VOI, the decision maker's attitude toward risk is included as a fundamental part of the assessment.

The current study aims to develop a complete VOI methodology that is applicable to projects in the oil and gas industry; the distinctive elements of this approach, as compared with the classical one, are the integration of theories and methodologies such as DOE, fuzzy logic and fuzzy inference systems for assessing VOI decision problems. In the design of the proposed VOI methodology, the decision maker's risk attitude is included as part of the workflow.

1.3 VALUE OF INFORMATION IN THE OIL AND GAS INDUSTRY

The classical approach to VOI has been applied in several domains. The oil and gas industry, characterised as making a large number of decisions involving huge amounts of resources, is one domain that can obtain great benefits from the use of this methodology.

In most of the decision problems, several or all input variables have uncertain values, which produces uncertainty in the outcomes of the problems; those uncertainties are present because of the limitation of the information either due to scarcity, randomness or vagueness. In that sense, acquiring additional data is a way to reduce the uncertainty. However, acquiring data has a negative impact in the project's value either due to the cost of the data or because of the delay in implementing the project. VOI is the methodology for assessing if acquiring data is worthwhile: it compares the value of the project with the current information with the value the project could have if additional data are acquired.

In the oil and gas industry typically, wells are separated by hundreds of metres to several kilometres, and that is the distance between points of hard data, e.g. porosity, saturation, etc.; inferred data are estimated between points with hard data, which makes the inferred data have uncertainty; in addition, the way in which reservoir properties distributed in nature is very complex, which makes that their description is based on randomness. Uncertainties due to measurements and interpretations are also present. All the mentioned uncertainties make that taking additional data could be, in particular cases, a convenient manner to reduce uncertainty and increase the value of the project. However, due to the cost associated with the acquisition of data and the potential delay in the project, getting more data does not always increase the value of the project. VOI is the methodology that will tell the decision maker whether acquiring more data is worthwhile.

In this research, it is proposed to make several changes to the classical VOI approach with the aim of optimising its processes and outcomes.

1.4 THE CLASSICAL METHODOLOGY FOR VALUE OF INFORMATION

Through the years, several authors have explained and used methodologies for VOI (Clemen, 1996; Newendorp and Schuyler, 2000; Koninx, 2000; Coopersmith and Cunningham, 2002; Bratvold, Bickel and Lohne, 2007); however, in most of the applications published, the methodology for VOI is not clearly stated and explained.

During the literature review made by the authors for this research (Chapter 2), several gaps were found, which if properly addressed will generate a more robust and consistent methodology for VOI.

The gaps found are associated with: i) whether the objective of VOI assessment is to prove that one specific data acquisition improves the value of the project or to find the data acquisition action that optimises the value of the project, ii) acknowledging that, in addition to the uncertainties due to lack of information and randomness (see Chapter 2 for a discussion of those terms), there could be also uncertainty due to the fuzziness of the data proposed to be acquired, and iii) recognising that decision are made by humans, and the tools, algorithms and methodologies helping in the decision-making process should follow a similar logic and terms as those used by the decision maker (otherwise, the methodology will be barely used).

In addition, during the research it was identified that the important concept of utility value (see Chapter 2 for a discussion on this topic) is scarcely used in the published applications of VOI, although its significance was proved by von Neumann and Morgenstern in the 1940s (1944).

1.5 RESEARCH SCOPE

In this section it will be discussed the justification for making this research work, the research questions which we target to answer in this thesis and the contribution to knowledge resulting from this investigation.

1.5.1 Research justification

The three most important justifications of this research work are anticipated as follows:

1) The classical methodology for VOI is activity-based instead of project-based: it assesses whether it is worth acquiring a specific information, but it does not search for which acquisition of information maximize the value of the project.
2) The classical methodology for VOI assumed the data is crisp with no uncertainty due to fuzziness; however, data can have uncertainty associated to fuzziness.
3) The classical methodology for VOI uses Boolean logic for assessing decisions which is different from the human logic which characterizes by vagueness.

In this research, a methodology is developed for carrying out VOI assessments (Chapter 3) that can fill the research gaps discussed in Sections 1.2 and 1.4.

1.5.2 Research questions

Two research questions are raised in the work:

1) How can the VOI methodology be converted from an activity-based process to a project-based process?

This question is motivated by the observation that the focus of VOI is typically on an assessment of the potential increase in the value of the project due to specific data acquisition actions impacting on one specific input data; there is no formal and systematic methodology for searching, within the project, for several competing data acquisition actions with the objective of maximising the project's value once all the potential alternatives of data acquisition have been considered. In the case of interactions between parameters, a given data acquisition action can impact on more than one parameter, and this one-to-many relationship should form part of the analysis for VOI optimisation.

2) How can the fuzzy nature of data and the fuzzy characteristics of the decision process be integrated into the classical VOI methodology?

This question is prompted by the observation that data in general and reservoir data in particular can be fuzzy, and that the uncertainty related to this fuzziness is not captured within the framework of the classical VOI.

In addition, the decision process followed by human beings is based on linguistic variables that obey the fuzzy logic, rather than the crisp logic observed in the classical VOI; an FIS can be developed to mimic human logic in the VOI assessment.

1.5.3 Contribution to knowledge

This section shows how the research presented in this thesis has generated a robust set of findings that contribute to the body of knowledge in the field of VOI, especially for applications in the oil and gas industry.

The contribution to knowledge of this research work is three-fold:

1) It integrates the uncertainty associated with the fuzziness of the subsurface data into the VOI decision assessment; the classical VOI methodology considers the uncertainty in the data associated with randomness and lack of knowledge but does not include the uncertainty due to fuzziness. It is proved that the failure to take into account the fuzziness of the data overestimates its value, and under certain circumstances, this can affect the final decision on whether to acquire subsurface data.
2) It develops a system for making VOI decisions that resembles a decision maker's logic; this approach is different from the one used in the existing methodology for VOI, which is based on

crisp threshold criteria. In addition, the proposed system permits the integration of more than one decision criterion into a single criterion used for making the decision.

3) It develops a holistic approach to determine which of the uncertain input variables have the most impact on the project's value and ranks them; this allows for selecting the data acquisition actions that provide more value to the project and assesses the benefits associated with those data acquisition actions; this approach pushes forward the classical VOI methodology, which focuses on the evaluation of isolated data acquisition activities.

1.6 RESEARCH AIM AND OBJECTIVES

The aim of this project is to develop a complete methodology for decision making that can assess the value of acquiring data in a holistic manner in the context of the oil and gas industry, integrating both the fuzzy nature of the data and the fuzzy decision criteria used by the decision maker.

The objectives of this research are to:

A. Integrate in the VOI assessment all the uncertain input parameters that impact on the value of the project;
B. Include the fuzzy nature of the data in the VOI assessment;
C. Develop a VOI decision-making assessment that uses the logical rules followed by the (human) decision maker.

1.7 RESEARCH METHODOLOGY

Based on a review of prior work carried out on the VOI methodology applied in the oil and gas industry, as well as in other domains, several gaps in the methodology are identified and some theories and techniques are proposed which, when appropriately integrated within the classical VOI methodology, will provide a much more robust assessment.

The identification and ranking of the main uncertain parameters that impact reservoir performance and the decision criteria is carried out through the application of statistical methods, DOE techniques, on the different production forecasts, which result from smart combinations of the uncertain parameters.

Fuzzy logic is used to characterise the uncertainty associated with the fuzziness of the data; the mathematical model describing the proposed VOI assessment for fuzzy data is discussed in this work and it is compared with the classical VOI assessment.

In this research an FIS is developed for assessing the VOI decision problem using more than one financial parameter, the utilities of NPV and IRR.

This new methodology for VOI is applied to a case study, in order to compare the classical approach with the proposed alternative.

1.8 OUTLINE OF THE THESIS

The thesis is divided into five chapters. The first chapter is the introduction of and justification for the research and describes the general background, emphasising the gaps found in the classical VOI approach and identifying the theories and methods proposed in this research to fill these gaps. In this chapter, the research questions are also discussed, as well as the methodology used in the current study, which is compared to the classical methodology for assessing VOI in the oil and gas industry. The theories and methodologies integrated within the VOI assessment have been previously used in other domains and with other objectives; it is shown that they can be successfully used to fill the gaps found in the classical VOI assessment.

A literature review is presented in Chapter 2; this draws on the literature related to VOI, describes the limitations of existing studies and reviews the theories and methodologies that are used in this research to fill the gaps in the classical methodology. This chapter provides the theoretical background supporting the VOI assessment and the theories of DOE, fuzzy logic and FIS that are used in this research; in addition, it also discusses the importance of including the decision maker's attitude to risk in the proposed methodology.

Chapter 3 discusses the current VOI methodology and acknowledge its limitations; then, it builds the proposed new methodology for VOI using methods and techniques develop on other domains.

Chapter 4 presents a case study of a VOI assessment based on an oil and gas project. The proposed methodology is fully applied in this case study and the results of both the classical and proposed VOI methodologies are reported, compared and critically discussed, and explanations for the reasons underpinning these results are included, where necessary.

Chapter 5 revisits the research questions formulated in Chapter 1 on the basis of the results of this research; discussion on future researches relevant to the field that were identified during this research is presented in this chapter; a final summary of this research is presented at the end.

Chapter Two

Literature Review

2.1 INTRODUCTION

This chapter presents the literature review for this research. It draws on the existing academic and practical applications of the classical VOI theory to identify the gaps in this methodology and inform other theories that can be integrated with the classical approach for generating a more reliable theory for data acquisition.

This research work is founded on four pillars: the discipline of decision analysis; the classical methodology for valuing the gathering of information; the design of experiments; and fuzzy logic and fuzzy inference systems; additionally, it assesses risk attitudes using utility functions. These theories are integrated with the aim of generating a robust methodology for VOI. This section is dedicated to discussing in more detail certain aspects of these disciplines and theories that are especially important in our research. Section 2.2 discusses the development of decision analysis theory, the origin of expected value as a decision criterion for making decisions and the concepts and theory supporting the assessment of the risk attitude of the decision maker by means of the utility functions; although this is a fundamental part of decision theory, it is rarely used in the VOI applications reported in the oil and gas domain. Section 2.3 discusses the development of the value of information theory, in particular in the oil and gas industry, and develops the main equations required to make assessments regarding data acquisition. Section 2.4 discusses important concepts used in this research, such as uncertainty, risk, probability and decisions. Section 2.5 reviews the theory of DOE and the main aspects of this methodology that are used in this research. Section 2.6 explains fuzzy logic, fuzzy data and how a fuzzy inference system is implemented in practical applications for making decisions; it also includes an update of the equations for assessing value of information (discussed in Section 2.3 for crisp data) for the case when the data is fuzzy.

2.2 DECISION ANALYSIS, RISK AVERSION AND UTILITY FUNCTIONS

A decision is choosing what to do or not do, resulting in a desirable outcome (Tang, 2006); Howard (1966) defines a decision as an absolute allocation of resources, absolute meaning that it is not possible or it is very costly to change back to the original situation before making the decision; so, a decision is the actual search for the course of action, not a mental obligation.

Decision analysis defines the procedures for logically trading off the factors influencing a decision, including uncertainties, values and preferences, in a design that models the decision. The last objective of decision analysis is to support decision makers to make better decisions. Decision analysis is a normative discipline that describes how people should logically make decisions (McNamee and Celona, 2008); normative models are distinct from descriptive models, which describe how people make decisions, regardless of rationality (Howard, 1988). Howard (1968) described decision analysis as the formal introduction of logic and preferences into the decisions through a combination of philosophy, methods, practice and applications. Decision making is the most important activity of a manager and, very often, it is a difficult task (Taghavifard, Khalili and Tavakkoli, 2009). In decision problems, the decision maker has at least two or more alternatives from which to choose which one is more desirable according to his/her preferences and values.

In 1654, Pierre de Fermat and Blaise Pascal proposed using the Expected Value (EV) method to choose between alternatives when the value and probability of each alternative are known. According to Schoemaker (1982), EV has been the major paradigm and the ruling method of making decisions since the 1940s. EV incorporates the chances of realizing the alternatives and, according to Hansson (2005), it is the approach used most often for decision making under risk.

Using EV, the decision maker chooses between risky alternatives by comparing their EV, i.e., multiplying the value of all the possible outcomes by their probabilities (Newendorp and Schuyler, 2000). In this definition, value is what matters to the decision maker and, when the value is monetary value, it is called Expected Monetary Value (EMV). When EMV is the decision criterion, the decision maker is impartial to money (Newendorp and Schuyler, 2000); however, most of the time, people are not impartial to money; this limitation of the theory of EV in real case situations was discussed by mathematicians back in the seventeenth century, giving rise to the concept of utility.

There are other methods, besides EV, which suit more simple applications and have been proposed for alternative selection in the presence of uncertainty, some of which are (Aguiar, 2004):

- Maximin or Wald criterion: no information on probabilities, select the alternative with the higher minimum. This is a conservative approach for choosing between alternatives and does not use all the information available.
- Maximax criterion: no information on probabilities, select the alternative with the higher maximum. This is an optimistic approach for choosing between alternatives and does not use all the information available.
- Hurwicz criterion: the weighted sum of the alternatives with extreme values; the weighted factors should sum to one; similarly, to Maximin and Maximax, not all the available information is used.
- Laplace’s sufficient reason criterion: assign the same probability for each alternative and sum the values times probability.

The first three criteria have important limitations because the chances of realizing the different alternatives are not incorporated into the analysis; this means that, because there is no criterion to decide which of these three criteria is more suitable, the final assessment will be influenced by the subjectivity of the decision maker in selecting one method over the other.

Laplace’s criterion uses the same probability for each alternative, which is unlikely to occur in a real situation. However, all these methods can be considered objective in the sense that once the method is decided, the assessment produces the same outcome.

In the earlier application of EV, the value was monetary value and the recommendation was to decide positively if the expected wealth increased; the probabilities were based on objective frequencies, similar to those observed in dice games (Hansson, 2005).

The expected value of a lottery A with outcomes x_i and probabilities of occurrence p_i is

$$E(A) = \sum_i x_i p_i \qquad (1)$$

For the case in which the outcomes form a continuous range of values

$$E(A) = \int_{-\infty}^{\infty} x \, dp \qquad (2)$$

The expected value is the average value of a lottery played many times. Until the middle of the 20th century, this concept was used for deciding between lotteries, whose uncertainty is represented by the probabilities. This criterion assumes that the decision maker's only concern is the value of the money.

Another criterion for choosing between alternatives is the Bayesian which combines the Hurwicz and Laplace criteria, using the information of the value of all alternatives and assigning, subjectively, probabilities to each alternative. The result of the action is the sum of the value of each alternative times their probabilities. The name Bayesian refers to the mathematician Thomas Bayes, who developed the theory for updating probabilities based on new information. The Bayesian criterion was not in fact developed by Bayes, even though it takes his name.

In 1713, the mathematician Nicolas Bernoulli posed a problem, now known as the Saint Petersburg paradox, in which outcomes using the EV method contradict common sense. This paradox was solved years later by his cousin, Daniel Bernoulli in 1738, as cited by D. Bernoulli (1954), using for the first time the concepts of utility value (instead of value) and expected utility value (EUV) (instead of expected value) maximization for assessing gamblers' preferences; he realized that the contradiction between the maximization of the EV criterion and the decision maker's common sense can be solved if it is assumed that the value that people assign to money, their preferences, is not a linear function of money but can be represented by a logarithmic function that describes a diminishing marginal utility for money.

The Saint Petersburg paradox consists of a lottery with a fixed entranced fee, in which a fair coin is tossed repeatedly until a "tail" appears, which ends the game. If the number of times the coin is tossed until a "tail" appears is k and the bidder wins 2^{k-1} units of money, how much would the decision maker be willing to pay as an entrance fee to play this lottery?

Following the criterion of expected value, the decision maker should be willing to pay an entrance fee equal to or less than the expected value. The probability that a fair coin shows a tail after k times not having shown a tail in the previous k-1 times is

$$P_k = (\tfrac{1}{2})^k \tag{3}$$

Consequently, the expected value is

$$E(A) = \sum_{k=1}^{\infty} x_k p_k = \sum_{k=1}^{\infty} 2^{k-1} (\tfrac{1}{2})^k = \sum_{k=1}^{\infty} \tfrac{1}{2} = +\infty \tag{4}$$

This means that, according to the expected value criterion, the decision maker should be willing to pay an arbitrarily large amount of money to participate in the lottery; however, the probability that the decision maker will win $128 = 2^7$ units of money is less than 1%. Therefore, it is very unlikely to win any important amount of money because the probability of occurrence is very low. Thus, contrary to the results of using the expected value of money as a decision criterion, most people refuse to pay more than just a few units of money to participate in this lottery.

Daniel Bernoulli realised that the expected monetary value criterion assumes that the same amount of money gained or lost might have a different meaning to a person, depending on several factors, which means that the appetite for money is not a linear function, e.g., if you have one million dollars your preference for winning another million dollars is higher than if you have one billion dollars, or not always is doubling the money twice as good. Based on these discussions, Bernoulli suggested that the parameter for making a decision should not be the value of money but the utility that the decision maker assigns to that money, which is a measure of the desirability or usefulness that the decision maker has.

This reasoning means that instead of maximising the expected monetary value, what should be maximised is the expected utility value, where the utility is a function u that assigns to every monetary value a number that represents the person's utility of that value; the monetary value of a project is its value in money terms while the utility value of a project is its value considering the risk attitude of the decision maker. The expected utility is

$$E(u) = \sum_i u(x_i)p_i \tag{5}$$

or for continuous outcomes

$$E(u) = \int_{-\infty}^{\infty} u(x)dp \tag{6}$$

Let us assume that the utility function has a logarithmic form, $u(x) = \ln(x)$, then for the Saint Petersburg lottery

$$E(u) = \sum_k u(x_k)p_k = \sum_k \ln\left(2^{k-1}\right)\left(\frac{1}{2}\right)^k = (ln2)\sum_k \frac{k-1}{2^k} < +\infty \tag{7}$$

This proves that using a utility function with a diminishing value of money ($\ln(x)$ grows more slowly as x increases) generates an expected value for the Saint Petersburg paradox that is closer to the value that a person assigns to this problem.

Similar results can be obtained with other concave functions, such as squared root or exponential functions. In general, a function $u: R \to R$ is concave on an interval (a, b) $if\ for\ all\ x_1, x_2\ \in (a, b)\ and\ \lambda \in (0,1)$, the following inequality holds:

$$\lambda u(x_1) + (1 - \lambda)u(x_2) \leq u(\lambda x_1 + (1 - \lambda)x_2) \tag{8}$$

u is strictly concave if the above inequality is always strict.

Similarly, a function $u: R \to R$ is convex on an interval (a, b) $if\ for\ all\ x_1, x_2\ \in (a, b)\ and\ \lambda \in (0,1)$, the following inequality holds:

$$\lambda u(x_1) + (1 - \lambda)u(x_2) \geq u(\lambda x_1 + (1 - \lambda)x_2) \tag{9}$$

u is strictly convex if the above inequality is always strict.

A person is risk-averse if he prefers the expected value of the lottery or less over the lottery itself, and a person is risk-seeking if he prefers every lottery over its expected value or more.

The theory for decision making based on Bernoulli's ideas is called the expected utility theory (EUT), also known as the subjective expected utility theory to stress that in some cases the probabilities are subjective rather than objective numbers.

Most people's risk attitude is described as risk-averse and the corresponding utility functions are concave, such as logarithm, exponential and square root functions; however, under specific circumstances, people's attitude can be described as risk-seeking or risk-neutral (for more details see this section below).

In his work, Bernoulli assumed that: 1) the utility function exists (the issue being the functional form or shape of it), and 2) probabilities are objective, resulting from the chances that each option has of occurring in a gamble or lottery. EUV combines profitability estimates with quantitative estimates of the degree of risk, producing a risk-adjusted decision criterion (Newendorp and Campbell, 1971) that is useful for comparing the desirability of different investment alternatives available to the decision maker. The Expected Utility Theory (EUT) can be understood as an extension of the Expected Value Theory (EVT), including the preferences of the decision maker.

It was more than two centuries later when Bernoulli's ideas about risk attitudes were formalized by John von Neumann and Oskar Morgenstern (1944) as part of a broader study about the theory of strategies against an opponent (Game Theory). Von Neumann and Morgenstern (vNM) developed a mathematical theory consisting of four axioms; completeness, transitivity, continuity and independence, which are the necessary and sufficient conditions for ensuring that the decision maker's attitude towards risk can be described quantitatively by a utility function that completely describes his/her value system with respect to money. When the decision maker's preferences follow those axioms, the decision maker is called a rational decision maker; in the case that the set of outcomes is infinite, the sure-thing axiom should be added to the four axioms, to ensure the existence of the utility function. vNM theory was a significant step in the theory of decision making because it ensures the existence of a utility function as long as the decision maker's preferences follow a set of axioms.

What vNM developed was a set of minimum axioms which, if satisfied by the decision maker's preference system, secure the existence of a utility function, with the properties described by Bernoulli.

The vNM theory lies on the four axioms described below:

1) *Completeness axiom*: For every pair of possible alternatives, A and B, either $A \prec B, A \succ B \; or \; A \sim B$.

This axiom ensures that the decision maker always has some opinion when deciding between alternatives. EUT satisfices this axiom as long as the utility function has a finite value.

2) *Transitivity axiom*: For every alternative, A, B, C with $A \preccurlyeq B \; and \; B \preccurlyeq C$, it has $A \preccurlyeq C$

This axiom provides order to the alternatives. EUT satisfies transitivity axioms because utility functions are real number functions satisfying transitivity.

3) *Independence axiom*: Let A and B be two lotteries with $A \succ B$, and let $\lambda \in (0,1]$, then for every lottery C, it must hold that

$$\lambda A + (1 - \lambda)C \succ \lambda B + (1 - \lambda)C \tag{10}$$

This axiom implies that when a rational decision maker chooses between two lotteries that are partially identical, the decision depends on the difference between the two lotteries, not on the identical part.

This third axiom makes use of the notion of combining lotteries: let A and B be lotteries and $\lambda \in [0,1]$, then $\lambda A + (1 - \lambda)B$ is a new lottery where A has probability λ and B has probability $1 - \lambda$.

4) *Continuity axiom* (or Archimedean axiom): Let A, B, C be lotteries with $A \succcurlyeq B \succcurlyeq C$, then there exists a probability p such that $B \sim pA + (1 - p)C$

This axiom makes a reasonable statement that, having three lotteries with an established order of preferences, there should be a way to mix between the most and the least preferred lotteries with a similar preference as the medium preference lottery.

The first two axioms of the vNM theory are not related with lotteries, while the other two are.

In view of the vNM axioms, the EUT can be formulated as follows:

Theorem: A preference relation that satisfies the completeness, transitivity, independence and continuity axioms can be represented by an EUT functional.

Because the four axioms are very reasonable and it is typically said that a reasonable person obeys them, EUT is a good prescriptive theory of decisions under risk.

The achievement of the vNM theory is that it progresses EUT from the special and concrete ideas of Bernoulli to a very general and abstract formulation.

Bayes (1764) developed a method, Bayes' theorem, for updating probabilities based on new information. However, Bayes' theorem does not specify how to estimate the original or prior probabilities.

The utility function described by Bernoulli and vNM theories contains the decision maker's attitude towards risk. Let us assume that the utility function is an increasing function $u: R \rightarrow R$, which represents the decision maker's preferences over lotteries on his or her wealth, where a lottery is a set of outcomes or consequences with their corresponding probabilities of occurrence $F: X \rightarrow [0,1]$. It will be assumed that u is differentiable as needed and, given a continuous lottery, the expected utility is:

$$E_F(u) \equiv \int u(x)dF(x) \tag{11}$$

Similarly, the expected wealth for a given F is:

$$E_F(x) \equiv \int xdF(x) \tag{12}$$

A comparison of Equations (48) and (49) defines the risk attitude of the decision maker. A decision maker whose preferences are described by the inequality in Equation (13) is called a risk-averse decision maker:

$$E_F(u) \leq u(E_F(x)) \tag{13}$$

Similarly, if the decision maker's preferences are described by Equation (14), he or she is called a risk-neutral decision maker:

$$E_F(u) = u(E_F(x)) \tag{14}$$

Finally, a decision maker with preferences described by the inequality in Equation (15) is called a risk-seeker:

$$E_F(u) \geq u(E_F(x)) \tag{15}$$

These criteria can also be described in terms of the concavity of the utility function, using a set of conditions relating risk attitude to the shape of the curve:

A) A decision maker is risk-averse if and only if u is concave;

B) A decision maker is risk-neutral if and only if u is linear;

C) A decision maker is a risk-seeker if and only if u is convex.

The risk attitude can be represented using utility functions and, for a risk-averse decision maker, the utility function is curved and opens downward (a concave shape), which converts money units ($) into utility units. Figure 2.1 represents a risk-averse utility function.

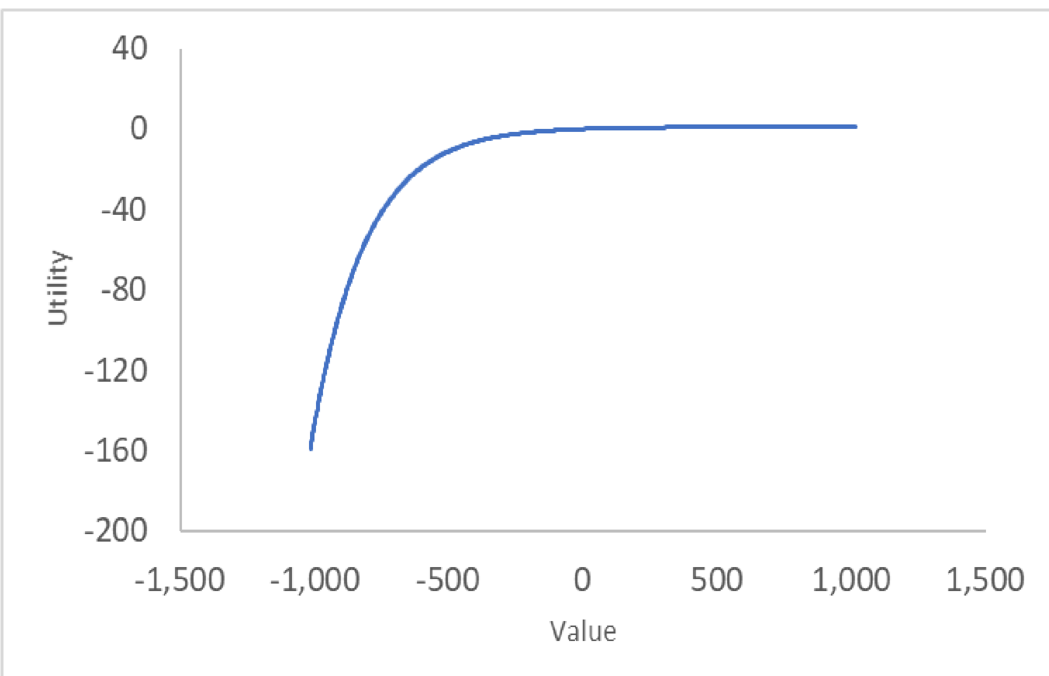

Figure 2.1. Risk-averse function

The utility function can be specified using a table of data, a graph (as in Figure 2.1) or a mathematical expression. In the case of a mathematical expression, a typical risk aversion function has the form of an exponential, square root or logarithm:

$$U(x) = 1 - e^{-x/R}\text{, exponential} \tag{16}$$

$$U(x) = \sqrt{x}\text{, square root} \tag{17}$$

$$U(x) = \log{(x)}\text{, logarithm} \tag{18}$$

All of these functions are characterised by an upward slope and a concave shape. The upward slope indicates that an increase in value corresponds to an increase in utility, while concavity implies that the individual is risk averse.

In practical applications, a decision maker is risk-averse when the value he or she assigns to a lottery is lower than the expected value of that lottery; in other words, a sure thing is preferred for a lower value rather than the lottery for its expected value. The secure value that the decision maker is willing to pay for the lottery is the certainty equivalent; the difference between the expected value of the lottery and the certainty equivalent that the decision maker assigns to the lottery is called the risk premium, which is the amount that the decision maker is willing to pay to avoid the risk of the lottery. Figure 2.2 shows how expected utility, certainty equivalent, expected value and risk premium all tie together; by definition, the utility function of the certain equivalent is the expected utility of the gamble (the decision maker is indifferent to the choice between them).

Following Equations (11) to (15), the certainty equivalent is:

$$CE = u^{-1}(E_F(u)) \tag{19}$$

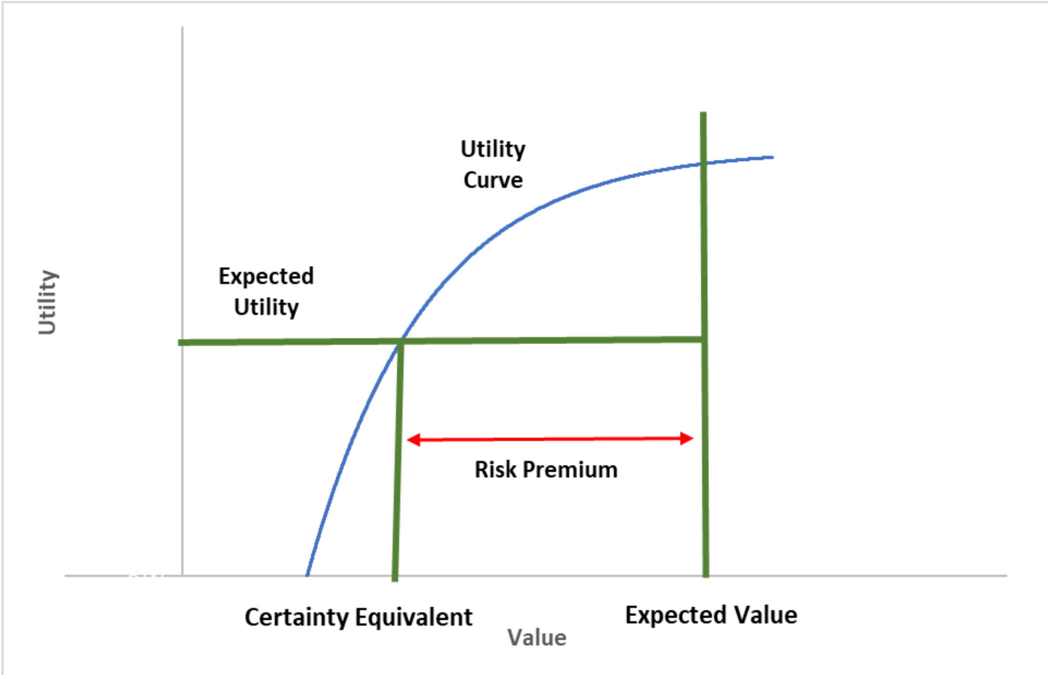

Figure 2.2. Certainty equivalent, utility function and risk premium

However, although most people show a risk-averse attitude most of the time, there are cases in which their attitudes are risk-seeking, meaning that a decision maker is willing to pay more for a lottery than its expected value; this is typically the risk behaviour shown by lottery gamblers. In general, risk attitude is not a fixed characteristic of individuals but depends on the circumstances of everyone.

Figure 2.3 shows a typical curve representing the behaviour of risk-averse, risk-seeking and risk-neutral individuals.

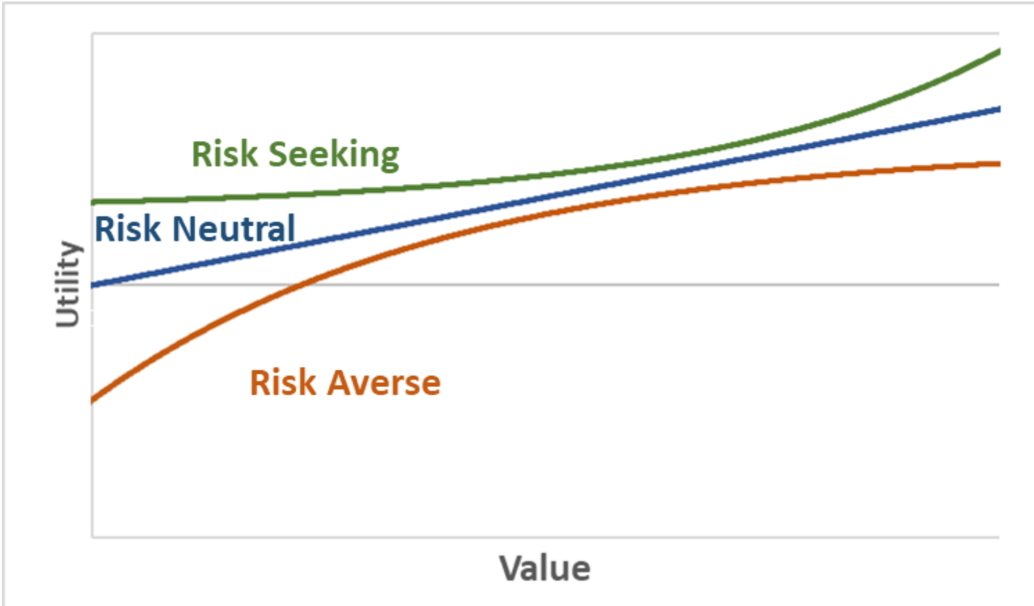

Figure 2.3. Risk attitudes for individuals

The certainty equivalent can be used to rank projects: a project with a higher certainty equivalent has a higher value according to the decision maker's risk preferences.

Based on the previous definition of certainty equivalents and risk premium, it can be easily understood that for a risk-averse attitude, the risk premium is positive, and for a risk-seeker, it is negative. The set of conditions relating risk attitude to the certainty equivalent are:

1) A decision maker is risk-averse if and only if $CE \leq E_F(x)$ for all F
2) A decision maker is risk-neutral if and only if $CE = E_F(x)$ for all F
3) A decision maker is risk-seeking if and only if $CE \geq E_F(x)$ for all F

One important measure of risk aversion is the absolute risk aversion, or the Arrow–Pratt coefficient of absolute risk aversion (Pratt, 1964), defined by:

$$r_A(x) = -\frac{u''(x)}{u'(x)} \tag{20}$$

where u''measures the concavity of the utility function and u' normalises the concavity, since the utility representation is unique under any affine transformation; the larger the r_A the more risk-averse is the person. For $u,$ a monotone increasing function, negative values of r correspond to risk-seeking persons while positive values of r correspond to risk-averse persons.

The general characteristics of the utility function in describing different risk attitudes have been discussed above. However, there is no mathematical method for computing these utility functions. There are two semi-quantitative methods for determining the decision-making utility function: (i) an assessment based on certainty equivalents, which involves an iterative calculation of the certainty equivalents for several lotteries until the utility function is constructed for a sufficient number of points, using a fixed probability distribution; and (ii) assessments using fixed amounts and varying probabilities to construct the utility function.

The exponential utility function is a particular case, as it has several unique characteristics. It depends on the tolerance factor, RT, which has a simple interpretation in terms of lotteries: RT is the largest value a decision maker is willing to risk in a gamble with a 50% probability of winning (Y) $ and 50% probability of losing (Y/2) $. This allows us to estimate the utility function via a process other than the two discussed in the previous paragraph. Pratt (1964) proved that, in this case, the certainty equivalent can be approximated by:

$$CE \approx \mu - \frac{0.5\sigma^2}{RT} \tag{21}$$

where μ and σ are the expected value and the variance of the outcomes.

In an exponential utility function, the larger the RT, the more risk-seeking the decision maker; conversely, the smaller the RT, the more risk-averse the decision maker.

Howard (1988) suggests, based on experience, that, for a company, a reasonable estimate value for RT is 1.24 times company sales or 6.4% of company total sales or 15.7% of company equity.

As discussed previously, even though the concept of utility values has been acknowledged before the development of the VOI methodology, most of the reported applications do not use it. This research recognises the importance of using utility values and includes it in the workflow for evaluating VOI decisions; the risk attitude of the decision maker is an integral part of the VOI methodology developed in the case study discussed in Chapter 4 of this thesis.

vNM theory is known as expected utility theory under risk; this theory is concerned with the assessment of risky prospects where the outcomes (or utility values) and the probabilities of the outcomes are known. In decision analysis nomenclature, this theory assumes that the lottery is known. The assumption that probabilities are known is an important characteristic of this theory and also a limitation in assessing real decision problems. This limitation gave rise to further developments that contributed to the formulation of the expected utility theory under uncertainty based on subjective probabilities, and to the Bayesian theory, which is described below.

Jacques Bernoulli (uncle of Nicolas and Daniel Bernoulli) (1713) defined the concept of subjective probabilities for the first time (in the book "Ars conjectandi" published 8 years after his death) as the degree of confidence a person has for the alternatives available, as cited by Hansson (2005). It was Ramsey (1926) who claimed that for decision under uncertainty, it is feasible for the decision maker to estimate the subjective probabilities for different alternatives associated with that decision. Years later, De Finetti (1937) discussed subjective probabilities and proposed that probability may not exist in any substantial sense, being just a numerical artefact of the uncertain property created by the observer (Mongin, 1997).

Founded on the works of Bernoulli, Ramsey, De Finetti and von Neumann and Morgenstern, Savage (1954) established a set of seven postulates (Karni, 2014) that are the necessary and sufficient conditions for representing the decision maker's preference by the expectations of a utility function on the set of possible consequences weighted with a set of subjective probabilities, where the utility function is a real-value and bounded function, unique up to positive affine transformation, and the probability is a unique, nonatomic and finitely additive probability measure (existence and uniqueness of utility and probability). The key point to understand is that Savage's theorem does not assume the existence of probabilities, which vNM does. Savage's theory is based on the idea that options can be valued using subjective probabilities and utilities that can be derived as long as the decision maker's preferences follow Savage's postulates.

Savage's theory proposes to infer from the decision maker's choices the prior probabilities (his beliefs) and utility function (his attitude toward risk) involved in his decision-making process. What Savage's model means is that if a decision maker follows the seven postulates shown below, then it secures the existence of the prior probabilities and utility function for that decision maker.

Savage's model is based on three sets: S or the set of states, C or the set of consequences and F or the set of choices which map from the set of states to the set of consequences and are called acts. Decisions are characterised by a preference relation ≥ on F: f ≥ g means the act f is preferred or as preferred as act g. The seven postulates supporting the theory of Savage are:

Postulate 1, weak order: the preference relation is a transitive and complete binary relation of F.

Postulate 2, sure-thing principle: for all acts, f, f', h and h' and for every event E, $f_E h \geq f'_E h$ if and only if $f_E h' \geq f'_E h'$.

Postulate 3, ordinal event independence: for every non-null event E and all constant acts, x and y, $x \geq y$ if and only if $x_E f \geq y_E f$ for every act f.

Postulate 4, comparative probability: for all events E and E' and constant acts x, y, x' and y' such that $x > y$ and $x' > y'$, $x_E y \geq x_{E'} y$ if and only if $x'_E y' \geq x'_{E'} y'$.

Postulate 5, non-degeneracy: for some constant acts x and x', $x > x'$

Postulate 6, small-event continuity: for all acts f, g and h, satisfying $f > g$, there is a finite partition $(E_i)_{i=1}^{n}$ of the state space such that, for all i , $f > h_{E_i} g$ and $h_{E_i} f > g$

Postulate 7, dominance: for every event E and all acts f and f', if $f >_E f'(s)$ for all s in E then $f \geq_E f'$ and if $f'(s) >_E f$ for all s in E then $f' \geq_E f$

Savage's theorem formulates that the postulates 1 to 7 are the necessary and sufficient conditions for the representation of the decision maker's preferences by the expectations of a utility function on the set of consequences with respect to a probability measure on the set of events.

In most situations in the real world, objective probabilities are not known and, for this kind of uncertainty, vNM theorem does not work. However, Savage's theorem provides us with a consistent method that can be used for making decisions, even in these situations.

Savage's model, once it is proven that the decision maker's preferences satisfy the seven axioms, is named the expected utility value under uncertainty, and it uses the following steps:

1) Assign subjective probabilities to the available alternatives;
2) Assign utility values to the consequence of the actions;
3) Estimate the expected utility of each lottery;
4) Compare numerically different lotteries and select the one that maximizes the value.

For practical applications, the use of subjective or Bayesian probability is a modification of Laplace's criteria and vNM theory, improved by the knowledge and experience of the decision maker or analyst.

In this way, there are two versions of the EUT: 1) EUT under risk, also known as von Neumann-Morgenstern theory (vNMT), where the probabilities associated with the outcomes are known and the unknown information is the decision maker's attitude towards risk, which is described with relations of the type "more desirable than" and is quantified via the utility function, and 2) EUT theory under uncertainty, also known as Subjective Expected Utility Theory (SEUT), which is supported by Savage's work, where both the decision maker's attitude towards risk and the associated probabilities, relations of the type "more likely than", describing the decision maker's beliefs in the likelihood of the events, are unknown; SEUT is also known as Bayesian decision theory. Any application of the Bayesian approach for decision making should follow the four principles of Bayes, described below:

1) The decision maker should have a coherent set of probabilistic beliefs, which means formal coherence or compliance with the mathematical law of probability.
2) The decision maker has a complete set of probabilistic beliefs, which means he assigns a subjective probability to each proposition.
3) When exposed to new evidence, the decision maker changes his beliefs in accordance with Bayes' theorem, which is discussed below.
4) A rational decision maker chooses the option with the highest expected utility.

The probability theory is founded on several axioms:

1) The probabilities of each outcome must lie between 0 and 1, $0 \leq p_A \leq 1$ where p_A is the probability of outcome A.
2) The sum of the probabilities of all outcomes must add up: for two mutually exclusive outcomes, the probability that both one and the other occur is the sum of the individual probabilities.
3) The total probability (the probability of the set consisting of all the outcomes) must equal 1: for a set of mutually exclusive and collectively exhaustive outcomes, the probability of that set is 1.
4) Conditional probability: given two outcomes A and B, the probability of both occurring at the same time is called joint probability and is written as $P(A \; and \; B)$. The conditional probability of outcome A given that the outcome B occurs is written as $P(A|B)$ and is given by:

$$P(A|B) = \frac{P(A \; and \; B)}{P(B)} \tag{22}$$

where $P(B)$ is the probability of outcome B.

5) Independence: two events X and Y, with outcomes $A_1, \dots, A_n$ and outcomes $B_1, \dots, B_m$, respectively, are independent if and only if

$$P(A_i|B_j) = P(A_i) \tag{23}$$

for all possible outcomes A_i and B_j.

6) Conditional independence: two events X and Y (with outcomes, $A_1, \dots, A_n$ and $B_1, \dots, B_m$, respectively) are conditionally independent given an event Z (with outcomes $C_1, \dots, C_l$) if and only if

$$P\left(A_i \middle| B_j, C_k\right) = P(A_i | C_k) \tag{24}$$

for all possible outcomes A_i, B_j and C_k.

7) Complements: Let $\bar{B}$ be the outcome that is the complement of B, that is, $\bar{B}$ occurs if and only if B does not occur.

8) Total probability of an event: for any two events A and B, the following is true:

$$P(A) = P(A\ and\ B) + P(A\ and\ \bar{B}) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B}) \tag{25}$$

9) Bayes' theorem: because of the symmetry of the definition of conditional probability, the following is true:

$$P(B|A)P(A) = P(A|B)P(B) \tag{26}$$

This can be rearranged to have

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} \tag{27}$$

which, after expanding the total probability as described in 7), can be written as

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|\bar{B})P(\bar{B})} \tag{28}$$

This equation is referred to as Bayes' theorem and is the basis for value of information assessment.

SEUT is a central theory of the present economic theory; it explains how people should behave under uncertain conditions and, in that sense, it is a normative theory. However, as a descriptive theory of how people actually behave, there have been long debates based on counterexamples that show contradictions.

One of the critiques of the SEUT approach was posed by Allais in 1953, as cited by van de Kuilen and Wakker (2006); Allais, working on the economic feasibility of exploring assets in Algeria, proposed examples of decision problems where the common-sense choices contradict the SEUT approach, violating the independence axiom (Hansson, 2005) (see Allais's paradox below in this section).

The development of a normative theory is highly important to advise the decision maker on how to make decisions; however, it is important to understand how people make decisions in reality, whether those decisions are rational and how they impact the benefits (Welsh and Begg, 2008).

Kahneman and Tversky (1979) analysed the responses in surveys taken on different groups of people that were exposed to a set of decision questions. They concluded that: 1) people have the tendency to make decisions in terms of deviations from a reference point, while EUT is defined in terms of a net wealth; 2) the utility functions that people use to make valuations are different for losses than for gains (risk-averse for gains and risk-seeking for losses) and the "feel" of loss is stronger than the "feel" of gain for equivalent amounts, and 3) people tend to weight outcomes according to a "decision weight", not the probabilities (most people overweight low probabilities and underweight high and medium probabilities). For solving these issues, they proposed a descriptive decision theory known as Prospect Theory.

The vNM EUT is a prescriptive theory in the sense that it shows how decisions should be made; however, it is not necessarily the way people decide. This is the reason for the development of theories for describing how people deviate from EUT or descriptive theories.

The vNM EUT was tested against decision problems posed in surveys that question the validity of the EUT for representing actual decision making. Here two examples are shown, in the form of surveys, explaining why the EUT is not able to capture the decisions taken by humans.

<u>Asian disease paradox</u>: for a country expecting to have an outbreak of an unusual disease that is expected to kill 600 people, two alternatives have been proposed to combat the pandemic, Table 2.1.:

Table 2.1. Asian disease outbreak – alternatives 1 and 2.

ALTERNATIVE 1		ALTERNATIVE 2	
Number of people saved from the disease	Probability (%)	Number of people saved from the disease	Probability (%)
200	100	600	33
		0	66

This question was posed in a survey and 72% preferred alternative 1.

For the same disease outbreak, two other alternatives were questioned, Table 2.2.:

Table 2.2. Asian disease outbreak – alternatives 3 and 4.

ALTERNATIVE 3		ALTERNATIVE 4	
Number of deaths from the disease	Probability (%)	Number of deaths from the disease	Probability (%)
400	100	0	33
		600	66

This question was posed in the same survey, with 78% preferring alternative 4.

However, this is a contradictory result because alternatives 1 and 2 are the same as 3 and 4, respectively. This contradictory result can be explained because of the way the problem was framed; this is the "framing effect". In the first example, two cases are compared in positive terms or gains: 200 people saved versus a 33% probability of saving 600 people; in these situations, people tend to be risk averse. The second example is the same as the first but posed in negative terms or losses: 400 people to die versus a 66% probability that 600 people will die; in these situations, people tend to be risk-seeking.

The Asian disease paradox is an example of the "framing effect", which means people decide by making a comparison with a "frame" or point of reference and the choice of the frame can be influenced by the way the problem is phrased.

Another case is the well-known Allais paradox that has been presented in several ways, one of which is included here: the decision maker has two decisions (1 and 2) to make each with two alternatives (A, B, C and D), Tables 2.3. and 2.4.:

Decision 1:

Table 2.3. Allais paradox – alternatives A and B.

ALTERNATIVE A		**ALTERNATIVE B**	
Win (MMUS$)	Probability (%)	Win (MMUS$)	Probability (%)
1	100	5	10
		1	89
		0	1

Decision 2:

Table 2.4. Allais paradox – alternatives C and D.

ALTERNATIVE C		**ALTERNATIVE D**	
Win (MMUS$)	Probability (%)	Win (MMUS$)	Probability (%)
1	11	5	10
0	89	0	90

Surveys indicated that 82% of people prefer A over B and 83% prefer D over C. This means that the perception of the probability of winning nothing is larger between A and B than the small difference between alternatives C and D. This is an example of people overweighting small probabilities.

Choosing A for the first decision and D for the second decision is in contradiction to the EUT.

Let us assume that, for the best and the worst outcomes, $U(\$0) = 0$ and $U(\$5\ million) = 1$, then

$$EU(A) = U(\$1\ million) \tag{29}$$

$$EU(B) = 0.10 + 0.89U(\$1\ million) \tag{30}$$

If A is preferred to B

$$EU(A) > EU(B) \tag{31}$$

or

$$U(\$1\ million) > 0.91 \tag{32}$$

From decision 2

$$EU(C) = 0.11U(\$1\ million) \tag{33}$$

$$EU(D) = 0.10 \tag{34}$$

If D is preferred to C

$$U(\$1\ million) < 0.91 \tag{35}$$

These contradictory results show that these decisions are not consistent with the EUT. Kahneman and Tversky (1981) attributed the lack of consistency to the certainty effects, by which they mean the tendency of people to increase the value of the certain outcomes versus the uncertain ones.

There is other experimental evidence that shows that people make decisions that do not always follow the EUT; however, this theory remains the most reliable one for how people should make decisions.

There are several ways to assess utility functions. Two of the most frequently used are: 1) based on the certainty equivalents, and 2) based on probabilities (for more details on this topic see Clemen, 1996). These methods are dependent on the decision maker's preferences. Similarly, because of the nature of probability assignment, different analysts will get different results depending on their assignment of probabilities (Aguiar, 2004).

All the previous progress toward building a theoretical framework to make decisions that include the preferences of the decision maker has not significantly permeated the world of real applications. In the analysis made by Corner and Corner (1995) of the published decision analysis applications from 1985–1995, 67% of those applications used the expected value as a decision-making criterion instead of the expected utility value. Howard (1988) reported that the decision maker's attitude towards preferences is of real practical concern in only 5–10% of business decision analyses, even though the concept is a fundamental part of decision-making theory.

However, Keefer (1991) showed that, in some cases, including the risk aversion attitude of the decision maker can change the assessment. Similarly, Kirkwood (2002) concluded, based on a set of artificial cases, that expected utility analysis led to different conclusions than expected value analysis for many cases, and suggested a straightforward way of identifying when that might happen.

In 1999, Hammond proposed a variant of Savage's utility function approach by considering the case where the domain of consequences is state dependent ("constant acts" mapping states and consequences may not be always true) and deriving a method to estimate subjective probabilities and utilities.

The oil and gas industry should be one of the domains that benefit more from all these developments, considering the large number of decisions and their associated cost. Grayson (1962) discussed and compared, for the first time, the workflow of the classical and Bayesian approach in a decision associated with the drilling of an exploration well, highlighting the advantages of the latter approach. Newendorp (1967) presented the pioneering application of utility theory concepts to a decision problem consisting of deciding whether to drill an oil production well; in his work, Newendorp recognized that obtaining probability estimates for the outcomes that might occur from drilling a well is a complex problem in itself, and he decided to focus on the selection of the value, keeping the chances of the alternatives occurring as a separate problem to be addressed in future work. Newendorp argued that EUT provides a value system that is superior to EMV because it includes the risk preferences of the decision maker in addition to losses or gains, turning it into a better approach for making drilling investment decisions. Newendorp pointed out that constructing a functional representation of a decision maker's risk preference is the main obstacle to using utility functions to value a decision problem, which is the reason why he dedicated his PhD thesis work to developing a method, based on surveys of drilling decision makers, to build representative utility functions. Utility theory allows the decision maker's preferences and biases regarding money to be included qualitatively in the expected value criterion, producing an EUV for each alternative (Newendorp and Campbell, 1971). In the meantime, Arps and Arps (1974) explored decisions concerning oil and gas explorations without using either utility or subjective probabilities in favour of a more traditional approach for decision making based on a "break even" and "gambler's ruin" criterion to make decisions, concluding that the greater the available risk capital, the greater the risk that the operator can take.

Most applications of decision analysis in the oil and gas industry are related to western countries; however, Hirakawa and Kato (1972) discussed the need to include the uncertainty in project valuation through the use of decision analysis tools and how new information (perfect information) can impact the valuation of the project.

Even though the oil and gas industry is characterized by taking large numbers of very costly decisions in situations of great uncertainty, the number of reported cases using EUT is very limited. Applications of EUT can be found in Silbergh and Brons (1972), who explore different methods to quantify project profitability and include a section describing utility theory for considering decision maker preferences. Newendorp and Campbell (1971) briefly discussed using a utility function, although in most of the work EVT was used. Cozzolino (1977) proposed a simplified utility framework for risk representation supported by three axioms that uniquely suggest that the utility function should be an exponential function and used that utility function for describing decision maker preference for drilling an exploratory well in an oil field. In 1978 Cozzolino discussed the traditional way to introduce risk in project valuation: 1) through increasing the discount factor—the higher the risk of the project, the higher the interest rate, and/or 2) an excessively high cut-off rate of return; he elucidated the problems associated with including risk in the discounted value of money and presented the utility function theory from a practical point of view with several examples taken from the oil and gas industry. He introduced the term “risk-adjusted rate” or RAD, which is equivalent to the term certainty equivalent in utility theory nomenclature. MacKay (1995) acknowledged this development and introduced one of the first computer applications of these theories and applied it to several cases related to the oil and gas industry. Akindele and Shapiro (1978) used utility function concepts in the context of the problem associated with generating the probability distribution of the profitability index of a project; they acknowledge that the utility function describes the amount of risk someone would take involving money under conditions of uncertainty. Walls, Morahan and Dyer (1995) developed a proprietary software for ranking oil exploration projects based on the use of EUT, allowing managers to evaluate projects with a consistent risk attitude policy and to rank projects based on overall performance; sensitivities to risk parameters are applied during this process; the valuation measure was the certainty equivalent and they used the exponential utility function for its ease of use, good approximations to other general forms of utility and because it is useful for treating multiple independent projects separately. Begg, Bratvold and Campbell (2003) stated that the use of utility theory in the decision making allows for incorporating the risk-attitude of the decision maker and the relative values for incremental increases in money. Walls (2005) discussed how to estimate the utility functions’ parameters for ranking a portfolio of projects in the 50 largest US-based oil companies over a 20-year period; he concluded that the risk attitude for a company impacts its performance—highly risk-averse companies perform worse than poorly risk-averse ones; similarly, risk tolerance increases as companies become larger. Vilela, Oluyemi and Petrovski (2017) compared the VOI assessments using values and utility values and discussed the results of sensitivity analysis on the value of the exponential factor, concluding that the assessment is impacted depending on the degree of risk aversion of the decision maker.

Rose (2004) pointed out that there is a need to continue monitoring a firm's level of risk aversion because of the changing corporate and industry environment, as well as the changing technologies developed for the Exploration and Production (E&P) business. Routine comparison and analysis of a company's risk aversion results in a consistent and robust decision-making policy.

It has been seen that decision analysis and risk preference concepts have permeated the theory and application of decision making in the oil and gas industry. One of the key decisions is data acquisition and the value it provides to the project where it is inserted. In the next section, the VOI in the oil and gas industry is discussed.

2.3 VALUE OF INFORMATION IN THE OIL AND GAS INDUSTRY

This section discusses, first, the main milestones in developing the methodology of VOI; in the second part, the focus is on VOI in relation to the oil and gas industry and, in the third part it develops the mathematical formulation of the VOI theory. However, it is sometimes difficult to separate the development of the theory from the applications in the oil and gas industry because some of the key developments that drove the theory development were strongly related to applications in the oil and gas industry.

VOI is a prescriptive methodology to assess the value that gathering data in the future may add to the present value of a project. VOI gives no value to uncertainty reduction but aims to make the best decision based on the underlying uncertainties (Bratvold, Bickel and Lohne, 2007). VOI is part of the broader Decision Analysis discipline discussed in Section 2.2.

In a project, investment decisions are made at some point in time and those investments have subsequent monetary consequences. Data gathering is a possible investment decision. Data gathering has a cost (investment). Does this data gathering generate monetary benefits compared with the no-data gathering option? If so, it is worth gathering the data.

Raiffa and Schlaifer (1961), Raiffa (1968) and Schlaifer (1969) developed the fundamental concepts and tools of VOI in the context of business administration; their goal was to enable business administrators to make wiser decisions. Their approach consists of using statistical inference and sampling tools in practical problems of decision making under conditions of uncertainty, where additional information about the state of the world can be obtained through experimentation. Before them, Grayson (1960) published his dissertation (converted into a book the same year), applying the VOI methodology to drilling decisions to be made by oil and gas operators, where uncertainties are exceptionally great; Grayson's work is the first reference showing the use of utility theory and subjective probability theory applied to an oil and gas decision problem that the authors of this research are aware of. Grayson (1962) used statistical inference nomenclature to analyse the drilling decision and the value associated with gathering additional information.

The next milestone for VOI in the oil and gas industry was the PhD thesis of Newendorp (1967), who discussed the necessity of developing and using the risk attitude of the decision maker as part of the VOI assessment; he discussed, specifically, the use of the exponential utility function to capture the decision maker's risk attitude.

Newendorp (1972) discussed in detail the logic, mathematical proof and methodology of Bayes' theorem (developed by Thomas Bayes in 1763), which is a fundamental mathematical tool behind VOI (this reference to Bayes' theorem did not include comments on VOI). He also discussed the concept of sequential sampling or sequential data acquisition. Subsequently, further research and applications expanded the scope of the subject and provided more robustness to the methodology.

Even though Dougherty (1971) did not present any novelty to the theory of VOI, he discussed, in a concise manner, the tools of VOI for the oil and gas industry and included several realistic examples of applications.

Warren (1983) discussed a methodology for deciding between initiating or rejecting a project and deferring the decision until more information is acquired, using a field development decision problem as an example.

Lohrenz (1988) presented four examples in the petroleum engineering domain, using decision trees to value data acquisition.

Silbergh and Brons (1972) wrote their paper at a time when the use of decision analysis and VOI was still a novelty in the oil and gas industry. They reviewed standard methods of profitability valuation, such as Profit Discounted by the Cost of Money to the Firm, Discounted Cash Flow and Rate of Return, showing the limitations of these methods when valuing uncertain projects, and discussed concepts such as utility functions and VOI. The importance of this paper lies in bringing the development and application of, what at that time was a new theory of VOI, to real-world problems.

In Moras, Lesso and MacDonald (1987), a different type of VOI application is discussed; the aim of this paper is to determine the value associated with different numbers of observation wells to monitor underground gas storage reservoir pressure so as to avoid gas migration and estimating the optimum number of observation wells using reservoir simulation tools. It is one of the first applications found in the literature where VOI and reservoir modelling are used in conjunction.

Gerhardt and Haldorsen (1989) contributed to the use of decision analysis tools, especially VOI, showing in simple examples how these methodologies work, e.g. in a long term well test on a vertical well, a polymer pilot project and an extended production test on a horizontal well. Other applications, such as that of Dunn (1992) for the estimation of well log information, and Stibolt and Lehman (1993) on the value of seismic information in an exploration asset applied as a European call option, broadened the scope of the VOI methodology in the oil and gas industry. Many of the initial developments and applications of VOI in the oil and gas industry were in the subsurface exploration domain. Rose (1987) described exploration activities as a series of investment decisions, whether to develop a project, acquire additional data or modify the hydrocarbon interest sharing.

In the subsurface domain, appraisal activities are those consisting of information-gathering with the objective of reducing reservoir uncertainties that may affect the field development; consequently, VOI has an important contribution to the assessment of appraisal activities, which was discussed in detail by Demirmen (1996) for the two types of appraisal activities, screening and optimization. This is a major contribution to VOI which effectively enlarged the scope of VOI assessment to a broader audience in the oil and gas industry.

Newendorp and Schuyler (2000) developed fundamental ideas related to VOI, including examples from the exploration and appraisal of oil and gas projects. Koninx (2000) discussed VOI from a methodological perspective, adding examples related to the value of 3D seismic acquisition and appraisal to clearly define the hydrocarbon composition. In other research, Coopersmith and Cunningham (2002) proposed a step-wise methodology to facilitate VOI assessment and, through SPE-related publications, Bratvold, Bickel and Lohne (2007) showed that, although the use of systematic qualitative methods in VOI has increased in recent years, it is still far from being a standard application, even when large investments are involved.

In the more than 50 years since Grayson's work, very few real VOI applications have been published beyond some occasional reports, even in the case of large capital investments (Bratvold, Bickel and Lohne, 2007).

Begg and Bartvold (2002) introduced alternative concepts for assessing the value of an uncertain project, such as the value of flexibility, a complementary methodology to VOI. Kullawan, Bratvold and Bickel (2014) discussed an important application of VOI in a geosteering operation in which a large number of real-time operations are executed day to day, demonstrating the flexibility of the VOI methodology in adapting to challenging circumstances. Similarly, Clemen (1996) and Suslick and Schiozer (2004) discussed applications and methods which enriched the VOI process.

The first step in applying the classical VOI method is to define a set of n discrete states of nature (known as 'cases') $s_1, \ldots \ldots s_n$ that describe the range of all possible project outcomes. Each state has a probability of occurrence $p(s_i)$, where (Clemen, 1996):

$$\sum_{i=1}^{n} p(s_i) = 1 \tag{36}$$

The probabilities in Equation (36) are known as “prior probabilities”, since they represent the current belief (i.e. before the acquisition of new data) regarding the likelihood that a state will occur. Experts assign these probabilities based on their experience and judgment.

Let us now consider a decision problem in which m alternative solutions are included in a set A:

$$A = \{u_1 . u_2 . u_3 \dots \dots \dots u_m\} \tag{37}$$

For each pair (each alternative u_j and state of nature s_i) there is a value u_{ji} that will materialise in the future if u_j and s_i both occur.

The expected value (EV) corresponding to the j^{th} alternative is defined as:

$$EV(u_j) = \sum_{i=1}^{n} u_{ji} p(s_i) \tag{38}$$

The decision criterion that is most often used is the selection of the alternative with the maximum EV:

$$EV(u^*) = \underset{j}{maxEV(u_j)} \tag{39}$$

Equation (39) represents the value of the *project without information* (i.e. with the current information). In the subsurface domain, this typically includes several uncertainties in input parameters, which will in turn result in uncertainties in the outcomes.

There are situations in which additional data may be acquired (in the future) that could narrow the uncertainty in the input parameters responsible for the spread (uncertainties) in the outcomes. The acquisition of these data would affect the value of each discrete state and would also modify the probabilities assigned to each state. The net effect of the changes in the values and probabilities of the states (cases) is a change in the value of the project.

In general (Bratvold et al., 2007):

$$VOI = EV_{with\,information} - EV_{without\,information} \tag{40}$$

The values $EV_{with\ information}$ and $EV_{without\ information}$ represent what it is believed the outcomes of the project would be in two different future situations.

Let us assume that the outcomes resulting from the acquired data are discretised in the following set X of l values:

$$X = \{x_1, \dots \dots \dots \dots, x_l\} \quad (41)$$

Here, the elements of set X, $x_1, \dots \dots x_l$ are the values that are measured or estimated during the data acquisition process; they may be values of porosity, permeability, pressure or depth, for example (with their corresponding units). The reliability probabilities $p(x_k|s_i)$ are assigned by experts in the same way as the prior probabilities in Equation (36). These reliability probabilities measure the likelihood that the data accurately identify the states of nature. Since real-world data are imperfect, the reliability probabilities are always less than one. In the Bayesian inference system, the concept of *imperfect data* is the opposite to that of *perfect data*, which represents an ideal (not a real-world) concept and assumes that these data can accurately predict the state of nature.

The reliability probabilities are flipped using Bayes' theorem to generate the posterior probabilities, as shown in Equation (42):

$$p(s_i|x_k) = \frac{p(x_k|s_i)p(s_i)}{p(x_k)} \quad (42)$$

The denominator in Equation (42) is the marginal probability of the new data $p(x_k)$, which is defined using the total probability theorem given in Equation (43):

$$p(x_k) = \sum_{i=1}^{n} p(x_k\,|s_i)p(s_i) \quad (43)$$

Given a data outcome x_k, the EV for the j^{th} alternative is:

$$EV\left(u_j \middle| x_k\right) = \sum_{i=1}^{n} u_{ji} p(s_i\,|x_k) \quad (44)$$

where $EV(u_j|x_k)$ is the expected value of the project for the j^{th} alternative and the data outcome x_k.

The optimum alternative is that which maximises the EV:

$$EV(u^*|x_k) = \max_j EV(u_j|x_k) \tag{45}$$

The unconditional maximum EV (i.e. the EV of the project considering the data acquisition outcomes) is the sum of the conditional EV weighted with the corresponding marginal probabilities:

$$EV(u_x^*) = \sum_{k=1}^{m} EV(u^*|x_k)p(x_k) \tag{46}$$

Finally, the VOI is the difference between the EV of the project *with information* and the EV of the project *without information* (Bratvold et al., 2007), as given in Equations (46) and (39):

$$VOI = EV(u_x^*) - EV(u^*) \tag{47}$$

If there were no uncertainties in a problem, making a decision concerning data acquisition would be a straightforward problem consisting of assessing the optimum course of action. However, uncertainty is everywhere.

Uncertainty reduction through data acquisition is worthwhile only if it can change a decision and the expected benefit of uncertainty reduction, mitigating the downside risk and capturing the upside opportunities, these being higher than its cost. Data acquisition is worthwhile when its outcomes changed our knowledge of the uncertain variables, increasing the value of the project (Bratvold, Bickel and Lohne, 2007).

Uncertainty, risk and probability are terms used frequently in this research; understand their meaning and significance is very important to properly frame our research problem and the solutions proposed.

2.4 UNCERTAINTY, RISK, PROBABILITY AND DECISIONS

Uncertainty is an adjective used to characterise a quantity that is not known accurately; it is said that a quantity carries uncertainty when it is not possible to assign to it an accurate value without doubt.

At the beginning of his well-known book, Knight (1921) proposed to reserve the term uncertainty for a non-quantifiable or unmeasurable quantity and the term risk for a quantity susceptible to measurement or quantifiable. This statement has been criticised (Begg, Bratvold and Welsh, 2014), arguing that all uncertainty is quantifiable. However, a few chapters later, Knight, expands his dissertation on subjective probabilities and makes a further distinction between risk and uncertainty—risk means that the values of the alternatives and their objective probabilities are known, and uncertainty means that, while knowing the values of the alternatives, an objective value for their probabilities is not known.

Luce and Raiffa (1957) stated that decisions are taken in one of the following states: 1) certainty, when each action is known to result in a specific outcome; 2) risk, when each action is known to result in one of a set of outcomes with a known probability; 3) uncertainty, when each action results in one of a set of outcomes where the probabilities are unknown, and 4) ignorance, when each action results in an unknown outcome with unknown probability (Hansson, 2005).

Probability is how the likelihood of an event occurring is measured (Clemen, 1996). Probability theory has two main branches, the frequentist and the Bayesian approaches.

The frequentist inference approach of probability draws conclusions from sample data and figures defined by its frequency. The origins of this approach to probability can be found in Aristotle and, later, in Poisson and Gauss (Simon, 2006). This is the framework on which methodologies of significance testing (Fisher, 1973), statistical hypothesis testing (Neyman, 1961 and Pearson, 1895) and confidence intervals (Neyman, 1961) are based. In this approach, the information resulting from testing is used in analysis and, consequently, there are no subjectivities on the parameters.

The Bayesian inference approach of probability draws conclusions based on the beliefs or knowledge of the analyst. The “prior” probability about the states of nature is assigned and, once new information is added, the “posterior” probability of the state can be estimated (updated probability), which is used to make inferences. The term Bayesian derives from the extensive use of Bayes’ theorem, which was introduced by Thomas Bayes in 1763. The use of the Bayesian approach was further developed by Wald (1949) and Savage (1954). Megill (1977) argued that in no other industry does the subjective probability play such an important role as in the oil and gas industry.

These two branches of probability theory, frequentist and Bayesian, serve to assess two different terms, variability and uncertainty. Variability describes the multiple values that a quantity can have, and uncertainty describes our inability to characterize a quantity by a number due to lack of knowledge, random nature of the quantity or its vagueness. These two terms are frequently confused, affecting good decision assessment (Begg, Bratvold and Welsh, 2014).

Uncertainty is a result of our incomplete knowledge about the world or incumbent system (McNamee and Celona, 2008). When there is uncertainty about something, that means it is not known if an event is true or false (Begg, Bratvold and Welsh, 2014).

Lack of knowledge is an expression used to characterise a state when part or all of the information about a quantity is unknown; e.g. in the description of an oil and gas field, data are acquired in the wells, such as porosity values; however, between wells, porosity has an unknown value which is characterised by saying that there is lack of knowledge about porosity values between wells.

Risk considerations involve the size of the investment regarding budget, potential gain or loss, and outcome probability. Properties of nature can be measured or determined within some degree of accuracy, which makes them uncertain by nature with a degree of uncertainty that could vary from very small to very large and all the intermediate values; subsequently, any quantities derived from them carry uncertainty too. However, following Taghavifard, Khalili and Tavakkoli (2009), business decision making is always made in conditions of uncertainty. Authors such as Rose (1987), Newendorp and Schuyler (2000), Macmillan (2000) and Simpson et al. (2000) agree that uncertainty is part of the decision-making process and it is a major issue for effective capital investment.

Uncertainty is implicit everywhere in the oil and gas industry (Bratvolt and Begg, 2006), and it is the reason behind the underperformance of many projects (Begg, Bratvold and Campbell, 2003). Bashear, Becker and Gabriel (1999) analysed the performance of E&P companies from 1977–1996. They concluded that, although many companies screen projects based on an internal rate of return between 15% and 25%, the average investment return for the 20-year period was 5.5% with a maximum of 7.6% and minimum of 3.4%. That low performance occurs even through the high oil prices from 1986–1996. Begg and Bratvold (2002) discussed statistics taken during the period 1990–2001, which show that both major and independent oil and gas companies underperformed when compared with the standard share index in the USA.

McVay and Dossary (2012) showed, using probabilistic modelling, that a moderate overconfidence bias (underestimating of uncertainty) and optimism bias can result in an expected disappointment of between 30–35% of the estimated Net Present Value (NPV), while greater degrees of overconfidence and optimism can produce more than 100% deviation with respect to estimated NPV. The modelling results agree with the industrial performance in the 1990s; these deviations are attributed to poor project assessment and selection due to uncertainty bias.

Randomness occurs when there is no pattern or law that governs the outcomes of an event and those random events are difficult to predict; in cases when a process is too considerably complex to be explained by simple equations, it is described as random process; e.g. in oil and gas reservoirs, several complex processes are described as random, such as the sand distribution in a reservoir.

Caers (2011) predicted that there are two sources of uncertainties:

A) uncertainties due to randomness: because of the randomness of nature, some processes are unpredictable and chaotic.

B) uncertainties due to limited understanding: limited or lacking knowledge of the analyst; depending upon what information is available , whether it has happened or not before and how that relates to the future.

Probability theory provides the tools and techniques for quantifying the degree of these kinds of uncertainty. It measures our degree of belief in the event occurring; probabilities are assigned based on judgment rather than on measure, as in quantities like length, porosity or temperature (Begg, Bratvold and Welsh, 2014).

However, randomness and lack of knowledge are not the only sources of uncertainty. According to Bellman and Zadeh (1970), there is another source of uncertainty that is associated with fuzziness, which is one of the main sources of uncertainty. Fuzziness can be present in the goals, the constraints and the consequences of a problem.

The tool to manage fuzziness is not probability, which assigns a value quantifying the likelihood that an event belongs to a set, but fuzzy theory, which assigns a value quantifying the degree to which an event belongs to a set.

Quantifying uncertainty in the initial phases of a project makes it possible to assess risk and plan for a range of probable project outcomes (Peake, Abadah and Skander, 2005), capturing the outcome uncertainties. There are different manners to classify the reservoir uncertainties.

For subsurface oil and gas appraisal projects, Demirmen (2001) classified the uncertainties into three types, uncertainty in hydrocarbon-in-place, uncertainty in well or reservoir productivity and uncertainty in fluid properties. However, a more complete classification of uncertainties is discussed by Corre, de Feraudy and Vincent (2000), who define five types of subsurface uncertainties:

Geophysical uncertainty

Geophysical uncertainty is related to any of the geophysical processes: acquisition, processing and interpretation (Corre, de Feraudy and Vincent, 2000). Tyler et al. (1996) and Sandsdalen et al. (1996) included the following as the sources of the more significant uncertainties:

A) Uncertainties and errors due to depth conversion methods;
B) Uncertainties and errors due to the seismic-to-well tie;
C) Uncertainties and errors in horizon picking;
D) Differences of geological interpretation;
E) Uncertainties in pre-processing and seismic migration;
F) Uncertainties in the top reservoir amplitude map

Geological uncertainties

Geological models carry uncertainties due to different geological schemes, the nature of reservoir rocks, extent and properties, and the sedimentary concept (Corre, de Feraudy and Vincent, 2000). The most significant sources of geological uncertainties are:

A) Uncertainty in gross rock volume;
B) Uncertainty in the extension and orientation of sedimentary bodies;
C) Uncertainty in the net-to-gross of the different layers;
D) Uncertainty in the fluid contacts;
E) Uncertainty in the porosity values and distribution;
F) Uncertainty in distribution, shape and limits of the reservoir rock types;

G) Uncertainty in facies distribution;

H) Uncertainty in permeability values and distribution

Petrophysical uncertainties

Petrophysical data comes from wells drilled, which even in a developed field, account for less than 0.1% of the total reservoir volume. On top of that, reservoirs typically have different degrees of heterogeneity, which makes reservoir properties highly uncertain, especially away from the wells. There is also uncertainty associated with measuring, intrinsic difficulties due to borehole and borehole fluids, and changes in the samples from the hole to the laboratory.

Dynamic uncertainties

Dynamic uncertainties are those affecting the flow of fluids inside the reservoir, such as relative permeability, fault transmissibility, horizontal barriers, well injectivity, well damage, well productivity index, and the vertical-to-horizontal permeability ratio, among others (Corre, de Feraudy and Vincent, 2000).

Geochemical uncertainties

These uncertainties are related with the identification of different paleoenvironments, the ages and maturity of hydrocarbons, migration and accumulation processes, and reservoir continuity; appropriate geochemical assessment of hydrocarbons can enhance petroleum discovery rates and improve the efficiency of reservoir development plans.

Geomechanical uncertainties

Knowing the stress regime (normal, strike-slip, or reverse) is a key element for well direction placement and fracture design; another important variable is the mud weight required to drill the well during the different phases; mud weight increases with deviation when horizontal stresses are low; however, mud weights are higher in the strike-slip regime, and there could be a large variation with drilling direction, especially at higher deviations. When no wells have been drilled in new exploration areas, estimation of mud weights can have considerable uncertainties. Currently, field development uses reservoir characterisation techniques that include mechanical properties of the field and initial stress distribution together with dynamic models for assessing the dynamic stress evolution of the field when production and injection occur.

Fluid uncertainties

Uncertainties in oil, gas and water characterization are very common in the reservoir, especially in the initial stages of development. The most significant are:

A) Uncertainties in fluid characteristics in different areas of the reservoir;
B) Uncertainties in the compositional analysis;
C) Uncertainties in the fluids' interfacial tension;
D) Uncertainties in volumetric measurements in the PVT analysis;
E) Uncertainties due to sampling inaccuracies;
F) Laboratory uncertainties and equipment calibration

Bratvold and Begg (2006) refer to the research done by Merrow (2003), who studied more than 1,000 exploration and production projects with investment between 1 million – 3 billion US$, observing that more than 10% of them showed at least two of the following forms of underperformance compared with the original plan: cost growth (more than 40%), time slippage (more than 40%) and 1st year operability (less than 50%), which they argued are a consequence of poor understanding of the uncertainties, which leads to overestimating returns or underestimating the risk of loss.

Uncertainty is the source of the need for data acquisition; however, uncertainty by itself is not enough to gather more data, rather the increase in the value of the project associated with reduced uncertainty is the criterion. The fact that most of the decision problems carry uncertainties makes it particularly important to properly assess which parameters carry uncertainty and which are more important than others in the context of the full project; this is the subject of the following section.

2.5 DESIGN OF EXPERIMENTS FOR SUBSURFACE ANALYSIS

Most of the references discussed so far on the methodology to calculate VOI consider "isolated" data gathering activities in the context of a project's value. However, references for a holistic assessment of VOI activities in the scope of a project are limited. Typically, in the examples shown, a data acquisition action impacting one of the uncertainties of the problem is identified and the VOI of that data acquisition is computed.

Koninx (2000) described a three-step process for VOI that is representative of most of the assessments reported. In 2002, Coopersmith and Cunningham described a twelve-step process for VOI, which, although considering the project in its completeness, lacks a comprehensive methodology for the identification and quantification of uncertainties and their interaction.

However, this approach fails the target and locates the centre of interest in the wrong place; indeed, the centre of interest of the VOI is the complete project and the identification and quantification of all the uncertainties impacting the project's value; this failure can be overcome using the methodology discussed in this thesis.

According to Rose (2004), the bias in estimating the parameters controlling the E&P projects is the main reason causing the oil companies delivering only about half of the predicted reserves from 1982–2002. This statement reflects the importance that the correct identification and valuation of the project's parameters have for the estimation of the value of the project, which impacts the decision-making process, such as project investments, development strategy, data acquisition, etc. The biases are present to some extent in all decisions and sometimes they interfere with our ability to make decisions consistent with our objectives (Begg, Bratvold and Campbell, 2003).

Design of Experiments (DOE), also called Experimental Design (ED), is a structured and organized method to conduct and analyse controlled tests to assess the factors affecting a response variable. Factors are varied simultaneously and independently of each other, making it possible to obtain a causal predictive model. By applying DOE, a series of experiments are done while systematically changing the process variables, observing and quantifying the changes in the output (Montgomery, 2005). DOE is designed to produce the maximum information from the least number of experiments. DOE can improve the performance of a process and reduce its variability or production cost (Telford, 2007).

Until the beginning of the nineteenth century, scientists used one-factor-at-a-time (OFAT) when conducting experiments. In OFAT, one factor is varied at a time while the other factors remain fixed at their original values (Davin, 2012); this process continues until all factors are varied. OFAT has two main drawbacks: 1) it needs a large number of experiments (time and money) and 2) it is not able to capture the interaction between parameters. However, testing several variables at the same time can provide information about possible interactions between factors (Durakovic, 2017), which is one of the great advantages of DOE.

DOE was invented by statistician Ronald Fisher (1935) at the Rothamsted Experimental Station, an agricultural research station near London; he conducted research with the aim of increasing crop yield in the UK and showed how valid conclusions can be drawn efficiently from experiments with natural fluctuations in the presence of nuisance variables.

There are three main approaches for DOE: Classical, Taguchi and Shainin (Tanco, Viles and Pozueta, 2009).

1) <u>Classical approach</u>

Montgomery (2005) affirms that the classic approach to DOE was developed in four stages:

<u>Stage 1</u> started in the 1920s with the experimentation carried out by Fisher that characterizes the introduction of scientific thinking and the development and application of factorial designs, fractional factorial designs and the analysis of variance (ANOVA) in experimental scientific research. Fractional

factorial designs were introduced during the 1930s and 1940s as a solution to the large number of experiments required by factorial designs. DOE was initially used on agricultural problems, and it was later applied in other domains (military and industry by the 1940s):

Stage 2 started with Box and Wilson (1951), who developed the Response Surface Method (RSM). Box also developed DOE for optimizing chemical processes. These authors noticed that industrial experiments differ from agricultural experiments in two aspects: *Immediacy* (the answer can be observed very quickly as opposed to agricultural experiments that need long times) and *Sequentially* (experiments can be planned after evaluating other ones). During this stage, several designs, with limited numbers of experiments, were developed to estimate quadratic functions, which allow interaction effects between factors to be computed: central composite designs (CCD), face central composite (FCD) and the Box–Behnken design (Ilzarbe et al., 2007). In this period, Deming brought DOE concepts to Japan, resulting in improving product quality by following Deming's 14 principles (Deming, 1982).

Stage 3 started at the end of the 1970s with the increasing interest from industries in improving their processes. In the 1970s and 1980s, Box, Hunter and Hunter made key contributions in the area of ED for physical experiments in chemistry, chemical engineering and industrial engineering. In this period, the approaches from Taguchi and Shainin were developed, which are characterized by their simplicity and efficiency.

Stage 4 started in the 1990s with the development of optimal designs and several software tools for DOE analysis, increasing the use of DOE in almost every industry. The computer-aided automatization of calculus and plots allow technique simplification.

2) Taguchi's approach

Taguchi's main focus was the development of robust design and method for quality improvement that helped spread the interest and use of DOE in several areas (electronics, aerospace, etc.). Taguchi made his developments in Japan during the 1940s and 1950s; however, it was in the 1980s that this approach was introduced in the USA and Europe. Taguchi's approach was more engineering than theoretical and statistical; he believed that a product of quality is one that causes a minimum loss during its lifetime.

Taguchi developed what is known as Quality Engineering, which consists of three phases: system design, parameter design and tolerance design. In system design, the objective is to know the system factors and the levels to which the system should operate. In parameter design, the aim is to improve the performance of a process or product by adjusting the factor levels (this if the phase related with DOE). During tolerance design, the objective is to determine the control parameters for each factor. Taguchi introduced the loss function which was an important contribution to statistics.

Taguchi's emphasis was to develop tools for the easy application of DOE, and prepared standard DOE, graphical tools to assign factors in the design, guides to easy results interpretation, methods to study uncontrollable factors using robust design techniques, etc.

3) Shainin approach

This approach was protected by Dorian Shainin with intellectual property rights and it was known only by his clients. This circumstance means that this approach is rarely discussed in technical forums.

In 2000, Keki Bhote and Adi Bhote were authorized to publish information on Shainin's approach because Motorola (where Bhote worked) won the 2000 American Management Association's Malcolm Baldrige National Quality Award (the first corporation to do so) and the award obliged the winner to share their methodologies with other US companies. Shainin's approach combines new and old techniques in a coherent manner that, if applied as suggested, can improve process efficiency. Shainin's approach proposed 1) use Pareto's principle (identify the factor with the main contribution to a process), 2) prefer factorial instead of fractional factorial design (identify factors influencing the response variability and reduce the variation to reduce the number of factors) and 3) highlight the importance of discussing the design with those ultimately responsible for the data assessment (Tanco, Viles and Pozueta, 2009).

In summary, the classical approach is the general methodology of DOE which leaves to the analyst the selection of the design, variables, levels, etc. using its own criteria and judgements; the freedom of the way to perform the analysis comes together with a high level of complexity which is the main limitation of this approach: indeed, this was the approach preferred by analysts with strong mathematical or statistical backgrounds.

The Taguchi and Shainin methods look for obtaining products of "quality" which include the DOE methodology as part of the assessment; both procedures consist of clearly stated recipes that target generating products with low variability or robustness. The Taguchi and Shainin methods were developed independently, one in Japan during 1940s and the other in USA during 1950s; both methods try to be simple and consider a standard set of experimental designs to make the evaluation be easy. Taguchi and Shainin developed their own set of plots for making a quick interpretation of the results. The main controversies about the Taguchi approach were about the use of the signal-to-noise ratio as the response, the tools proposed for analysing the experiments and the selection of experiments. In the case of Shainin's approach, several tools proposed for this method have been strongly criticised, such as Variable Search™ and Pre-Control™.

The Shainin method is considered a method for improving the efficiency. It was stablished as a three-stage process characterised by its high efficiency; first, the method identifies all the variables of the problem, and several tools (Multy-Vary, Component Search™, Paired Comparison™) are used to reduce the number of variables involved in the process; these tools use DOE and response variation; during the second stage, the tool Variable Search™ is used for sequential experimentation which allows to reduce even more the number of variables and permit using the factorial design; other tools (B vs. C™, Response Surface, Scatterplots) are used to confirm and optimise the results; in the third and last stage, tools such as Positrol™, Process certification and pre-control are used to secure sustainable results.

Each project has a metric that measures the value of the project under different assumptions of the input parameters. The value of the project is typically measured by the net present value (NPV) or internal rate of return (IRR) that the project is expected to deliver over its duration. In the oil and gas industry, these financial parameters or objective functions depend on the expected production and price of hydrocarbons over the period plus the anticipated investments, operating costs and contractual elements.

As previously discussed in this chapter, the uncertainty in hydrocarbon production is a function of various subsurface parameters such as reservoir permeability, water saturation, etc.

DOE is a technique for carrying out a series of tests in which specific changes are made to the input variables of a system or process and the effects of these changes are measured in terms of the response variables (Telford, 2007). During experimentation, the effects of the parameters on the response are measured.

DOE is used to understand a system or process through experimentation. Figure 2.4 illustrates the methodology of DOE.

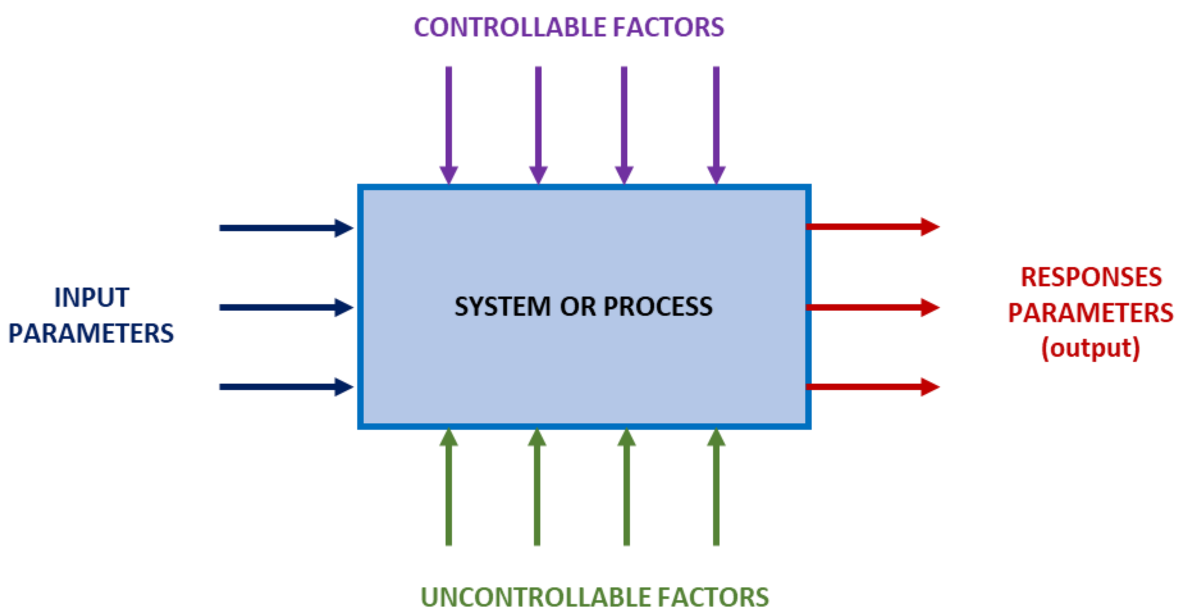

Figure 2.4. Diagram of the design of experiments approach

The following notation is used in this diagram:

- System or process: This is the function, model or manufacturing process used to transform input into output;
- Input: The parameters, variables or material subject to the system;
- Response or output: the product derived from the system or process;
- Controllable factors: the factors that experimenters can control;
- Uncontrollable factors: the factors that experimenters cannot control.

In DOE, factors can be (White and Royer, 2003):

i) Controllable: can be varied for the analyst;

ii) Observable: can be measured relatively accurately, but cannot be controlled;

iii) Uncertain: can neither be controlled nor measured accurately.

The response or objective function is the basis for decisions, and is related to the value of the project, e.g. the cumulative oil production at a defined time, the oil rate target, the time to water breakthrough, etc.

The goal of DOE is to maximise the amount of information obtained from a study with the minimum number of experiments.

DOE can be used for various purposes, including:

1) Comparison, i.e. to determine from two or more sets of inputs (e.g. raw materials for making a final product, or parameters in an experiment) which set produces the optimum results. For comparison purposes, the most frequently used approach is one-factor design.
2) Variable screening, i.e. to identify the most important variables affecting the performance of a product or system. For variable screening, the most widely used methods are two-level factorial design (full factorial or fractional factorial; the full factorial method is used for variable screening in our case study in Chapter 4), the Taguchi orthogonal array and Plackett–Burman design.

3) Transfer function exploration, i.e. to assess the value of the most influential variables in terms of the system performance. This application has been especially fruitful in the use of response surface methods to model complex systems by proxy or surrogate models using linear and quadratic functions.
4) System optimisation, i.e. when the objective is to improve the performance of the system, such as its quality, efficiency or reliability.

For identification of the transfer function and optimisation of the system, the approaches most often used are central composite design (CCD) and Box–Behnken design.

5) System robustness, i.e. when the objective is to make the system robust against "noise" such as uncontrollable or environmental factors.

For system robustness, the most widely used approach is Taguchi robust design.

In the case study discussed in this research, DOE is used for the purposes of (i) comparison and (ii) variable screening. Hence, the discussion in this section primarily relates to the methods associated with these two purposes, although methods used for other purposes are briefly described.

In the context of this research, the DOE technique is used to assess which input variables have the greatest impact on the project's value; the use of DOE differentiates the proposed VOI from the classical VOI methodology which focuses on one selected input variable without regard to other input variables that may have a higher impact on the project value. In this research, instead of assessing whether a specific data acquisition action adds value to the project, DOE is used for searching amongst all the uncertain input variables and ranking them according to their impact on the uncertain value of the project; this procedure means that the data acquisition action is selected wisely, the one that optimises the value of the project.

Methodologically, DOE is implemented in seven steps (Guo and Mettas, 2012):

A) Clarify and state objectives: In this step, the objective of the experiment is defined. This step helps in preparing the list of problems that will be addressed by the experiment.

B) Choose responses: This refers to the outcome of the experiments for which the responses should be measurable.

C) Choose factors and levels: This refers to the variables that are studied in the experiments in order to measure the effects on the responses. The value range of these factors is defined based on expert knowledge, and the values used in the experiments (the factor levels) are chosen within this value range.

D) Choose the DOE: The analyst selects the factors, the factor levels and the design type. The selection of the design depends on the number of parameters, the levels considered, the purpose of the study, the time frame for delivery of results, and whether interactions between parameters are defined.

E) Perform the experiments: The design matrix (i.e. the matrix of experiments to be performed) should guide the experiment. Each experiment includes one level per variable.

F) Analyse the data: Statistical methods, ANOVA and the analyst's knowledge of system performance are the tools used to analyse the outcome of the DOE.

G) Draw conclusions and deliver recommendations: Conclusions are drawn from an analysis of the data and recommendations for decision making are made in the final step of the design of the experimental process.

When choosing factors, it is important to know whether there are interdependencies between factors, as these can be used to reduce their number; for example, in some cases, uncertainty in remaining oil saturations (s_{or}), irreducible water saturation (s_{wir}) and the shape of the relative permeability curve can be combined into a single "relative permeability" uncertainty. However, this practice may reduce the possibility of assessing the value of the individual parameters in the overall reservoir assessment.

Once the parameters have been selected, their ranges must be determined; if wrongly chosen, these can negatively affect the selection of the most influential parameters (the 'heavy hitters').

The main objective of the screening study is to determine the most influential parameters, i.e. those that cause the largest changes in the response of interest when their defined ranges are varied. For screening purposes, a one-factor-at-a-time (OFAT) design is used. In this approach, in order to estimate the effect of each of the possible k factors, the response is evaluated using the two extreme values of the parameter of interest, maintaining the remaining k-1 factors at a certain set of values. The difference in the response functions at those values of the parameter k then represents the effect of this parameter in the response function. The whole process is then repeated to assess each of the other factors individually. This strategy is inefficient in terms of the number of simulation runs required; it does not allow for the measurement of any interactions (and in fact assumes that there are no such interactions).

A Pareto chart is used to visualise the impact of the factor on the response. This shows the absolute values of the standardised effects, from the largest to the smallest. The standardised effects are t-statistics used to test the null hypothesis that the effects are zero; the vertical dashed line shows which effects are larger than the reference line. A Pareto chart determines the magnitude and the importance of the effects but cannot determine which of them increase or decrease the response, as it represents the absolute values of these effects.

Factorial design is an experimental strategy in which variables are varied simultaneously, rather than one at a time. A full factorial design assesses the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable. A full two-level factorial design with k factors requires 2^k runs, which for large values of k can be prohibitive. Each set of values is called a design point. In cases with n levels, the number of experiments is n^k, and it is therefore clear that the number of experiments required increases quickly with n. However, full factorial design has the great advantage that it accounts not only for the main effects (i.e. those arising from each parameter), but also for interaction effects (i.e. those resulting from the influences between parameters).

When the number of variables is large, and time and resources are limited, fractional factorial design can be used. This involves a subset of the experimental runs of a full factorial design, and hence requires a smaller number of simulation runs but confounding or aliasing some factor interactions. Fractional factorial design was developed by Fisher (1935).

In order to optimise (reduce) the number of experiments, further developments in DOE have given rise to other designs, such as: (i) Plackett–Burman design, which is a low-resolution two-level design that is adequate for a screening study in which the objective is to identify the ‘heavy hitters’ but not to compare them (PB has the advantage that it requires a relatively low number of simulation runs); and (ii) optimal designs that give greater flexibility in terms of factor selection and interactions and that increase the number of levels of the factors (Leiviska, 2013).

For a three-level design, a composite design consists of a 2^k design, with $2\ x\ n$ start points and one centre point. The $2\ x\ n$ start points correspond to each parameter at its highest and lowest values, keeping the remainder at the most likely value.

For a reservoir with “n” uncertain parameters, $2(n+1)$ models need to be run: one run with all the parameters at their most likely value, and two runs for each other parameter, one for the lowest value and one for the highest.

The design can be represented in a compact, tabular form called a design matrix, which facilitates calculations of the factor effects and their interactions.

The visual analysis of results is carried out using a normal plot generated by the tool called ANOVA.

The significance level is calculated as one minus the confidence level for the analysis.

The main effects plot shows the differences between the mean levels for each factor. The main effects occur when different levels of a factor affect the response differently; in the main effects plot, a steeper slope for the line indicates a larger magnitude for the effect.

Contour plots are designed to display a two-dimensional view in which points with the same response are connected through a contour map. These contour maps are constructed based on a model that is generated based on the matrix design and the input response. In a contour plot of two variables, the rest of the variables are held constant, meaning that these plots are valid only for fixed levels of the other variables.

An ANOVA normal plot of the effects shows the standardised effects of the factors, relative to a distribution fit line that corresponds to the case where all the effects are zero. The standardised effects are t-statistics testing the null hypothesis, which assumes that the effects are zero. For each factor, the farther from zero the value on the vertical axis, the larger the statistical significance of that factor. The effects are positive if the response increases when settings change from a lower value to a higher one, and vice versa.

ANOVA is a statistical method that allows us to detect the most significant factors in a multi-factor model.

In brief, ANOVA is used to determine whether there are statistically significant differences between the outputs of experiments resulting from different selections of input parameters. When several sets of experiments are available, a t-test can be used to check whether the experiments produce different results due to random variation or due to changes in the input parameters. The null and the alternative hypotheses used in this process are as follows:

$$H_0: \mu_1 = \mu_2 = \cdots . = \mu_k = 0 \tag{48}$$

$$H_1: \mu_j \neq 0 \; for\ at\ least\ one\ j\ different\ than\ zero \tag{49}$$

In order to carry out the test, an ANOVA estimation is first generated, and an F-test is then performed based on this. The observed value is the ratio of the experimental mean squares (MS_{Tr}) and error mean squares MS_E (error variance):

$$F_0 = \frac{MS_{Tr}}{MS_E} = \frac{\frac{SS_{Tr}}{a-1}}{\frac{SS_E}{a(n-1)}} \tag{50}$$

where SS_{Tr} is the sum of squares of the experiments, SS_E is the sum of squares of the errors, $(a-1)$ is the degrees of freedom of the experiment, $a(n-1)$ is the degrees of freedom of the error, a is the number of experiments, and n is the number of observations for each experiment. The total sum of squares (Ss_T) is the sum of the sum of squares for experiments and sum of squares for error:

$$SS_T = SS_{Tr} + SS_E = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (51)$$

where $\hat{y}_i$ is the predicted value for the i^{th}test, $\bar{y}$ is the mean of the response variable observations, and y_i is the i^{th} observed value of the response variable.

Based on the value of F_0, H_0 is accepted or rejected as follows:

$$H_0 \; is \; rejected \; if \; F_0 > F_{\alpha,(a-1),a(n-1)} \quad (52)$$

$$H_0 \; is \; accepted \; if \; F_0 < F_{\alpha,(a-1),a(n-1)} \quad (53)$$

H_0 is rejected if the observed value of F_0 is greater than its critical value $F_{\alpha,(a-1),a(n.1)}$. The critical value is taken from the statistical table for significance level α, the degrees of freedom for the numerator $(a - 1)$, and the degrees of freedom for the denominator $a(n - 1)$. H_0 is accepted if the observed value is lower than the critical value.

When the DOE is used to generate the transfer function, a general linear regression model on the k input parameters should be estimated:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots \ldots\ldots + \beta_k x_k + \varepsilon \quad (54)$$

where y is the response variable, β_j. $j = 0,1, \ldots, k$ are the regression coefficients representing the change in response variable per unit change in input variable, x_j is the input variable, and Ɛ is a random error that is assumed to be normally distributed with $N(0, \sigma^2)$.

The least squares method is used to estimate the parameters in multiple linear regression models. For a system with k input variables in which n experiments are conducted, there exists a system of n equations of the form:

$$y_i = \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i \; ; i = 1,2, \dots, n \tag{55}$$

The β parameters can be calculated by minimising the sum of squares for error, ε^2. This equation is a surrogate model representing the system or process.

Lenth's analysis

Lenth's PSE, also called pseudo standard error, is a method used for calculating critical values for the effects when there are no replicates, which is the case in computer simulation experiments. For this calculation, it is assumed that the standard deviation of a sample from a normal distribution $N(0, \sigma)$ with 0 mean and standard deviation σ may be estimated as 1.5 times the median (absolute values). When some effects are non-null, a two-step process should be followed to delete the effects that exceed 2.5 times this estimate and to compute again. The critical value for the effects is reached by multiplying PSE by the appropriate critical value for t with $m/3$ degree of freedom, where m is the number of effects assessed.

DOE was originally developed for real-world experimentation and classical applications are found in Box and Wilson (1951), Hunter and Hunter (1978) and Box and Draper (1987). Law and Kelton (1991) and Myers and Montgomery (2002) include sections on DOE for simulation purposes but as part of a broader scope. The applications of DOE have expanded to several domains such as the chemical industry (Yang, Bi and Mao, 2002; Sjoblom et al., 2005; Ruotolo and Gubulin, 2005), materials (Suffield, Dillman and Haworth, 2004; Liao, 2003; Hoipkemeier-Wilson et al., 2004), industrial engineering (Tong, Kwong and Yu, 2004; Galantucci, Percoco and Spina, 2003; Du et al., 2002), electronic (Ogle and Hornberger, 2001) or mechanical engineering (Passmore, Patel and Lorentzen, 2001; Nataraj, Arunachalam and Dhandapani, 2005; Farhang-Mehr and Azann, 2005; Cervantes and Engstrom, 2004), aerospace (Zang and Green, 1999) and the analysis and optimization of nonlinear systems (Sacks et al., 1989).

Computational simulations and reservoir simulations especially differ from experiments in agriculture or medicine in that they do not have random error, so the same input will always produce the same output. In simulation work, the objective of DOE is to determine the factors that have the most impact on the response and get that result with the least number of simulation runs (Law, 2015), which is called factor screening or sensitivity analysis. Subsequently, using statistical techniques, response surface models can be obtained to build a surrogate model of the original complex model representing the system.

In the oil and gas industry, DOE was first introduced in reservoir engineering assessment in the early nineties by Damsleth, Hage and Volden (1992), Egeland et al. (1992) and Larsen, Kristoffersen and Egeland (1994) and, since then, it has been used in the petroleum industry for many purposes, such as: for identifying the main geological parameters responsible for oil recovery (White et al., 2001); for uncertainty integration to quantify their impact on original oil in place, recoverable reserves and production profiles (Corre, de Feraudy and Vincent, 2000); for assessing uncertainties in production profiles (Venkataraman, 2000); investigating the impact of geologic heterogeneities and uncertainties in different development schemes (Wang and White, 2002); and for defining the minimum number of reservoir simulation runs needed to identify and quantify the factors accountable for the uncertainties of the reservoir performance (Peake, Abadah and Skander, 2005). Additionally, studies on production forecasting and ultimate recovery estimates representing the numerical reservoir simulation by a surrogate response surface model are discussed by Friedmann, Chawathe and Larue (2001) and Murtha et al. (2009), while Dejean and Blanc (1999) discussed DOE, dividing the uncertain factors into uncontrollable and controllable and adapting DOE accordingly, and Law (2017) discussed the workflow for applying DOE to simulation modelling.

In our research work, the interest in DOE is not for building a response surface model but for the identification of the parameters that most impact the project value, the first step before assessing any data acquisition.

2.6 FUZZY LOGIC, FUZZY DATA AND FUZZY INFERENCE SYSTEMS

Crisp data are data that can be unequivocally characterised by either totally membership in a set or no membership at all; in this section of the thesis, this concept is contrasted with fuzzy data which describe data with a partial degree of membership in several sets. Crisp data use single values in their description, and fuzzy data use a set of numbers in their description. Fuzziness is the quality of a quantity to be fuzzy or unclear.

In Section 2.4, *uncertainty, risk, probability and decisions*, it is stated that, in addition to uncertainties due to randomness and lack of knowledge, uncertainty due to fuzziness also exists. It is also mentioned that fuzzy logic theory, described in this section, is the tool to manage that kind of uncertainty.

Classical logic is supported by two fundamental laws (Garrido, 2012):

1) The principle of excluded-middle: propositions are true or false with no other possibility
2) The principle of non-contradiction: a statement cannot be true and false simultaneously

This dichotomy between true and false generates several paradoxes that highlight the limitations of classical logic and the need for finding other forms of logic. Aristotle (380 B.C.) and Plato, the Greek philosopher, were the pioneers in considering that things do not have to be of one kind or the other but in an intermediate range; so, they said that there are different degrees of truth or falsity. The American philosopher Charles Sander Peirce (1902) was the first to consider vagueness instead of the true-false dichotomy, to understand how the world and humans work and their relationship with uncertain language human habits (Garrido, 2012).

At the beginning of the 20^{th} century, Bertrand Russell (1908) discussed the limitations of classical logic and used the paradox of Epimenides for illustration—when Epimenides, the Cretan, says that all Cretans lied, is he lying or telling the truth? These paradoxes cannot be solved using classical logic. Russell argued that classical logic will inevitably result in contradictions and concluded that vagueness, which is part of language, is a matter of degrees. Russell also explored other paradoxes, such as the one arising from the contradiction that the set of all sets does not contain itself, or the paradox of the barber, which similarly exposes the contradictions of the classical logic.

The theory of “vague sets” originates in the work of Heisenberg and Black. Heisenberg (1927) introduced the uncertainty principle of quantum mechanics, which state that there is a fuzziness in nature, a form of fundamental limit to what can be known about the behaviour of quantum particles; he said that the most that can be expected is to calculate probabilities for where things are and how they will behave. Max Black (1937), as cited by Garrido (2012), analysed modelling vagueness using classical logic and discussed the vagueness of the terms a human being used and the application of a profile or curve for the analysis of ambiguity, which is the beginning of what came to be known as the membership function in the frame of fuzzy logic.

Ludwig Wittgenstein (1953) studied different meanings of the same word, concluding that in language, the same word expresses different modes and manners that represent the vagueness of the terms.

Jan Lukasiewicz (1920), as cited by Garrido (2012), proposed a logic of three values, or trivalent logic (later extended to multi-value logic), which, in addition to the true and false values, accepts a value of indeterminate truth, which was assigned a value or grade of membership of 0.5. This was the first logic of vagueness; the elements of an asset have a possible degree of belonging not restricted to 0 and 1.0; Lukasiewicz’ works give origin to possibility theory.

Lotfi Zadeh published the paper Fuzzy Set in 1965, describing the mathematics of fuzzy numbers and how fuzzy logic can be used to describe events that have a partial degree of belonging to sets. In his work, Zadeh applied the logic developed by Lukasiewicz to the objects of a set and developed the algebra of fuzzy sets. While Russell and Black used the term vagueness to refer to the new logic, Zadeh preferred the term fuzzy.

The German mathematician Dieter Klaua presented, in 1965 and 1966, two versions of a cumulative hierarchy of what he named many-valued sets, of which Zadeh’s sets are a particular case (Gottwald, 2010).

While at the beginning, Zadeh’s ideas were rejected by the western scientific community, Zadeh and other scientists such as Bellman and Zadeh (1970), Lakoff (1978), Dunn (1992), Bezdek (1993, 2014), Negoita and Ralescu (1977), Goguen (1967), Bandler and Kohout (1978), Sugeno and Murofushi (1987), Sugeno and Kang (1988), Mizumoto and Tanaka (1976, 1981), Tanaka, Taniguchi and Wang (1999), Zimmermann and Sebastian (1994), Zimmermann (1996), etc. continued the development of the new theory. Zadeh (1968) showed how fuzzy events can be described using fuzzy set theory. In 1971, Zadeh published "Quantitative Fuzzy Semantics", where he developed the formal elements for the use of fuzzy logic and its applications, as this theory is known.

Fuzzy logic adapts to human reasoning and expressions more effectively than Boolean logic and achieves this through the use of quantifiers of the "degree of belonging" that elements (data values, whether measured or estimated) have on a set of categories (linguistic terms such as qualifiers or decisions).

Fuzzy logic deals with linguistic terms such as "high", "low", or "little", and allows us to replicate human reasoning capabilities.

Fuzzy set theory is a generalisation of a two-value crisp set to a membership function fuzzy set, and is an extension of multi-valued logic, which can be implemented for approximate reasoning problems. It involves the use of fractional truth, where the values of truth range between fully false and fully true.

Formally, a fuzzy set consists of a universe of discourse and a membership function that maps each element in the universe of discourse into a membership value, in the interval zero to one. It is assumed that X is the universe of discourse and x is one element of X; then the fuzzy set A is characterised by the mapping:

$$\mu_A(x): X \rightarrow [0, 1] \tag{56}$$

where the membership function $\mu_A(x)$ represents the degree of belonging of x to A and can be any value between zero and one. In contrast, in the two-value set, the value of $\mu_A(x)$ can be either zero or one.

Classical set theory, developed by Georg Cantor in the 19^{th} century, defines how crisp sets are related by logic operators such as intersection (AND), union (OR) and complement (NOT) operating based on two-value logic. These operators are the same as those used in fuzzy sets, but in this case are applied for all possible fuzzy values, which are real values between zero and one.

Let X be the course of discourse and A and B two fuzzy sets defined over X; the set operators are defined as follows:

A) The intersection (AND) operator

In classical set theory, the intersection is defined as the elements that belong to both A and B simultaneously. In fuzzy set theory, the intersection represents how many of the elements are in both sets. This is defined by:

$$\mu_{A \cap B}(x) = \min\big(\mu_A(x), \mu_B(x)\big) \quad \forall\, x \in X \tag{57}$$

B) The union (OR) operator

In the classical set theory, the union operator represents those elements that belong to either set. In fuzzy set theory, the union operator measures how much of each element is in either set, and it is represented by:

$$\mu_{A \cup B}(x) = \max\big(\mu_A(x), \mu_B(x)\big) \quad \forall\, x \in X \tag{58}$$

C) Complement (NOT) operator: Let $\bar{A}$ denote the complement of the fuzzy set A.

In classical set theory, the complement operator defines those elements that do not belong to the set. In fuzzy set theory, the complement measures the extent to which an element does not belong to the set. This operator is mathematically formulated as follows:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \qquad \forall\, x \in X \tag{59}$$

D) Containment operator: A fuzzy set A is a subset of fuzzy set B if and only if

$$\mu_A(x) \leq \mu_B(x) \quad \forall\, x \in X \tag{60}$$

In classical set theory, elements of a subset belong entirely to that set; however, in fuzzy theory, they may only partially belong.

The membership function can be of any shape and type. It should represent the expert's opinion and must satisfy two basic conditions: (i) a lower limit of zero and an upper limit of one; and (ii) for each $x \in X, \mu_A$ has to be sole.

Membership functions may be symmetrical or asymmetrical, and have three regions: the core, support and boundary.

The core is the region of the membership function in which the elements have a value of one:

$$core(A) = \{x \in X \mid \mu_A(x) = 1\} \tag{61}$$

The support is the region of the membership function in which the value of the elements is greater than zero:

$$support(A) = \{x \in X \mid \mu_A(x) > 0\} \tag{62}$$

The boundary is the region of the membership function in which the elements have a value between zero and one:

$$boundary(A) = \{x \in X \mid 0 < \mu_A(x) < 1\} \tag{63}$$

The membership function is characterised by a height, which represents its maximum value:

$$height(A) = \{x \in X \mid max(\mu_A(x))\} \tag{64}$$

For each fuzzy set, there is a belonging function that quantifies the degree to which each element belongs to the set; these functions are typically, linear, triangular, trapezoid etc.

Zadeh (1973) introduced the main ideas underlying the analysis of complex systems, in which human knowledge is captured within a set of fuzzy rules. A fuzzy rule is a conditional statement in the form:

$$IF\ x\ is\ A\ THEN\ y\ is\ B \tag{65}$$

The IF part of the rule is the antecedent, and the THEN part is the consequence.

These rules are defined based on the knowledge and experience of experts within the relevant domain.

More than anywhere else, Zadeh's ideas were welcome in Japan, South Korea, China and India. Since 1973, fuzzy logic started to be used and applied by several Japanese universities. Terano and Shibata in Tokyo and Tanaka and Asai in Osaka made important contributions to the development of fuzzy logic and its applications (Garrido, 2012).

The classical VOI assumes that the outcome of a data acquisition action is a data value without fuzziness or imprecision, which is not typically the situation in the real world; the imprecision occurs due to the measurement made during the data acquisition, and due to the linguistic variables used to describe the state of nature on the project of interest. In the proposed VOI methodology, fuzzy variables are used instead of crisp variables, to account for the imprecision in the data.

In the oil and gas industry, uncertainty may be the result of a *lack of information*, *inaccuracy of measurement* or *lexical vagueness*, as discussed in Chapter 2. Typical examples of a *lack of information* are the porosity and permeability values used to populate the reservoir models between existing wells. Probability techniques are used to manage uncertainties due to a lack of complete information.

Inaccuracy of measurement relates to the measuring tools used and to the classification and interpretation of the measurement. For example, pressure gradient measurements have uncertainties associated with the measured value and depth; similarly, the saturation values of the remaining oil usually carry uncertainties related to the methods, conditions and accuracy of the logs and laboratory experiments. The subjectivity of the interpreter also adds uncertainty to the values resulting from the data, since the interpretation of results often results in categories with *lexical vagueness,* such as "large", "profitable", "small", etc. Uncertainties arising from *inaccuracy of measurement* and *lexical vagueness* introduce imprecision into the data. In this thesis, it is shown how fuzzy logic can be used to manage uncertainty in VOI assessments in the oil and gas industry.

There are two approaches that can be used in understanding the outcome of data acquisition involving crisp or fuzzy data. In the crisp approach, the outcome of the data acquisition falls into only one of the discrete intervals into which the range of possible outcomes of the data acquisition is divided; in the fuzzy approach (fuzzy VOI), the outcome of the data may fall into more than one of these discrete intervals.

In classical logic, an element or event either "belongs" to a set of outcomes or does not, and this can be described using the binary representation of zero or one. In classical set theory, this is referred to as the characteristic function for the set of events. Fuzzy logic extends the concept of the characteristic function to a membership function, which represents the meaning of "belonging" as a continuous value between zero and one. In this way, the degree to which an event belongs to these sets of outcomes is represented by the membership function of the event on those sets.

Fuzzy logic captures the concept of vagueness via the membership function, which is a mapping from a given universe of discourse X to a unit interval containing the membership values.

In a crisp set of events M, the probability of occurrence of the events in the set is:

$$P(M) = \sum_{x \epsilon X} p(x) \mu_M = \sum_{x_k \in M} p(x_k) \tag{66}$$

where: $M \subset X$ (67)

In Equation (66), $p(x)$ is the probability of the occurrence of event x, μ_M is the characteristic function (defined in Equation (68) below), and $p(x_k)$ are the probabilities of the events for which the characteristic function is one.

The characteristic function is (Zadeh, 1965):

$$\mu_M = \begin{cases} 1, & x_k \in M \\ 0, & otherwise \end{cases} \tag{68}$$

For a fuzzy set, the probability of a fuzzy event $\widetilde{M}$ is:

$$P(\widetilde{M}) = \sum_{k=1}^{r} \mu_{\widetilde{M}}(x_k) p(x_k) \tag{69}$$

where $\mu_{\widetilde{M}}(x_k)$ is the membership function $\mu_{\widetilde{M}}$ evaluated for the value x_k.

The posterior probabilities of the states of nature, given a fuzzy event $\widetilde{M}$, are given by Equation (41), assuming that the reliability, prior probabilities and membership functions of the fuzzy events are known (Ross, 2010):

$$P(s_i|\widetilde{M}) = \frac{\sum_{k=1}^{r} p(x_k|s_i)\mu_{\widetilde{M}}(x_k)p(s_i)}{P(\widetilde{M})} = \frac{P(\widetilde{M}|s_i)p(s_i)}{P(\widetilde{M})} \tag{70}$$

where the fuzzy reliability probabilities are:

$$P(\widetilde{M}|s_i) = \sum_{k=1}^{r} p(x_k|s_i)\mu_{\widetilde{M}}(x_k) \tag{71}$$

An orthogonal fuzzy system is a set $\emptyset$ of fuzzy sets, $\emptyset = \{\widetilde{M}_1, \widetilde{M}_2, \ldots\ldots \widetilde{M}_l\}$, satisfying the condition:

$$\sum_{f=1}^{l} \mu_{\widetilde{M}_f}(x_m) = 1 \qquad \{\text{for all } x_m \in X\} \tag{72}$$

For fuzzy events, if the fuzzy system is an orthogonal set and the data outcome is represented by the fuzzy set $\widetilde{M}_k$, then the EV of the j^{th} alternative and membership function $\widetilde{M}_f$ is given by:

$$EV(u_j|\widetilde{M}_f) = \sum_{i=1}^{n} u_{ij}p(s_i|\widetilde{M}_f) \tag{73}$$

The optimum alternative for the fuzzy set $\widetilde{M}_k$ is that which maximises the EV:

$$EV(u^*|\widetilde{M}_f) = \max_j EV(u_j|\widetilde{M}_f) \tag{74}$$

The unconditional maximum EV takes the form:

$$EV(u_\emptyset^*) = \sum_{f=1}^{l} EV(u^*|\widetilde{M}_f)p(\widetilde{M}_f) \tag{75}$$

Finally, the VOI is the difference between the *EV with information* and the *EV without information*, from Equations (75) and (39):

$$VOI = EV(u_\emptyset^*) - EV(u^*) \tag{76}$$

An important milestone in the development of fuzzy logic was the construction of the first fuzzy controller for a steam engine by Assilian and Mamdani (1974); in this paper, fuzzy logic is used to convert heuristic control rules into an automatic control strategy. However, the first real implementation of a fuzzy controller was made by the Danish engineers Lauritz Peter Holmbland and Jens Jurgen Ostergaard (1980), who developed the commercial system of fuzzy control working for F.L, Smidth & Co. in a cement factory in Denmark (Larsen, 1980; Umbers and King, 1980), which resulted in one of the first successful test runs on a full-scale industrial process.

Fuji Company developed a fuzzy logic controller for chemical injection in water treatment plants for the first time in Japan (Yagishita, Itho and Sugeno, 1984). Researchers at Bell Laboratories developed the first fuzzy chip (1985), which was later used in several products, such as camcorders, cameras, etc.

In 1988, Yamakawa published the article "Fuzzy Controller Hardware System", in which the first fuzzy controller in integrated circuits was developed. Hitachi introduced the first-time fuzzy controller to control Sendai's underground in 1987 by applying predictive fuzzy theory and succeeded in making the train operate better than with the traditional Proportional-Integral-Derivative control method, and it has been used since then (Yasunobu, Miyamoto and Ihara, 1983); this is one of the more spectacular fuzzy control systems built. In 1985, Omron electronics developed the first fuzzy computer.

Simultaneously with all these successful applications of fuzzy logic for real-world problems, researchers continued theoretical developments following Mamdani's path. In 1985, Takagi and Sugeno developed a tool to build a fuzzy model of a system where both fuzzy implications and reasoning are used.

Neural network and fuzzy systems share similarities and that has resulted in the development of neuro-fuzzy systems, which use learning methods based on neural networks to identify and optimize their parameters. Finally, genetic algorithms, together with neural networks and fuzzy systems, are very strong tools in control systems.

Although fuzzy systems and probability work in the same range ([0.0, 1.0]), it is important to distinguish them. Probability refers to our ignorance regarding a statement, while fuzziness refers to the degree to which something happens, or a condition exists.

In classical logic, there exists a mapping function, named the characteristic function, which associates the elements of the universe of discourse with one of the elements of the set $\{0,1\}$; fuzzy logic generalizes this concept by defining a membership function for mapping the universe of discourse with the closed interval $[0,1]$. In this sense, fuzzy set theory is a generalization of classical set theory.

The main objective of fuzzy logic is to build a system based on the behaviour and thinking of humans; it is based on experts' experience and knowledge. From the practical perspective, fuzzy logic is implemented using a Fuzzy Inference System (FIS), which is the actual process of mapping from a given input to an output using fuzzy logic. As stated by Negnevitsky (2005), "Fuzzy logic is not logic that is fuzzy, but a logic that is used to describe fuzziness".

Fuzzy inference is the process of mapping from a given input to an output, using fuzzy logic. There are two types of FIS, the Mamdani type and the Sugeno type, and these vary in terms of the way in which the outputs are determined.

The most frequently used fuzzy inference method was proposed by Mamdani and Assilian (1975) for the design of a system to control a steam engine and boiler combination. It is based on Zadeh's work on fuzzy algorithms to control complex systems. The output of a Mamdani method is a fuzzy set for each output, and these are aggregated and finally defuzzified. In certain situations, the output of the membership function is a single spike rather than a fuzzy set; this type of output is called a singleton membership function and is a pre-defuzzified fuzzy set. Singletons are used to enhance the efficiency of the defuzzification process, since, rather than computing the centroid of a two-dimensional function, the weighted average of only a few points can be calculated.

The Mamdani fuzzy inference system is a process consisting of four steps: fuzzification of input variables, rule evaluation, aggregation of outputs and defuzzification.

Step1: Fuzzification of the input variables

This involves determining the degree to which the input variables belong to the appropriate fuzzy sets described by the membership functions. The input is a crisp number defined in the universe of discourse, with ranges determined by expert judgment; the output of the fuzzification process is a fuzzy degree of membership, in the range zero to one, in the fuzzy set. This fuzzification process is followed for each of the input variables against the fuzzy sets.

Step 2: Rule evaluation

This step consists of applying the fuzzified inputs to the antecedents of the fuzzy rules. When the fuzzy rules have more than one antecedent, fuzzy operators (AND or OR) are used to obtain a single number representing the result of the antecedent evaluation, and this number is then applied to the consequence membership function. To evaluate the disjunction in the rule antecedents, the OR fuzzy operation (Equation 58) can be used; similarly, to evaluate the conjunction in the rule antecedents, the AND fuzzy operator (Equation 57) can be applied. The antecedents are the degree of the input variables after fuzzification and, regardless of whether the antecedent of a rule has one or more parts, the fuzzy operator results in a single number that represents the outcome of the antecedents for that rule.

Each rule has a weight that is applied to the number given by the antecedent. These weights represent the relative importance of each rule and lie in the range zero to one. The results of the antecedent evaluation can be applied to the membership function of the consequence. In this manner, the consequence membership function is clipped or reshaped to the level of the truth value of the rule antecedent. The consequence membership is typically cut at the level of the antecedent truth, and this method is known as clipping or the correlation minimum. Since this method involves the loss of some information, an alternative method is to use the scaling or correlation product, which preserves the original shape of the fuzzy set. In this case, the rule consequence is adjusted by multiplying the membership degrees by the truth value of the rule antecedent.

Step 3: Aggregation of outputs

Decisions should be based on the result of applying all the rules simultaneously. Aggregation refers to the process in which the rules are compiled to produce a single fuzzy set. During aggregation, the fuzzy sets representing the output of each rule are combined into a single fuzzy set. The input of the aggregation is the list of truncated output functions returned by the implication process of each rule, and the output is a single fuzzy set for each output variable. The most common methods for aggregation are MAX (maximum), PROBOR (probabilistic OR) and SUM (simple sum of the output of each rule).

Step 4: Defuzzification

The input of the defuzzification process is a fuzzy set, and the output is a single number. Although fuzziness helps in the rule evaluation step, the final desired output for each variable is generally a number that can be calculated using the defuzzification process.

The most popular defuzzification method is the centroid calculation, which returns the point at which a vertical line divides the aggregated set into two equal masses. Mathematically, this centroid, or centre of gravity (COG), technique is expressed by

$$COG = \frac{\int_a^b \mu_A(x)x\,dx}{\int_a^b \mu_A(x)\,dx} \tag{77}$$

Equation (77) is the continuous version of the COG technique, which can also be discretised for finite intervals. Other possible methods include bisector, middle of maximum, largest of maximum and smallest of maximum approaches.

The Sugeno fuzzy inference system was developed 10 years after the Mamdani´s method (Sugeno, 1985); it is very similar to Mamdani's and uses the same steps. The main difference between Sugeno´s and

Mandani´s methods is that, instead of the rule consequence being a fuzzy set, it is a function that is generally a singleton; this is the zero order Sugeno fuzzy model. In the Sugeno system, instead of clipping the fuzzy set, the spike of the singleton is cut at the value of the antecedent evaluation. Aggregation is carried out based on a weighted sum of the cut spikes.

In practice, fuzzy logic is implemented using an FIS. This is a nonlinear procedure that derives its output based on fuzzy reasoning and a set of IF-THEN rules. It performs approximate reasoning in the same way as the human brain, albeit in a much more primitive manner.

The FIS is one of the most widely used applications of fuzzy logic, and has been applied in very different contexts and within various problem domains such as the assessment of water quality in rivers (Ocampo, 2008); improvements in the quality of image expansion (Sakalli, Yan and Fu, 1999); the differential diagnosis of non-toxic thyropathy (Guo and Ling, 2008); the development of a fuzzy logic controller for a traffic junction (Pappis and Mamdani, 1997); the design of a sensor-based fire monitoring system for coal mines using fuzzy logic (Muduli, Jana and Mishra, 2018); estimation of the impact of tax legislation reforms on potential tax (Musayev, Madatova and Rustamov, 2016); pipeline risk assessment (Jamshidi et al., 2013); the diagnosis of depression (Chattopadhyay, 2014); the assessment of predicted river discharge (Jayawardena et al., 2014); calculation of geological strength indices and slope stability assessments (Sonmez, Gokceoglu and Ulusay, 2004); regulation of industrial reactors (Ghasem, 2006); the use of a fuzzy logic approach for file management and organisation (Gupta, 2011).

In the domain of the oil and gas industry, several applications of FIS have been reported, such as the streamline-based fuzzy logic workflow to redistribute water injection by accounting for operational constraints and number of supported producers in a pattern (Bukhamseen et al., 2017), the identification of horizontal well placement (Popa, 2013), estimating strength of rock using FIS (Sari, 2016), and predicting the rate of penetration in shale formations (Ahmed et al., 2019). Fuzzy logic has been used in combination with other Artificial Intelligence techniques such as Adaptative Neuro-Fuzzy Inference System (ANFIS) in practical applications, e.g. to predict the inflow performance of vertical wells producing two-phase flow

(Basfar et al., 2018) or to predict geomechanical failure parameters (Alloush et al., 2017); FIS has also been used in conjunction with Analytical Hierarchical processes to evaluate the water injection performance in heterogeneous reservoirs (Oluwajuwon and Olugbenga, 2018) and to make decisions in the application of fuzzy inference systems for VOI in the oil and gas industry (Vilela, Oluyemi and Petrovski, 2019).

From a methodological perspective, an FIS can be understood as a general procedure that transforms a set of input variables into a set of outputs, following the workflow shown in Figure 2.5.

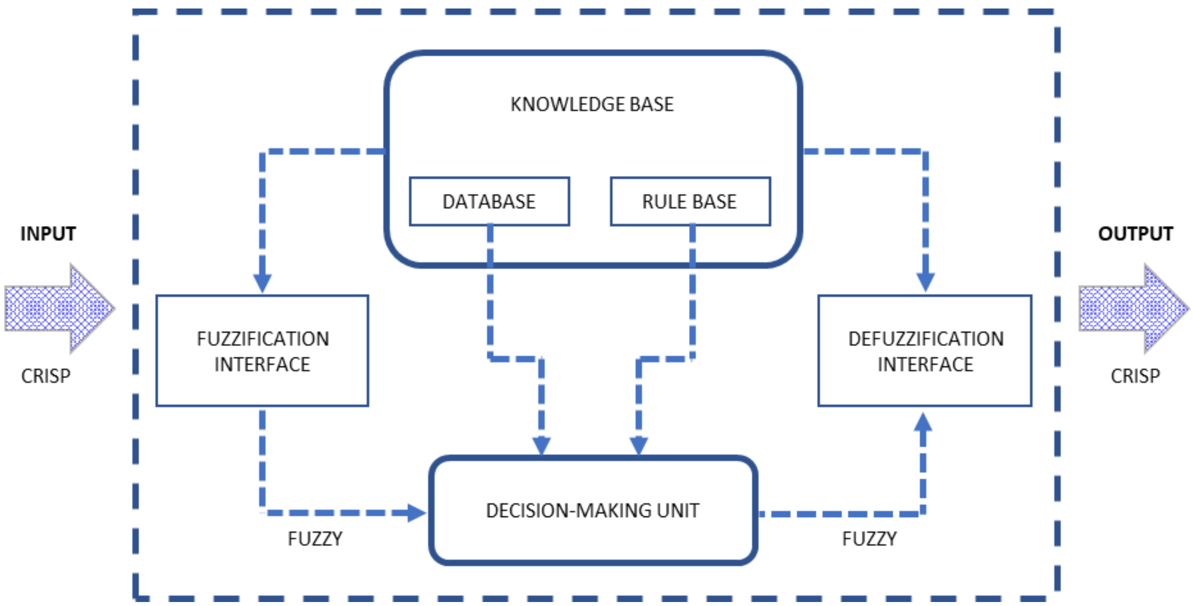

Figure 2.5. Fuzzy inference system workflow

Figure 2.5 shows the FIS as a procedure consisting of four blocks (or steps) in which the inputs and outputs are in crisp form (although in some cases these may be fuzzy).

The variables describing decisions (e.g. endorse or reject a project) are fuzzy variables, suited for the human logic but not for the Boolean logic used in the classical VOI methodology; in this context, an FIS is developed in this research work for replicating the human decision process for data acquisition.

For capturing the imprecision in the data and to evaluate the criteria to accept/reject a VOI assessment, it is proposed to integrate fuzzy theory into the classical methodology; Vilela, Oluyemi and Petrovski (2018) discussed a novel application to manage the fuzziness of subsurface data into the framework of VOI assessment.

2.7 SUMMARY

Decision analysis and VOI techniques have been applied to several domains, including the oil and gas industry where, due to the large number of decisions and monetary capital investment involved, it is an especially rich domain for exploiting their benefits.

While decision analysis and VOI were theoretically formulated before the 1970s, their application in the oil and gas industry was very limited until the 1980s. Even today, the application of these techniques is not as frequent as it should be, considering the potential benefits and the research that has been carried out on the link between the use of those techniques and good business performance (MacMillan, 2000).

Fuzzy logic burst onto the scene during the 1970s and 1980s and its applications were mostly related with control systems and automation. The relationship between decisions and fuzzy logic comes through FIS, with applications in areas as dissimilar as medicine and engineering. In the oil and gas industry domain, the applications of FIS have mostly been used for the optimisation of field operations.

Experiments that were initially designed, developed and applied in the 1920s to agricultural problems began to be applied to computational problems with the appearance of high-speed computers in the 1980s; this made it possible to use DOE in problems related to the computational modelling of hydrocarbon fields for the screening of relevant parameters as well as in reservoir optimisation studies and uncertainty analysis (when applied in conjunction with response surface modelling).

According to our research questions (Section 1.5), several issues remain in need of improvement in the classical approach for VOI and, for each, this research proposes a technique or methodology to solve the corresponding issue, integrated into a new methodology for VOI development in this research.

Our research work proposes the use of fuzzy logic to integrate the fuzziness of the data in the assessments, as well as a fuzzy inference system to aggregate the typically imprecise terms used in the assessments and DOE techniques, to provide an optimum understanding of the more relevant parameters of data acquisition; it also goes back to the sources of SEUT theory, recognising the importance of including the decision maker's attitude towards risk in the assessment.

Chapter Three

Methodology for Value of Information: classical and proposed

3.1 INTRODUCTION

This chapter reviews the classical methodology for VOI and discusses the weakness observed; then, several techniques and methods that can be used to improve the classical methodology for VOI are discussed; the chapter concludes with a description, step by step, of the new VOI methodology proposed in this research and how the new techniques and methods are integrated in the proposal methodology.

3.2 CLASSICAL METHODOLOGY FOR VALUE OF INFORMATION

The classical methodology of VOI (which is well described by Clemen 1996, Newendorp and Schuyler 2000, Koninx 2000, Coopersmith and Cunningham 2002, Bratvold, Bickel and Lohne 2007) has been applied in several case studies in the oil and gas industry and it differs from application to application; however, it can be broadly described by 12 steps, as follows:

1) Define a discrete set (usually three) of states of nature (each state of nature is defined by a selected set of values for the uncertain input parameters);
2) Calculate the value (monetary values, e.g. Net Present Value, Internal Rate of Return, etc.) of each state of nature;
3) Estimate the (Bayesian) prior probabilities associated with each state of nature;
4) Calculate the expected monetary value of the project, assuming the no data acquisition;
5) Identify one uncertain parameter that affects the value of the project;
6) Select one data acquisition action which can impact on the uncertainty of the parameter identified in Step 5;
7) Calculate the cost associated with the data acquisition proposed in Step 6, i.e. the investment needed to acquire the data and the associated production losses (if any);
8) Estimate the reliability probabilities associated with the data acquisition;

9) Apply Bayes' theorem (Bayes' theorem is discussed in Chapter 2) to obtain the posterior probabilities (using the prior probabilities and the reliability probabilities);
10) Calculate the expected value of the project, assuming the proposed data acquisition;
11) Subtract the expected value of the project with the present information from the expected value of the project with the proposed data acquisition (i.e. the value calculated in Step 10 minus the value in Step 4);
12) If the difference in Step 11 is higher than the associated cost of data acquisition in Step 7, it is worthwhile gathering the data; otherwise, gathering the new data is not recommended.

3.3 LIMITATIONS IN THE CLASSICAL METHODOLOGY FOR VALUE OF INFORMATION

Based on the analysis made in the Literature Review (Chapter 2), authors of this research conclude that the classical methodology for VOI has the following weaknesses:

1) Typically, the classical approach for VOI assessment is carried out when it has been identified that the value of the project depends on an uncertain input variable that may be better defined if a specific piece of data is acquired. This approach lacks a complete analysis of the project uncertainties and the impact that the different inputs and their interactions have on the project's value. This procedure to assess the value for acquiring data can limit the opportunities to improve the project's value;
2) The classical approach to VOI does not provide an integral assessment of the impact that a specific data gathering activity may have on the uncertainty of more than one variable;
3) VOI does not consider that the data to be acquired may carry uncertainties that are due not only to randomness but also to fuzziness;
4) Although the utility value is a well-known concept, it is not typically used in VOI assessments;

5) The criteria used by decision makers for making decisions (e.g. to reject a project or to accept a data acquisition proposal) are fuzzy. However, the results from the classical VOI assessment are crisp numbers; the handling of this dichotomy requires different tools from the ones used in the classical approach for VOI;

In many cases, more than one objective function is used to make decisions; in these cases, they are treated separately, and the results from the different objective functions may be contradictory, depending on the outcome resulting from the assessment made for each objective function. An integrated result for the different objective functions is required.

3.4 TECHNIQUES FOR IMPROVING THE CLASSICAL METHODOLOGY FOR VALUE OF INFORMATION

Once the limitations in the classical methodology for VOI have been identified, several techniques and methods are suggested for improving the methodology.

For moving from an activity-based assessment to a project-based assessment, the authors propose to use the technique of DOE (described in Chapter 2, Literature Review); DOE allows a consistent and coherent analysis of all the project input variables and their uncertainties and concludes with a ranking of variables and their interactions in terms of their impact on the project's value; this analysis can be subsequently used for the identification of the most valuable data acquisition actions for the specific project under consideration. This analysis allows also to compare two or more data acquisition actions in terms of their value to the project and decide accordingly.

As has been discussed in the Literature Review, the uncertainty in the project is not only due to lack of data and randomness, but it can be also due to fuzziness in the data proposed to be acquired. In the classical methodology for VOI, the data is assumed crisp, and the assessment is based on that assumption; Section

2.6, equations 69 to 76 show the modifications that have to be made to equations 41 to 47 (Section 2.3) to account for the fuzziness of the data in the framework of VOI assessment.

Decisions in the real world are taken using linguistic variables, such as large, high, and good, which are fuzzy or vague in meaning; however, the classical methodology for VOI is crisp. This dichotomy can be solved if the methodology is built using a logic that follows human reasoning; this can be achieved by means of a Fuzzy Inference system, which the authors of this research develop as an example for the case study discussed in this research work.

3.5 NEW METHODOLOGY FOR VALUE OF INFORMATION

To remove the limitations found in the classical VOI methodology, described on Section 3.3, a new methodology for VOI is proposed in this research work, which consists of the 17 steps described below and shown in Figure 1.1.

1) Define a discrete set (usually three) of states of nature (each state of nature is defined by a selected set of the values for the uncertain input parameters);
2) Calculate the values of the decision criteria (monetary values, e.g. Net Present Value, Internal Rate of Return, etc.) of each state of nature;
3) Estimate the utility-value for each of the decision criteria (e.g. utility values of parameters calculated in Step 2) for the different state of nature;
4) Estimate the (Bayesian) prior probabilities associated with each state of nature;
5) Calculate the expected value of the project for each of the selected utility-value decision criteria parameters, assuming the no data acquisition;
6) Develop an FIS using decisions rules over the utility-value decision criteria and the “linguistic variables” decisions options for assessing the optimum decision;

7) Calculate the expected utility value of the project for the no data acquisition case using the FIS built in step 6.
8) Identify and rank all uncertain parameters and their interactions by their impact on the value of the project using DOE techniques;
9) Identify data acquisition actions that can change the perception of the uncertainty of the main parameters and their interactions;
10) Select one of the data acquisition actions identified in Step 9 and calculate the cost associated with it, i.e. the investment needed to acquire the data and the production losses (if any);
11) Assess the imprecision of the new data outcomes: the imprecision of the data outcomes is captured via the membership functions in the fuzzy logic framework;
12) Estimate the reliability probability associated with the data acquisition;
13) Apply Bayes' theorem to obtain posterior probabilities (using the prior probabilities, the reliability probabilities and the membership functions);
14) Calculate the expected value of the project for each of the selected utility-value decision criteria parameters, assuming the proposed data acquisition action;
15) Using the FIS already develop in step 6, calculate the expected value of the project for the data acquisition selected in step 10;
16) Repeat Steps 10, 11, 12, 13, 14 and 15 for each of the possible data acquisition actions;
17) Carry out the VOI assessment: choose the alternative with the highest defuzzified value, between the no data acquisition and the data acquisition alternatives.

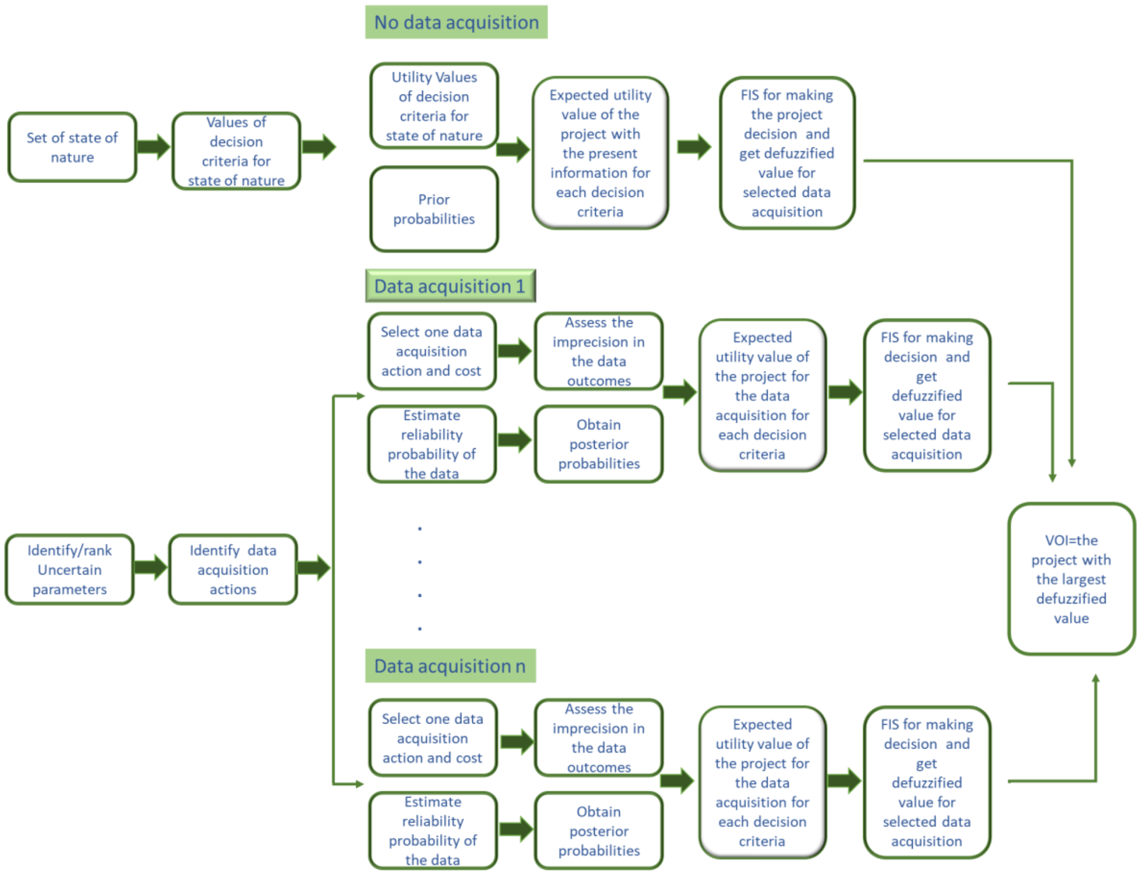

Figure 3.1. Proposed value of information methodology

3.6 SUMMARY

The proposed methodology for VOI successfully removes the limitations found in the classical VOI; it logically integrates methodologies from other domains within the VOI assessment to conclude with a more complete methodology. The case study discussed in Chapter 4 is a practical application of the proposed methodology; of course, a real case has its own limitations and may not allow to show all the benefits of the methodology proposed; however, in this research work, the case study illustrates very well the main aspect of the new methodology for VOI.

Chapter Four

Case Study

4.1 CASE STUDY

The case study was carried out using an actual field. The field is not identified for confidentiality reasons. This project was at the early appraisal phase when the operating company decided to terminate it due to the risk of failure.

Here, reservoir data and a dynamic simulation model are used to illustrate how the methodology developed in this research work for assessing the VOI could have been applied in this case for making data acquisition decisions. In addition to reservoir data, company financial information is used, such as the cost of wells and facilities; financial criteria for the same company are also used to evaluate the benefits of their projects, such as the net present value and internal rate of return.

4.2 RESERVOIR DESCRIPTION

The case study is on a clastic reservoir, a nearly flat structure located at 1,050 m TVDSS. The reservoir thickness is between 15 and 20 m, and the area of the reservoir is 12.5 km by 5.4 km. Based on the seismic information and the wells that have already been drilled, there is no indication of fractures or any other anomalies. The sedimentary environment is marine.

Fluid surface samples taken from three of the exploration and appraisal wells indicate that the reservoir is filled with dead oil with a bubble point pressure (pressure at which the gas in solution into the oil begins to come out of solution) of 1,194 psi and a dissolved gas concentration (R_s) (portion of the gas dissolved in the crude oil) of 0.5681 MSCF/STB. The oil viscosity (measure of the amount of resistance that oil experience to flow in the porous media) was measured in all the samples as 1.08 cp.

The discovery well was *P01*, which was followed by appraisal wells *P02*, *P03* and *P04*. Although the first three wells show good petrophysical properties, *P04* has very poor characteristics. An analysis of the porosity (measure of the void space in a material) –permeability (measure of the number of inches per hour that a fluid moves in a rock) relationship shows that the Timur's correlation reflects the relationship between these two parameters well and s_{wirr} (that is the minimum water saturation, reached when the hydrocarbon content is at maximum). Three lab measured values of s_{wirr} resulted in 15%, which is assumed constant throughout the field until more information is gathered. It has the functional form:

$$\kappa = 0.136 \times \emptyset^{4.4} / s_{wirr}^2 \tag{78}$$

Tables A1-1 and A1-2 in Appendix 1 show the values of the average porosity and permeability measured in the four wells in the nine layers used for simulation modelling.

The reason for the contrasting values observed for porosity and permeability between the three wells located to the west (*P01*, *P02* and *P03*) and the appraisal well to the east (*P04*) is not clear; however, two approaches are considered to be equally valid in explaining these differences:

1) A diagenetic phenomenon acting over the reservoir degrades its properties from west to east;

2) The values of these properties measured for the fourth well are due to a local effect in this location or are the result of measurement issues that make them unrepresentative of the distribution of the properties of the reservoir.

The irreducible water saturation s_{wirr} was measured for two plugs, and the average value was 15%.

4.3 RESERVOIR MODEL

A simulation model is used to forecast the performance of the field and wells. The dynamic model was built in Eclipse (Schlumberger™) for the authors of this research, based on a static model provided by the operator company. Figure 4.1 shows the location of the four appraisal wells on the depth map extracted from the model.

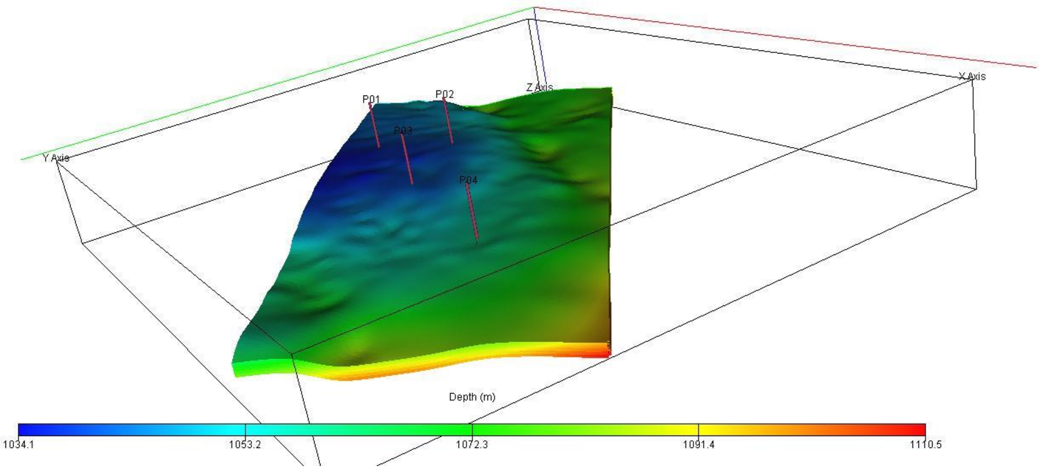

Figure 4.1. Structural map of the field with the exploration and appraisal wells

The depth of the reservoir datum (reservoir reference depth) is 1,010 m TVDSS with a corresponding initial reservoir pressure of 3,626 psi (pressure=3,626 psi @datum).

The dynamic model that was built to predict development scenarios contains 234,000 cells, with 180*150 areal cells and nine layers. The average values of DX and DY are 69.4 and 36.3 m respectively, and the average layer thickness is 1.75 m, with a minimum of 0.61 m and a maximum of 3.8 m.

Tables A1-3 and A1-4 in Appendix 1 show the mean porosity and permeability values per layer for the low, medium and high models.

4.4 MODEL UNCERTAINTIES

Based on the information gathered during the exploration and appraisal phases, and experience gained from analogue fields, the technical team members estimate that the forecasted reservoir performance has six major sources of uncertainty:

1) horizontal permeability distribution (PXY);
2) vertical permeability (PZE);
3) oil and water relative permeability (REP);
4) aquifer support, volume and productivity index (AQU);
5) oil/water contact (OWC);
6) well productivity index (WPI).

These uncertainties are described in more detail below.

1) The permeability values measured in wells *P01*, *P02* and *P03* are high, in the range 320 to 1,650 mD; however, the permeability values measured in well *P04* are exceptionally low, at less than 1.0 mD. In addition, the well test conducted on *P04* was unsuccessful, and no fluid was measured at the surface, thus confirming the very low permeability measured for this well. The degradation in porosity and permeability around well *P04* is attributed to: i) diagenesis in this area of the reservoir or ii) a local effect (see Section 4.2). Neither the area of the reservoir affected by diagenesis nor the extent of these poor properties is known, meaning that permeability distribution is one of the uncertainties impacting on the performance of this field. To represent the uncertainty in the horizontal permeability, three scenarios are considered:

    1.1. <u>Large area of good permeability</u>: This case assumes that the low values of porosity and permeability in well *P04* are the result of a local effect around this well, meaning that these values are not used to populate the model.

1.2. Small area of good permeability: This case assumes that the low values of porosity and permeability observed for well *P04* are a consequence of a diagenetic phenomenon affecting the reservoir, in which the values measured for the first three wells are gradually reduced towards the fourth well; in this case, porosity is reduced from the values observed for *P01*, *P02* and *P03* to the values measured for *P04*.

1.3. Medium area of good permeability: This case assumes that the low values extend beyond the *P04* well but not as far as in the low case.

Figures A1-1, A1-2 and A1-3 in Appendix 1 show the permeability distribution for the high, medium and low cases.

2) The vertical permeability was not measured for the core samples taken from the exploration and appraisal wells. Based on analogue fields, it is assumed that the $\frac{k_v}{k_h}$ is between 0.01 and 10.0 (analogue fields show this range of values between vertical and horizontal permeability).

3) Relative permeability experiments were not conducted in the lab, since neither core was available for these tests; however, the irreducible water saturation (s_{wirr}), critical water saturation (s_{wcr}) and initial water saturation (s_{wi}) were measured, and their values were 0.15, 0.21 and 0.18 respectively. These figures and the Corey's exponent relationship were used to build the relative permeability curves for this two-phase fluid. The uncertainty in the remaining oil saturation is estimated using values of 0.15, 0.17 and 0.20 for the high, medium and low cases, respectively. Figure A1-4 in Appendix 1 shows the relative permeability curve for the high, medium and low scenarios.

4) The aquifer strength is another source of uncertainty. While some fields in the same basin experienced an important aquifer influx after a few weeks of production, others experienced the opposite. In the dynamic model, the aquifer was modelled using the analytical Fetkovich aquifer, and its strength is driven by two parameters:

   i) volume: between $2.52e^9$ and $2.52e^{13}$ STB and,

   ii) productivity index : between 217 and 868 STB/d/psi

5) The oil–water contact measured using electric logging is different for wells *P01*, *P02* and *P03*, with values in the range 1,070 to 1,080 m TVDSS.
6) The history matching of the well test conducted on wells *P01*, *P02* and *P03* requires changes in the productivity index (PI) of each well. These adjustments to the PIs are justified on the basis of the uncertainties in the static and dynamic parameters that are not properly captured in the model, mainly because of the limited data available. The uncertainty in the PI is implemented by a PI multiplier applied to future wells: for the low case, the PI multiplier is 0.90, for the medium case 8.90 and for the high case 18.40, which are the factors used to match the well test results obtained for the wells *P01*, *P02* and *P03* respectively.

Table 4.1 summarises the low, medium and high values of each of the six uncertain parameters.

Table 4.1. Uncertain parameters: low, medium and high values

UNCERTAIN PARAMETERS	LOW	MEDIUM	HIGH
Horizontal permeability	Extended diagenesis	Medium case diagenesis	Local diagenesis
Vertical permeability (mD)	0.01	0.50	10.00
Relative permeability	Co=3.1 / Cw=3.3 / Sorw= 0.15	Co=2.5 / Cw=4.4 / Sorw=0.17	Co=1.8 / Cw=5.5 / Sorw=0.20
Aquifer strength, AQU Vol. / AQI PI STB/(STB/d/psi)	$2.52e^{9}$ / 217	$2.52e^{11}$ / 434	$2.52e^{13}$ / 868
Oil/water contact (m)	1,070	1,075	1,080
Well PI multiplier	0.90	8.90	18.40

4.5 FIELD DEVELOPMENT PLAN

Analogue fields located close to this field have been developed using horizontal wells. For this reason, and due to the intention to use exploration and appraisal wells for future field development, all the exploration and appraisal wells drilled were horizontal, with an average horizontal section of between 600–700 *m* in the reservoir section; this is the estimated horizontal section planned for the development phase.

To maximise the oil recovery and due to the hydrocarbon characteristics, the development strategy assumes that for each oil producer well, a nearby parallel water injector well will be drilled for sweeping the oil and supporting the reservoir pressure. Of the four wells already drilled, *P01*, *P02* and *P03* can be utilised for the field development; however, due to its poor rock properties, *P04* is not feasible for the exploitation phase and the plan is to abandon it.

The objective of the project is to produce the first oil in January 2021, with six producer wells on stream (three already existing wells and three additional ones to be drilled during 2020); at the beginning of 2022, water flooding will start with six water injection wells, each paired with one of the producer wells. Six months after starting water injection (July 2022), depending on the static scenario, two or four new producers will be drilled and completed. Six months later (Jan 2023), the corresponding water injectors will start injecting.

The areal extension of the reservoir is determined by the extension of the good permeability and by the depth of the OWC. The number of wells in each case is designed in such a way as to avoid well interference. Table 4.2 shows the number of oil producers and water injectors for each scenario of permeability.

Table 4.2. Number of oil producers and water injectors for the three scenarios of good permeability areal extension

WELL TYPE	LARGE	SMALL	MEDIUM
Producers	10	6	8
Injectors	10	6	8

Here, *Large* means that the measured permeability values in *P04* are a local effect and that the good permeability extends over the entire reservoir except for a very limited area around *P04*; *Small* means that the *P04* value is the result of a distribution trend involving high values for *P01*, *P02* and *P03* and a low value for *P04*; and *Medium* is an intermediate case between the *Large* and the *Small* scenarios.

The depth of the OWC is also a parameter limiting the number of wells when developing the field. Table 4.3 summarises the number of wells for each OWC scenario.

Table 4.3. Number of oil producers and water injectors for the three scenarios of OWC

WELL TYPE	DEEP	SHALLOW	MEDIUM
Producers	10	6	8
Injectors	10	6	8

Here, *Deep* means that the OWC is the deepest possible (1,080 m); *Shallow* means that the OWC is the shallowest possible (1,070 m); and *Medium* is the arithmetic mean of the two extreme cases (1,075 m).

These two parameters, the extension of good permeability and OWC, limit the number of development wells: the number of wells in each case is the minimum allowed between the two parameters.

In the cases involving small and medium areas of extension of good permeability, two dry wells are included in the economic analysis. Similarly, in the case of the shallow OWC, two dry wells are used. The reason for including these dry wells is that before any other wells are drilled, there is no information that indicates the underlying conditions, meaning that drilling in unproductive locations is possible.

If an additional well is drilled for data acquisition, only one dry well is required rather than two. Figure 4.2 shows the schedule for connection of the oil producers (OP) and water injectors (WI) to the stream.

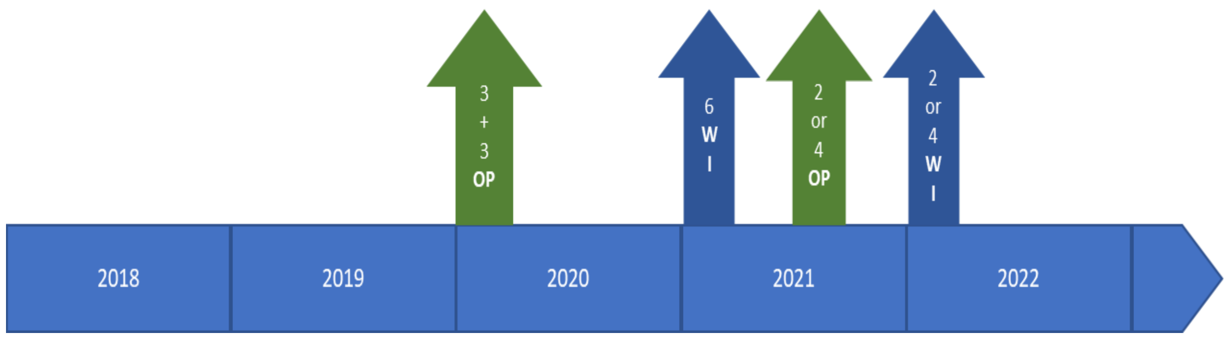

Figure 4.2. Schedule for oil producers and water injectors

4.6 DECISION PROBLEM

The decision that the operator company must take is whether to proceed with or to terminate the project. However, the acquisition of data can change the value of the project, thereby impacting the decision problem.

In the case where the operator decides to acquire additional data, this action carries a cost and possible delay in the project start; these negative impacts may be worthwhile if compensated by the positive impact of risk reduction and an increase in the project's value.

In Section 4.7, an uncertainty analysis is conducted to determine which uncertain parameters have a larger effect on the response. This assessment will be used to steer the data acquisition actions required to maximise the project's value.

4.7 UNCERTAINTY ANALYSIS: DESIGN OF EXPERIMENTS

DOE methodology is used here to carry out an uncertainty analysis and identify those uncertain parameters that have the most impact on the value of the project and can steer the data acquisition needs.

The objective functions for the uncertainty analysis are the NPV and the utility of the NPV (UNPV).

With the six factors discussed in Section 4.4, a full factorial design requires 64 simulation runs. Despite this large number of runs, this design is selected because it allows us to determine the main and interaction effects using a design that is still manageable from the computational side. Dynamic reservoir simulations are performed using Eclipse software (a commercial application from Schlumberger).

The six uncertain reservoir parameters vary between their maximum and minimum, following the design matrix shown in Tables 4.5 and 4.6. The cumulative forecast for oil for the 64 runs is shown in Figure 4.3.

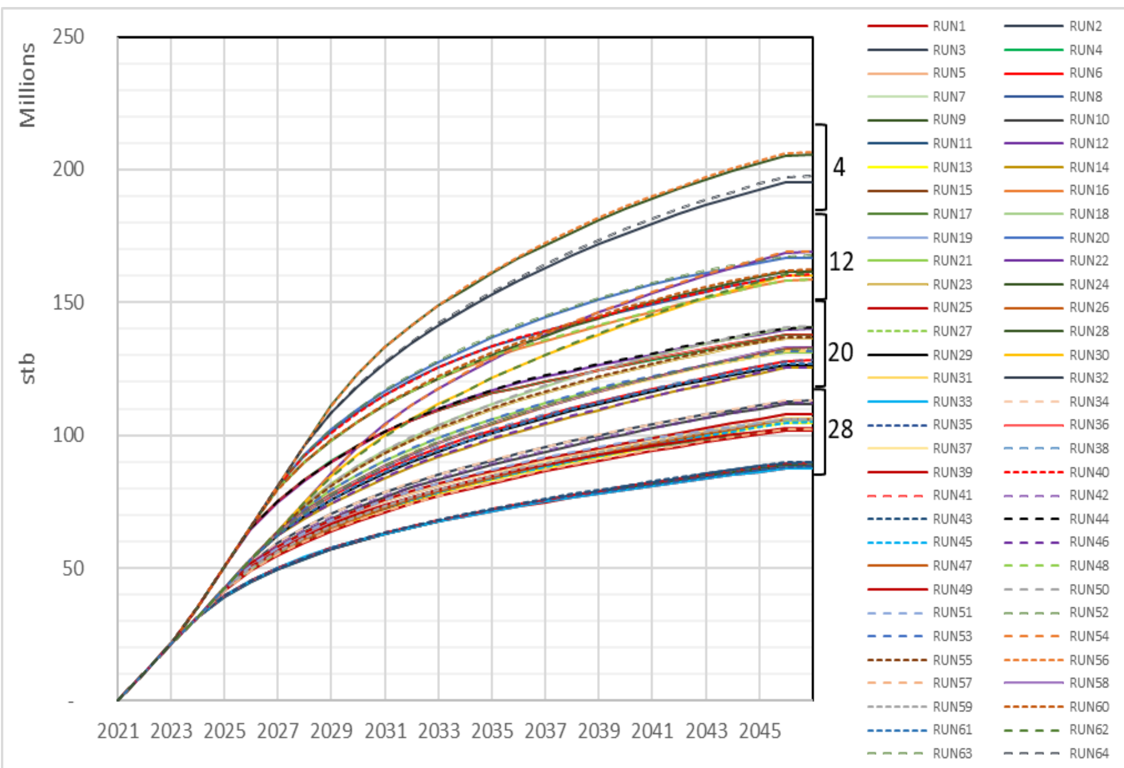

Figure 4.3. Uncertainty in cumulative oil production

Tables A1-5 and A1-6 in Appendix 1 show the results for cumulative oil production for the 64 dynamic simulation runs. The maximum and minimum values are 206 MM STB (run 56) and 88 MM STB (run 1), and the dispersion in the objective function is large (118 MM STB).

In Figure 4.3 the range of cumulative oil has been divided into four intervals, and Table 4.4 shows the number of runs with cumulative oil within each interval.

Table 4.4. Number of runs with cumulative oil cumulative within selected intervals

INTERVAL	MINIMUM INTERVAL, MM STB	MAXIMUM INTERVAL, MM STB	NUMBER OF RUNS
1	86	118	28
2	118	150	20
3	150	182	12
4	182	214	4

The financial model used to evaluate the project benefits, NPV, is built using Excel software (Windows Office). This model includes the oil production forecast resulting from the simulation runs and the CAPEX (Capital Expenditure or investment), OPEX (Operational expenditure), oil price forecast and tax, as shown in Tables A1-7, A1-8 and A1-9 in Appendix 1. Because at the time of this analysis there was no information available on royalties and rights to minerals, those were excluded from the calculations.

Following Walls (2005) and as suggested in Section 2.2, the utility function used is the exponential, which in this study case will have a tolerance factor (TF) of US$ 4,000 MM. This TF is representative of the company's historic attitude toward risk for projects. Figure 4.4 shows the resultant utility function for the uncertainty analysis of this case study.

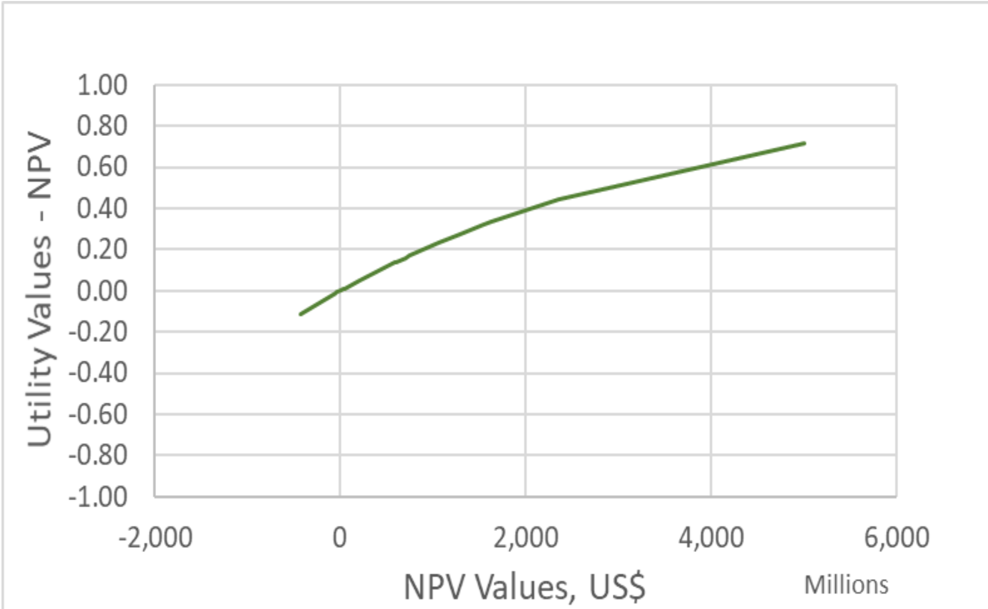

Figure 4.4. Utility value function

The matrix design is a compact representation of the values assigned to each variable for each case: when the variable is assigned to the highest value, +1 is used; when the variable is assigned to the lowest value, −1 is used; and when the variables is assigned to the middle value, 0 is used. Tables 4.5 and 4.6 show the matrix design, NPV and UNPV for each of these cases.

Table 4.5. Full factorial design matrix (first 32 runs)

RUN	OWC	PXY	REP	PZE	AQU	WPI	VALUE, MM US$	UNPV
Run 1	-1	-1	-1	-1	-1	-1	- 514.6	- 0.14
Run 2	1	-1	-1	-1	-1	-1	174.3	0.04
Run 3	-1	1	-1	-1	-1	-1	- 502.2	- 0.13
Run 4	1	1	-1	-1	-1	-1	984.8	0.22
Run 5	-1	-1	1	-1	-1	-1	16.1	0.00
Run 6	1	-1	1	-1	-1	-1	552.1	0.13
Run 7	-1	1	1	-1	-1	-1	44.1	0.01
Run 8	1	1	1	-1	-1	-1	1,566.8	0.32
Run 9	-1	-1	-1	1	-1	-1	- 499.5	- 0.13
Run 10	1	-1	-1	1	-1	-1	123.6	0.03
Run 11	-1	1	-1	1	-1	-1	- 489.8	- 0.13
Run 12	1	1	-1	1	-1	-1	1,015.2	0.22
Run 13	-1	-1	1	1	-1	-1	- 76.4	- 0.02
Run 14	1	-1	1	1	-1	-1	452.3	0.11
Run 15	-1	1	1	1	-1	-1	- 54.8	- 0.01
Run 16	1	1	1	1	-1	-1	1,463.3	0.31
Run 17	-1	-1	-1	-1	1	-1	- 69.2	- 0.02
Run 18	1	-1	-1	-1	1	-1	854.5	0.19
Run 19	-1	1	-1	-1	1	-1	19.1	0.00
Run 20	1	1	-1	-1	1	-1	1,693.0	0.35
Run 21	-1	-1	1	-1	1	-1	662.8	0.15
Run 22	1	-1	1	-1	1	-1	1,406.7	0.30
Run 23	-1	1	1	-1	1	-1	784.5	0.18
Run 24	1	1	1	-1	1	-1	2,490.6	0.46
Run 25	-1	-1	-1	1	1	-1	- 133.8	- 0.03
Run 26	1	-1	-1	1	1	-1	642.0	0.15
Run 27	-1	1	-1	1	1	-1	- 41.7	- 0.01
Run 28	1	1	-1	1	1	-1	1,510.6	0.31
Run 29	-1	-1	1	1	1	-1	507.1	0.12
Run 30	1	-1	1	1	1	-1	1,211.1	0.26
Run 31	-1	1	1	1	1	-1	631.4	0.15
Run 32	1	1	1	1	1	-1	2,256.0	0.43

Table 4.6. Full factorial design matrix (last 32 runs)

RUN	OWC	PXY	REP	PZE	AQU	WPI	VALUE, MM US$	UTILITY
Run 33	-1	-1	-1	-1	-1	1	- 508.7	- 0.14
Run 34	1	-1	-1	-1	-1	1	181.8	0.04
Run 35	-1	1	-1	-1	-1	1	- 498.1	- 0.13
Run 36	1	1	-1	-1	-1	1	985.3	0.22
Run 37	-1	-1	1	-1	-1	1	24.6	0.01
Run 38	1	-1	1	-1	-1	1	558.0	0.13
Run 39	-1	1	1	-1	-1	1	47.0	0.01
Run 40	1	1	1	-1	-1	1	1,566.0	0.32
Run 41	-1	-1	-1	1	-1	1	- 480.2	- 0.13
Run 42	1	-1	-1	1	-1	1	135.3	0.03
Run 43	-1	1	-1	1	-1	1	- 477.4	- 0.13
Run 44	1	1	-1	1	-1	1	1,029.0	0.23
Run 45	-1	-1	1	1	-1	1	- 57.3	- 0.01
Run 46	1	-1	1	1	-1	1	460.9	0.11
Run 47	-1	1	1	1	-1	1	- 43.4	- 0.01
Run 48	1	1	1	1	-1	1	1,468.3	0.31
Run 49	-1	-1	-1	-1	1	1	- 56.5	- 0.01
Run 50	1	-1	-1	-1	1	1	860.9	0.19
Run 51	-1	1	-1	-1	1	1	29.9	0.01
Run 52	1	1	-1	-1	1	1	1,707.4	0.35
Run 53	-1	-1	1	-1	1	1	672.0	0.15
Run 54	1	-1	1	-1	1	1	1,409.8	0.30
Run 55	-1	1	1	-1	1	1	792.5	0.18
Run 56	1	1	1	-1	1	1	2,506.2	0.47
Run 57	-1	-1	-1	1	1	1	- 119.8	- 0.03
Run 58	1	-1	-1	1	1	1	645.3	0.15
Run 59	-1	1	-1	1	1	1	- 31.1	- 0.01
Run 60	1	1	-1	1	1	1	1,532.4	0.32
Run 61	-1	-1	1	1	1	1	520.9	0.12
Run 62	1	-1	1	1	1	1	1,212.0	0.26
Run 63	-1	1	1	1	1	1	640.8	0.15
Run 64	1	1	1	1	1	1	2,290.7	0.44

The ANOVA technique allows us to estimate the parameters that most affect the objective function (UNPV). In this study, ANOVA is implemented using the commercial software MiniTab17.3.1.

Figure 4.5 shows a Pareto chart of the effects of these parameters on the utility function, with a significance level of 5%. The magnitudes of these effects indicate that the most important parameters are those labelled A, E, C, B, AB and AC, which correspond to the reservoir parameters OWC, AQU, REP, PXY and the interaction terms OWC/PXY and OWC/REP. Other factors that are much less important are D or PZE, the three-term interaction ABE or OWC/PXY/AQU and the two-term interactions CD, DE and CE corresponding to the factors REP/PZE, PZE/AQU and REP/AQU.

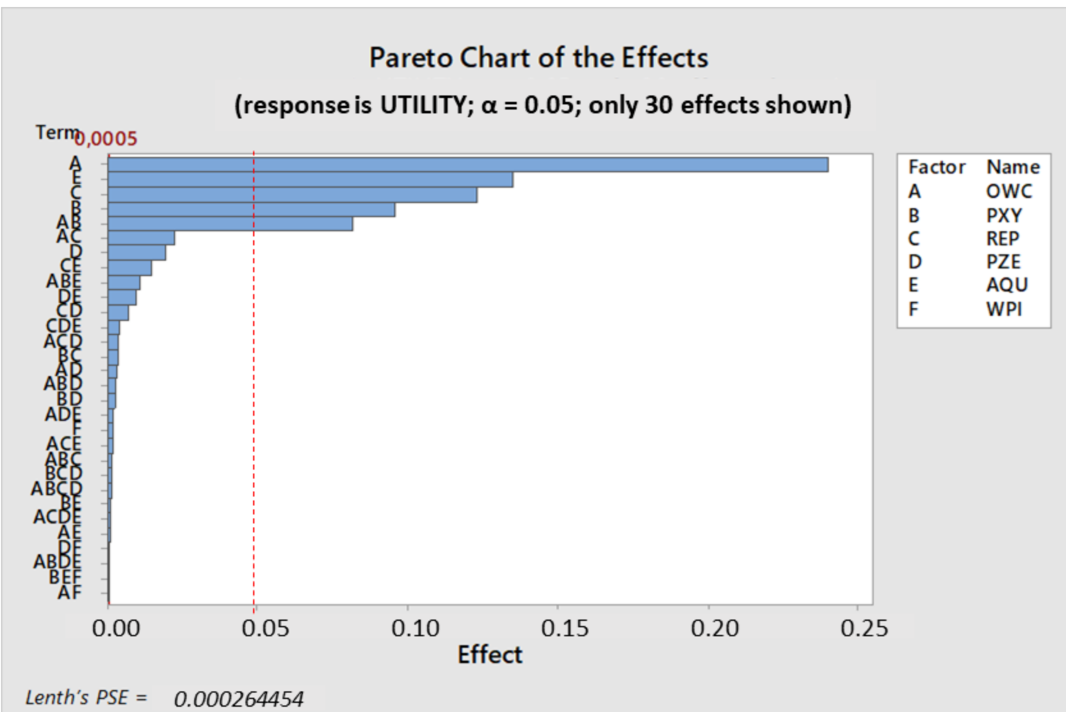

Figure 4.5. Pareto chart of the effects of the parameters, with a significance level of 0.05

In computational experiments there are no replications (all experiments done with the same inputs produce the same result; the estimation of standard error is done using the Lenth approach (see Chapter 2 for more details); Figure 4.5 shows at the left bottom the calculated Lenth's PSE (pseudo standard error).

Figure 4.6 shows a normal plot of the effects for the utility function with a significance level of 5%. The normal plot shows that, while an increase in the values of OWC, AQU, REP, PXY and the interaction terms OWC/PXY and REP/AQU increases the utility function, an increase in the values of PZE or the interaction terms OWC/REP, OWC/PXY/AQU, OWC/AQU and PZE/AQU decreases the response utility value.

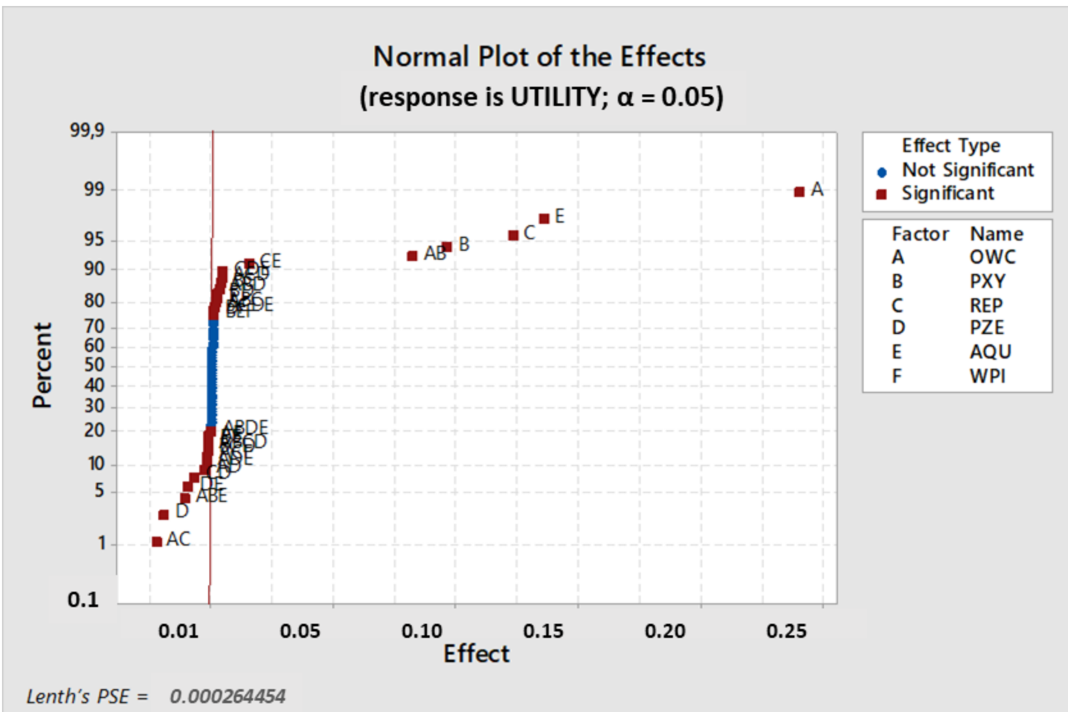

Figure 4.6. Normal plot of the effects of parameters, with a significance level of 0.05

The normal plot also confirms that the main factors impacting the response function are OWC, PXY, REP and AQI.

The slope of the plot of the effects in Figure 4.7 confirms that OWC is the factor with the highest impact on the response, followed by AQU and REP, which have very similar impacts on the response. PXY also has an important effect, while PZE and WPI have much less relevance in terms of their effect in the response.

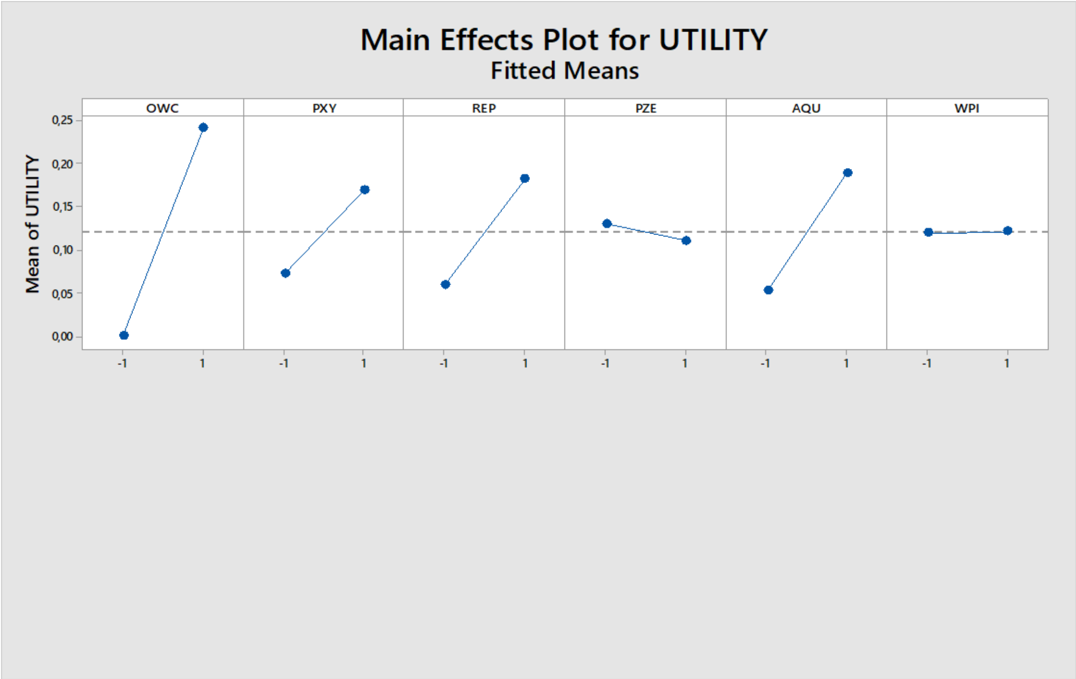

Figure 4.7. Main plot of utility values

The interaction effects are shown in Figure 4.8, from which it can be seen that the effect of OWC on the response depends on the PXY, REP and AQU values and is independent of the PZE and WPI values. The PXY effect is impacted by the level selected for REP and AQU but is independent of the levels of PZE and WPI. The effect of REP depends on the AQU values but is independent of the values of PZE and WPI. PZE depends on the AQU level but not on the WPI values and, finally, the AQU values are independent of the WPI figures.

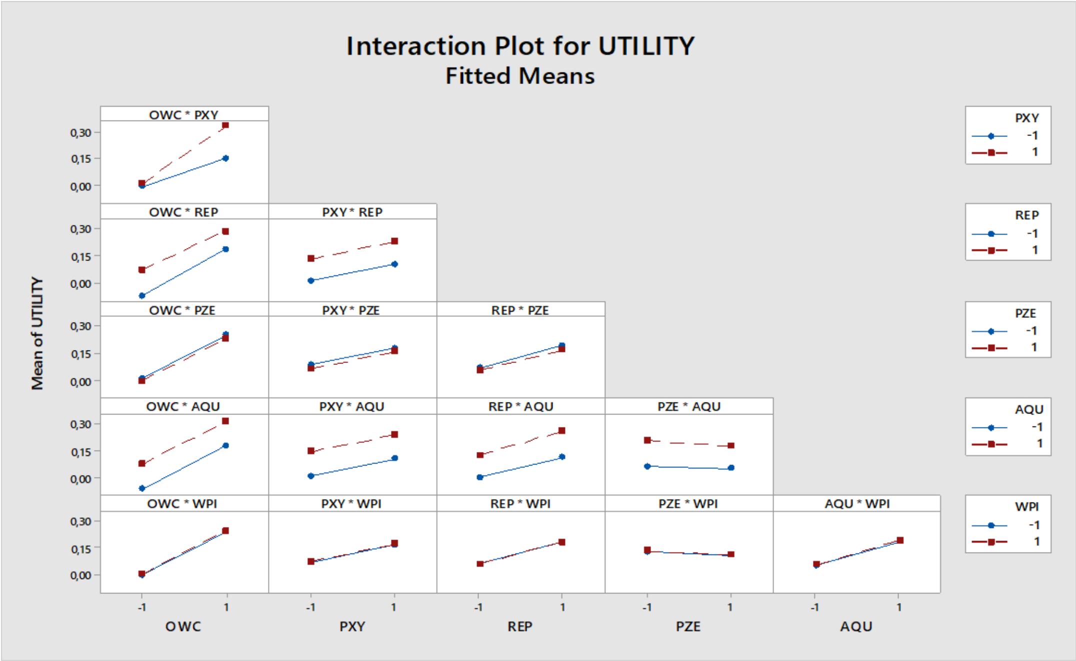

Figure 4.8. Interaction plot of utility values

Figure 4.9 shows the cube plot, which is a 3D view of the utility values for all combinations of the 64 runs.

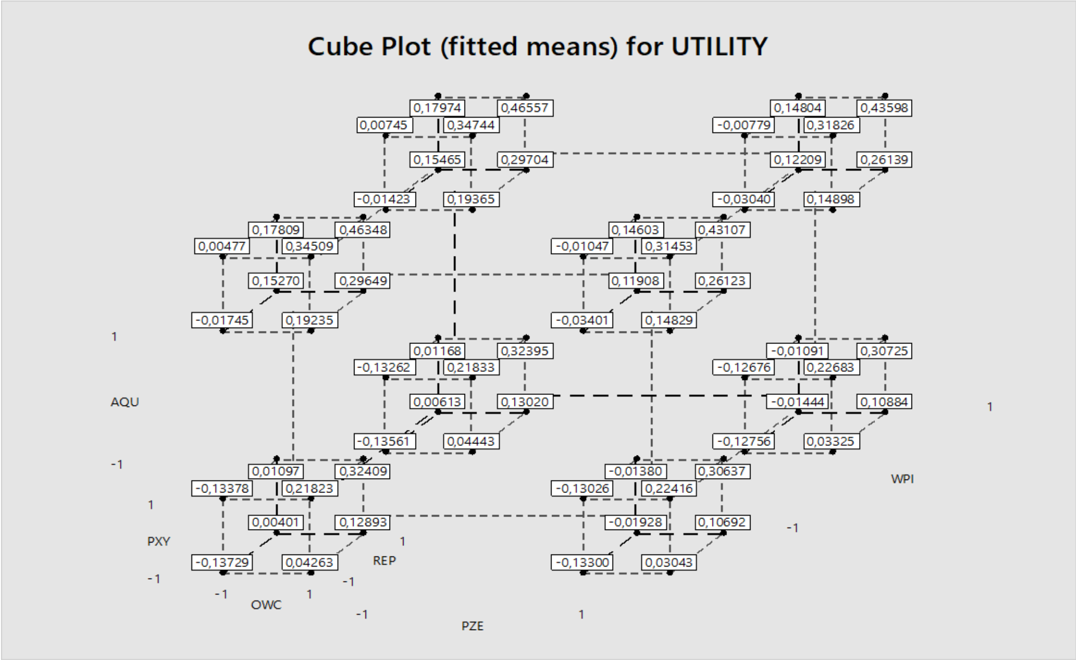

Figure 4.9. Cube plot (fitted means) of utility

The results of the uncertainty analysis indicate that the four uncertainty parameters that impact the most on the UNPV are as follows, in order of importance: OWC, AQU, REP and PXY. There are also interaction terms which are important, i.e. combinations of the previous four factors (OWC/PXY and OWC/REP).

4.8 PROJECT ASSESSMENT WITH CURRENT DATA

In Section 4.7, it was shown that the four parameters OWC, AQU, REP and PXY are the main responsible for the uncertainty in the value of the project.

To estimate the expected value of the project with the current data and uncertainty, simulation models were run for each combination of *high* (1) and *low* (-1) values for these four parameters, keeping the other two parameters, WPI and PZE, at their medium values. This means that 16 dynamic simulation models need to be run, according to the matrix design shown in Table 4.7.

Table 4.7. Matrix design for the assessment of the project

CASE	OWC	AQU	REP	PXY
S1	1	1	1	1
S2	1	1	1	-1
S3	1	1	-1	1
S4	1	1	-1	-1
S5	1	-1	1	1
S6	1	-1	1	-1
S7	1	-1	-1	1
S8	1	-1	-1	-1
S9	-1	1	1	1
S10	-1	1	1	-1
S11	-1	1	-1	1
S12	-1	1	-1	-1
S13	-1	-1	1	1
S14	-1	-1	1	-1
S15	-1	-1	-1	1
S16	-1	-1	-1	-1

The value of the project is the expected value of the metrics used to value the project; in this case, these are the utility values of the NPV and the internal rate of return (for the sensitivity analysis in Section 4.7, only the NPV is used).

Technical experts in this reservoir (geologists, geophysicists, reservoir engineers, etc.) estimate the prior probabilities of occurrence of each of these 16 cases; these probabilities are assigned based on the experience and knowledge that the experts have on this reservoir and analogue ones in the same basin.

The utility assessment is done using the utility function discussed in Section 4.7 and shown in Figure 4.4, with a TF of US$ 4,000 MM for the NPV and 12% for the IRR. These TFs represent the company's attitude toward risk for projects with similar investment needs. The utility values of the NPV and IRR are called the UNPV and UIRR respectively.

Figures 4.10 and 4.11 show the utility values versus NPV and IRR values for this case study.

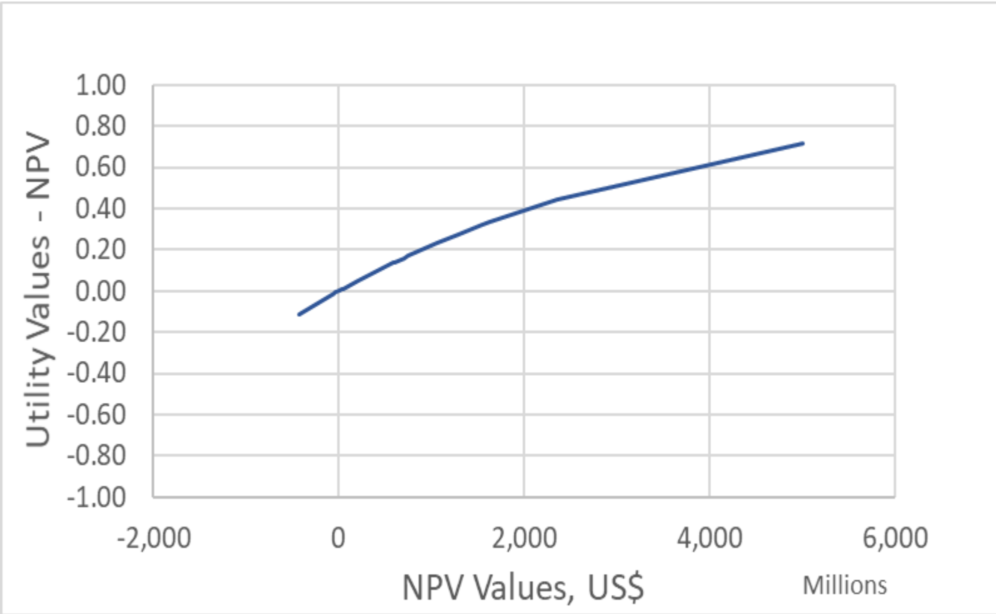

Figure 4.10. Utility values against NPV for the project

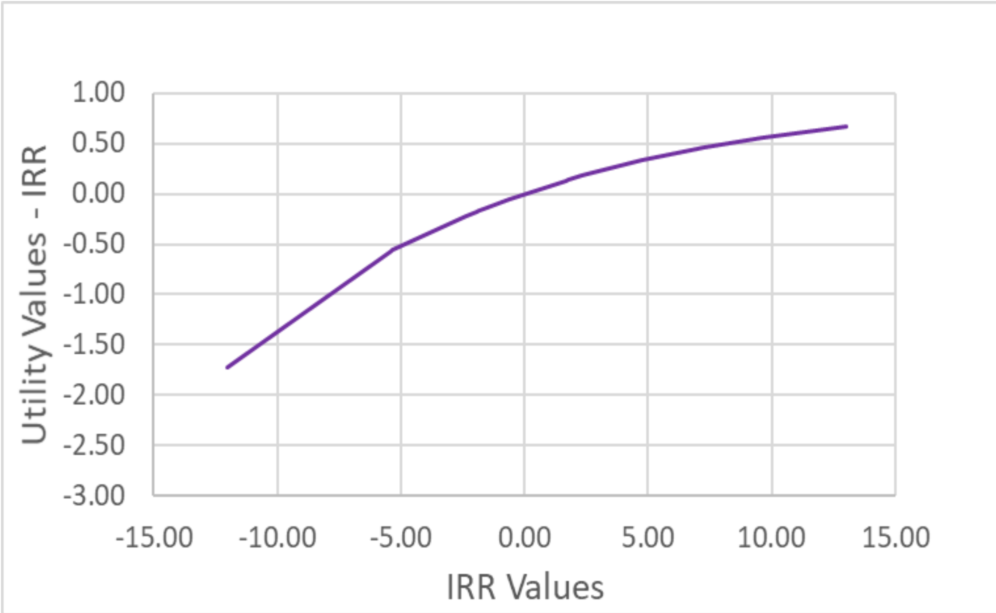

Figure 4.11. Utility values against IRR for the project

Taking into consideration the CAPEX of the project and typical figures for bank interest rates, it can be seen that the company shows a moderate risk aversion attitude which is, in fact, close to risk neutrality.

The 16 simulation runs are built and run, resulting in a yearly cumulative forecast. These figures are evaluated with the financial model, resulting in an estimation of the cash flow, NPV and IRR, and in each case, the respective utility values are calculated.

Table 4.8 shows the prior probabilities, cumulative oil production, NVP, IRR and the respective utility values for the 16 cases described above.

Table 4.8. Prior probability, cumulative oil production, NPV, IRR and utility values for each case

CASE	PRIOR PROBABILITY	OIL CUM, MM *STB*	NPV, US$ MM	UNPV	IRR, %	UIRR
S1	0.0042	198	2,361	0.446	22	0.551
S2	0.0098	162	1,287	0.275	17	0.328
S3	0.0078	164	1,631	0.335	19	0.452
S4	0.0182	135	754	0.172	15	0.190
S5	0.0168	159	1,540	0.320	19	0.440
S6	0.0392	128	596	0.138	14	0.136
S7	0.0312	139	1,058	0.232	17	0.327
S8	0.0728	113	205	0.050	11	-0.052
S9	0.0168	133	706	0.162	14	0.175
S10	0.0392	127	584	0.136	14	0.130
S11	0.0312	107	45	0.011	10	-0.150
S12	0.0728	103	-47	-0.012	10	-0.215
S13	0.0672	106	26	0.006	10	-0.164
S14	0.1568	105	7	0.002	10	-0.176
S15	0.1248	90	-426	-0.112	7	-0.557
S16	0.2912	90	-431	-0.114	7	-0.566

These outcomes are used in Section 4.12 to assess the value of the project for the case where no data are acquired.

4.9 DATA ACQUISITION PROPOSAL FOR THE MAIN UNCERTAIN VARIABLES

The uncertainty analysis shows that the parameters with the largest effect on the response are as follows, in order of importance:

A) OWC: oil/water contact

B) AQU: aquifer support, volume and productivity index

C) REP: oil and water relative permeability

D) PXY: horizontal permeability distribution

4.9.1 Description of the main uncertain parameters

OWC: As shown in Table 4.1 in Section 4.4, in our case study, the OWC varies between 1,070 and 1,080 m with a mean value of 1,075 m. This range of uncertainty of 10 m results because in three of the wells (*P01*, *P02* and *P03*) different values were measured, and in the fourth well (*P04*) the OWC could not be identified. To better characterise this parameter and thus reduce the uncertainty, data acquisition is suggested in terms of drilling a new well and defining the OWC using logs and the Drill Stem Test (DST). Most of the cost associated with this data acquisition is related to the drilling operation, while logs and DST represent only a marginal cost.

AQU: Defining the strength of an aquifer is a difficult task if wells have not been in production for enough time to receive the aquifer influx. It was estimated by the technical team that, based on the characteristics of the reservoir, a well should be tested over at least four months to determine the aquifer strength. This extended well testing can be done on a new well or on one of the existing wells. The latter option is much cheaper than the former; however, if an existing well is used for the extended well test, the only uncertain parameter that will be impacted is the aquifer strength, leaving the other three parameters unchanged. There are several logistical issues that need to be solved regarding the handling of production fluids, oil, water and gas during this testing. If an existing well is used, most of the cost is associated with the handling of fluids during the testing operation.

REP: Due to the lack of a core that would enable a Special Core Analysis Lab (SCAL) test to determine the relative permeability curve and end-point saturations, the uncertainty reduction in this data requires the acquisition of new cores in different sections of the reservoir interval and, for this, a new well must be drilled. Most of the cost associated with this data acquisition action is related to the drilling operation, while the core and lab analysis represents only a marginal cost.

PXY: The areal extension of the good permeability zone can be measured only by a new well located in the area between the good permeability zone (wells *P01*, *P02* and *P03*) and the bad permeability zone (*P04*). The closer the new well is to well *P04*, the higher the risk of finding bad permeability; this analysis requires a core to be taken and a Conventional Core Analysis Lab (CCAL) test to be performed. Most of the cost associated with this data acquisition action is related to the drilling operation, while the core and lab analysis are only a marginal cost.

4.9.2 Data acquisition alternatives

Two alternative data acquisition activities are discussed: (i) *drilling a new well and performing an extended well test*; and (ii) *performing an extended well test on an existing well*.

i. Drilling a new well and performing an extended well test for data acquisition

Drilling a new well can affect the four uncertain parameters. The well should be in a location between the area of proven good permeability (*P01*, *P02*, *P03*) and the location of bad permeability (*P04*). This well de-risks the PXY distribution (Section 4.9.1) and can simultaneously be used to measure the OWC. Several cores can be taken from this well at different depths and can be subsequently analysed in the laboratory to estimate a reliable range for the relative permeability curve. Finally, an extended well test can be carried out to capture the strength of the aquifer. It has been estimated that the well test will take between three and six months; if the impact of the aquifer (high water cut in the well) can be measured over a short time, this means that the aquifer is strong, and otherwise it is weak.

ii. Performing an extended well test on an existing well

Using an existing well for an extended well test incurs only the cost associated with the test itself. In either data acquisition action, the extended well test involves several logistical challenges such as disposal of the fluids produced during the well test (since no infrastructure is available at present) and continuous test monitoring in an isolated location. A temporary setup can be arranged during the test period at an extra cost.

4.10 CLASSICAL VALUE OF INFORMATION

In this study, fuzzy logic tools are used to integrate the imprecision in the data in the methodology to assess the value of acquiring data. For comparison purposes, the results of both the classical methodology and the proposed fuzzy methodology are shown.

4.10.1 Classical value of information for drilling a new well and performing an extended well test

As discussed in Chapter 2, the analyst needs to estimate the probability of each state occurring for each case or reliability probability. In this case study, there are 16 cases (as shown in the matrix in Table 4.7, Section 4.8).

For each of the four parameters (OWC, AQU, REP and PXY), two possible outcomes are assumed: *high* and *low*. The number of reliability probabilities to estimate is 256, which results from an estimation for each state (each of the 16 runs) of the probability of occurrence of the 16 combinations of *high* and *low* for the parameters.

Bayes' theorem is used to transform the reliability probabilities into posterior probabilities, which are then used as input to estimate the value of each data outcome. These values are combined with the residual probabilities to estimate the expected value of the data acquisition alternative.

In Appendix 1, Tables A1-10 to A1-15 show the reliability probabilities and Tables A1-16 to A1-21 show the posterior probabilities. The residual probabilities are shown in Table A1-22.

The expected values of each possible data acquisition outcome using NPV, IRR, UNPV and UIRR are summarised in Tables 4.9 and 4.10.

Table 4.9. Expected value for drilling a new well and performing an extended well test with classical VOI methodology (first eight cases)

EXPECTED VALUE DATA OUTCOME	NPV, MM US$	IRR, %	UNPV	UIRR
Exp (Dev, hhhh)	148.1	-1.3	0.0280	-0.1610
Exp (No Dev, hhhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hhhh)	**148.1**	**-1.3**	**0.0280**	**-0.1610**
Exp (Dev, hhhl)	97.3	-1.6	0.0168	-0.1896
Exp (No Dev, hhhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hhhl)	**97.3**	**-1.6**	**0.0168**	**-0.1896**
Exp (Dev, hhlh)	51.7	-2.0	0.0041	-0.2370
Exp (No Dev, hhlh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hhlh)	**51.7**	**-2.0**	**0.0041**	**-0.2370**
Exp (Dev, hhll)	12.0	-2.3	-0.0040	-0.2537
Exp (No Dev, hhll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hhll)	**12.0**	**-2.3**	**-0.0040**	**-0.2537**
Exp (Dev, hlhh)	70.2	-1.8	0.0091	-0.2176
Exp (No Dev, hlhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hlhh)	**70.2**	**-1.8**	**0.0091**	**-0.2176**
Exp (Dev, hlhl)	29.6	-2.1	0.0008	-0.2348
Exp (No Dev, hlhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hlhl)	**29.6**	**-2.1**	**0.0008**	**-0.2348**
Exp (Dev, hllh)	-2.7	-2.4	-0.0085	-0.2715
Exp (No Dev, hllh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hllh)	**-2.7**	**-2.4**	**-0.0075**	**-0.2715**
Exp (Dev, hlll)	-36.8	-2.6	-0.0154	-0.2837
Exp (No Dev, hlll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,hlll)	**-15.0**	**-2.6**	**-0.0075**	**-0.2837**

Table 4.10. Expected value for drilling a new well and performing an extended well test with classical VOI methodology (last eight cases)

EXPECTED VALUE DATA OUTCOME	NPV, MM US$	IRR, %	UNPV	UIRR
Exp (Dev, lhhh)	17.3	-2.2	0.0533	-0.1039
Exp (No Dev, lhhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,lhhh)	17.3	-2.2	0.0533	-0.1039
Exp (Dev, lhhl)	-8.0	-2.4	-0.0076	-0.2551
Exp (No Dev, lhhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,lhhl)	-8.0	-2.4	-0.0075	-0.2551
Exp (Dev, lhlh)	-60.5	-2.8	-0.0214	-0.3024
Exp (No Dev, lhlh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,lhlh)	-15.0	-2.8	-0.0075	-0.3024
Exp (Dev, lhll)	-79.8	-2.9	-0.0252	-0.3086
Exp (No Dev, lhll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,lhll)	-15.0	-2.9	-0.0075	-0.3086
Exp (Dev, llhh)	-41.6	-2.6	-0.0163	-0.2824
Exp (No Dev, llhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,llhh)	-15.0	-2.6	-0.0075	-0.2824
Exp (Dev, llhl)	-58.6	-2.7	-0.0195	-0.2868
Exp (No Dev, llhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,llhl)	-15.0	-2.7	-0.0075	-0.2868
Exp (Dev, lllh)	-118.0	-3.2	-0.0354	-0.3458
Exp (No Dev, lllh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,lllh)	-15.0	-3.2	-0.0075	-0.3458
Exp (Dev, llll)	-149.9	-3.4	-0.0427	-0.3661
Exp (No Dev, llll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*,llll)	-15.0	-3.4	-0.0075	-0.3661

These figures are used in Section 4.13.1 to assess the value of *drilling a new well and performing an extended well test* using classical VOI methodology.

4.10.2 Classical value of information for performing an extended well test on an existing well

In this case, the data acquisition activity involves performing an extended well test on one of the existing wells (appraisal wells *P01*, *P02* or *P03*). The outcome of this data acquisition affects only one of the uncertain variables, that is, the strength of the aquifer. In the 16 cases shown in Table 4.7 (Section 4.8), eight of them have a high level of strength for the aquifer (aquifer volume and PI) and the other eight cases have a low level.

In the same way as shown in Section 4.9.1, the reliability probability for each case is estimated and the posterior probabilities are computed using Bayes' theorem.

Tables A1-23 to A1-35 in Appendix 1 show the reliability, posterior and residual probabilities. These probabilities and the project values are used to estimate the expected value of each possible data acquisition outcome using NVP, IRR, UNPV and UIRR, as shown in Tables 4.11 and 4.12 below.

Table 4.11. Expected value for performing an extended well test on an existing well with classical VOI methodology (first eight cases)

EXPECTED VALUE DATA OUTCOME	NPV, MM US$	IRR, %	UNPV	UIRR
Exp (Dev, hhhh)	267.6	-0.3	0.0558	-0.0791
Exp (No Dev, hhhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hhhh)	**267.6**	**-0.3**	**0.0558**	**-0.0791**
Exp (Dev, hhhl)	267.6	-0.3	0.0558	-0.0791
Exp (No Dev, hhhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hhhl)	**267.6**	**-0.3**	**0.0558**	**-0.0791**
Exp (Dev, hhlh)	267.6	-0.3	0.0558	-0.0791
Exp (No Dev, hhlh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hhlh)	**267.6**	**-0.3**	**0.0558**	**-0.0791**
Exp (Dev, hhll)	267.6	-0.3	0.0558	-0.0791
Exp (No Dev, hhll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hhll)	**267.6**	**-0.3**	**0.0558**	**-0.0791**
Exp (Dev, hlhh)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, hlhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hlhh)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**
Exp (Dev, hlhl)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, hlhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hlhl)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**
Exp (Dev, hllh)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, hllh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hllh)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**
Exp (Dev, hlll)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, hlll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, hlll)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**

Table 4.12. Expected value for performing an extended well test on an existing well with classical VOI methodology (last eight cases)

EXPECTED VALUE DATA OUTCOME	NPV, MM US$	IRR, %	UNPV	UIRR
Exp (Dev, lhhh)	267.6	-0.3	0.0930	0.0121
Exp (No Dev, lhhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, lhhh)	**267.6**	**-0.3**	**0.0930**	**0.0121**
Exp (Dev, lhhl)	267.6	-0.3	0.0558	-0.0791
Exp (No Dev, lhhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, lhhl)	**267.6**	**-0.3**	**0.0558**	**-0.0791**
Exp (Dev, lhlh)	267.6	-0.3	0.0558	-0.0791
Exp (No Dev, lhlh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, lhlh)	**267.6**	**-0.3**	**0.0558**	**-0.0791**
Exp (Dev, lhll)	267.6	-0.3	0.0558	-0.0791
Exp (No Dev, lhll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, lhll)	**267.6**	**-0.3**	**0.0558**	**-0.0791**
Exp (Dev, llhh)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, llhh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, llhh)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**
Exp (Dev, llhl)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, llhl)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, llhl)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**
Exp (Dev, lllh)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, lllh)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, lllh)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**
Exp (Dev, llll)	-16.2	-2.4	-0.0081	-0.2470
Exp (No Dev, llll)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, llll)	**-15.0**	**-2.4**	**-0.0075**	**-0.2470**

These figures are used in Section 4.13.1 to assess the value of performing an extended well test on an existing well using the classical VOI methodology.

4.11 VALUE OF INFORMATION WITH FUZZY DATA

In this section, a fuzzy data acquisition methodology is developed to include the imprecision in the data that it is proposed to be acquired.

4.11.1 Value of information with fuzzy data for drilling a new well and performing an extended well test

In order to include the imprecision in the data, three membership functions, M_1 or *high*, M_2 or *medium*, M_3 or *low* are used, which correspond to the data acquisition outcome described by the linguistic variables *high*, *medium* and *low* respectively. Here, *high* means that the compound effect of data acquisition on the four parameters is high, although one or more parameters may not have this effect, and a similar description applies for *medium* and *low*. The value assigned to each compound state for each membership function describes the degree of membership that the compound state has in the respective membership function.

The degree of membership is defined based on the author's experience and understanding of the field and subsurface data acquisition. The compound value of the four parameters in the membership function is the average value. Table 4.13 shows the membership functions, M_1 M_2 and M_3, for each potential data outcome.

Table 4.13. Membership functions for each compound parameter

	(hhhh)	(hhhl)	(hhlh)	(hhll)	(hlhh)	(hlhl)	(hllh)	(hlll)
$\boldsymbol{M_1}$	0.638	0.550	0.525	0.438	0.500	0.413	0.388	0.300
$\boldsymbol{M_2}$	0.250	0.263	0.275	0.288	0.250	0.263	0.275	0.288
$\boldsymbol{M_3}$	0.113	0.188	0.200	0.275	0.250	0.325	0.338	0.413

	(lhhh)	(lhhl)	(lhlh)	(lhll)	(llhh)	(llhl)	(lllh)	(llll)
$\boldsymbol{M_1}$	0.525	0.438	0.413	0.325	0.388	0.300	0.275	0.188
$\boldsymbol{M_2}$	0.263	0.275	0.288	0.300	0.263	0.275	0.288	0.300
$\boldsymbol{M_3}$	0.213	0.288	0.300	0.375	0.350	0.425	0.438	0.513

Based on these membership functions and the reliability probabilities, the probability of the membership functions associated with each state can be estimated and then the posterior probabilities and the residual probabilities, as summarised in Tables A1-36, A1-37 and A1-38 in Appendix 1.

The above information is used to estimate the expected value of the project using NPV, IRR and the utility values of NPV and IRR.

Table 4.14 summarises the expected values with NVP, IRR and utility values.

Table 4.14. Expected values for each membership function for drilling a new well and performing an extended well test

	NPV, MM US$	IRR	UNPV	UIRR
Exp (Dev, M_1)	-1.6	-2.3	-0.0072	-0.2604
Exp (No Dev, M_1)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, M_1)	**-1.6**	**-2.3**	**-0.0072**	**-0.2604**
Exp (Dev, M_2)	-25.3	-2.5	-0.0128	-0.2764
Exp (No Dev, M_2)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, M_2)	**-15.0**	**-2.5**	**-0.0075**	**-0.2764**
Exp (Dev, M_3)	-43.0	-2.6	-0.0169	-0.2883
Exp (No Dev, M_3)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, M_3)	**-15.0**	**-2.6**	**-0.0075**	**-0.2883**

In Section 4.12, the results of this assessment are discussed.

4.11.2 Value of information with fuzzy data for performing an extended well test on an existing well

In the same way as for the VOI with fuzzy data for the new well alternative discussed in the previous section, the imprecision in the data for the extended well test in an existing well is included using three membership functions, M_1, M_2, M_3 , for the *high*, *medium* and *low* data acquisition outcomes. These outcomes correspond to the only parameter that is evaluated with the data acquisition, that is, the aquifer strength. The value assigned to each membership function, shown in Table 4.15, describes the degree of membership that the state has for the respective membership function.

Table 4.15. Membership functions for each compound parameter

	(hhhh)	(hhhl)	(hhlh)	(hhll)	(hlhh)	(hlhl)	(hllh)	(hlll)
M_1	0.700	0.700	0.700	0.700	0.150	0.150	0.150	0.150
M_2	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
M_3	0.100	0.100	0.100	0.100	0.650	0.650	0.650	0.650

	(lhhh)	(lhhl)	(lhlh)	(lhll)	(llhh)	(llhl)	(lllh)	(llll)
M_1	0.700	0.700	0.700	0.700	0.150	0.150	0.150	0.150
M_2	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
M_3	0.100	0.100	0.100	0.100	0.650	0.650	0.650	0.650

Table 4.16 summarises the expected values for the NVP, IRR and utility.

Table 4.16. Expected values for each membership function for performing an extended well test on an existing well

	NPV, MM US$	IRR, %	UNPV	UIRR
Exp (Dev, M_1)	198.6	-0.8	0.0402	-0.1199
Exp (No Dev, M_1)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, M_1)	**198.6**	**-0.8**	**0.0402**	**-0.1199**
Exp (Dev, M_2)	97.3	-1.5	0.0174	-0.1798
Exp (No Dev, M_2)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, M_2)	**97.3**	**-1.5**	**0.0174**	**-0.1798**
Exp (Dev, M_3)	10.2	-2.2	-0.0022	-0.2314
Exp (No Dev, M_3)	-15.0	-12.0	-0.0075	-1.7183
Exp (*, M_3)	**10.2**	**-2.2**	**-0.0022**	**-0.2314**

In Section 4.12, the results of this assessment are discussed.

4.12 DEVELOPMENT OF FUZZY INFERENCE SYSTEM FOR THE UTILITY VALUES

MATLAB® R2015a software is used to build the FIS through the Fuzzy Logic Designer application. Figure 4.12 shows a schematic of the FIS developed in this work.

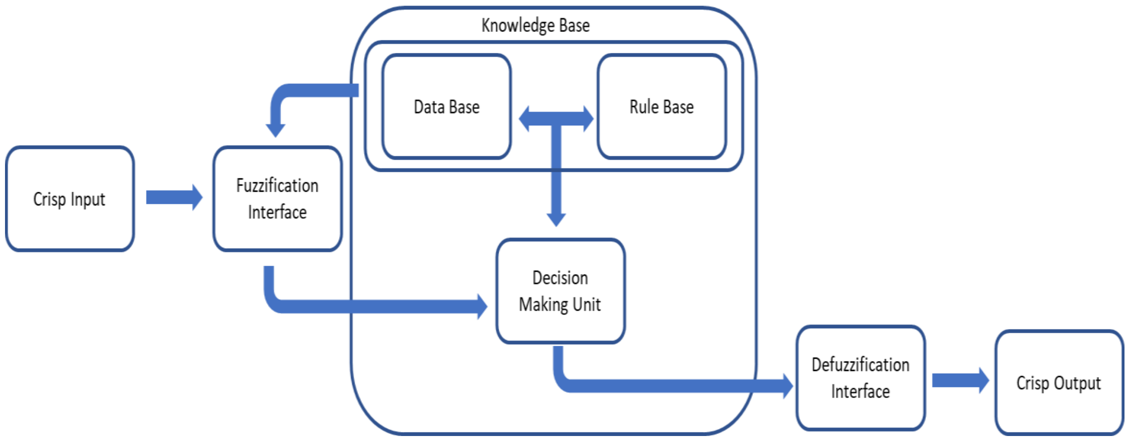

Figure 4.12. Fuzzy inference system for the value of information

The FIS is used to carry out the project assessment once the utility values are known. In this work, the UNPV and the UIRR are used to value the project, and each utility is assigned to the three membership functions corresponding to high, medium and low: UNPV_ High, UNPV_Medium, UNPV_Low, UIRR_High, UIRR_Medium and UIRR_Low.

In FIS, a crisp number is input into the system and it is fuzzified by the mapping of the number in the set of membership functions; then, the fuzzified inputs are applied to the antecedent of the fuzzy rules, and when the rule has multiple antecedents, operators are used to obtain a single number which is then applied over the consequent membership functions to obtain the rule evaluation, which is a set; the same procedure is repeated over all the rules; the sets resulting from rule evaluations are aggregated to obtain a resulting fuzzy set. In most cases, the final fuzzy set is defuzzified to obtain a single number which will be the final outcome of the FIS. The whole intention of the defuzzification process is to get a single number from the assessment which can be used for the decision.

In the proposed methodology for VOI, we use this method; however, the option to represent the final solution as a fuzzy set can also be considered; during defuzzification, information is lost, but it allows to make an assessment based on one number, which is easier to handle and interpret. More investigation can be carried out to propose solutions based on fuzzy sets instead of single numbers, which is outside the scope of this research work.

Triangular and truncated triangular functions are used to model the membership functions. These functions are built in such a way that for very high or very low utility values, the outcome belongs to only one membership function, while for utility values near zero there is an overlap between the three membership functions. The decision options are ‘to endorse’, ‘not to endorse’ or ‘to reframe the project’. Figures 4.13, 4.14 and 4.15 show the membership functions used.

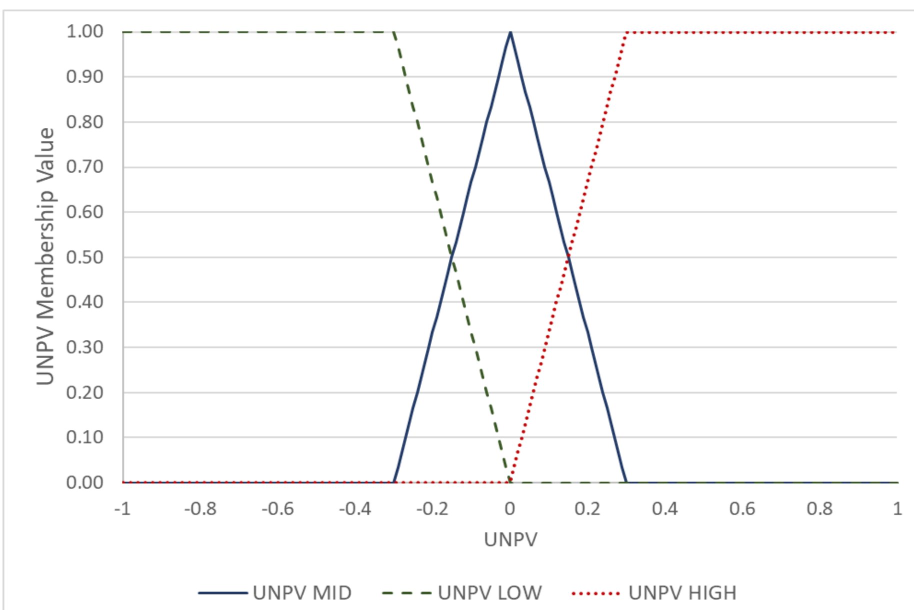

Figure 4.13. Membership function for UNPV

In this case study a wider membership function is used for the middle section of the UIRR compared with that of the UNPV to represent a larger fuzziness in this utility value, which is an author's selection.

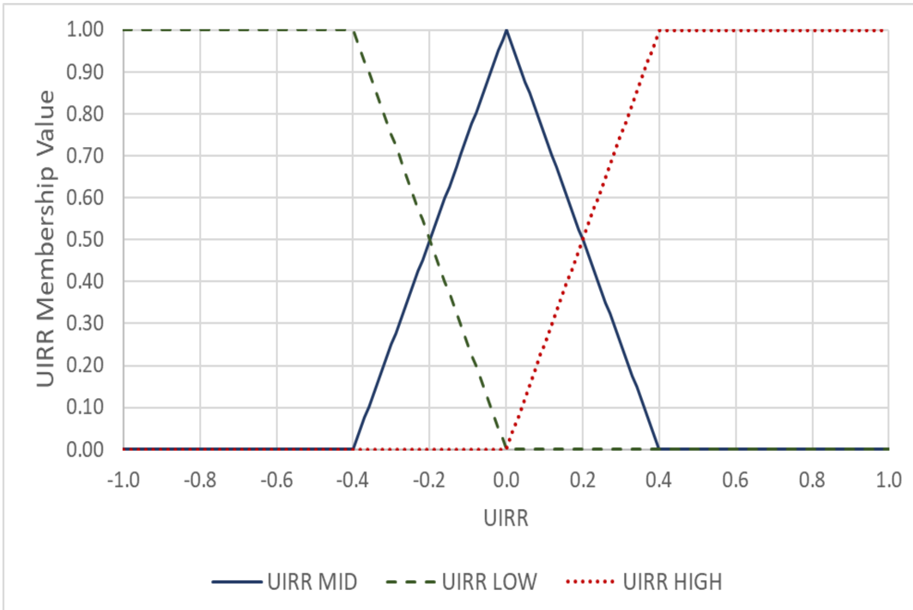

Figure 4.14. Membership function for UIRR

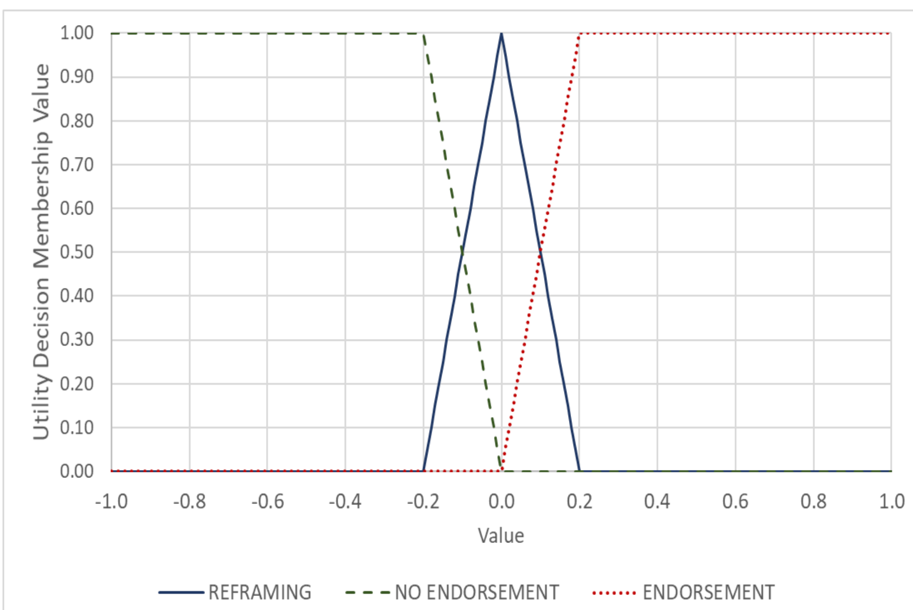

Figure 4.15. Membership function for decision

IF…..THEN rules are designed to reflect the imprecision in the decision process.

Nine decision rules have been built in the FIS for VOI decision making; this is not the only set of rules that can be built, but it represents previous decisions taken by this operator company (the decision maker). These rules are problem dependent and need to be constructed for each problem.

Three possibilities for rules consequences are considered: endorsement (accept the project), no endorsement (reject the project), and reframing (continue analysis until an alternative is found).

Two parameters, UNPV and UIRR, are used as antecedents in the decision rules, each one defined at three levels, high, medium and low. The combination of one level of one parameter and one level of the other resulted in one of the three possible consequences.

The review of confidential information regarding the decisions taken by the operator during the last 5 years suggested that the operator's decision system can be encapsulated in the proposed rules included in Table 4.17.

Table 4.17. Decision rules

RULE	IF	THEN
Rule 1	(UNPV is UNPV_LOW) AND (UIRR is UIRR_HIGH)	(DECISION IS REFRAMING)
Rule 2	(UNPV is UNPV_LOW) AND (UIRR is UIRR_MEDIUM)	(DECISION IS NO_ENDORSEMENT)
Rule 3	(UNPV is UNPV_LOW) AND (UIRR is UIRR_LOW)	(DECISION IS NO_ENDORSEMENT)
Rule 4	(UNPV is UNPV_MEDIUM) AND (UIRR is UIRR_HIGH)	(DECISION IS ENDORSEMENT)
Rule 5	(UNPV is UNPV_MEDIUM) AND (UIRR is UIRR_MEDIUM)	(DECISION IS REFRAMING)
Rule 6	(UNPV is UNPV_MEDIUM) AND (UIRR is UIRR_LOW)	(DECISION IS NO_ENDORSEMENT)
Rule 7	(UNPV is UNPV_HIGH) AND (UIRR is UIRR_HIGH)	(DECISION IS ENDORSEMENT)
Rule 8	(UNPV is UNPV_HIGH) AND (UIRR is UIRR_MEDIUM)	(DECISION IS ENDORSEMENT)
Rule 9	(UNPV is UNPV_HIGH) AND (UIRR is UIRR_LOW)	(DECISION IS REFRAMING)

4.13 CASE STUDY RESULTS

In the methodology proposed in this study, a decision is made using an FIS. Compared with the classical methodology, the use of the FIS has the advantage of including the imprecision in the terms used to decide between the possible options for the project, which is a characteristic of human reasoning. In this section, both methodologies, the classical approach (Section 4.13.1) and the proposed fuzzy VOI approach (Section 4.13.2) are discussed.

The results of these assessments are described below for both the classical and FIS methodologies: the expected values for the no data and data acquisition cases, using the classical and the fuzzy data approaches.

4.13.1 Expected value results

In Section 4.8.1, two data acquisition alternatives are discussed, i.e. *drilling a new well and performing an extended well test* and *performing the extended well test in an existing well*. Here, the results of these alternatives are presented.

4.13.1.1 Expected value for drilling a new well and performing an extended well test data acquisition

Based on the results presented in Tables 4.7-9 and 4.13, the NPV, IRR and utility values, in the case of *drilling a new well and performing an extended well test*, the outcomes of the assessment are shown in Table 4.18.

Table 4.18. Expected value assessment for drilling a new well and performing an extended well test data acquisition proposal

	NO DATA	CRISP DATA	FUZZY DATA
NPV (MM US$)	3.02	12.19	-9.78
IRR (%)	-2.30	-2.49	-2.49
UNPV	-0.0069	0.0006	-0.0074
UIRR	-0.2536	-0.2665	-0.2741

UNPV analysis:

Using UNPV as the decision criterion, Table 4.18 shows that when the classical methodology is used the value of the alternative *drilling a new well and performing an extended well test* is higher (0.0006) than the alternative *do not acquire data* (-0.0069). However, this conclusion is misleading, because when the fuzzy characteristics of the data are included in the analysis, the value of *drilling a new well and performing an extended well test* is reduced (to -0.0074); that is, it is below the value of the *do not acquire data* case. Overall, the best option is *do not acquire data*.

UIRR analysis:

When using UIRR as a decision criterion with the classical methodology, the alternative *drilling a new well and performing an extended well test* (-0.2665) has a lower value than the alternative *do not acquire data* (-0.2536). Concluding: using both crisp and fuzzy data, the value of the project *drilling a new well and performing an extended well test* is lower than the *do not acquire data* project, as shown in Table 4.18.

4.13.1.2 Expected value for performing an extended well test on an existing well data acquisition

Based on the results presented in Tables 4.8, 4.11, 4.12 and 4.16 for the option of *performing an extended well test on an existing well*, the values of the NPV, IRR and the respective utilities are shown in Table 4.19.

Table 4.19. Expected value assessment for performing an extended well test on an existing well

	NO DATA	CRISP DATA	FUZZY DATA
NPV (MM US$)	3.02	98.04	97.31
IRR (%)	-2.30	-1.53	-1.53
UNPV	-0.0069	0.0197	0.0174
UIRR	-0.2536	-0.1752	-0.1798

UNPV analysis:

Using UNPV as a decision criterion, for the data acquisition case of *performing an extended well test on an existing well*, the classical methodology shows that both the crisp and fuzzy values are higher than the case of *do not acquire data*, which makes this alternative (i.e. performing an extended well test on an existing well) the recommended decision.

UIRR analysis:

When the UIRR is used as a decision criterion to assess the case of *performing an extended well test on an existing well*, the classical methodology shows that the best project (i.e. the highest UIRR) is the *data acquisition* alternative, because both crisp and fuzzy data acquisition have higher values than the *do not acquire data* alternative. In this case, it is observed that when the fuzzy characteristics of the data are included in the analysis, the value of the data acquisition is reduced, as expected.

4.13.1.3 Expected value for the data acquisition alternatives

Using the UNPV as a decision criterion and considering that the data is fuzzy, Tables 4.18 and 4.19 show that a comparison between the two alternatives (i.e. *drilling a new well and performing an extended well test*, and *performing an extended well test on an existing well*) indicates that the best option is the latter. This is because the utility value of the *extended well test on an existing well* is 0.0174, which is higher than the utility value of *drilling a new well and performing an extended well test* of -0.0074; in the case when the data is assumed crisp, even though the outcomes are different, the same conclusion holds. In addition, the data acquisition alternatives (*drilling a new well and performing and extended well test* and *performing an extended well test on an existing well*) are better than the *do not acquire data* alternative.

On the other hand, using UIRR as a decision criterion and considering the data to be fuzzy, Tables 4.18 and 4.19 show that a comparison of both projects (i.e. *drilling a new well and performing an extended well test*, and *performing an extended well test on an existing well*) indicates that *performing an extended well test on an existing well* is the best project, since its value (-0.1798) is higher than that of the *drilling a new well* *and performing an extended well test* (-0.2741). The same conclusion holds in the case when the data is assumed to be crisp; however, the data acquisition alternative is only better than the *do not acquire data* alternative when the data is fuzzy; for the new well alternative with crisp data, the best option is *do not acquire data*.

4.13.2 Fuzzy inference system assessment

The FIS discussed in Section 4.11 is used in this work for the assessment of the two alternatives annotated: the *drilling a new well and performing an extended well test* alternative is discussed first, and then *performing an extended well test on an existing well*.

4.13.2.1 Fuzzy inference system for drilling a new well and performing an extended well test and performing an extended well test on an existing well using utility values

The FIS is used to assess the value of the project for each of the different alternatives.

Based on the utility values (which include the risk attitude of the decision maker), Table 4.20 shows the values for the cases of *do not acquire data*, *crisp data acquisition* and *fuzzy data acquisition*.

Table 4.20. Fuzzy inference system assessment for the case study using utility values

PROJECT	NO DATA	CRISP DATA	FUZZY DATA
New well	-0.274	-0.268	-0.289
Extended well test	-0.274	-0.171	-0.178

Considering the results shown in Table 4.20, it can be concluded that for the case of *drilling a new well and performing an extended well test*, the best decision is to endorse the project without acquiring data. For the case of *performing an extended well test on an existing well*, the best option is to acquire the data. In addition, a comparison of the *drilling a new well and performing an extended well test* project with the *performing an extended well test on an existing well* project shows that the best decision is to *perform an extended well test on an existing well*.

4.13.2.2 Fuzzy inference system for drilling a new well and performing an extended well test and performing an extended well test on an existing well using values

For completeness, the same analysis shown in Section 4.12.2.1, but in terms of values rather than utility figures, is included here. Table 4.21 shows the values for *do not acquire data*, *crisp data acquisition* and *fuzzy data acquisition*.

Table 4.21. Fuzzy inference system assessment for the case study using values

PROJECT	NO DATA	CRISP DATA	FUZZY DATA
New well	-0.217	-0.170	-0.359
Extended well test	-0.217	0.444	0.444

The results shown above indicate that for the case of *drilling a new well and performing an extended well test* data acquisition, the best option is *no data acquisition*. For *performing an extended well test on an existing well*, the optimum option is to acquire the data. A comparison of the two projects (*drilling a new well and performing an extended well test* and *performing an extended well test on an existing well*) shows that the best decision is to *perform a well test on one of the existing wells*.

4.13.3 Fuzzy inference system sensitivity analysis

In Section 4.12 the characteristics of triangular membership functions used in the FIS are discussed; the functional form and the parameters of the membership functions are chosen to model the degree of belonging of the utility values within the fuzzy sets describing outcomes and decisions. However, these selections are not unique, and other possibilities are also feasible. In this section two kind of sensitivities are made: 1) sensitivity to the functional form of the membership function, and 2) sensitivity to the parameters of the membership functions.

For assessing the sensitivity to the functional form, in addition to the triangular membership functions described in Section 4.12, the sigmoidal functions are used in the sensitivity analysis (see Chapter 2 for details of these membership functions). The parameters of triangular as well as sigmoidal functions are systematically changed to model different degrees of membership per each utility value into the several decision alternatives. Figures 4.16 and 4.17 show two examples of the sigmoidal membership function used in this analysis, where the former shows a sharp split of the range and the latter shows a blunt split of the range which is used to represent less or more fuzziness.

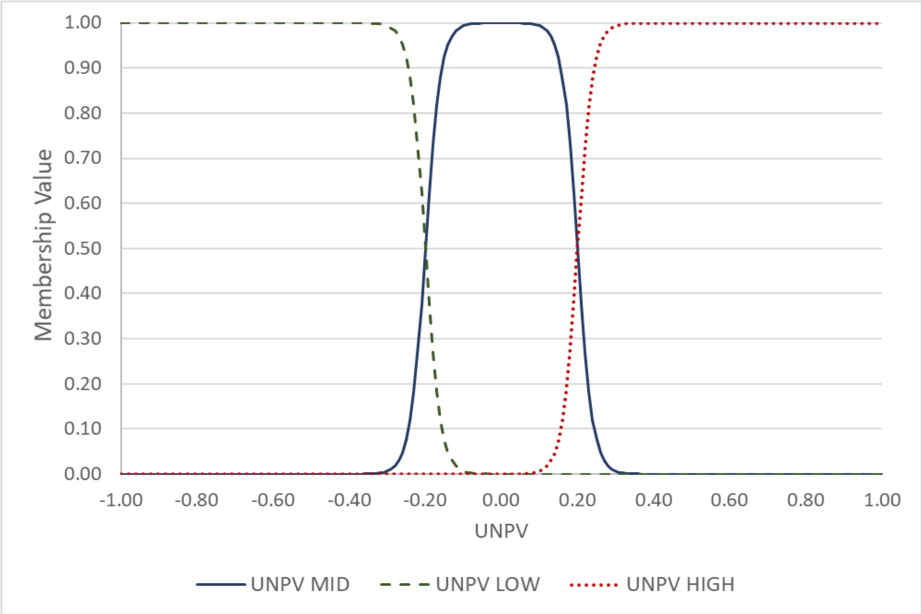

Figure 4.16. Sharp sigmoidal membership function

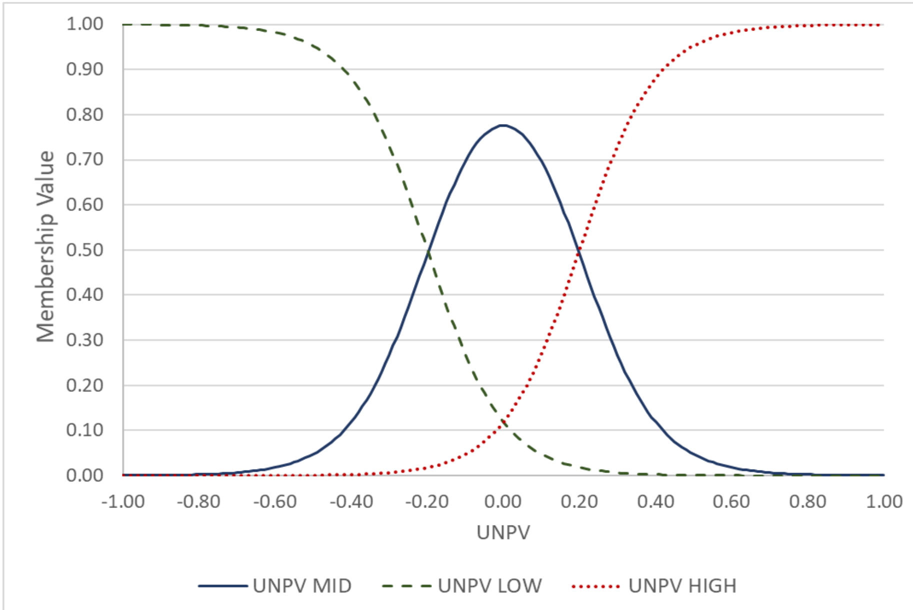

Figure 4.17. Flat sigmoidal membership function

Multiple evaluations of the utility values contained in Tables 4.18 and 4.19 (Sections 4.13.1.1 and 4.13.1.2) are made by varying the parameters of the triangular and sigmoidal functions; these parameter variations make the functions sharper or flatter; to simplify the notation, an index is defined to represent the level of sharpness in the function: the higher the index, the sharper is the function, and the lower the index, the flatter is the function (the index results from the combination of the parameters of the function whose effect is to make the function sharper or flatter). These evaluations assess how decisions change between the three alternatives discussed, *do not acquire data*, *performing the extended well test on an existing well* and *drilling a new well and performing an extended well test,* when the membership functions are changed from a sharper to a flatter function for the two membership functions, triangular and sigmoidal; the results of these assessments are shown in Figures 4.18 and 4.19.

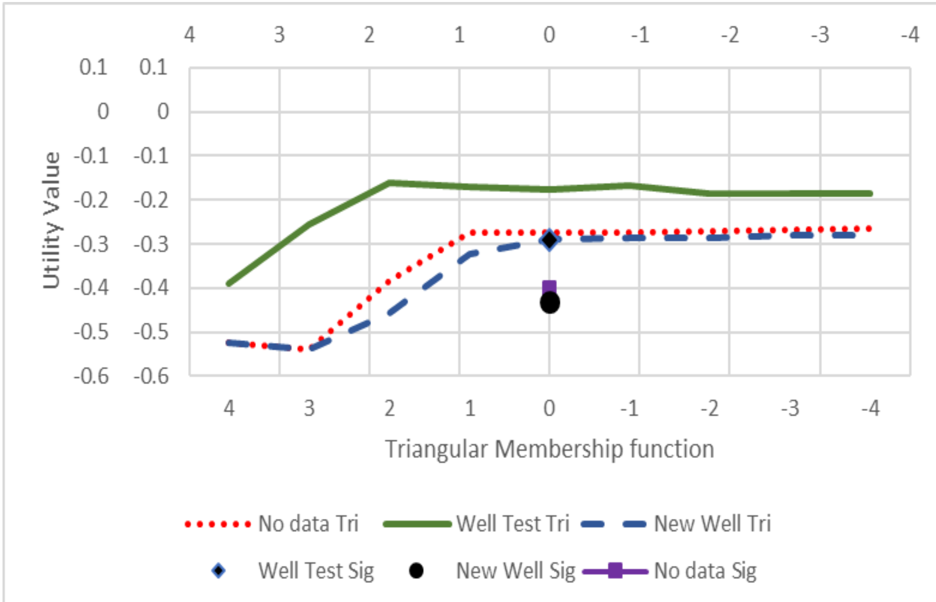

Figure 4.18. Assessment of the three alternatives with iterative selections of the triangular membership function's parameters

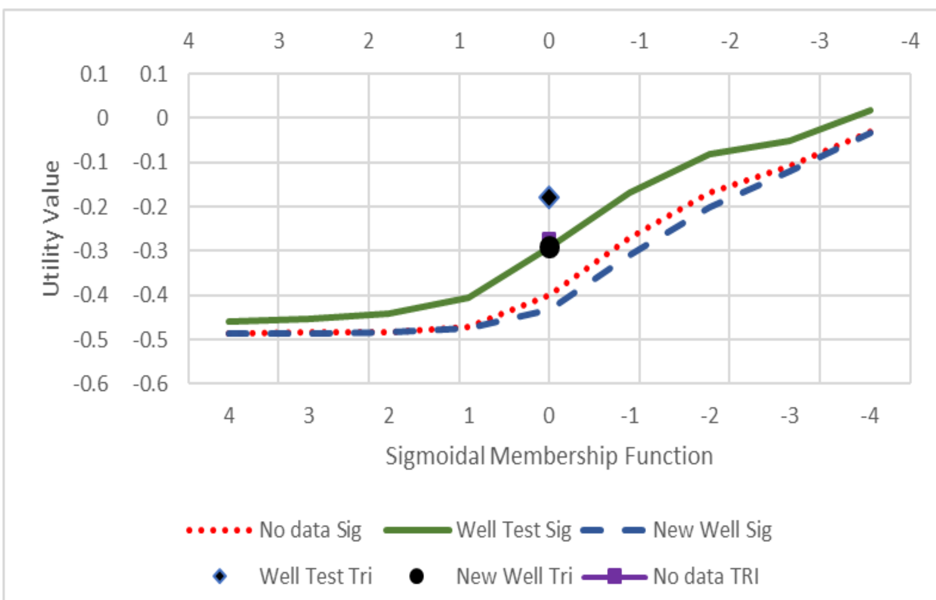

Figure 4.19. Assessment of the three alternatives with iterative selections of the sigmoidal membership function's parameters

For reference, in the plot of the triangular membership function, the values associated with the sigmoidal assessment of the three alternatives are included; the same applies for the sigmoidal membership function.

Figures 4.18 and 4.19 show that, consistently, in the range of variation of the parameters of both membership functions, triangular and sigmoidal, the best alternative is *performing an extended well test on an existing well*.

When comparing the two alternatives: i) *do not acquire data* and, ii) *drilling a new well and performing an extended well test*, the former seems to be a better option for most of the values of the parameters of both functions, but when the functions are very sharp or very flat, these two options produce similar outcomes.

4.14 DISCUSSION OF THE RESULTS

This section discusses the results of the case study presented in Section 4.1. In this research, even though it is recognized that the subsurface data discussed in the case study are fuzzy, an analysis assuming that the data are crisp (which is the assumption of the classical VOI) is included; this allow to have a better understanding of the results of this research by means of a comparison between the results of the crisp and the fuzzy approaches

4.14.1 Approaches results

The results of the case study presented in this chapter are discussed using two decision making approaches:

i) Crisp assessment for crisp and for fuzzy data;

ii) FIS assessment for crisp and for fuzzy data.

4.14.1.1 Crisp assessment for the crisp and fuzzy data

When using the crisp assessment approach, there are two separate cases: for the utility of NPV and the utility of IRR. The results corresponding to UNPV are shown in Figure 5.1, and the results of the UIRR are shown in Figure 5.2; in Figures 5.1 and 5.2, the relation “>” represents the preference relationship, e.g. A > B means A is preferred to B, or the relationship: Data > No data means acquire data is preferred to do not acquire data.

For UNPV: when crisp data are used, *drilling a new well and performing an extended well test* is a better alternative than the *do not acquire data* alternative, and the *extended well test on an existing well* is a better alternative than the *do not acquire data* alternative. However, when comparing the two projects (*drilling a new well and performing an extended well test* versus *performing an extended well test on an existing well*), the alternative *performing an extended well test on an existing well* is the best one.

When the data are assumed to be fuzzy, the *do not acquire data* option is better than *drilling a new well and performing an extended well test*, and the *extended well test on an existing well* option is better than the *do not acquire data* option. When comparing these two projects (*drilling a new well and performing an extended well test* versus *performing an extended well test on an existing well*), the best alternative is to make an *extended well test on an existing well*.

The analysis of the project *drilling a new well and performing an extended well test* produces different recommendations depending on whether the data are assumed to be crisp or fuzzy: *drilling a new well and performing an extended well test* is recommended when the data are assumed to be crisp, and *do not drill a well* when the data are assumed to be fuzzy. In the project *performing an extended well test on an existing well*, the recommendation is the same whether the data are crisp or fuzzy.

To summarise, using crisp or fuzzy data produces different recommendations for the *drilling a new well and performing an extended well test* data acquisition alternative and the same recommendation in the *performing an extended well test on an existing well* data acquisition alternative.

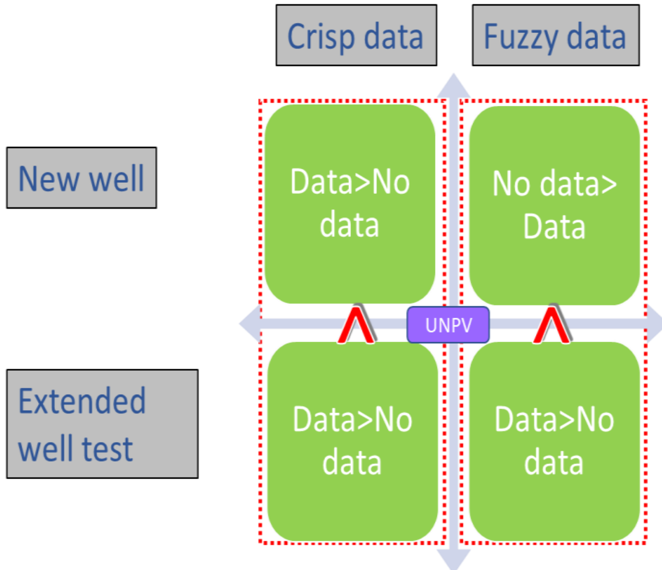

Figure 4.20. Results of the crisp criterion for the case study using UNPV

In Figure 4.20, for the utility UNPV and, as can be observed in the two upper squares, for the new well case using crisp data, data acquisition is preferred to no data acquisition, but, when using fuzzy data, no data acquisition is preferred to data acquisition; from the two lower squares, for the extended well test case, data acquisition is preferred to no data acquisition in both cases, crisp and fuzzy data. In addition, using crisp data, the extended well test is preferred to the new well, as highlighted by the preferred symbol in the vertical direction (in red in the figure); similarly, for fuzzy data, the extended well test is preferred to the new well.

For UIRR: when crisp data are used, the *do not acquire data* alternative is better than the *drilling a new well and performing an extended well test* data acquisition option, and *performing an extended well test on an existing well* is a better option than the *do not acquire data* alternative; similar recommendations result when the data are assumed to be fuzzy. When comparing these projects (*drilling a new well and performing* *an extended well test* versus *performing an extended well test on an existing well*), the best alternative is to *perform an extended well test on an existing well*.

In the project *drilling a new well and performing an extended well test*, the recommended action is *do not acquire data* (do not drill the well) in both cases, whether crisp or fuzzy data; in the project *performing an extended well test on an existing well*, the recommendation is to acquire the data whether the data are crisp or fuzzy.

The conclusion is that, whether the data are crisp or fuzzy, the conclusions are similar: *do not acquire data* is a better alternative than *drilling a new well and performing an extended well test*, and the alternative *performing an extended well test on an existing well* is a better option than the *do not acquire data* alternative.

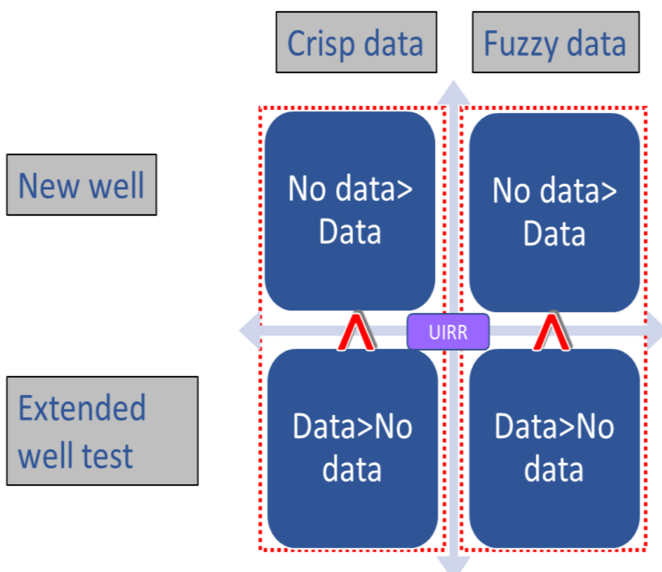

Figure 4.21. Results of the crisp criterion for the case study using UIRR

In Figure 4.21, for the utility UIRR and, as can be observed in the two upper squares, for the new well case using crisp and fuzzy data, data acquisition is preferred to no data acquisition; from the two lower squares, for the extended well test case, data acquisition is preferred to no data acquisition in both cases, crisp and fuzzy data. In addition, using crisp data, the extended well test is preferred to the new well, as highlighted by the preferred symbol in the vertical direction (in red in the figure); similarly, for fuzzy data, the extended well test is preferred to the new well.

These results mean that, in this case study, taking into account the fuzzy nature of the data has a great impact on the decision when the alternative is *drilling a new well and performing an extended well test*; however, in the alternative *performing an extended well test on an existing well*, the same decision is recommended whether the data are assumed crisp or fuzzy.

From the assessments presented in Tables 4.20 and 4.21, it is observed that the value of the data is higher when they are assumed to be crisp than when they are assumed fuzzy. The reduction in the value of the data depends on the membership function used to describe the fuzziness of the data.

VOI is a method for deciding whether acquiring new data is worthwhile; to do so, it compares the value of the project with and without the new data: when the former is higher than the latter, the methodology suggests that acquiring the data is worthwhile and not acquiring it otherwise.

In the cases where the reduction in the value of the data due to their fuzziness is higher than the VOI on the crisp assessment, the fuzzy data approach for VOI produces different results than the crisp approach for VOI. In these cases, the fuzzy assessment is more accurate or representative than the crisp assessment because it considers the fuzzy nature of the data, which is not done in the crisp approach.

In our case study, the project *performing an extended well test on an existing well* is much better than *do not acquire data*, which explains the reason why crisp and fuzzy data assessments produce the same recommendation; however, the project *drilling a new well and performing an extended well test*, which has a crisp assessment slightly higher than that of *do not acquire data*, produces different results when the data are assumed to be crisp than when they are assumed to be fuzzy.

Summarising, considering the fuzzy nature of the data in the VOI assessment reduces the value of the data. In the cases where the reduction in the value of the data due to fuzziness is higher than the classical VOI, the acknowledgement that the data are fuzzy changes the decision from acquire to do not acquire.

4.14.1.2 Crisp assessment for the crisp and fuzzy data

When FIS is used for assessing the decision criteria, only one 'aggregated' result is obtained, as shown in Figure 4.22.

For FIS: when crisp data is used, the projects *drilling a new well and performing an extended well test* and *performing an extended well test on an existing well* are better alternatives than *do not acquire data*. For the case assuming that the data are fuzzy, *do not acquire new data* is better than *drilling a new well and performing an extended well test*, and it is better to *perform an extended well test on an existing well* than *do not acquire data*. For both cases, whether the data are crisp or fuzzy, the best alternative is *performing an extended well test on an existing well*.

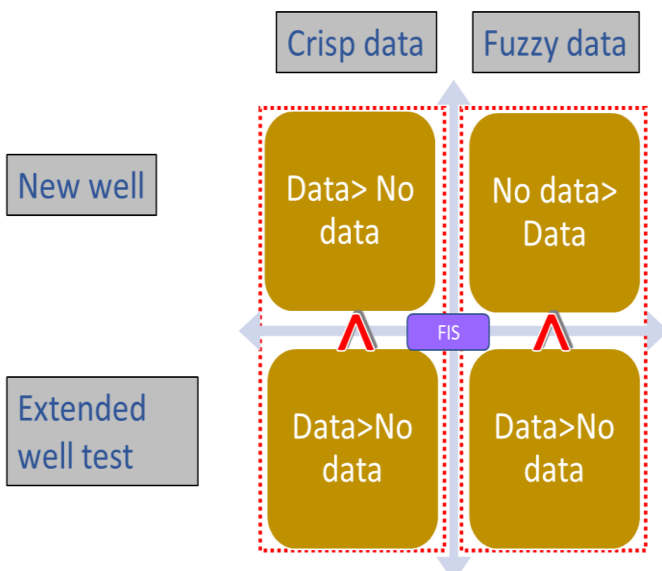

Figure 4.22. Results of the FIS criteria for the case study

In Figure 4.22, as can be observed in the two upper squares, for the new well case using crisp data, data acquisition is preferred to no data acquisition, but, when using fuzzy data, no data acquisition is preferred to data acquisition; from the two lower squares, for the extended well test case, data acquisition is preferred to no data acquisition in both cases, crisp and fuzzy data. In addition, using crisp and fuzzy data, the extended well test is preferred to the new well, as highlighted by the preferred symbol in the vertical direction (in red in the figure).

These results indicate that the evaluation of the *drilling a new well and performing an extended well test* alternative is affected whether the fuzzy nature of the data is included in the assessment. However, in the *performing an extended well test on an existing well* alternative, the same recommendation is obtained whether crisp or fuzzy data are assumed.

This FIS assessment has a great advantage over the crisp assessment in that it avoids the contradictory recommendations discussed above, i.e. those that occur between the assessments based on UNPV and UIRR for the *drilling a new well and performing an extended well test* alternative.

In all of the alternatives assessed in our case study, it was observed that the value of the data, when they are assumed to be fuzzy, is always lower than the value of the data when they are assumed to be crisp.

Based on previous discussions, it could be that, in some cases, while the value of the crisp data acquisition alternative is higher than the value of no data acquisition, when the fuzzy nature of the data is considered in the analysis, the results change the recommendation from do acquire data to do not acquire data (indeed, this happens in the UNPV in the *drilling a new well and performing an extended well test* data acquisition alternative, Table 4.17). There are also cases in which the decrease in the value of the data does not change the decision, as occurs with UIRR for the *drilling a new well and performing an extended well test* data acquisition (Table 4.17).

These results mean that, potentially, the consideration of the fuzzy nature of the data can change the decision, providing a more accurate evaluation of the data and, consequently, the fuzzy nature of the data should be included in the analysis for assessing the value that new data has in the value of the project.

The sensitivity analysis performed using triangular and sigmoidal membership functions shows that, independently of the level of flattening or sharpening of the functions, the alternative *performing an extended well test on an existing well* is always better than *drilling a new well and performing an extended well test* and *do not acquire new data*; however, the alternative *drilling a new well and performing an extended well test* is not always better than *do not acquire data*, which only occurs when the functions are neither very sharp nor very flat: for very flat or very sharp membership functions, the *drilling a new well and performing an extended well test* and the *do not acquire data* alternatives have similar utility values.

The sensitivity analysis also shows that the utility values of the triangular membership functions are higher than the utility values of the sigmoidal membership functions, even though efforts were made to calibrate both sets of functions to the same values at the “0” level.

4.14.2 Application of the VOI methodology in other fields

A request for data acquisition is not limited to the oil and gas industry, and in general it is needed in domains where uncertainty is present in the set of input variables, and that uncertainty translated to the outcomes.

To apply this methodology to other domains other than the oil and gas industry, it should be possible to build a model (mathematical or empirical) of the system that describes its future performance in terms of the uncertain variables; to do that, one objective function describing what matters to the decision maker should be defined.

Another requirement for applying the VOI methodology is that it should be feasible (economically, physically and timely) to acquire additional data, and experts have to be able to assign prior and reliability probability to the several combinations of probable cases. Finally, it should be possible to change the current decision based on the data acquisition outcome (otherwise, it does not make sense acquire the data). In case the previous conditions can be fulfilled, the methodology of VOI can be used to assess the value that new data have to the project.

In the agricultural domain, decisions associated with expected benefits for changing the type of seed, in the automotive business the best material for specific parts of the engine, in the civil engineering the process for selecting the best materials in terms of strength and durability for constructions materials, in the mining domain decisions concerning with the identification of mines larger than the minimum economical size are examples where the proposed methodology can be easily adapted.

4.14.3 Limitations of the proposed methodology

Even though the proposed method fills the gaps found in the classical method for VOI, it still has a few limitations, which are discussed below.

1) A complete evaluation of the main parameters impacting on the value of a project can be a very time-consuming task when the assessment is made on a complex reservoir: the resources and time required to complete the study can be greater than those available for the study. It is not uncommon to find simulation runs taking more than 6 hours to complete, which makes this assessment expensive and time-consuming when many dynamic simulation runs are required.
2) The identification and assessment of different data acquisition alternatives may be challenging in complex problems, and the difficulty increases when there are multiple alternatives. At present, there is no method that ensures that all the data acquisition alternatives related with the uncertainties of a given reservoir are considered in the VOI acquisition assessment.
3) The selection of the membership functions used for assessing the fuzziness of the data, and those used for assessing the decision criteria, are highly subjective, lacking a formal process for their definition and parametrization, which may result in biases in the selection of the membership functions. There is a need to have a consistent and formal process for selecting the membership functions in order to avoid bias in the decisions. An intelligent system can be developed to create

the membership functions associated with the fuzziness of several types of data, based on previous decisions.

4) The concept of fuzzy data is not a common topic for the professionals involved in subsurface evaluations, making it difficult to apply. The use of fuzzy logic and probability theory requires a special training for the specialist on subsurface topics.

5) In Section 2.2, the Asian disease and the Allais paradoxes were used to exemplify that when making decisions, in some cases, the decision maker does not follow the vNM theory, or how rational people should make decisions, but the Prospect Theory, which describes how people make decisions while impacted for several effects such as reflection and framing which condition the decision maker's assessment; the proposed methodology is a prescriptive theory which does not consider these effects.

Chapter Five

Conclusions and Recommendations

5.1 INTRODUCTION

This chapter answers the research questions posed in the first chapter. It also shows how the proposed theories, techniques and methodologies can enhance the classical VOI methodology, thus generating a more robust and complete methodology that is valuable to the decision makers and to oil and gas industry practitioners.

5.2 THE RESEARCH QUESTIONS REVISITED

In Chapter 1 two research questions were proposed that this study had aimed to address. Each question corresponds to a gap found in the classical VOI methodology. This section focuses on each question and shows how, using the combined set of theories, methodologies and techniques discussed in this research, those gaps can be filled, allowing a more robust methodology for VOI to emerge.

The first research question aimed at changing the strategy for applying VOI from an activity-based assessment to a project-based assessment. This question was motivated by the observation that most of the reported applications of VOI focus on identifying uncertain input variables and defining a data acquisition activity that can impact on our understanding of the associated uncertainty. Under this premise, the classical VOI methodology is applied to assess whether the proposed data acquisition activity is worthwhile. The objective function used in the reported assessments is mostly the cumulative hydrocarbon production or expected monetary value of the project. In addition, the risk attitude of the decision maker is rarely part of the assessment.

This approach lacks a methodological consideration of the complete set of uncertain input variables that the project has to deal with. In other words, it may be missing the opportunity to implement more beneficial data acquisition activities. Additionally, when several data acquisition activities are feasible, choosing the most beneficial one should be based on the ranking of the impact that each variable and their interactions has on the utility value of the project. The approach presented in this research assumes that the objective of any decision related with data acquisition is to optimize the value of the project, not just to improve its value.

To overcome this issue, DOE techniques are used for the identification and ranking of the uncertain input variables according to selected metrics. In the case study presented, a dynamic reservoir simulation model is used to predict field performance; all the profiles are subject to financial assessment to define the project value and utility value for each input selection. It is important to clarify that, in general, even in cases where dynamic simulation is not available, the methodology can be used inasmuch as different profiles can be assigned to different combinations of input parameters.

In this research, the definition and ranking of uncertain variables is carried out using DOE techniques; a full factorial design and ANOVA analysis are used for ranking all the variables and their main interactions. Depending on the number of variables, other designs (such as fractional factorial design or compositional design) can be used to produce quicker answers.

The second research question concerned two aspects related to fuzzy logic: i) assessing the impact of including the fuzzy nature of the reservoir data within the VOI and, ii) designing as well as developing a fuzzy decision system suitable for assessing VOI problems (a decision system that is more closely aligned to human logic than the classical decision approach based on crisp criteria).

The first aspect of the second research question aimed to establish whether the fuzzy nature of the subsurface data has an impact on the VOI assessment. Fuzziness in the data is one form of uncertainty that cannot be captured by using probabilistic tools and it is associated with the imprecision inherent in the data. In the classical VOI methodology, a set of production profiles corresponding to crisp values of the input parameters are evaluated assuming that the data are crisp, which means that the outcome of the data acquisition is a sharp value that corresponds exactly to one of the cases being evaluated. However, this does not correspond with what happens in reality. The data are always measured with imprecision and do not accurately match the cases being evaluated. In this research, fuzzy logic and fuzzy membership functions are used to describe and measure the imprecision in the data. The fuzzy VOI approach developed in this research uses the same mathematical description as classical VOI, replacing the crisp variables with fuzzy ones. The concept of fuzzy data acquisition is developed by integrating the fuzzy nature of the data proposed to be acquired within the VOI methodology, which can then be used in problems relating to the oil and gas industry.

The second aspect of the second research question attempted to address whether a decision system for making VOI assessment that follows the same logic as the human mind can be developed. This question is motivated by the recognition that human thinking uses fuzzy description criteria, whilst the classical VOI uses a Boolean logic; these differences make it difficult to reconcile the outcome of the assessment with the thinking of the decision maker.

The research presented in this thesis addressed this question by developing a FIS. The first advantage of the FIS is its capability to aggregate the effects of several decision criteria into one outcome and, secondly, the possibility to replicate the human decision process by the FIS through the use of membership functions. In this research work an FIS is developed to account for the assessment associated with data acquisition. The FIS has been used previously for several different applications in engineering and here the use of an FIS for the assessment of the VOI is developed. In this research, selected membership functions are used to capture the imprecision in the data that it is proposed to be gathered.

In this research, the importance is recognised of using utility values instead of values for capturing the decision maker’s attitude towards risk. The concept of utility value was used in VOI assessments for the oil and gas industry in the 1950s; however, since then, most of the reported cases of VOI use financial parameters (such as NPV, IRR, DPI or other utility values) instead of their utility values, which has the clear disadvantage of underestimating the importance that the risk attitude of the decision maker has on the

VOI decisions; even though it is not one of the research topics, the risk attitude of the decision maker is included in the fuzzy methodology for VOI.

5.3 RECOMMENDATIONS FOR FUTURE WORK

Whilst conducting the research underpinning this thesis, it is recognised that some areas of research were not explored that could be the subject of future work, as will be discussed in this section.

Due to the limitations discussed in Section 4.14.2 related to the large resources required for the assessment of complex reservoirs, it is important to find other ways to include dynamic modelling; one possible line of investigation is to use proxy models in VOI assessment for predicting the reservoir performance instead of using dynamic simulation models. This technique (proxy modelling) has been used before for probabilistic assessment and modelling optimization; however, it can also be used within the VOI framework for a quick identification of the significant parameters.

Another area of research that can be pursued is the generation of a consistent and comprehensive method for identifying the data acquisition actions associated with subsurface assessments. This is not a simple issue, but it can be addressed using sophisticated expert systems. This requires the building of a complete database of the uncertainties and data acquisition actions, including the mappings between them; the association rules can select and rank the data acquisition actions based on the uncertainties and type of reservoirs.

In relation to the observed limitation in the way the membership functions are selected for making VOI assessment, additional research can be conducted in the domain of intelligent systems to build, for VOI assessments, membership functions to i) describe the fuzziness associated with different types of subsurface data, and ii) capture the fuzziness associated with the decision criteria used to make decisions concerning data acquisition.

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APPENDICES

APPENDIX 1: CASE STUDY SUPPORTING INFORMATION

Table A1.1. Porosity values measured on the appraisal wells

LAYER	POROSITY %, P01	POROSITY %, P02	POROSITY %, P03	POROSITY %, P04
1	28	25	25	5
2	28	26	26	5
3	29	27	27	5
4	29	27	27	5
5	28	25	24	5
6	27	23	22	5
7	26	24	22	5
8	26	23	21	5
9	23	22	20	5

Table A1.2. Permeability values measured on the appraisal wells

LAYER	PERMEABILITY mD, P01	PERMEABILITY mD, P02	PERMEABILITY mD, P03	PERMEABILITY mD, P04
1	1415	860	850	1
2	1400	1030	1025	1
3	1655	1210	1190	1
4	1630	1190	1210	1
5	1395	840	730	1
6	1210	595	495	1
7	1025	710	490	1
8	1015	585	395	1
9	585	485	315	1

Table A1.3. Mean porosity values

LAYER	MEDIUM, %	HIGH, %	LOW, %
1	16.6	17.4	12.1
2	16.9	17.7	12.4
3	17.2	18.1	12.7
4	17.2	18.0	12.6
5	16.3	17.1	11.9
6	15.9	16.7	11.6
7	15.7	16.7	11.5
8	15.3	16.3	11.2
9	14.4	15.5	10.5

Table A1.4. Mean permeability values

LAYER	MEDIUM, mD	HIGH, mD	LOW, mD
1	337	376	284
2	370	412	313
3	413	461	353
4	403	450	342
5	301	341	251
6	275	306	230
7	253	289	208
8	229	262	187
9	159	193	119

Table A1.5. Cumulative Oil

RUN	OIL CUMULATIVE, MM STB	RUN	OIL CUMULATIVE, MM STB
1	87.5	**17**	102.4
2	113.0	**18**	140.5
3	88.1	**19**	105.7
4	137.8	**20**	166.7
5	106.5	**21**	131.3
6	128.2	**22**	168.9
7	107.9	**23**	136.5
8	160.3	**24**	205.4
9	89.1	**25**	101.9
10	111.9	**26**	132.9
11	89.7	**27**	105.5
12	140.0	**28**	161.5
13	104.8	**29**	126.6
14	125.5	**30**	160.3
15	105.9	**31**	131.8
16	158.5	**32**	195.3

Table A1.6. Oil Cumulative

RUN	OIL CUMULATIVE, MM STB	RUN	OIL CUMULATIVE, MM STB
33	87.5	**49**	102.7
34	113.1	**50**	140.9
35	88.2	**51**	106.1
36	137.9	**52**	167.5
37	106.5	**53**	131.7
38	128.2	**54**	169.3
39	107.9	**55**	137.0
40	160.4	**56**	206.4
41	89.4	**57**	102.3
42	112.1	**58**	133.2
43	89.9	**59**	105.9
44	140.5	**60**	162.4
45	104.9	**61**	127.1
46	125.7	**62**	160.7
47	106.0	**63**	132.3
48	158.5	**64**	197.5

Table A1.7. CAPEX, OPEX, oil price forecast (14 years)

YEAR	OIL PRICE, US$	OPEX, MM US$	CAPEX LOW, MM US$	CAPEX MEDIUM, MM US$	CAPEX HIGH, MM US$
2019	80	50	3330	3330	3330
2020	81	50	163	146	129
2021	82	50	0	100	166
2022	82	50	0	0	0
2023	83	50	0	0	0
2024	84	50	0	0	0
2025	85	50	0	0	0
2026	86	50	0	0	0
2027	87	50	0	0	0
2028	87	50	0	0	0
2029	88	50	0	0	0
2030	89	50	0	0	0
2031	90	50	0	0	0
2032	91	50	0	0	0

Table A1.8. CAPEX, OPEX, Oil Price Forecast (14 years)

YEAR	OIL PRICE, US$	OPEX, MM US$	CAPEX LOW, MM US$	CAPEX MEDIUM, MM US$	CAPEX HIGH, MM US$
2033	92	50	0	0	0
2034	93	50	0	0	0
2035	94	50	0	0	0
2036	95	50	0	0	0
2037	96	50	0	0	0
2038	97	50	0	0	0
2039	98	50	0	0	0
2040	99	50	0	0	0
2041	100	50	0	0	0
2042	101	50	0	0	0
2043	102	50	0	0	0
2044	103	50	0	0	0
2045	104	50	0	0	0
2046	105	300	0	0	0

Table A1.9. Cost estimates

CONCEPT	COST, MM US$
Cost producer well w completion	20.0
Cost producer well w/o completion	17.0
Cost injector well w completion	21.5
Cost injector well w/o completion	18.0
Facilities cost	3,270.0

Table A1.10. Data acquisition: one well. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s1)	0.110	**P(hhhh|s2)**	0.080	**P(hhhh|s3)**	0.080
P(hhhl|s1)	0.090	**P(hhhl|s2)**	0.140	**P(hhhl|s3)**	0.060
P(hhlh|s1)	0.090	**P(hhlh|s2)**	0.060	**P(hhlh|s3)**	0.140
P(hhll|s1)	0.060	**P(hhll|s2)**	0.080	**P(hhll|s3)**	0.080
P(hlhh|s1)	0.090	**P(hlhh|s2)**	0.060	**P(hlhh|s3)**	0.060
P(hlhl|s1)	0.060	**P(hlhl|s2)**	0.080	**P(hlhl|s3)**	0.040
P(hllh|s1)	0.060	**P(hllh|s2)**	0.040	**P(hllh|s3)**	0.080
P(hlll|s1)	0.040	**P(hlll|s2)**	0.060	**P(hlll|s3)**	0.060
P(lhhh|s1)	0.090	**P(lhhh|s2)**	0.060	**P(lhhh|s3)**	0.060
P(lhhl|s1)	0.060	**P(lhhl|s2)**	0.080	**P(lhhl|s3)**	0.040
P(lhlh|s1)	0.060	**P(lhlh|s2)**	0.040	**P(lhlh|s3)**	0.080
P(lhll|s1)	0.040	**P(lhll|s2)**	0.060	**P(lhll|s3)**	0.060
P(llhh|s1)	0.060	**P(llhh|s2)**	0.040	**P(llhh|s3)**	0.040
P(llhl|s1)	0.040	**P(llhl|s2)**	0.060	**P(llhl|s3)**	0.020
P(lllh|s1)	0.040	**P(lllh|s2)**	0.020	**P(lllh|s3)**	0.060
P(llll|s1)	0.010	**P(llll|s2)**	0.040	**P(llll|s3)**	0.040

Table A1.11. Data acquisition: one well. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s4)	0.060	**P(hhhh|s5)**	0.080	**P(hhhh|s6)**	0.060
P(hhhl|s4)	0.080	**P(hhhl|s5)**	0.060	**P(hhhl|s6)**	0.080
P(hhlh|s4)	0.080	**P(hhlh|s5)**	0.060	**P(hhlh|s6)**	0.040
P(hhll|s4)	0.140	**P(hhll|s5)**	0.040	**P(hhll|s6)**	0.060
P(hlhh|s4)	0.040	**P(hlhh|s5)**	0.140	**P(hlhh|s6)**	0.080
P(hlhl|s4)	0.060	**P(hlhl|s5)**	0.080	**P(hlhl|s6)**	0.140
P(hllh|s4)	0.060	**P(hllh|s5)**	0.080	**P(hllh|s6)**	0.060
P(hlll|s4)	0.080	**P(hlll|s5)**	0.060	**P(hlll|s6)**	0.080
P(lhhh|s4)	0.040	**P(lhhh|s5)**	0.060	**P(lhhh|s6)**	0.040
P(lhhl|s4)	0.060	**P(lhhl|s5)**	0.040	**P(lhhl|s6)**	0.060
P(lhlh|s4)	0.060	**P(lhlh|s5)**	0.040	**P(lhlh|s6)**	0.020
P(lhll|s4)	0.080	**P(lhll|s5)**	0.020	**P(lhll|s6)**	0.040
P(llhh|s4)	0.020	**P(llhh|s5)**	0.080	**P(llhh|s6)**	0.060
P(llhl|s4)	0.040	**P(llhl|s5)**	0.060	**P(llhl|s6)**	0.080
P(lllh|s4)	0.040	**P(lllh|s5)**	0.060	**P(lllh|s6)**	0.040
P(llll|s4)	0.060	**P(llll|s5)**	0.040	**P(llll|s6)**	0.060

Table A1.12. Data acquisition: one well. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s7)	0.060	**P(hhhh|s8)**	0.040	**P(hhhh|s9)**	0.080
P(hhhl|s7)	0.040	**P(hhhl|s8)**	0.060	**P(hhhl|s9)**	0.060
P(hhlh|s7)	0.080	**P(hhlh|s8)**	0.060	**P(hhlh|s9)**	0.060
P(hhll|s7)	0.060	**P(hhll|s8)**	0.080	**P(hhll|s9)**	0.040
P(hlhh|s7)	0.080	**P(hlhh|s8)**	0.060	**P(hlhh|s9)**	0.060
P(hlhl|s7)	0.060	**P(hlhl|s8)**	0.080	**P(hlhl|s9)**	0.040
P(hllh|s7)	0.140	**P(hllh|s8)**	0.080	**P(hllh|s9)**	0.040
P(hlll|s7)	0.080	**P(hlll|s8)**	0.140	**P(hlll|s9)**	0.020
P(lhhh|s7)	0.040	**P(lhhh|s8)**	0.020	**P(lhhh|s9)**	0.140
P(lhhl|s7)	0.020	**P(lhhl|s8)**	0.040	**P(lhhl|s9)**	0.080
P(lhlh|s7)	0.060	**P(lhlh|s8)**	0.040	**P(lhlh|s9)**	0.080
P(lhll|s7)	0.040	**P(lhll|s8)**	0.060	**P(lhll|s9)**	0.060
P(llhh|s7)	0.060	**P(llhh|s8)**	0.040	**P(llhh|s9)**	0.080
P(llhl|s7)	0.040	**P(llhl|s8)**	0.060	**P(llhl|s9)**	0.060
P(lllh|s7)	0.080	**P(lllh|s8)**	0.060	**P(lllh|s9)**	0.060
P(llll|s7)	0.060	**P(llll|s8)**	0.080	**P(llll|s9)**	0.040

Table A1.13. Data acquisition: one well. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s10)	0.060	**P(hhhh|s11)**	0.060	**P(hhhh|s12)**	0.040
P(hhhl|s10)	0.080	**P(hhhl|s11)**	0.040	**P(hhhl|s12)**	0.060
P(hhlh|s10)	0.040	**P(hhlh|s11)**	0.080	**P(hhlh|s12)**	0.060
P(hhll|s10)	0.060	**P(hhll|s11)**	0.060	**P(hhll|s12)**	0.080
P(hlhh|s10)	0.040	**P(hlhh|s11)**	0.040	**P(hlhh|s12)**	0.020
P(hlhl|s10)	0.060	**P(hlhl|s11)**	0.020	**P(hlhl|s12)**	0.040
P(hllh|s10)	0.020	**P(hllh|s11)**	0.060	**P(hllh|s12)**	0.040
P(hlll|s10)	0.040	**P(hlll|s11)**	0.040	**P(hlll|s12)**	0.060
P(lhhh|s10)	0.080	**P(lhhh|s11)**	0.080	**P(lhhh|s12)**	0.060
P(lhhl|s10)	0.140	**P(lhhl|s11)**	0.060	**P(lhhl|s12)**	0.080
P(lhlh|s10)	0.060	**P(lhlh|s11)**	0.140	**P(lhlh|s12)**	0.080
P(lhll|s10)	0.080	**P(lhll|s11)**	0.080	**P(lhll|s12)**	0.140
P(llhh|s10)	0.060	**P(llhh|s11)**	0.060	**P(llhh|s12)**	0.040
P(llhl|s10)	0.080	**P(llhl|s11)**	0.040	**P(llhl|s12)**	0.060
P(lllh|s10)	0.040	**P(lllh|s11)**	0.080	**P(lllh|s12)**	0.060
P(llll|s10)	0.060	**P(llll|s11)**	0.060	**P(llll|s12)**	0.080

Table A1.14. Data acquisition: one well. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s13)	0.060	**P(hhhh|s14)**	0.040	**P(hhhh|s15)**	0.040
P(hhhl|s13)	0.040	**P(hhhl|s14)**	0.060	**P(hhhl|s15)**	0.020
P(hhlh|s13)	0.040	**P(hhlh|s14)**	0.020	**P(hhlh|s15)**	0.060
P(hhll|s13)	0.020	**P(hhll|s14)**	0.040	**P(hhll|s15)**	0.040
P(hlhh|s13)	0.080	**P(hlhh|s14)**	0.060	**P(hlhh|s15)**	0.060
P(hlhl|s13)	0.060	**P(hlhl|s14)**	0.080	**P(hlhl|s15)**	0.040
P(hllh|s13)	0.060	**P(hllh|s14)**	0.040	**P(hllh|s15)**	0.080
P(hlll|s13)	0.040	**P(hlll|s14)**	0.060	**P(hlll|s15)**	0.060
P(lhhh|s13)	0.080	**P(lhhh|s14)**	0.060	**P(lhhh|s15)**	0.060
P(lhhl|s13)	0.060	**P(lhhl|s14)**	0.080	**P(lhhl|s15)**	0.040
P(lhlh|s13)	0.060	**P(lhlh|s14)**	0.040	**P(lhlh|s15)**	0.080
P(lhll|s13)	0.040	**P(lhll|s14)**	0.060	**P(lhll|s15)**	0.060
P(llhh|s13)	0.140	**P(llhh|s14)**	0.080	**P(llhh|s15)**	0.080
P(llhl|s13)	0.080	**P(llhl|s14)**	0.140	**P(llhl|s15)**	0.060
P(lllh|s13)	0.080	**P(lllh|s14)**	0.060	**P(lllh|s15)**	0.140
P(llll|s13)	0.060	**P(llll|s14)**	0.080	**P(llll|s15)**	0.080

Table A1.15. Data acquisition: one well. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s16)	0.020
P(hhhl|s16)	0.040
P(hhlh|s16)	0.040
P(hhll|s16)	0.060
P(hlhh|s16)	0.040
P(hlhl|s16)	0.060
P(hllh|s16)	0.060
P(hlll|s16)	0.080
P(lhhh|s16)	0.040
P(lhhl|s16)	0.060
P(lhlh|s16)	0.060
P(lhll|s16)	0.080
P(llhh|s16)	0.060
P(llhl|s16)	0.080
P(lllh|s16)	0.080
P(llll|s16)	0.140

Table A1.16. Data acquisition: one well. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s1|hhhh)	0.011	**P(s2|hhhh)**	0.019	**P(s3|hhhh)**	0.015
P(s1|hhhl)	0.008	**P(s2|hhhl)**	0.028	**P(s3|hhhl)**	0.009
P(s1|hhlh)	0.008	**P(s2|hhlh)**	0.012	**P(s3|hhlh)**	0.023
P(s1|hhll)	0.005	**P(s2|hhll)**	0.014	**P(s3|hhll)**	0.011
P(s1|hlhh)	0.007	**P(s2|hlhh)**	0.011	**P(s3|hlhh)**	0.009
P(s1|hlhl)	0.004	**P(s2|hlhl)**	0.013	**P(s3|hlhl)**	0.005
P(s1|hllh)	0.004	**P(s2|hllh)**	0.007	**P(s3|hllh)**	0.010
P(s1|hlll)	0.002	**P(s2|hlll)**	0.008	**P(s3|hlll)**	0.007
P(s1|lhhh)	0.007	**P(s2|lhhh)**	0.011	**P(s3|lhhh)**	0.009
P(s1|lhhl)	0.004	**P(s2|lhhl)**	0.013	**P(s3|lhhl)**	0.005
P(s1|lhlh)	0.004	**P(s2|lhlh)**	0.007	**P(s3|lhlh)**	0.010
P(s1|lhll)	0.002	**P(s2|lhll)**	0.008	**P(s3|lhll)**	0.007
P(s1|llhh)	0.004	**P(s2|llhh)**	0.006	**P(s3|llhh)**	0.005
P(s1|llhl)	0.002	**P(s2|llhl)**	0.007	**P(s3|llhl)**	0.002
P(s1|lllh)	0.002	**P(s2|lllh)**	0.003	**P(s3|lllh)**	0.006
P(s1|llll)	0.000	**P(s2|llll)**	0.004	**P(s3|llll)**	0.003

Table A1.17. Data acquisition: one well. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s4|hhhh)	0.027	**P(s5|hhhh)**	0.033	**P(s6|hhhh)**	0.057
P(s4|hhhl)	0.029	**P(s5|hhhl)**	0.020	**P(s6|hhhl)**	0.063
P(s4|hhlh)	0.031	**P(s5|hhlh)**	0.021	**P(s6|hhlh)**	0.033
P(s4|hhll)	0.046	**P(s5|hhll)**	0.012	**P(s6|hhll)**	0.042
P(s4|hlhh)	0.014	**P(s5|hlhh)**	0.044	**P(s6|hlhh)**	0.058
P(s4|hlhl)	0.017	**P(s5|hlhl)**	0.021	**P(s6|hlhl)**	0.088
P(s4|hllh)	0.018	**P(s5|hllh)**	0.022	**P(s6|hllh)**	0.039
P(s4|hlll)	0.021	**P(s5|hlll)**	0.014	**P(s6|hlll)**	0.045
P(s4|lhhh)	0.014	**P(s5|lhhh)**	0.019	**P(s6|lhhh)**	0.029
P(s4|lhhl)	0.017	**P(s5|lhhl)**	0.011	**P(s6|lhhl)**	0.038
P(s4|lhlh)	0.018	**P(s5|lhlh)**	0.011	**P(s6|lhlh)**	0.013
P(s4|lhll)	0.021	**P(s5|lhll)**	0.005	**P(s6|lhll)**	0.022
P(s4|llhh)	0.005	**P(s5|llhh)**	0.020	**P(s6|llhh)**	0.035
P(s4|llhl)	0.009	**P(s5|llhl)**	0.013	**P(s6|llhl)**	0.040
P(s4|lllh)	0.010	**P(s5|lllh)**	0.013	**P(s6|lllh)**	0.021
P(s4|llll)	0.012	**P(s5|llll)**	0.007	**P(s6|llll)**	0.026

Table A1.18. Data acquisition: one well. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s7|hhhh)	0.046	**P(s8|hhhh)**	0.071	**P(s9|hhhh)**	0.033
P(s7|hhhl)	0.025	**P(s8|hhhl)**	0.088	**P(s9|hhhl)**	0.020
P(s7|hhlh)	0.053	**P(s8|hhlh)**	0.092	**P(s9|hhlh)**	0.021
P(s7|hhll)	0.034	**P(s8|hhll)**	0.105	**P(s9|hhll)**	0.012
P(s7|hlhh)	0.046	**P(s8|hlhh)**	0.081	**P(s9|hlhh)**	0.019
P(s7|hlhl)	0.030	**P(s8|hlhl)**	0.093	**P(s9|hlhl)**	0.011
P(s7|hllh)	0.073	**P(s8|hllh)**	0.097	**P(s9|hllh)**	0.011
P(s7|hlll)	0.036	**P(s8|hlll)**	0.146	**P(s9|hlll)**	0.005
P(s7|lhhh)	0.023	**P(s8|lhhh)**	0.027	**P(s9|lhhh)**	0.044
P(s7|lhhl)	0.010	**P(s8|lhhl)**	0.047	**P(s9|lhhl)**	0.021
P(s7|lhlh)	0.031	**P(s8|lhlh)**	0.048	**P(s9|lhlh)**	0.022
P(s7|lhll)	0.018	**P(s8|lhll)**	0.062	**P(s9|lhll)**	0.014
P(s7|llhh)	0.028	**P(s8|llhh)**	0.043	**P(s9|llhh)**	0.020
P(s7|llhl)	0.016	**P(s8|llhl)**	0.055	**P(s9|llhl)**	0.013
P(s7|lllh)	0.033	**P(s8|lllh)**	0.057	**P(s9|lllh)**	0.013
P(s7|llll)	0.021	**P(s8|llll)**	0.064	**P(s9|llll)**	0.007

Table A1.19. Data acquisition: one well. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s10|hhhh)	0.057	**P(s11|hhhh)**	0.046	**P(s12|hhhh)**	0.071
P(s10|hhhl)	0.063	**P(s11|hhhl)**	0.025	**P(s12|hhhl)**	0.088
P(s10|hhlh)	0.033	**P(s11|hhlh)**	0.053	**P(s12|hhlh)**	0.092
P(s10|hhll)	0.042	**P(s11|hhll)**	0.034	**P(s12|hhll)**	0.105
P(s10|hlhh)	0.029	**P(s11|hlhh)**	0.023	**P(s12|hlhh)**	0.027
P(s10|hlhl)	0.038	**P(s11|hlhl)**	0.010	**P(s12|hlhl)**	0.047
P(s10|hllh)	0.013	**P(s11|hllh)**	0.031	**P(s12|hllh)**	0.048
P(s10|hlll)	0.022	**P(s11|hlll)**	0.018	**P(s12|hlll)**	0.062
P(s10|lhhh)	0.058	**P(s11|lhhh)**	0.046	**P(s12|lhhh)**	0.081
P(s10|lhhl)	0.088	**P(s11|lhhl)**	0.030	**P(s12|lhhl)**	0.093
P(s10|lhlh)	0.039	**P(s11|lhlh)**	0.073	**P(s12|lhlh)**	0.097
P(s10|lhll)	0.045	**P(s11|lhll)**	0.036	**P(s12|lhll)**	0.146
P(s10|llhh)	0.035	**P(s11|llhh)**	0.028	**P(s12|llhh)**	0.043
P(s10|llhl)	0.040	**P(s11|llhl)**	0.016	**P(s12|llhl)**	0.055
P(s10|lllh)	0.021	**P(s11|lllh)**	0.033	**P(s12|lllh)**	0.057
P(s10|llll)	0.026	**P(s11|llll)**	0.021	**P(s12|llll)**	0.064

Table A1.20. Data acquisition: one well. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(13|hhhh)	0.098	**P(s14|hhhh)**	0.153	**P(s15|hhhh)**	0.122
P(s13|hhhl)	0.054	**P(s14|hhhl)**	0.190	**P(s15|hhhl)**	0.050
P(s13|hhlh)	0.057	**P(s14|hhlh)**	0.066	**P(s15|hhlh)**	0.158
P(s13|hhll)	0.024	**P(s14|hhll)**	0.113	**P(s15|hhll)**	0.090
P(s13|hlhh)	0.100	**P(s14|hlhh)**	0.175	**P(s15|hlhh)**	0.139
P(s13|hlhl)	0.064	**P(s14|hlhl)**	0.200	**P(s15|hlhl)**	0.080
P(s13|hllh)	0.067	**P(s14|hllh)**	0.104	**P(s15|hllh)**	0.166
P(s13|hlll)	0.038	**P(s14|hlll)**	0.135	**P(s15|hlll)**	0.107
P(s13|lhhh)	0.100	**P(s14|lhhh)**	0.175	**P(s15|lhhh)**	0.139
P(s13|lhhl)	0.064	**P(s14|lhh)**	0.200	**P(s15|lhhl)**	0.080
P(s13|lhlh)	0.067	**P(s14|lhlh)**	0.104	**P(s15|lhlh)**	0.166
P(s13|lhll)	0.038	**P(s14|lhll)**	0.135	**P(s15|lhll)**	0.107
P(s13|llhh)	0.139	**P(s14|llhh)**	0.185	**P(s15|llhh)**	0.148
P(s13|llhl)	0.068	**P(s14|llhl)**	0.277	**P(s15|llhl)**	0.094
P(s13|lllh)	0.071	**P(s14|lllh)**	0.124	**P(s15|lllh)**	0.230
P(s13|llll)	0.045	**P(s14|llll)**	0.138	**P(s15|llll)**	0.110

Table A1.21. Data acquisition: one well. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s16|hhhh)	0.142
P(s16|hhhl)	0.236
P(s16|hhlh)	0.246
P(s16|hhll)	0.314
P(s16|hlhh)	0.217
P(s16|hlhl)	0.279
P(s16|hllh)	0.290
P(s16|hlll)	0.333
P(s16|lhhh)	0.217
P(s16|lhhl)	0.279
P(s16|lhlh)	0.290
P(s16|lhll)	0.333
P(s16|llhh)	0.258
P(s16|llhl)	0.294
P(s16|lllh)	0.307
P(s16|llll)	0.450

Table A1.22. Data acquisition: one well. Residual probabilities

COMPOUND STATE	RESIDUAL PROBABILITIES
P(hhhh)	0.041
P(hhhl)	0.049
P(hhlh)	0.047
P(hhll)	0.056
P(hlhh)	0.054
P(hlhl)	0.063
P(hllh)	0.060
P(hlll)	0.070
P(lhhh)	0.054
P(lhhl)	0.063
P(lhlh)	0.060
P(lhll)	0.070
P(llhh)	0.068
P(llhl)	0.079
P(lllh)	0.076
P(llll)	0.091

Table A1.23. Data acquisition: extended well test. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s1)	0.100	**P(hhhh|s2)**	0.100	**P(hhhh|s3)**	0.100
P(hhhl|s1)	0.100	**P(hhhl|s2)**	0.100	**P(hhhl|s3)**	0.100
P(hhlh|s1)	0.100	**P(hhlh|s2)**	0.100	**P(hhlh|s3)**	0.100
P(hhll|s1)	0.100	**P(hhll|s2)**	0.100	**P(hhll|s3)**	0.100
P(hlhh|s1)	0.025	**P(hlhh|s2)**	0.025	**P(hlhh|s3)**	0.025
P(hlhl|s1)	0.025	**P(hlhl|s2)**	0.025	**P(hlhl|s3)**	0.025
P(hllh|s1)	0.025	**P(hllh|s2)**	0.025	**P(hllh|s3)**	0.025
P(hlll|s1)	0.025	**P(hlll|s2)**	0.025	**P(hlll|s3)**	0.025
P(lhhh|s1)	0.100	**P(lhhh|s2)**	0.100	**P(lhhh|s3)**	0.100
P(lhhl|s1)	0.100	**P(lhhl|s2)**	0.100	**P(lhhl|s3)**	0.100
P(lhlh|s1)	0.100	**P(lhlh|s2)**	0.100	**P(lhlh|s3)**	0.100
P(lhll|s1)	0.100	**P(lhll|s2)**	0.100	**P(lhll|s3)**	0.100
P(llhh|s1)	0.025	**P(llhh|s2)**	0.025	**P(llhh|s3)**	0.025
P(llhl|s1)	0.025	**P(llhl|s2)**	0.025	**P(llhl|s3)**	0.025
P(lllh|s1)	0.025	**P(lllh|s2)**	0.025	**P(lllh|s3)**	0.025
P(llll|s1)	0.025	**P(llll|s2)**	0.025	**P(llll|s3)**	0.025

Table A1.24. Data acquisition: extended well test. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s4)	0.100	**P(hhhh|s5)**	0.100	**P(hhhh|s6)**	0.100
P(hhhl|s4)	0.100	**P(hhhl|s5)**	0.100	**P(hhhl|s6)**	0.100
P(hhlh|s4)	0.100	**P(hhlh|s5)**	0.100	**P(hhlh|s6)**	0.100
P(hhll|s4)	0.100	**P(hhll|s5)**	0.100	**P(hhll|s6)**	0.100
P(hlhh|s4)	0.025	**P(hlhh|s5)**	0.025	**P(hlhh|s6)**	0.025
P(hlhl|s4)	0.025	**P(hlhl|s5)**	0.025	**P(hlhl|s6)**	0.025
P(hllh|s4)	0.025	**P(hllh|s5)**	0.025	**P(hllh|s6)**	0.025
P(hlll|s4)	0.025	**P(hlll|s5)**	0.025	**P(hlll|s6)**	0.025
P(lhhh|s4)	0.100	**P(lhhh|s5)**	0.100	**P(lhhh|s6)**	0.100
P(lhhl|s4)	0.100	**P(lhhl|s5)**	0.100	**P(lhhl|s6)**	0.100
P(lhlh|s4)	0.100	**P(lhlh|s5)**	0.100	**P(lhlh|s6)**	0.100
P(lhll|s4)	0.100	**P(lhll|s5)**	0.100	**P(lhll|s6)**	0.100
P(llhh|s4)	0.025	**P(llhh|s5)**	0.025	**P(llhh|s6)**	0.025
P(llhl|s4)	0.025	**P(llhl|s5)**	0.025	**P(llhl|s6)**	0.025
P(lllh|s4)	0.025	**P(lllh|s5)**	0.025	**P(lllh|s6)**	0.025
P(llll|s4)	0.025	**P(llll|s5)**	0.025	**P(llll|s6)**	0.025

Table A1.25. Data acquisition: extended well test. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s7)	0.100	**P(hhhh|s8)**	0.100	**P(hhhh|s9)**	0.038
P(hhhl|s7)	0.100	**P(hhhl|s8)**	0.100	**P(hhhl|s9)**	0.038
P(hhlh|s7)	0.100	**P(hhlh|s8)**	0.100	**P(hhlh|s9)**	0.038
P(hhll|s7)	0.100	**P(hhll|s8)**	0.100	**P(hhll|s9)**	0.038
P(hlhh|s7)	0.025	**P(hlhh|s8)**	0.025	**P(hlhh|s9)**	0.088
P(hlhl|s7)	0.025	**P(hlhl|s8)**	0.025	**P(hlhl|s9)**	0.088
P(hllh|s7)	0.025	**P(hllh|s8)**	0.025	**P(hllh|s9)**	0.088
P(hlll|s7)	0.025	**P(hlll|s8)**	0.025	**P(hlll|s9)**	0.088
P(lhhh|s7)	0.100	**P(lhhh|s8)**	0.100	**P(lhhh|s9)**	0.038
P(lhhl|s7)	0.100	**P(lhhl|s8)**	0.100	**P(lhhl|s9)**	0.038
P(lhlh|s7)	0.100	**P(lhlh|s8)**	0.100	**P(lhlh|s9)**	0.038
P(lhll|s7)	0.100	**P(lhll|s8)**	0.100	**P(lhll|s9)**	0.038
P(llhh|s7)	0.025	**P(llhh|s8)**	0.025	**P(llhh|s9)**	0.088
P(llhl|s7)	0.025	**P(llhl|s8)**	0.025	**P(llhl|s9)**	0.088
P(lllh|s7)	0.025	**P(lllh|s8)**	0.025	**P(lllh|s9)**	0.088
P(llll|s7)	0.025	**P(llll|s8)**	0.025	**P(llll|s9)**	0.088

Table A1.26. Data acquisition: extended well test. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s10)	0.038	**P(hhhh|s11)**	0.038	**P(hhhh|s12)**	0.038
P(hhhl|s10)	0.038	**P(hhhl|s11)**	0.038	**P(hhhl|s12)**	0.038
P(hhlh|s10)	0.038	**P(hhlh|s11)**	0.038	**P(hhlh|s12)**	0.038
P(hhll|s10)	0.038	**P(hhll|s11)**	0.038	**P(hhll|s12)**	0.038
P(hlhh|s10)	0.088	**P(hlhh|s11)**	0.088	**P(hlhh|s12)**	0.088
P(hlhl|s10)	0.088	**P(hlhl|s11)**	0.088	**P(hlhl|s12)**	0.088
P(hllh|s10)	0.088	**P(hllh|s11)**	0.088	**P(hllh|s12)**	0.088
P(hlll|s10)	0.088	**P(hlll|s11)**	0.088	**P(hlll|s12)**	0.088
P(lhhh|s10)	0.038	**P(lhhh|s11)**	0.038	**P(lhhh|s12)**	0.038
P(lhhl|s10)	0.038	**P(lhhl|s11)**	0.038	**P(lhhl|s12)**	0.038
P(lhlh|s10)	0.038	**P(lhlh|s11)**	0.038	**P(lhlh|s12)**	0.038
P(lhll|s10)	0.038	**P(lhll|s11)**	0.038	**P(lhll|s12)**	0.038
P(llhh|s10)	0.088	**P(llhh|s11)**	0.088	**P(llhh|s12)**	0.088
P(llhl|s10)	0.088	**P(llhl|s11)**	0.088	**P(llhl|s12)**	0.088
P(lllh|s10)	0.088	**P(lllh|s11)**	0.088	**P(lllh|s12)**	0.088
P(llll|s10)	0.088	**P(llll|s11)**	0.088	**P(llll|s12)**	0.088

Table A1.27. Data acquisition: extended well test. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s13)	0.038	**P(hhhh|s14)**	0.038	**P(hhhh|s15)**	0.038
P(hhhl|s13)	0.038	**P(hhhl|s14)**	0.038	**P(hhhl|s15)**	0.038
P(hhlh|s13)	0.038	**P(hhlh|s14)**	0.038	**P(hhlh|s15)**	0.038
P(hhll|s13)	0.038	**P(hhll|s14)**	0.038	**P(hhll|s15)**	0.038
P(hlhh|s13)	0.088	**P(hlhh|s14)**	0.088	**P(hlhh|s15)**	0.088
P(hlhl|s13)	0.088	**P(hlhl|s14)**	0.088	**P(hlhl|s15)**	0.088
P(hllh|s13)	0.088	**P(hllh|s14)**	0.088	**P(hllh|s15)**	0.088
P(hlll|s13)	0.088	**P(hlll|s14)**	0.088	**P(hlll|s15)**	0.088
P(lhhh|s13)	0.038	**P(lhhh|s14)**	0.038	**P(lhhh|s15)**	0.038
P(lhhl|s13)	0.038	**P(lhhl|s14)**	0.038	**P(lhhl|s15)**	0.038
P(lhlh|s13)	0.038	**P(lhlh|s14)**	0.038	**P(lhlh|s15)**	0.038
P(lhll|s13)	0.038	**P(lhll|s14)**	0.038	**P(lhll|s15)**	0.038
P(llhh|s13)	0.088	**P(llhh|s14)**	0.088	**P(llhh|s15)**	0.088
P(llhl|s13)	0.088	**P(llhl|s14)**	0.088	**P(llhl|s15)**	0.088
P(lllh|s13)	0.088	**P(lllh|s14)**	0.088	**P(lllh|s15)**	0.088
P(llll|s13)	0.088	**P(llll|s14)**	0.088	**P(llll|s15)**	0.088

Table A1.28. Data acquisition: extended well test. Reliability probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	RELIABILITY PROBABILITY
P(hhhh|s16)	0.038
P(hhhl|s16)	0.038
P(hhlh|s16)	0.038
P(hhll|s16)	0.038
P(hlhh|s16)	0.088
P(hlhl|s16)	0.088
P(hllh|s16)	0.088
P(hlll|s16)	0.088
P(lhhh|s16)	0.038
P(lhhl|s16)	0.038
P(lhlh|s16)	0.038
P(lhll|s16)	0.038
P(llhh|s16)	0.088
P(llhl|s16)	0.088
P(lllh|s16)	0.088
P(llll|s16)	0.088

Table A1.29. Data acquisition: extended well test. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s1|hhhh)	0.008	**P(s2|hhhh)**	0.020	**P(s3|hhhh)**	0.016
P(s1|hhhl)	0.008	**P(s2|hhhl)**	0.020	**P(s3|hhhl)**	0.016
P(s1|hhlh)	0.008	**P(s2|hhlh)**	0.020	**P(s3|hhlh)**	0.016
P(s1|hhll)	0.008	**P(s2|hhll)**	0.020	**P(s3|hhll)**	0.016
P(s1|hlhh)	0.001	**P(s2|hlhh)**	0.003	**P(s3|hlhh)**	0.003
P(s1|hlhl)	0.001	**P(s2|hlhl)**	0.003	**P(s3|hlhl)**	0.003
P(s1|hllh)	0.001	**P(s2|hllh)**	0.003	**P(s3|hllh)**	0.003
P(s1|hlll)	0.001	**P(s2|hlll)**	0.003	**P(s3|hlll)**	0.003
P(s1|lhhh)	0.008	**P(s2|lhhh)**	0.020	**P(s3|lhhh)**	0.016
P(s1|lhhl)	0.008	**P(s2|lhhl)**	0.020	**P(s3|lhhl)**	0.016
P(s1|lhlh)	0.008	**P(s2|lhlh)**	0.020	**P(s3|lhlh)**	0.016
P(s1|lhll)	0.008	**P(s2|lhll)**	0.020	**P(s3|lhll)**	0.016
P(s1|llhh)	0.001	**P(s2|llhh)**	0.003	**P(s3|llhh)**	0.003
P(s1|llhl)	0.001	**P(s2|llhl)**	0.003	**P(s3|llhl)**	0.003
P(s1|lllh)	0.001	**P(s2|lllh)**	0.003	**P(s3|lllh)**	0.003
P(s1|llll)	0.001	**P(s2|llll)**	0.003	**P(s3|llll)**	0.003

Table A1.30. Data acquisition: extended well test. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s4|hhhh)	0.036	**P(s5|hhhh)**	0.034	**P(s6|hhhh)**	0.078
P(s4|hhhl)	0.036	**P(s5|hhhl)**	0.034	**P(s6|hhhl)**	0.078
P(s4|hhlh)	0.036	**P(s5|hhlh)**	0.034	**P(s6|hhlh)**	0.078
P(s4|hhll)	0.036	**P(s5|hhll)**	0.034	**P(s6|hhll)**	0.078
P(s4|hlhh)	0.006	**P(s5|hlhh)**	0.006	**P(s6|hlhh)**	0.013
P(s4|hlhl)	0.006	**P(s5|hlhl)**	0.006	**P(s6|hlhl)**	0.013
P(s4|hllh)	0.006	**P(s5|hllh)**	0.006	**P(s6|hllh)**	0.013
P(s4|hlll)	0.006	**P(s5|hlll)**	0.006	**P(s6|hlll)**	0.013
P(s4|lhhh)	0.036	**P(s5|lhhh)**	0.034	**P(s6|lhhh)**	0.078
P(s4|lhhl)	0.036	**P(s5|lhhl)**	0.034	**P(s6|lhhl)**	0.078
P(s4|lhlh)	0.036	**P(s5|lhlh)**	0.034	**P(s6|lhlh)**	0.078
P(s4|lhll)	0.036	**P(s5|lhll)**	0.034	**P(s6|lhll)**	0.078
P(s4|llhh)	0.006	**P(s5|llhh)**	0.006	**P(s6|llhh)**	0.013
P(s4|llhl)	0.006	**P(s5|llhl)**	0.006	**P(s6|llhl)**	0.013
P(s4|lllh)	0.006	**P(s5|lllh)**	0.006	**P(s6|lllh)**	0.013
P(s4|llll)	0.006	**P(s5|llll)**	0.006	**P(s6|llll)**	0.013

Table A1.31. Data acquisition: extended well test. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s7|hhhh)	0.062	**P(s8|hhhh)**	0.146	**P(s9|hhhh)**	0.013
P(s7|hhhl)	0.062	**P(s8|hhhl)**	0.146	**P(s9|hhhl)**	0.013
P(s7|hhlh)	0.062	**P(s8|hhlh)**	0.146	**P(s9|hhlh)**	0.013
P(s7|hhll)	0.062	**P(s8|hhll)**	0.146	**P(s9|hhll)**	0.013
P(s7|hlhh)	0.010	**P(s8|hlhh)**	0.024	**P(s9|hlhh)**	0.020
P(s7|hlhl)	0.010	**P(s8|hlhl)**	0.024	**P(s9|hlhl)**	0.020
P(s7|hllh)	0.010	**P(s8|hllh)**	0.024	**P(s9|hllh)**	0.020
P(s7|hlll)	0.010	**P(s8|hlll)**	0.024	**P(s9|hlll)**	0.020
P(s7|lhhh)	0.062	**P(s8|lhhh)**	0.146	**P(s9|lhhh)**	0.013
P(s7|lhhl)	0.062	**P(s8|lhhl)**	0.146	**P(s9|lhhl)**	0.013
P(s7|lhlh)	0.062	**P(s8|lhlh)**	0.146	**P(s9|lhlh)**	0.013
P(s7|lhll)	0.062	**P(s8|lhll)**	0.146	**P(s9|lhll)**	0.013
P(s7|llhh)	0.010	**P(s8|llhh)**	0.024	**P(s9|llhh)**	0.020
P(s7|llhl)	0.010	**P(s8|llhl)**	0.024	**P(s9|llhl)**	0.020
P(s7|lllh)	0.010	**P(s8|lllh)**	0.024	**P(s9|lllh)**	0.020
P(s7|llll)	0.010	**P(s8|llll)**	0.024	**P(s9|llll)**	0.020

Table A1.32. Data acquisition: extended well test. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s10|hhhh)	0.029	**P(s11|hhhh)**	0.023	**P(s12|hhhh)**	0.055
P(s10|hhhl)	0.029	**P(s11|hhhl)**	0.023	**P(s12|hhhl)**	0.055
P(s10|hhlh)	0.029	**P(s11|hhlh)**	0.023	**P(s12|hhlh)**	0.055
P(s10|hhll)	0.029	**P(s11|hhll)**	0.023	**P(s12|hhll)**	0.055
P(s10|hlhh)	0.046	**P(s11|hlhh)**	0.036	**P(s12|hlhh)**	0.085
P(s10|hlhl)	0.046	**P(s11|hlhl)**	0.036	**P(s12|hlhl)**	0.085
P(s10|hllh)	0.046	**P(s11|hllh)**	0.036	**P(s12|hllh)**	0.085
P(s10|hlll)	0.046	**P(s11|hlll)**	0.036	**P(s12|hlll)**	0.085
P(s10|lhhh)	0.029	**P(s11|lhhh)**	0.023	**P(s12|lhhh)**	0.055
P(s10|lhhl)	0.029	**P(s11|lhhl)**	0.023	**P(s12|lhhl)**	0.055
P(s10|lhlh)	0.029	**P(s11|lhlh)**	0.023	**P(s12|lhlh)**	0.055
P(s10|lhll)	0.029	**P(s11|lhll)**	0.023	**P(s12|lhll)**	0.055
P(s10|llhh)	0.046	**P(s11|llhh)**	0.036	**P(s12|llhh)**	0.085
P(s10|llhl)	0.046	**P(s11|llhl)**	0.036	**P(s12|llhl)**	0.085
P(s10|lllh)	0.046	**P(s11|lllh)**	0.036	**P(s12|lllh)**	0.085
P(s10|llll)	0.046	**P(s11|llll)**	0.036	**P(s12|llll)**	0.085

Table A1.33. Data acquisition: extended well test. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s13|hhhh)	0.050	**P(s14|hhhh)**	0.118	**P(s15|hhhh)**	0.094
P(s13|hhhl)	0.050	**P(s14|hhhl)**	0.118	**P(s15|hhhl)**	0.094
P(s13|hhlh)	0.050	**P(s14|hhlh)**	0.118	**P(s15|hhlh)**	0.094
P(s13|hhll)	0.050	**P(s14|hhll)**	0.118	**P(s15|hhll)**	0.094
P(s13|hlhh)	0.078	**P(s14|hlhh)**	0.183	**P(s15|hlhh)**	0.146
P(s13|hlhl)	0.078	**P(s14|hlhl)**	0.183	**P(s15|hlhl)**	0.146
P(s13|hllh)	0.078	**P(s14|hllh)**	0.183	**P(s15|hllh)**	0.146
P(s13|hlll)	0.078	**P(s14|hlll)**	0.183	**P(s15|hlll)**	0.146
P(s13|lhhh)	0.050	**P(s14|lhhh)**	0.118	**P(s15|lhhh)**	0.094
P(s13|lhhl)	0.050	**P(s14|lhhl)**	0.118	**P(s15|lhhl)**	0.094
P(s13|lhlh)	0.050	**P(s14|lhlh)**	0.118	**P(s15|lhlh)**	0.094
P(s13|lhll)	0.050	**P(s14|lhll)**	0.118	**P(s15|lhll)**	0.094
P(s13|llhh)	0.078	**P(s14|llhh)**	0.183	**P(s15|llhh)**	0.146
P(s13|llhl)	0.078	**P(s14|llhl)**	0.183	**P(s15|llhl)**	0.146
P(s13|lllh)	0.078	**P(s14|lllh)**	0.183	**P(s15|lllh)**	0.146
P(s13|llll)	0.078	**P(s14|llll)**	0.183	**P(s15|llll)**	0.146

Table A1.34. Data acquisition: extended well test. Posterior probabilities

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P(s16|hhhh)	0.218
P(s16|hhhl)	0.218
P(s16|hhlh)	0.218
P(s16|hhll)	0.218
P(s16|hlhh)	0.340
P(s16|hlhl)	0.340
P(s16|hllh)	0.340
P(s16|hlll)	0.340
P(s16|lhhh)	0.218
P(s16|lhhl)	0.218
P(s16|lhlh)	0.218
P(s16|lhll)	0.218
P(s16|llhh)	0.340
P(s16|llhl)	0.340
P(s16|lllh)	0.340
P(s16|llll)	0.340

Table A1.35. Data acquisition: extended well test. Residual probabilities

COMPOUND STATE	RESIDUAL PROBABILITIES
P(hhhh)	0.050
P(hhhl)	0.050
P(hhlh)	0.050
P(hhll)	0.050
P(hlhh)	0.075
P(hlhl)	0.075
P(hllh)	0.075
P(hlll)	0.075
P(lhhh)	0.050
P(lhhl)	0.050
P(lhlh)	0.050
P(lhll)	0.050
P(llhh)	0.075
P(llhl)	0.075
P(lllh)	0.075
P(llll)	0.075

Table A1.36. Data acquisition: one well. Reliability probability for the membership functions

COMPOUND STATE CONDITIONAL PROBABILITY	FUZZY RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	FUZZY RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	FUZZY RELIABILITY PROBABILITY
P(M_1|s1)	0.4825	**P(M_2|s1)**	0.2650	**P(M_3|s1)**	0.2525
P(M_1|s2)	0.4650	**P(M_2|s2)**	0.2675	**P(M_3|s2)**	0.2675
P(M_1|s3)	0.4600	**P(M_2|s3)**	0.2700	**P(M_3|s3)**	0.2700
P(M_1|s4)	0.4425	**P(M_2|s4)**	0.2725	**P(M_3|s4)**	0.2850
P(M_1|s5)	0.4550	**P(M_2|s5)**	0.2750	**P(M_3|s5)**	0.2700
P(M_1|s6)	0.4375	**P(M_2|s6)**	0.2775	**P(M_3|s6)**	0.2850
P(M_1|s7)	0.4325	**P(M_2|s7)**	0.2800	**P(M_3|s7)**	0.2875
P(M_1|s8)	0.4150	**P(M_2|s8)**	0.2825	**P(M_3|s8)**	0.3025
P(M_1|s9)	0.4600	**P(M_2|s9)**	0.2675	**P(M_3|s9)**	0.2725
P(M_1|s10)	0.4425	**P(M_2|s10)**	0.2700	**P(M_3|s10)**	0.2875
P(M_1|s11)	0.4375	**P(M_2|s11)**	0.2725	**P(M_3|s11)**	0.2900
P(M_1|s12)	0.4200	**P(M_2|s12)**	0.2750	**P(M_3|s12)**	0.3050
P(M_1|s13)	0.4325	**P(M_2|s13)**	0.2775	**P(M_3|s13)**	0.2900
P(M_1|s14)	0.4150	**P(M_2|s14)**	0.2800	**P(M_3|s14)**	0.3050
P(M_1|s15)	0.4100	**P(M_2|s15)**	0.2825	**P(M_3|s15)**	0.3075
P(M_1|s16)	0.3925	**P(M_2|s16)**	0.2850	**P(M_3|s16)**	0.3225

Table A1.37. Data acquisition: one well. Fuzzy Posterior probability

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
$P(s1\|M_1)$	0.4825	$P(s1\|M_2)$	0.2650	$P(s1\|M_3)$	0.2525
$P(s2\|M_1)$	0.4650	$P(s2\|M_2)$	0.2675	$P(s2\|M_3)$	0.2675
$P(s3\|M_1)$	0.4600	$P(s3\|M_2)$	0.2700	$P(s3\|M_3)$	0.2700
$P(s4\|M_1)$	0.4425	$P(s4\|M_2)$	0.2725	$P(s4\|M_3)$	0.2850
P(s5|M_1)	0.4550	$P(s5\|M_2)$	0.2750	$P(s5\|M_3)$	0.2700
$P(s6\|M_1)$	0.4375	$P(s6\|M_2)$	0.2775	$P(s6\|M_3)$	0.2850
$P(s7\|M_1)$	0.4325	$P(s7\|M_2)$	0.2800	$P(s7\|M_3)$	0.2875
$P(s8\|M_1)$	0.4150	$P(s8\|M_2)$	0.2825	$P(s8\|M_3)$	0.3025
$P(s9\|M_1)$	0.4600	$P(s9\|M_2)$	0.2675	$P(s9\|M_3)$	0.2725
$P(s10\|M_1)$	0.4425	$P(s10\|M_2)$	0.2700	$P(s10\|M_3)$	0.2875
$P(s11\|M_1)$	0.4375	$P(s11\|M_2)$	0.2725	$P(s11\|M_3)$	0.2900
$P(s12\|M_1)$	0.4200	$P(s12\|M_2)$	0.2750	$P(s12\|M_3)$	0.3050
$P(s13\|M_1)$	0.4325	$P(s13\|M_2)$	0.2775	$P(s13\|M_3)$	0.2900
$P(s14\|M_1)$	0.4150	$P(s14\|M_2)$	0.2800	$P(s14\|M_3)$	0.3050
$P(s15\|M_1)$	0.4100	$P(s15\|M_2)$	0.2825	$P(s15\|M_3)$	0.3075
$P(s16\|M_1)$	0.3925	$P(s16\|M_2)$	0.2850	$P(s16\|M_3)$	0.3225

Table A1.38. Data acquisition: one well. Residual probability of the membership functions

MEMBERSHIP FUNCTION	**RESIDUAL PROBABILITY**
p(M_1)	0.3906
P(M_2)	0.2770
P(M_3)	0.3324

Table A1.39. Data acquisition: extended well test. Reliability probability for the membership functions

COMPOUND STATE CONDITIONAL PROBABILITY	FUZZY RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	FUZZY RELIABILITY PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	FUZZY RELIABILITY PROBABILITY
P(M_1|s1)	0.5900	**P(M_2|s1)**	0.2000	**P(M_3|s1)**	0.2100
P(M_1|s2)	0.5900	**P(M_2|s2)**	0.2000	**P(M_3|s2)**	0.2100
P(M_1|s3)	0.5900	**P(M_2|s3)**	0.2000	**P(M_3|s3)**	0.2100
P(M_1|s4)	0.5900	**P(M_2|s4)**	0.2000	**P(M_3|s4)**	0.2100
P(M_1|s5)	0.5900	**P(M_2|s5)**	0.2000	**P(M_3|s5)**	0.2100
P(M_1|s6)	0.5900	**P(M_2|s6)**	0.2000	**P(M_3|s6)**	0.2100
P(M_1|s7)	0.5900	**P(M_2|s7)**	0.2000	**P(M_3|s7)**	0.2100
P(M_1|s8)	0.5900	**P(M_2|s8)**	0.2000	**P(M_3|s8)**	0.2100
P(M_1|s9)	0.3150	**P(M_2|s9)**	0.2000	**P(M_3|s9)**	0.4850
P(M_1|s10)	0.3150	**P(M_2|s10)**	0.2000	**P(M_3|s10)**	0.4850
P(M_1|s11)	0.3150	**P(M_2|s11)**	0.2000	**P(M_3|s11)**	0.4850
P(M_1|s12)	0.3150	**P(M_2|s12)**	0.2000	**P(M_3|s12)**	0.4850
P(M_1|s13)	0.3150	**P(M_2|s13)**	0.2000	**P(M_3|s13)**	0.4850
P(M_1|s14)	0.3150	**P(M_2|s14)**	0.2000	**P(M_3|s14)**	0.4850
P(M_1|s15)	0.3150	**P(M_2|s15)**	0.2000	**P(M_3|s15)**	0.4850
P(M_1|s16)	0.3150	**P(M_2|s16)**	0.2000	**P(M_3|s16)**	0.4850

Table A1.40. Data acquisition: extended well test. Fuzzy posterior probability

COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY	COMPOUND STATE CONDITIONAL PROBABILITY	POSTERIOR PROBABILITY
P($s1\|M_1$)	0.0067	**P($s1\|M_2$)**	0.0042	**P($s1\|M_3$)**	0.0021
P($s2\|M_1$)	0.0156	**P($s2\|M_2$)**	0.0098	**P($s2\|M_3$)**	0.0048
P($s3\|M_1$)	0.0124	**P($s3\|M_2$)**	0.0078	**P($s3\|M_3$)**	0.0038
P($s4\|M_1$)	0.0290	**P($s4\|M_2$)**	0.0182	**P($s4\|M_3$)**	0.0089
P(s5|M_1)	0.0268	**P($s5\|M_2$)**	0.0168	**P($s5\|M_3$)**	0.0082
P($s6\|M_1$)	0.0625	**P($s6\|M_2$)**	0.0392	**P($s6\|M_3$)**	0.0191
P($s7\|M_1$)	0.0498	**P($s7\|M_2$)**	0.0312	**P($s7\|M_3$)**	0.0152
P($s8\|M_1$)	0.1161	**P($s8\|M_2$)**	0.0728	**P($s8\|M_3$)**	0.0356
P($s9\|M_1$)	0.0143	**P($s9\|M_2$)**	0.0168	**P($s9\|M_3$)**	0.0189
P($s10\|M_1$)	0.0334	**P($s10\|M_2$)**	0.0392	**P($s10\|M_3$)**	0.0442
P($s11\|M_1$)	0.0266	**P($s11\|M_2$)**	0.0312	**P($s11\|M_3$)**	0.0352
P($s12\|M_1$)	0.0620	**P($s12\|M_2$)**	0.0728	**P($s12\|M_3$)**	0.0821
P($s13\|M_1$)	0.0572	**P($s13\|M_2$)**	0.0672	**P($s13\|M_3$)**	0.0758
P($s14\|M_1$)	0.1335	**P($s14\|M_2$)**	0.1568	**P($s14\|M_3$)**	0.1769
P($s15\|M_1$)	0.1062	**P($s15\|M_2$)**	0.1248	**P($s15\|M_3$)**	0.1408
P($s16\|M_1$)	0.2479	**P($s16\|M_2$)**	0.2912	**P($s16\|M_3$)**	0.3284

Table A1.41. Data acquisition: extended well test. Residual probability of the membership functions

MEMBERSHIP FUNCTION	**RESIDUAL PROBABILITY**
p(M_1)	0.3700
P(M_2)	0.2000
P(M_3)	0.4300

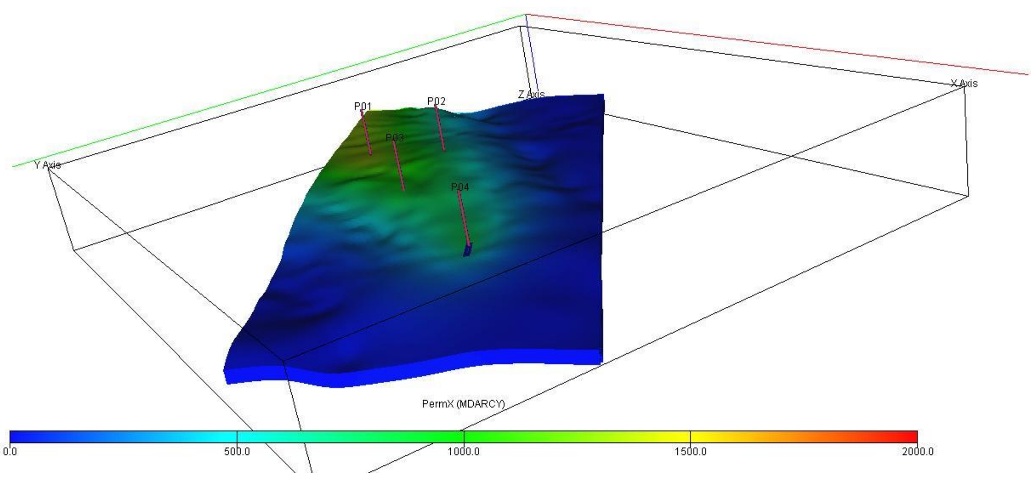

Figure A1.1. Permeability distribution for the PXY high case

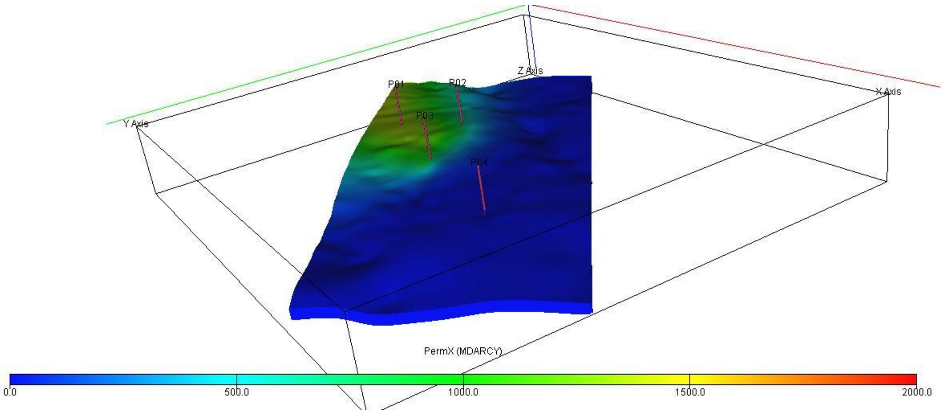

Figure A1.2. Permeability distribution for the PXY low case

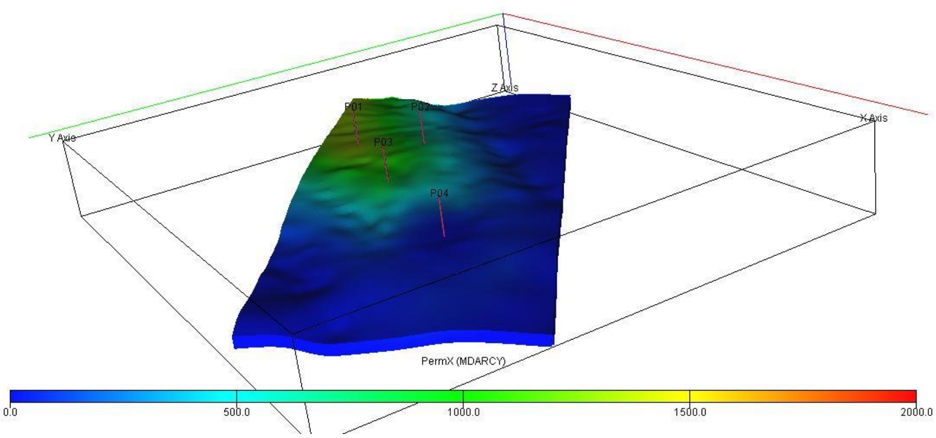

Figure A1.3. Permeability distribution for the PXY medium case

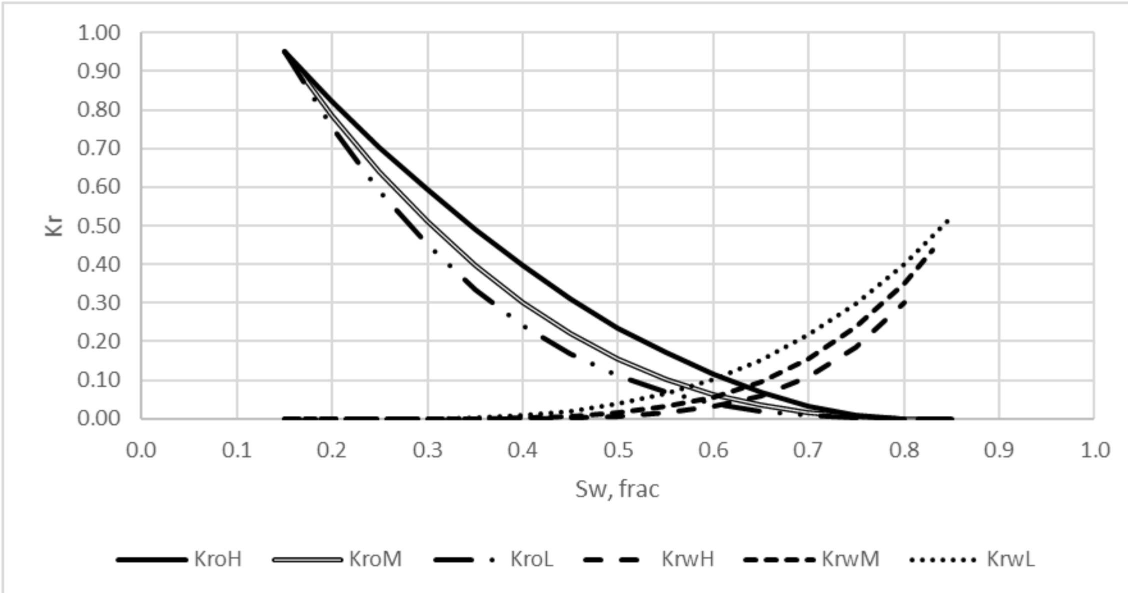

Figure A1.4. Relative permeability: high, medium and low cases

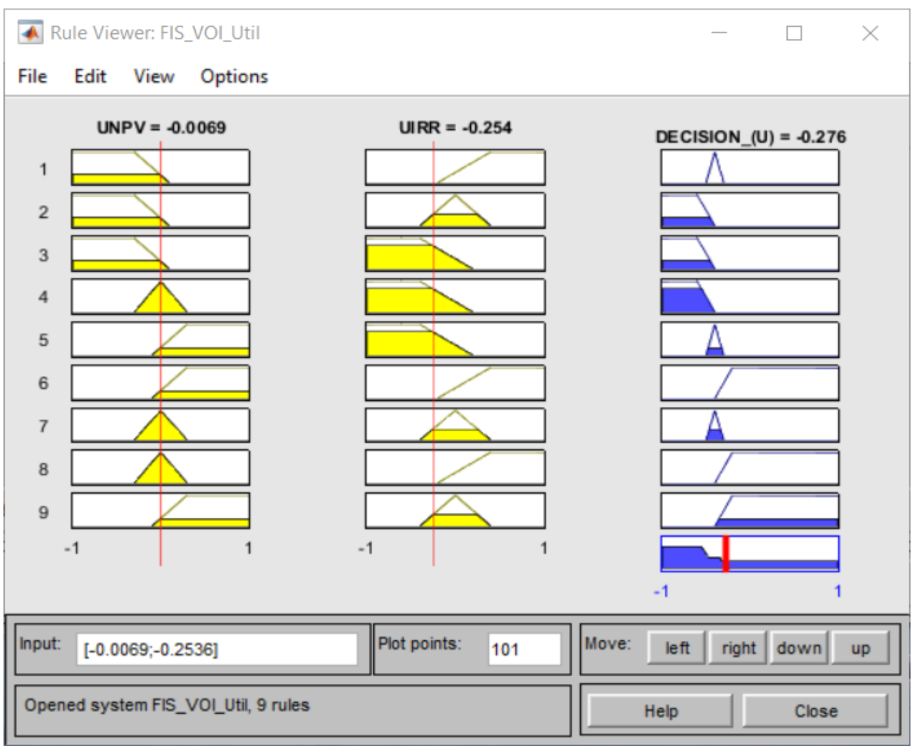

Figure A1.5. FIS evaluation for the no data acquisition alternative

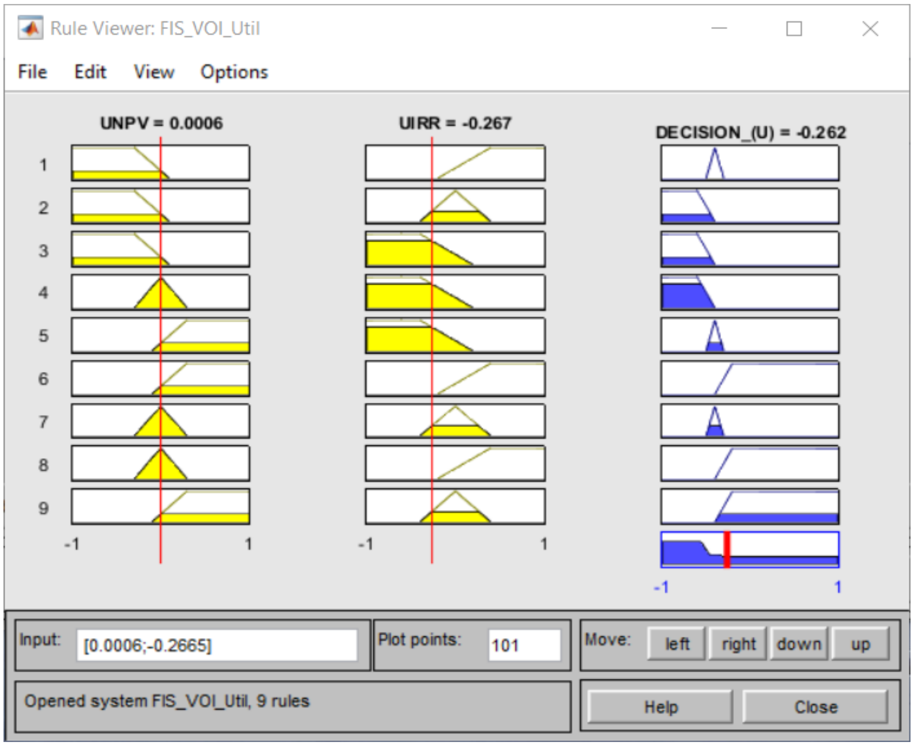

Figure A1.6. FIS evaluation for the data acquisition alternative: one well

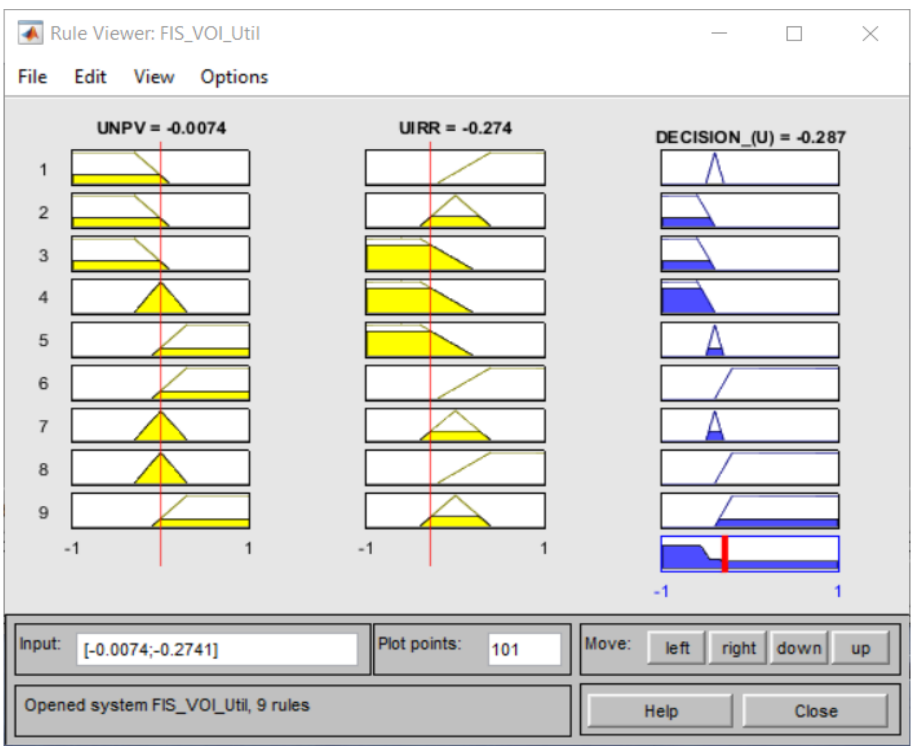

Figure A1.7. FIS evaluation for the data acquisition alternative: one well fuzzy data

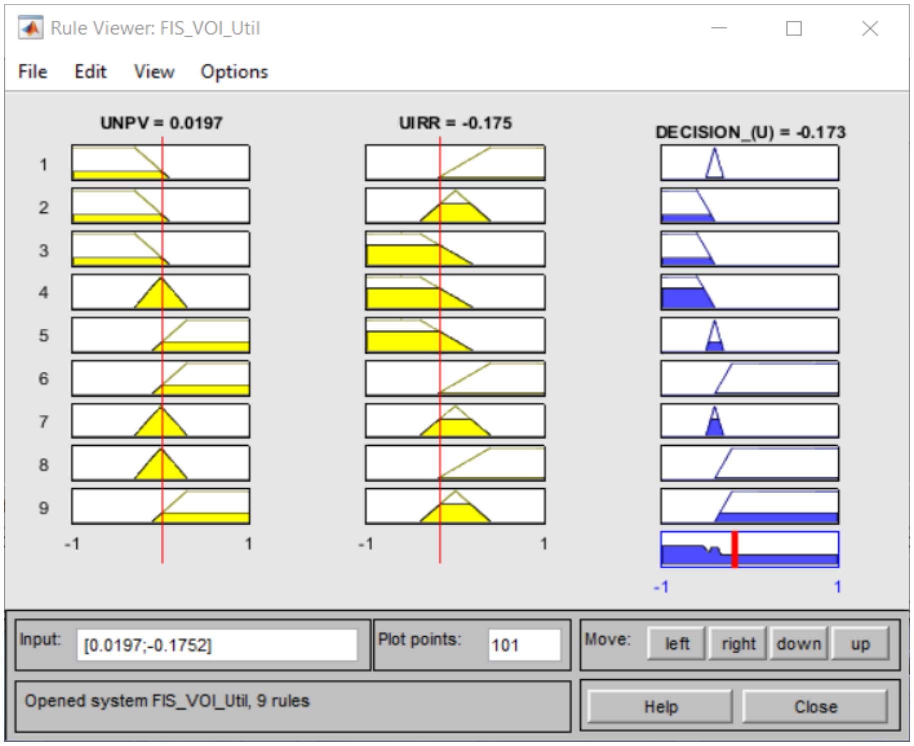

Figure A1.8. FIS evaluation for the data acquisition alternative: extended well test

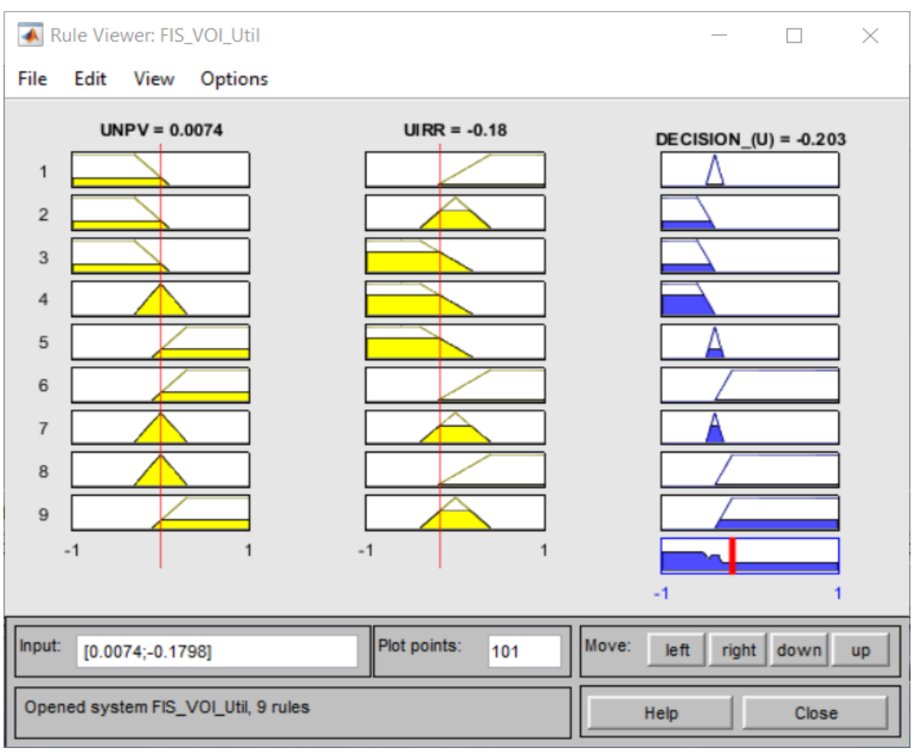

Figure A1.9. FIS evaluation for the data acquisition alternative: extended well test fuzzy data

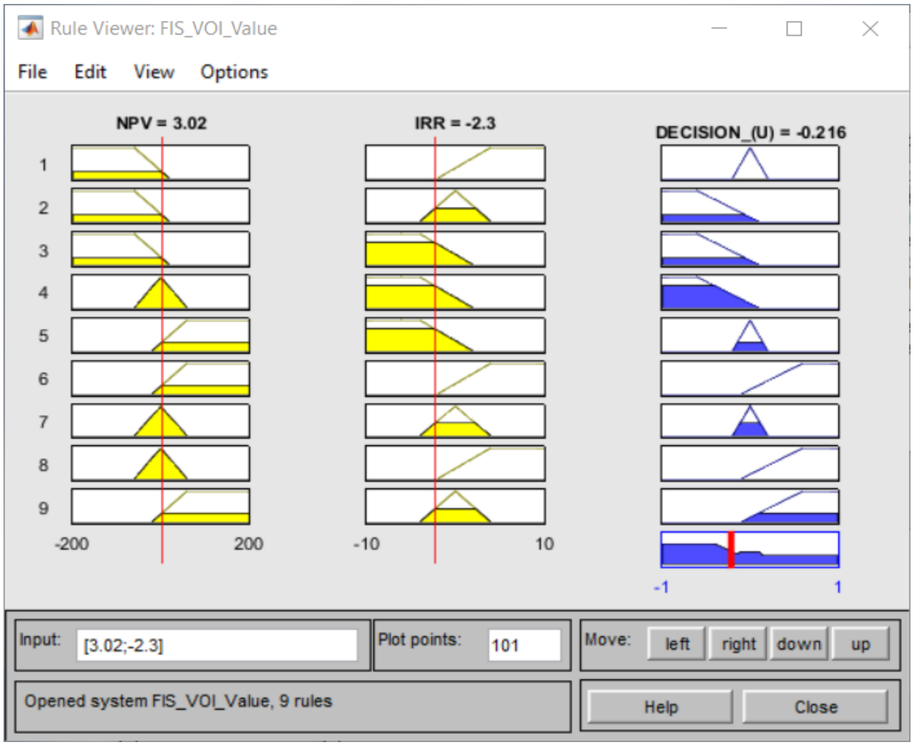

Figure A1.10. FIS evaluation for the no data acquisition values

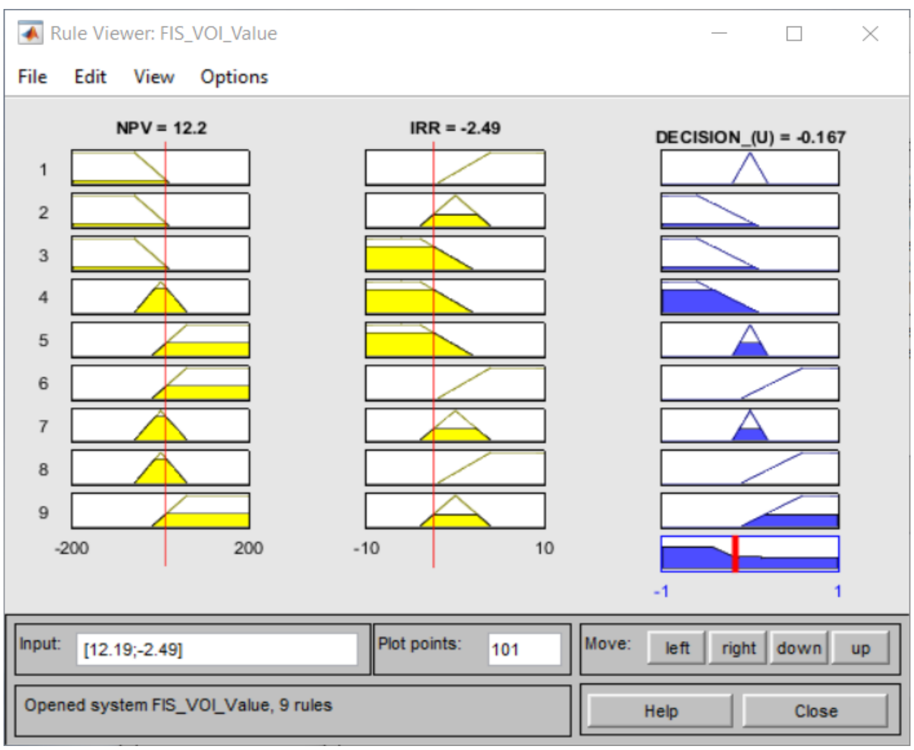

Figure A1.11. FIS evaluation for the data acquisition values: one well

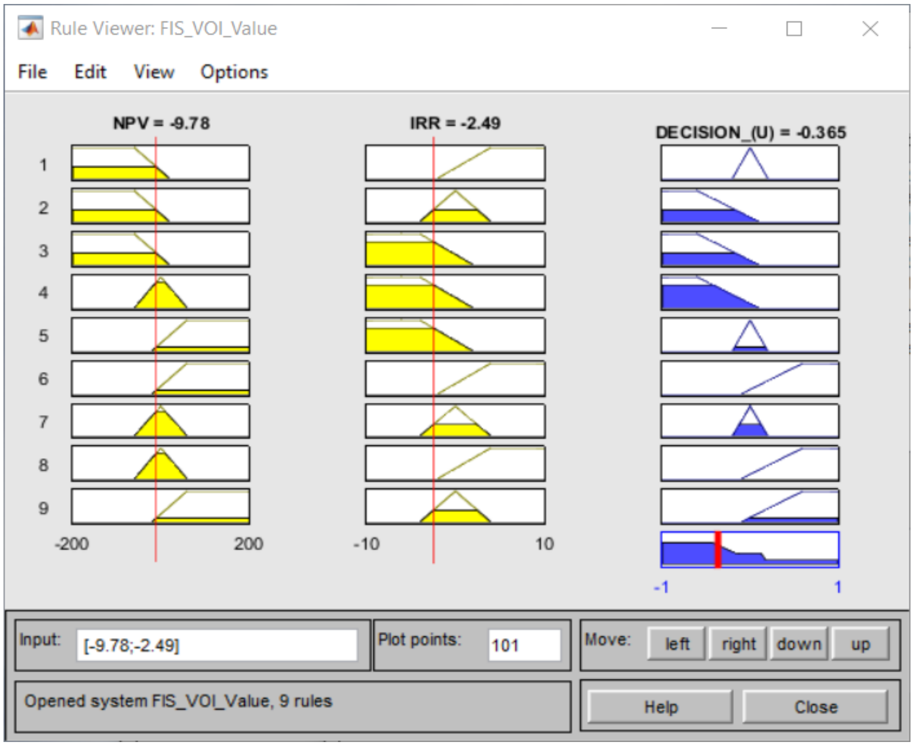

Figure A1.12. FIS evaluation for the data acquisition values: one well fuzzy

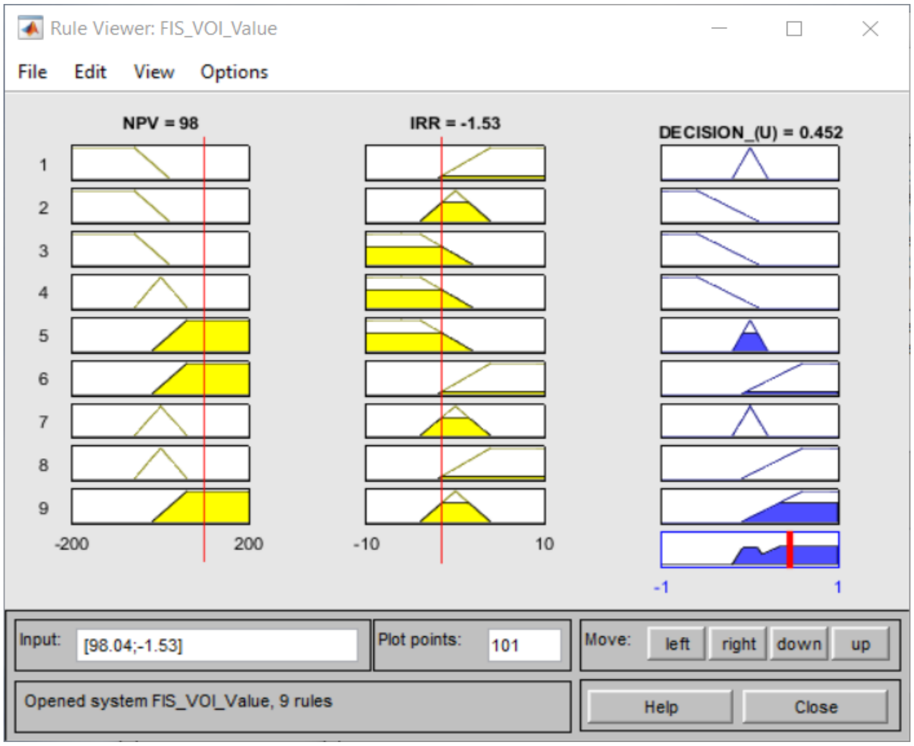

Figure A1.13. FIS evaluation for the data acquisition values well test

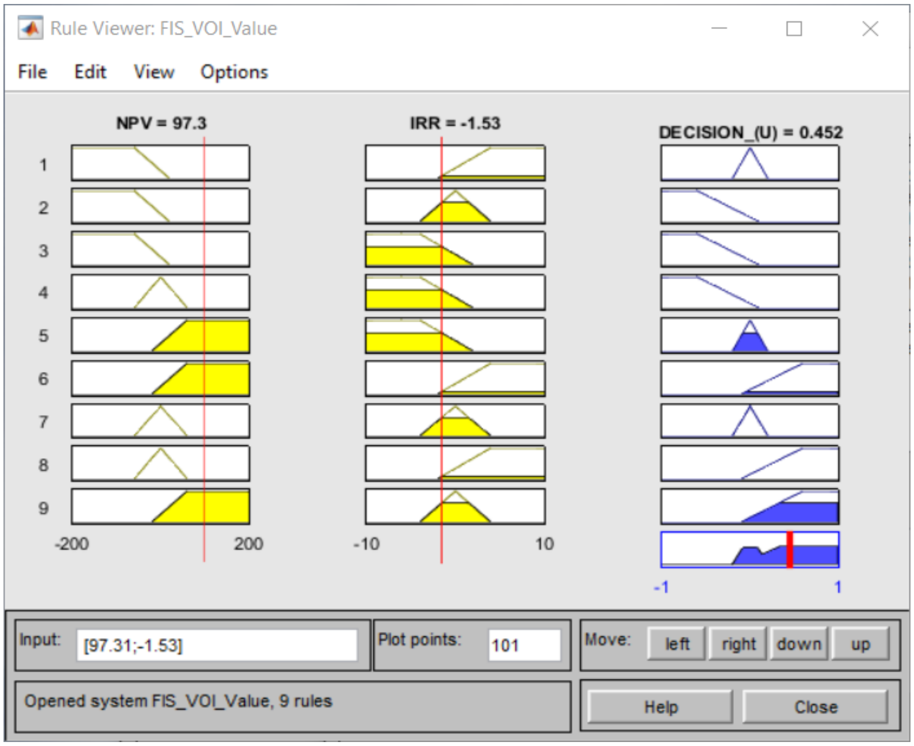

Figure A1.14. FIS evaluation for the data acquisition values fuzzy well test

APPENDIX 2: PAPERS

The papers that have been generated during the current research work are listed below.

VILELA, M., OLUYEMI, G. and PETROVSKI, A. (2017). Value of information and risk reference in oil and gas exploration and production projects. In: *Society of Petroleum Engineers, Annual Technical Conference and Exhibition*, 1–3 November 2017, Baku, Azerbaijan. SPE 189044-MS

VILELA, M., OLUYEMI, G. and PETROVSKI, A. (2018). Fuzzy data analysis methodology for the assessment of value of information in the oil and gas industry. In: *2018 IEEE International Conference on Fuzzy Systems*, pp. 1540–1546.

VILELA, M., OLUYEMI, G. and PETROVSKI, A. (2019). Fuzzy logic applied to value of information assessment in oil and gas projects. In: Petroleum Science. Article not assigned to an issue yet. https://doi.org/10.1007/s12182-019-0348-0.

VILELA, M., OLUYEMI, G. and PETROVSKI, A. (2019). A fuzzy inference system applied to value of information assessment for oil and gas industry. *In: Decision Making: Applications in Management and Engineering*, 2(2), pp. 1–18.

VILELA, M., OLUYEMI, G. and PETROVSKI, A. (2019). Sensitivity analysis applied to fuzzy inference on the value of information in the oil and gas industry. In: *International Journal of Applied Decision Sciences,* Vol. X, No Y, 2019 (approved waiting publication).